CUET (UG) Exam Paper 2024

National Testing Agency
MATHEMATICS/APPLIED MATHEMATICS
(Solved)

[This includes Questions pertaining to Domain Specific Subject only]

Time Allowed: 60 Mins. **Maximum Marks: 200**

General Instructions :

(i) *Section A will have 15 questions covering both i.e., Mathematics/Applied Mathematics which will be compulsory for all candidates.*
(ii) *Section B1 will have 35 questions from Applied Mathematics out of which 25 questions need to be attempted. Section B2 will have 35 questions purely from Mathematics out of which 25 questions will be attempted.*
(iii) *Correct answer or the most appropriate answer : Five marks (+ 5)*
(iv) *Any incorrect option marked will be given minus one mark (– 1).*
(v) *Unanswered/Marked for Review will be given no mark (0).*
(vi) *If more than one option is found to be correct then Five marks (+5) will be awarded to only those who have marked any of the correct options.*
(vii) *If all options are found to be correct then Five marks (+5) will be awarded to all those who have attempted the question.*
(viii) *If none of the options is found correct or a Question is found to be wrong or a Question is dropped then all candidates who have appeared will be given five marks (+5).*
(ix) *Calculator / any electronic gadgets are not permitted.*

Section - A

Mathematics/Applied Mathematics

1. If A and B are symmetric matrices of the same order, then $AB - BA$ is a:

(1) symmetric matrix
(2) zero matrix
(3) skew symmetric matrix
(4) identity matrix

Ans. Option (3) is correct.

Explanation:

$$(AB - BA)^T = B^TA^T - A^TB^T$$
$$(AB)' = B'A'$$
$$= BA - AB = -(AB - BA).$$

confirming it is skew symmetric.

2. If A is a square matrix of order 4 and $|A| = 4$, then $|2A|$ will be :

(1) 8 **(2)** 64
(3) 16 **(4)** 4

Ans. Option (2) is correct.

Explanation:

The determinant of a scalar multiple of a matrix kA is given by $|kA| = k^n|A|$, where n is the order of the matrix A. In this case,

$$n = 4 \text{ and } |A| = 4.$$

Hence, $|2A| = 2^4 \times 4 = 16 \times 4 = 64.$

3. If $[A]_{3\times2}\,[B]_{x\times y} = [C]_{3\times1}$, then:

(1) $x = 1, y = 3$ **(2)** $x = 2, y = 1$
(3) $x = 3, y = 3$ **(4)** $x = 3, y = 1$

Ans. Option (2) is correct.

Explanation:

Given the matrix multiplication

$[A]_{3\times2}\,[B]_{x\times y} = [C]_{3\times1}$,

The values of x and y such that the multiplication is valid and results in a 3×1 matrix.

1. The number of columns in matrix A must equal the number of rows in matrix B for the multiplication to be defined. Matrix A has 2 columns, so matrix B must have 2 rows. Therefore,
$$x = 2$$
2. The result of the multiplication, matrix C, is a 3×1 matrix. This means the product of A and B must result in a matrix with 3 rows and 1 column. Since A is 3×2, B must have 1 column to match the 1 column of C.
Therefore, $y = 1$.

4. If a function $f(x) = x^2 + bx + 1$ is increasing in the interval $[1, 2]$, then the least value of b is:

(1) 5 **(2)** 0 **(3)** – 2 **(4)** – 4

Ans. Option (3) is correct.

Explanation:

If a function $f(x) = x^2 + bx + 1$ is increasing in the interval $[1, 2]$, then the least value of b is:

To determine the least value of b for which $f(x) = x^2 + bx + 1$ is increasing in the interval $[1, 2]$, we analyze the first derivative of the function.

The first derivative $f'(x)$ is:

$$f'(x) = \frac{d}{dx}(x^2 + bx + 1) = 2x + b$$

The function $f(x)$ is increasing on [1, 2] if $f'(x) \geq 0$ for x in [1, 2].

So, we require:

$$2x + b \geq 0$$

For $x = 1$:

$$2{\cdot}1 + b \geq 0$$
$$2 + b \geq 0$$
$$b \geq -2$$

For $x = 2$:

$$2{\cdot}2 + b \geq 0$$
$$4 + b \geq 0$$
$$b \geq -4$$

The more restrictive condition from $x = 1$ gives us:

$$b \geq -2$$

Therefore, the least value of b for which $f(x) = x^2 + bx + 1$ is increasing in the interval [1, 2] is

$$b = -2.$$

5. Two dice are thrown simultaneously. If X denotes the number of fours, then the expectation of X will be:

(1) $\frac{5}{9}$ **(2)** $\frac{1}{3}$ **(3)** $\frac{4}{7}$ **(4)** $\frac{3}{8}$

Ans. Option (2) is correct.

Explanation:

– X denotes the number of fours that appear when two dice are thrown.

– X values 0, 1, or 2.

– Probability that a single die shows a four:

$$\text{P(four)} = \frac{1}{6}$$

Probability that a single die does not show a four:

$$\text{P(not four)} = \frac{5}{6}$$

P(X = 0): Both dice do not show a four.

$$P(X = 0) = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$$

(P(X = 1): One die shows a four and the other does not.

$$P(X = 1) = 2\times\left(\frac{1}{6}\times\frac{5}{6}\right) = \frac{10}{36} = \frac{5}{18}$$

(P(X = 2): Both dice show a four.

$$P(X = 2) = \left(\frac{1}{6}\right)^2 = \frac{1}{36}$$

$$\text{Expectation } E(X) = \sum_x x_i{\cdot}P(X_i)$$

$$E(X) = 0.\frac{25}{36} + 1.\frac{5}{18} + 2.\frac{1}{36}$$

$$E(X) = \frac{5}{18} + \frac{1}{18} = \frac{6}{18} = \frac{1}{3}$$

6. For the function $f(x) = 2x^3 - 9x^2 + 12x - 5, x \in [0, 3]$, match **List-I** with **List-II**:

	List-I		List-II
A.	Absolute maximum value	**I.**	3
B.	Absolute minimum value	**II.**	0
C.	Point of maxima	**III.**	– 5
D.	Point of minima	**IV.**	4

Choose the **correct** answer from the options given below :

(1) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

(2) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

(3) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(4) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

Ans. Option (4) is correct.

Explanation:

The given endpoints are [0, 3] and critical points will be find by taking derivative and equate with zero.

$$f'(x) = 6x^2 - 18x + 12$$
$$0 = 6x^2 - 18x + 12$$
$$0 = x^2 - 3x + 2$$
$$x = 1, 2$$

1. Calculate $f(x)$ at critical points and endpoints:
   - Endpoints:
     $f(0) = -5$
     $f(3) = 2(3)^3 - 9(3)^2 + 12(3) - 5 = 4$
   - Critical points:
     $f(1) = 2(1)^3 - 9(1)^2 + 12(1) - 5 = 0$
     $f(2) = 2(2)^3 - 9(2)^2 + 12(2) - 5 = -1$

   Apply second derivative test as:
   $f''(x) = 12x - 18$
   Substitute the critical points value and end points values in second derivative test:
   $f''(1) = 12*1 - 18 = -6$
   $f''(2) = 12*2 - 18 = 6$
   $f''(0) = 12*0 - 18 = -18$
   $f''(3) = 12*3 - 18 = 18$
2. Match characteristics with numerical values:
   - A Absolute maximum value → (IV) 4 (Occurs at (x = 3)
   - B Absolute minimum value → (III) – 5 (Occurs at (x = 2)
   - C Point of maxima → (I) 3
   - D Point of minima → (II) 0

7. An objective function $Z = ax + by$ is maximum at points (8, 2) and (4, 6). If $a \geq 0$ and $b \geq 0$ and $ab = 25$, then the maximum value of the function is equal to:

(1) 60 **(2)** 50 **(3)** 40 **(4)** 80

Ans. Option (2) is correct.

Explanation:

1. Set up equations using the points where Z is maximized:
   At (8, 2): $Z = 8a + 2b$
   At (4, 6): $Z = 4a + 6b$

2. Use the given condition ($ab = 25$):

From $Z = 8a + 2b$:

$$b = \frac{Z - 8a}{2}$$

Substitute $b = \frac{Z-8a}{2}$ into $ab = 25$:

$$a\left(\frac{Z-8a}{2}\right) = 25$$

$$aZ - 8a^2 = 50$$

From $Z = 4a + 6b$:

$$b = \frac{Z-4a}{6}$$

Substitute $b = \frac{Z-4a}{6}$ into $ab = 25$:

$$a\left(\frac{Z-4a}{6}\right) = 25$$

$$a(Z - 4a) = 150$$

$$aZ - 4a^2 = 150$$

3. Solve the system of equations:

Solving $aZ - 8a^2 = 50$ and $aZ - 4a^2 = 150$:

$$-4a^2 = -100$$

$$a^2 = 25$$

$$a = 5 \text{ since } a \geq 0$$

Substitute $a = 5$ into $ab = 25$:

$$5b = 25$$

$$b = 5$$

4. Calculate Z at (8, 2) and (4, 6):

At (8, 2): $Z = 8(5) + 2(5) = 40 + 10 = 50$

At (4, 6): $Z = 4(5) + 6(5) = 20 + 30 = 50$

Therefore, the maximum value of Z is 50.

8. The area of the region bounded by the lines $x + 2y = 12$, $x = 2$, $x = 6$ and x-axis is:

(1) 34 sq units **(2)** 20 sq units

(3) 24 sq units **(4)** 16 sq units

Ans. Option (4) is correct.

Explanation:

$$\text{Area} = \left|\int_2^6 y\,dx\right|$$

$$x + 2y = 12$$

$$2y = 12 - x$$

$$y = 6 - \frac{x}{2}$$

Thus, $\left|\int_2^6 \left(6 - \frac{x}{2}\right)dx\right| = \left|6x - \frac{x^2}{4}\right|_2^6$

$$= [(36 - 9) - (12 - 1)]$$

$$= 16 \text{ sq. units}$$

9. A die is rolled thrice. What is the probability of getting a number greater than 4 in the first and the second throw of dice and a number less than 4 in the third throw?

(1) $\frac{1}{3}$ **(2)** $\frac{1}{6}$ **(3)** $\frac{1}{9}$ **(4)** $\frac{1}{18}$

Ans. Option (4) is correct.

Explanation:

1. Probability of getting a number greater than 4 in the first and second throw:
   - Each die roll has numbers from 1 to 6.
   - Numbers greater than 4: 5 and 6.
   - Probability for each throw to be greater than 4: $\frac{2}{6} = \frac{1}{3}$

   Since the events are independent, the probability that both the first and second throws result in numbers greater than 4 is:

$$\left(\frac{1}{3}\right) \times \left(\frac{1}{3}\right) = \frac{1}{9}$$

2. Probability of getting a number less than 4 in the third throw:
   - Numbers less than 4: 1, 2, and 3.
   - Probability for the third throw to be less than 4: $\frac{3}{6} = \frac{1}{2}$

3. Combine the probabilities:

   The events of interest are independent, so multiply their probabilities together:

$$\frac{1}{9} \times \frac{1}{2} = \frac{1}{18}$$

   The probability of getting a number greater than 4 in the first and second throw of dice, and a number less than 4 in the third throw, is $\frac{1}{18}$.

10. The corner points of the feasible region determined by $x + y \leq 8$, $2x + y \geq 8$, $x \geq 0$, $y \geq 0$ are $A(0, 8)$, $B(4, 0)$ and $C(8, 0)$. If the objective function $Z = ax + by$ has its maximum value on the line segment AB, then the relation between a and b is:

(1) $8a + 4 = b$ **(2)** $a = 2b$

(3) $b = 2a$ **(4)** $8b + 4 = a$

Ans. Option (2) is correct.

Explanation:

$$Z_{\max a} = Z_{\max b}$$

$$a.0 + b.8 = a.4 + b.0$$

$$8b = 4a$$

$$a = 2b$$

11. If $t = e^{2x}$ and $y = \log_e t^2$, then $\frac{d^2y}{dx^2}$ is:

(1) 0 **(2)** $4t$

(3) $\frac{4e^{2t}}{t}$ **(4)** $\frac{e^{2t}(4t-1)}{t^2}$

Ans. Option (1) is correct.

Explanation:

Given $t = e^{2x}$ and $y = \log_e(t^2)$, find $\frac{d^2y}{dx^2}$.

1. Express y in terms of t

Given $t = e^{2x}$,

$$t^2 = (e^{2x})^2 = e^{4x}$$

And, $\quad y = \log_e(t^2) = \log_e(e^{4x}) = 4x$

So, $\quad y = 4x.$

2. First derivative $\frac{dy}{dx}$

Since $\quad y = 4x,$

$$\frac{dy}{dx} = 4$$

3. Second derivative $\frac{d^2y}{dx^2}$

Since the first derivative $\frac{dy}{dx}$ is constant,

$$\frac{d^2y}{dx^2} = 0$$

12. $\int \frac{\pi}{x^{n+1} - x} dx =$

(1) $\frac{\pi}{n}\log_e\left|\frac{x^n - 1}{x^n}\right| + C$ **(2)** $\log_e\left|\frac{x^n + 1}{x^n - 1}\right| + C$

(3) $\frac{\pi}{n}\log_e\left|\frac{x^n + 1}{x^n}\right| + C$ **(4)** $\pi\log_e\left|\frac{x^n}{x^n - 1}\right| + C$

Ans. Option (1) is correct.

Explanation:

By the constant multiple rules for integrals, which states:

$$\int c\, f(x)dx = c\int f(x)dx$$

Setting, $c = \pi$ and $f(x) = \frac{1}{x^{n+1} - x}$

$$\int \frac{\pi}{x^{n+1} - x} dx = \pi\int \frac{1}{x^{n+1} - x} dx$$

$$\pi\int \frac{1}{x^{n+1} - x} dx = \pi\int \frac{1}{x(x^n - 1)} dx$$

$$u = x^n$$

$$du = nx^{n-1}dx$$

$$x^{n-1}dx = \frac{du}{n}$$

Rewrite the integral using u:

$$\pi\int \frac{1}{x(x^n - 1)} dx = \pi\int \frac{1}{n}.\frac{1}{u(u-1)} du$$

$$= \frac{\pi}{n}\int \frac{1}{u(u-1)} du$$

Integrate each term separately:

$$\frac{\pi}{n}\int\left(\frac{1}{u-1} - \frac{1}{u}\right)du = \frac{\pi}{n}\left(\int \frac{1}{u-1}du - \int \frac{1}{u-1}du\right)$$

Integrate each part:

$$\frac{\pi}{n}(\ln|u-1| - \ln|u|) + C$$

$$= \frac{\pi}{n}\ln\left|\frac{u-1}{u}\right| + C$$

Substituting back

Since $\quad u = x^n:$

$$\frac{\pi}{n}\ln\left|\frac{x^n - 1}{x^n}\right| + C$$

13. The value of $\int_0^1 \frac{a - bx^2}{\left(a + bx^2\right)^2} dx$

(1) $\frac{a-b}{a+b}$ **(2)** $\frac{1}{a-b}$ **(3)** $\frac{a+b}{2}$ **(4)** $\frac{1}{a+b}$

Ans. Option (4) is correct.

Explanation:

Let, $\quad I = \int \frac{a - bx^2}{(a + bx^2)^2} dx$

$$= \int \frac{a - bx^2}{(bx^2 + a)^2} dx$$

$$= -\int \frac{bx^2 - a}{(bx^2 + a)^2} dx$$

Writing $bx^2 - a$ as $bx^2 + a - 2a$

$$= -\int\left[\frac{bx^2 + a}{(bx^2 + a)^2} - \frac{2a}{(bx^2 + a)^2}\right]dx$$

$$= -\left[\int\left(\frac{1}{bx^2 + a} - \frac{2a}{(bx^2 + a)^2}\right)dx\right]$$

$$= -\left[\int \frac{1}{bx^2 + a}dx - 2a\int \frac{1}{(bx^2 + a)^2}dx\right]$$

Now, take

$$I_1 = \int \frac{1}{bx^2 + a} dx$$

Put $\quad u = \frac{\sqrt{b}x}{\sqrt{a}} \Rightarrow du = \frac{\sqrt{b}}{\sqrt{a}}dx$

$$I_1 = \int \frac{\sqrt{a}}{\sqrt{b}(au^2 + a)} du$$

$$= \frac{1}{\sqrt{a}\sqrt{b}}\int \frac{1}{u^2 + 1} du = \frac{1}{\sqrt{a}\sqrt{b}}\tan^{-1}u$$

$$= \frac{1}{\sqrt{a}\sqrt{b}}\tan^{-1}\frac{\sqrt{b}x}{\sqrt{a}}$$

Now, take

$$I_2 = \int \frac{1}{(bx^2 + a)^2} dx$$

Put $\quad x = \frac{\sqrt{a}\tan(u)}{\sqrt{b}}$

$\Rightarrow \quad u = \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)$

Also, $\quad dx = \frac{\sqrt{a}\sec^2(u)}{\sqrt{b}} du$

$$I_2 = \int \frac{\sqrt{a}\sec^2(u)}{\sqrt{b}(a\tan^2(u)+a)^2}du$$

$$= \frac{1}{a^{\frac{3}{2}}\sqrt{b}}\int \frac{1}{\sec^2(u)}du$$

[Since, $a\tan^2(u) + a = a\sec^2(u)$]

Now, take

$$\int \frac{1}{\sec^2(u)}du = \int \cos^2(u)du$$

Using reduction formula,

$$\int \cos^n(u)du$$

$$= \frac{\cos^{n-1}(u)\sin(u)}{n} + \frac{n-1}{n}\int \cos^{n-2}(u)du$$

we get, $$= \frac{\cos(u)\sin(u)}{2} + \frac{1}{2}\int 1du$$

[For $n = 2$]

$$= \frac{\cos(u)\sin(u)}{2} + \frac{u}{2}$$

Therefore, $$I_2 = \frac{1}{a^{\frac{3}{2}}\sqrt{b}}\int \frac{1}{\sec^2(u)}du$$

$$= \frac{\cos(u)\sin(u)}{2a^{\frac{3}{2}}\sqrt{b}} + \frac{u}{2a^{\frac{3}{2}}\sqrt{b}}$$

Resubstitute $u = \tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)$,

$$= \frac{\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}\sqrt{b}} + \frac{x}{2a^2\left(\frac{bx^2}{a}+1\right)}$$

$$\left[\begin{aligned} &\text{Since, } \sin\left[\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\right] = \frac{\sqrt{bx}}{\sqrt{a}\sqrt{\frac{bx^2}{a}+1}} \\ &\cos\left[\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)\right] = \frac{1}{\sqrt{\frac{bx^2}{a}+1}} \end{aligned}\right]$$

Now,

$$I = -\left[\int \frac{1}{bx^2+a}dx - 2a\int \frac{1}{(bx^2+a)^2}dx\right]$$

$$I = -[I_1 - I_2]$$

$$= -\left[\begin{aligned} &\frac{1}{\sqrt{a}\sqrt{b}}\tan^{-1}\frac{\sqrt{bx}}{\sqrt{a}} \\ &-2a\left\{\frac{\tan^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}\sqrt{b}} + \frac{x}{2a^2\left(\frac{bx^2}{a}+1\right)}\right\} \end{aligned}\right]$$

$$= -\left[-\frac{x}{a\left(\frac{bx^2}{a}+1\right)}\right]$$

$$= \frac{x}{a\left(\frac{bx^2}{a}+1\right)}$$

Thus, we have

$$\int \frac{a-bx^2}{(bx^2+a)^2}dx = \frac{x}{a\left(\frac{bx^2}{a}+1\right)} + C$$

$$= \frac{x}{bx^2+a} + C$$

Hence, $$I = \int_0^1 \frac{a-bx^2}{(a+bx^2)^2}dx$$

$$= \left[\frac{x}{bx^2+a}\right]_0^1 = \frac{1}{a+b}$$

14. The second order derivative of which of the following functions is 5^x ?

(1) $5^x \log_e 5$ **(2)** $5^x (\log_e 5)^2$

(3) $\frac{5^x}{\log_e 5}$ **(4)** $\frac{5^x}{(\log_e 5)^2}$

Ans. Option (2) is correct.

Explanation:

Taking log on both sides in the function:

$$\log_e(y) = \log_e(5^x)$$
$$= x\log_e(5)$$

The first derivative is:

$$\frac{y'}{y} = \log_e(5)$$

$$y' = y\log_e(5)$$
$$y' = 5^x \log_e(5)$$

The second derivative is:

$$y'' = \log_e(5).5^x \log_e(5)$$
$$y'' = 5^x(\log_e(5))^2$$

15. The degree of the differential equation $\left(1-\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}} = k\frac{d^2y}{dx^2}$ is:

(1) 1 **(2)** 2 **(3)** 3 **(4)** $\frac{3}{2}$

Ans. Option (2) is correct.

Explanation:

The given differential equation,

$$\left(1-\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}} = k\frac{d^2y}{dx^2}$$

Now, raise both sides to the power of $\frac{2}{3}$ to clear the exponent:

$$\left(\left(1-\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}\right)^{\frac{2}{3}} = \left(k\frac{d^2y}{dx^2}\right)^{\frac{2}{3}}$$

Simplifying the above equation by cubing:

$$\left[1-\left(\frac{dy}{dx}\right)^2\right]^3 = \left(k\frac{d^2y}{dx^2}\right)^2$$

The degree of the given differential $\left(1-\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}} = k\frac{d^2y}{dx^2}$ is 2.

Section - B1

Mathematics

16. Let R be the relation over the set A of all straight lines in a plane such that $l_1\,R\,l_2 \Leftrightarrow l_1$ is parallel to l_2. Then R is:

(1) Symmetric
(2) An equivalence relation
(3) Transitive
(4) Reflexive

Ans. Option (2) is correct.

Explanation:

The relation R over the set of all straight lines in a plane, where (l_1 R l_2) means that (l_1) is parallel to (l_2), is: An Equivalence relation

17. The probability of not getting 53 Tuesdays in a leap year is:

(1) $\frac{2}{7}$ **(2)** $\frac{1}{7}$ **(3)** 0 **(4)** $\frac{5}{7}$

Ans. Option (4) is correct.

Explanation:

Leap year = 366 days. 364 is divisible by 7 and hence there will be two excess weekdays in a leap year.
The two weekdays may be (Sunday, Monday), (Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday), (Saturday, Sunday).
So, the sample pair S has 7 pairs of excess weekdays that is

$$n(\text{S}) = 7$$

Now we want the desired event E to have not getting 53 Tuesdays. E consists of 5 pair in S which is be (Sunday, Monday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday), (Saturday, Sunday).

So, $n(\text{E}) = 5$

The probability that a leap year not getting 53 Tuesdays

$$= \frac{n(\text{E})}{n(\text{S})} = \frac{5}{7}$$

18. The angle between two lines whose direction ratios are propotional to $1, 1, -2$ and $(\sqrt{3}-1), (-\sqrt{3}-1), -4$ is:

(1) $\frac{\pi}{3}$ **(2)** π **(3)** $\frac{\pi}{6}$ **(4)** $\frac{\pi}{2}$

Ans. Option (1) is correct.

Explanation:

The ratios are $(1, 1, -2)$ and $(\sqrt{3}-1, -\sqrt{3}-1, -4)$

Angle between them is given by

$$\cos\theta = \frac{1(\sqrt{3}-1)+1(-\sqrt{3}-1)-2\times -4}{\sqrt{1^2+1^2+(-2)^2}\sqrt{(\sqrt{3}-1)^2+(-\sqrt{3}-1)^2+(-4)^2}}$$

$$\cos\theta = \frac{6}{\sqrt{6}\times 2\sqrt{6}}$$

$$\cos\theta = \frac{6}{12}$$

$$\theta = \frac{\pi}{3}$$

19. If $(\vec{a}-\vec{b})\cdot(\vec{a}+\vec{b}) = 27$ and $|\vec{a}| = 2|\vec{b}|$, then $|\vec{b}|$ is:

(1) 3 **(2)** 2 **(3)** $\frac{5}{6}$ **(4)** 6

Ans. Option (1) is correct.

Explanation:

$$(\vec{a}-\vec{b}).(\vec{a}+\vec{b}) = 27, |\vec{a}| = 2|\vec{b}|$$

$$|\vec{a}|^2-|\vec{b}|^2 = 27$$

$$4|\vec{b}|^2-|\vec{b}|^2 = 27$$

$$3|\vec{b}|^2 = 27$$

$$|\vec{b}|^2 = 9$$

$$|\vec{b}| = 3$$

20. If $\tan^{-1}\left(\frac{2}{3^{-x}+1}\right) = \cot^{-1}\left(\frac{3}{3^x+1}\right)$, then which one of the following is true?

(1) There is no real value of x satisfying the above equation.
(2) There is one positive and one negative real value of x satisfying the above equation.
(3) There are two real positive values of x satisfying the above equation.
(4) There are two real negative values of x satisfying the above equation.

Ans. Option (2) is correct.

Explanation:

Given the equation:

$$\tan^{-1}\left(\frac{2}{3^{-x}+1}\right) = \cot^{-1}\left(\frac{3}{3^{x}+1}\right)$$

Convert $\cot^{-1}(y)$ to $\tan^{-1}\left(\frac{1}{y}\right)$:

$$\tan^{-1}\left(\frac{2}{3^{-x}+1}\right) = \tan^{-1}\left(\frac{3^{x}+1}{3}\right)$$

Set the arguments equal:

$$\frac{2}{3^{-x}+1} = \frac{3^{x}+1}{3}$$

$$2\cdot 3 = (3^x + 1)(3^{-x} + 1)$$

$$4 = 3^x + 3^{-x}$$

By $\quad y = 3^x$

$$4y = y^2 + 1$$

$$y^2 - 4y + 1 = 0$$

By simplifying the quadratic equation:

$$y = 2 \pm \sqrt{3}$$

Recall $\quad y = 3^x$

$$3^x = 2+\sqrt{3} \quad \text{(positive value)}$$

$$3^x = 2-\sqrt{3} \quad \text{(negative value)}$$

21. If A, B and C are three singular matrices given by $A=\begin{bmatrix} 1 & 4 \\ 3 & 2a \end{bmatrix}, B=\begin{bmatrix} 3b & 5 \\ a & 2 \end{bmatrix}$ and $C=\begin{bmatrix} a+b+c & c+1 \\ a+c & c \end{bmatrix}$, then the value of abc is:

(1) 15 **(2)** 30 **(3)** 45 **(4)** 90

Ans. Option (3) is correct.

Explanation:

Determinant of A:

$$A = \begin{bmatrix} 1 & 4 \\ 3 & 2a \end{bmatrix}$$

The determinant of A is:

$$\det(A) = 1 \times 2a - 4 \times 3 = 2a - 12$$

$$2a - 12 = 0$$

$$a = 6$$

Determinant of B:

$$B = \begin{bmatrix} 3b & 5 \\ a & 2 \end{bmatrix}$$

The determinant of B is:

$$\det(B) = 3b \times 2 - 5 \times a = 6b - 5a$$

$$6b - 5a = 0$$

$$b = 5$$

Determinant of C:

$$C = \begin{bmatrix} a+b+c & c+1 \\ a+c & c \end{bmatrix}$$

The determinant of C is:

$$\det(C) = (a + b + c) \times c - (c + 1) \times (a + c)$$

$$\det(C) = ac + bc + c^2 - ac - c^2 - a - c$$

$$bc - a - c = 0$$

From $\quad a = 6$ and $b = 5$:

$$5c - 6 - c = 0$$

$$c = \frac{3}{2}$$

Value of abc:

$$abc = 6 \times 5 \times \frac{3}{2}$$

$$abc = 45$$

22. The value of the integral $\int_{\log_e 2}^{\log_e 3} \frac{e^{2x}-1}{e^{2x}+1}dx$ is:

(1) $\log_e 3$ **(2)** $\log_e 4 - \log_e 3$

(3) $\log_e 9 - \log_e 4$ **(4)** $\log_e 3 - \log_e 2$

Ans. Option (2) is correct.

Explanation:

$$\int_{\log_e 2}^{\log_e 3} \frac{e^{2x}-1}{e^{2x}+1}dx = \int \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}dx \quad \text{...(i)}$$

Let $\quad (e^x + e^{-x}) = t$

By taking the derivative,

$(e^x + (-1) \times e^{-x})dx = dt$

$\therefore \quad dt = e^x - e^{-x}\, dx$

From equation (i):

$$= \int \frac{1}{t}dt = \log(t)$$

$$= \left[\log(e^x + e^{-x})\right]_{\log_e 2}^{\log_e 3}$$

$$= \log\left|3+\frac{1}{3}\right| - \log\left|2+\frac{1}{2}\right|$$

$$= \log\frac{10}{3} - \log\frac{5}{2}$$

$$= \log_e\left(\frac{4}{3}\right)$$

$$= \log_e 4 - \log_e 3$$

23. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c} = 0$, where $\vec{a}$ and $\vec{b}$ are unit vectors and $|\vec{c}| = 2$, then the angle between the vectors $\vec{b}$ and $\vec{c}$ is:

(1) 60° **(2)** 90° **(3)** 120° **(4)** 180°

Ans. Option (4) is correct.

Explanation:

From $\quad \vec{a}+\vec{b}+\vec{c} = 0,$

we have: $\quad \vec{c} = -\vec{a}-\vec{b}$

First, let's calculate $\vec{b}.\vec{c}$:

$$\vec{b}.\vec{c} = -\vec{b}.\vec{a} - \vec{b}.\vec{b}$$

Since $\vec{b}$ is a unit vector:

$$\vec{b}.\vec{b} = |\vec{b}|^2 = 1$$

So, $\quad \vec{b}.\vec{c} = -\vec{b}.\vec{a} - 1$

Using the magnitude of $\vec{c}$:

$$|\vec{c}| = \sqrt{\vec{c}.\vec{c}}$$

$$= \sqrt{(-\vec{a}-\vec{b}).(-\vec{a}-\vec{b})}$$

$$2 = \sqrt{2+2(\vec{a}.\vec{b})}$$

Square both sides:

$$2 = 2(\vec{a}.\vec{b})$$

$$\vec{a}.\vec{b} = 1$$

Now, calculate the dot product $(\vec{b}.\vec{c})$ again:

$$\vec{b}.\vec{c} = -\vec{b}.\vec{a}-1$$

$$\vec{b}.\vec{c} = -2$$

By dot product formula for the angle between b and c:

$$\vec{b}.\vec{c} = |\vec{b}||\vec{c}|\cos(\theta)$$

$$-2 = 1.2.\cos(\theta)$$

$$\cos(\theta) = -1$$

$$\theta = 180°$$

24. Let $[x]$ denote the greatest integer function Then match **List-I** with **List-II**·

List-I		List-II	
A.	$\lvert x-1\rvert + \lvert x-2\rvert$	**I.**	is differentiable everywhere except at $x = 0$
B.	$x - \lvert x\rvert$	**II.**	is continuous everywhere
C.	$x - [x]$	**III.**	is not differentiable at $x = 1$
D.	$x\lvert x\rvert$	**IV.**	is differentiable at $x = 1$

Choose the **correct** answer from the options given below:

(1) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)
(2) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)
(3) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)
(4) (A)-(II), (B)-(IV), (C)-(III), (D)-(I)

Ans. Option (3) is correct.

Explanation:

(A) $|x-1| + |x-2|$

- This function involves absolute values, which cause points of non-differentiability at $(x = 1)$ and $(x = 2)$.
- Absolute value functions are continuous everywhere, even though they may not be differentiable at some points.

(B) $x - |x|$

This function is not continuous at $x = 0$, and it is differentiable everywhere except at $x = 0$.

Correct match: (1) is differentiable everywhere except at $x = 0$.

(C) $x - [x]$

It represents the fractional part of x which is continuous and differentiable everywhere except at integer points where

$$x = [x].$$

(3) is not differentiable at

$$x = 1$$

(D) $x|x|$

This function is differentiable at $x = 1$ because it is quadratic, but it is not differentiable at

$$x = 0.$$

Correct match: (4) is differentiable at

$$x = 1.$$

25. The rate of change (in cm^2/s) of the total surface area of a hemisphere with respect to radius r at $r = \sqrt[3]{1.331}$ cm is:

(1) 66π **(2)** 6.6π **(3)** 3.3π **(4)** 4.4π

Ans. Option (2) is correct.

Explanation:

Total surface area of a hemisphere

$$s = 2\pi r^2 + \pi r^2$$

$$s = 3\pi r^2$$

$$\frac{ds}{dr} = 6\pi r$$

$$r = \sqrt[3]{1.331}$$

$$r = 1.1$$

Thus, $\frac{ds}{dr} = 6\pi \times 1.1 = 6.6\pi$

26. The area of the region bounded by the lines $\frac{x}{7\sqrt{3}a} + \frac{y}{b} = 4$, $x = 0$ and $y = 0$ is:

(1) $56\sqrt{3}ab$ **(2)** $56a$ **(3)** $\frac{ab}{2}$ **(4)** $3ab$

Ans. Option (1) is correct.

Explanation:

Let's solve the problem step by step.

$$\frac{x}{7\sqrt{3}a} + \frac{y}{b} = 4 \Rightarrow \frac{x}{28\sqrt{3}a} + \frac{y}{4b} = 1$$

By the area of the triangle:

The area (A) of a triangle with vertices at

$$A = \frac{1}{2} \times \text{length} \times \text{height}$$

$$A = \frac{1}{2} \times 28\sqrt{3}a \times 4b$$

$$A = \frac{1}{2} \times 112\sqrt{3}a \times b$$

$$A = 56\sqrt{3}ab$$

27. If A is a square matrix and I is an identity matrix such that $A^2 = A$, then $A(I-2A)^3 + 2A^3$ is equal to:

(1) $I + A$ **(2)** $I + 2A$ **(3)** $I - A$ **(4)** A

Ans. Option (4) is correct.

Explanation:

1. $A^2 = A$

This means that A is idempotent.

2. Simplifying $(2A^3)$:

$$A^2 = A$$

$$A^3 = A \times A^2 = A \times A = A$$

Therefore: $2A^3 = 2A$

3. Simplifying $(I-2A)^3$

$$(I-2A)^3 = I-4A+4A^2$$
$$= I-4A+4A$$
$$= I-4A+4A$$
$$= I-4A+4A = I$$
$$(I-2A)^3 = (I-2A)\cdot(I-2A)^2$$
$$= (I-2A)\cdot I = I-2A$$

Substituting above

$$A(I-2A)^3 + 2A^3 = A(I-2A) + 2A$$
$$A(I-2A) + 2A = A\cdot I - A\cdot 2A + 2A$$
$$= A - 2A^2 + 2A$$

As we know $A^2 = A$

$$= A - 2A + 2A = A$$

28. Match **List-I** with **List-II**:

	List-I		List-II
A.	Integrating factor of $xdy-(y+2x^2)dx=0$	**I.**	$\frac{1}{x}$
B.	Integrating factor of $(2x^2-3y)dx = xdy$	**II.**	x
C.	Integrating factor of $(2y+3x^2)dx+xdy=0$	**III.**	x^2
D.	Integrating factor of $2xdy+(3x^3+2y)dx=0$	**IV.**	x^3

Choose the **correct** answer from the options given below:

(1) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)
(2) (A)-(I), (B)-(IV), (C)-(III), (D)-(II)
(3) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)
(4) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Ans. Option (2) is correct.

Explanation:

(A) $xdy-(y+2x^2)dx = 0$

On rewriting, we get

$$\frac{dy}{dx} - \left(\frac{y}{x}\right)dx = -2x$$

Comparing it by $\frac{dy}{dx}+Py = Q$, we get $P = -\frac{1}{x}$

$$I.F. = e^{\int Pdx} = e^{\int\left(-\frac{1}{x}\right)dx}$$
$$= e^{-\log x}$$
$$= \frac{1}{x}$$

Therefore, integrating factor is $\frac{1}{x}$.

(B) $(2x^2-3y)dx = xdy$

On rewriting, we get

$$\frac{dy}{dx}+\frac{3y}{x} = 2x$$

Here, integrating factor is x^3.

(Apply same steps to calculate)

(C) $(2y+3x^2)dx+xdy=0$

On rewriting, we get

$$\frac{dy}{dx}+\frac{2}{x}y = -3x$$

Here, integrating factor is x^2.

(Apply same steps to calculate)

(D) $2xdy+(3x^3+2y)dx = 0$

On rewriting, we get

$$\frac{dy}{dx}+\frac{1}{x}y = -\frac{3}{2}x^2$$

Here, integrating factor is x.

(Apply same steps to calculate)

29. If the function $f:\mathbb{N}\to\mathbb{N}$ is defined as

$$f(n) = \begin{cases} n-1, & \text{if } n \text{ is even} \\ n+1, & \text{if } n \text{ is odd} \end{cases}, \text{ then}$$

(A) f is injective **(B)** f is into
(C) f is surjective **(D)** f is invertible

Choose the **correct** answer from the options given below:

(1) (B) only **(2)** (A), (B) and (D) only
(3) (A) and (C) only **(4)** (A), (C) and (D) only

Ans. Option (4) is correct.

Explanation:

1. Injectivity:
 A function f is injective (one-to-one) if
 $f(a) = f(b)$ implies $a = b$.
 For n even: $f(n) = n-1$
 For n odd: $f(n) = n+1$
 If n_1 and n_2 are different:
 If n_1 is even and n_2 is odd:
 $(f(n_1) = n_1 - 1)$
 and $(f(n_2) = n_2 + 1)$
 If n_1 is odd and n_2 is even:
 $(f(n_1) = n_1 + 1)$
 and $(f(n_2) = n_2 - 1)$
 In both cases $(f(n_1) \neq f(n_2))$
 Therefore, f is injective.
2. Surjectivity:
 A function f is surjective (onto) if for every element $m \in \mathbb{N}$, there exists an $n \in \mathbb{N}$ such that $f(n) = m$.
 For even $m, f(m+1) = m$.
 For odd $m, f(m-1) = m$.
 Thus, every $m \in \mathbb{N}$ has a pre-image under f.
 Therefore, f is surjective.
3. Invertibility:
 A function f is invertible if it is both injective and surjective.
 Since f is injective and surjective, f is invertible.

30. $\int_0^{\frac{\pi}{2}} \frac{1-\cot x}{\operatorname{cosec} x + \cos x}dx =$

(1) 0 **(2)** $\frac{\pi}{4}$ **(3)** ∞ **(4)** $\frac{\pi}{12}$

Ans. Option (1) is correct.

Explanation:

$$\int_0^{\frac{\pi}{2}} \frac{1-\frac{\cos x}{\sin x}}{\frac{1}{\sin x}+\cos x}dx = \int_0^{\frac{\pi}{2}} \frac{\frac{\sin x-\cos x}{\sin x}}{\frac{1+\sin x\cdot\cos x}{\sin x}}dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} \quad \text{...(1)}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right) - \cos\left(\frac{\pi}{2} - x\right)}{1 + \left(\sin\frac{\pi}{2} - x\right).\cos\left(\frac{\pi}{2} - x\right)dx}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1 + \cos x \cdot \sin x}dx \quad \text{...(2)}$$

(1) + (2)

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x + \sin x - \cos x}{1 + \sin x \cos x}dx$$

$$I = 0$$

31. If the random variable X has the following distribution:

X	0	1	2	otherwise
$P(X)$	k	$2k$	$3k$	0

Match **List-I** with **List-II**:

	List-I		List-II
A.	k	**I.**	$\frac{5}{6}$
B.	$P(X < 2)$	**II.**	$\frac{4}{3}$
C.	$E(X)$	**III.**	$\frac{1}{2}$
D.	$P(1 \le X \le 2)$	**IV.**	$\frac{1}{6}$

Choose the **correct** answer from the options given below:

(1) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)
(2) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)
(3) (A)-(I), (B)-(II), (C)-(IV), (D)-(III)
(4) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Ans. Option (2) is correct.

Explanation:

The sum of all probabilities must equal 1.

$$k + 2k + 3k = 1$$

(A) $$k = \frac{1}{6}$$

(B) $P(X < 2)$

$$P(X < 2) = P(X = 0) + P(X = 1)$$
$$= \frac{1}{6} + \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$$

(C) Expected value, $E(X)$:

$$E(X) = \Sigma X.P(X)$$
$$= 0.\frac{1}{6} + 1.\frac{2}{6} + 2.\frac{3}{6}$$
$$= 0 + \frac{2}{6} + \frac{6}{6}$$
$$= \frac{8}{6} = \frac{4}{3}$$

(D) $P(1 \le X \le 2)$:

$$P(1 \le X \le 2) = P(X = 1) + P(X = 2)$$
$$= \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$$

32. For a square matrix $A_{n \times n}$

(A) $|\text{adj } A| = |A|^{n-1}$ **(B)** $|A| = |\text{adj } A|^{n-1}$

(C) $A(\text{adj } A) = |A|$ **(D)** $|A^{-1}| = \frac{1}{|A|}$

Choose the **correct** answer from the options given below:

(1) (B) and (D) only **(2)** (A) and (D) only
(3) (A), (C) and (D) only **(4)** (B), (C) and (D) only

Ans. Option (2) is correct.

Explanation:

(A) $(|\text{adj}(A)| = |A|^{n-1})$

Given is correct because the determinant of the adjugate (or adjoint) of A, denoted as (adj(A)) is given by:

$$|\text{adj}(A)| = |A|^{n-1}$$

(D) $\left(|A^{-1}| = \frac{1}{|A|}\right)$

Given is correct because the determinant of the inverse of a matrix A, if A is invertible, is the reciprocal of the determinant of A:

$$|A^{-1}| = \frac{1}{|A|}$$

33. The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a:

(A) Scalar matrix **(B)** Diagonal matrix
(C) Skew-symmetric matrix **(D)** Symmetric matrix

Choose the **correct** answer from the options given below:

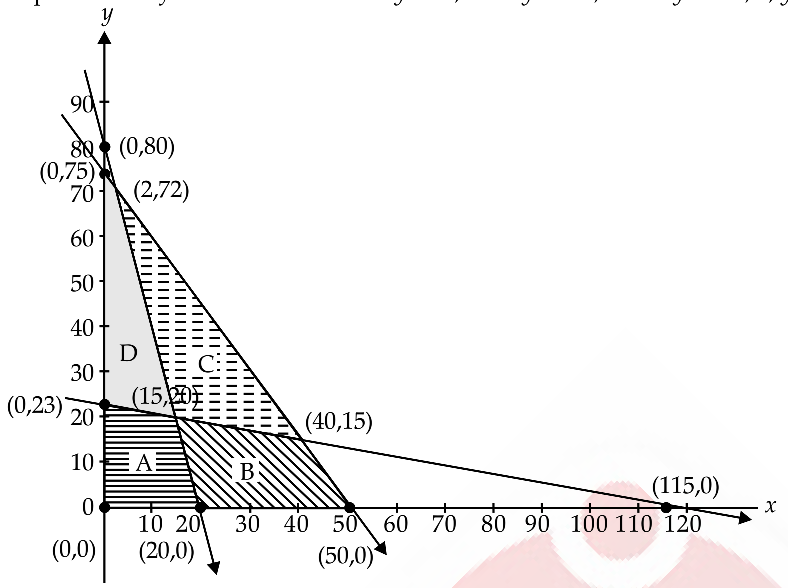
(1) (A), (B) and (D) only **(2)** (A), (B) and (C) only
(3) (A), (B), (C) and (D) **(4)** (B), (C) and (D) only

Ans. Option (1) is correct.

Explanation:

Scalar Matrix:
A scalar matrix is diagonal matrix where all the diagonal elements are equal. The given matrix has all diagonal elements equal to 1. so it is a scalar matrix.
Diagonal Matrix:
A diagonal matrix is matrix in which the entries outside the main diagonal are all zero. The given matrix satisfies this condition, so it is a diagonal matrix.
Symmetric Matrix:
A symmetric matrix is a square matrix that is equal to its transpose that is $A = A^T$. The given matrix satisfies this condition, so it is a symmetric matrix.

34. The feasible region represented by the constraints $4x + y \geq 80$, $x + 5y \geq 115$, $3x + 2y \leq 150$, $x, y \geq 0$ of an LPP is:



(1) Region A **(2)** Region B **(3)** Region C **(4)** Region D

Ans. Option (3) is correct.

Explanation:

$$4x + y \geq 80 \Rightarrow (0, 80), (20, 0)$$
$$x + 5y \geq 115 \Rightarrow (0, 23), (115, 0)$$

This coordinates eliminates the regions of A,B,D.

35. The area of the region enclosed between the curves $4x^2 = y$ and $y = 4$ is:

(1) 16 sq. units **(2)** $\frac{32}{3}$ sq. units

(3) $\frac{8}{3}$ sq. units (4) $\frac{16}{3}$ sq. units

Ans. Option (4) is correct.

Explanation:

Set the equations equal to each other to find the points where the curves intersect.

$$4x^2 = 4$$

Solving for x

$$x^2 = 1$$

$\Rightarrow$ $x = \pm 1$

The points of intersection are (1, 4) and (– 1, 4).

$$\int_a^b (y_2 - y_1)dx$$

Here, ($y_2 = 4$) (the upper curve) and ($y_1 = 4x^2$) (the lower curve).

$$\text{Area} = \int_{-1}^{1} (4 - 4x^2)dx$$
$$= \left[4x - \frac{4}{3}x^3\right]_{-1}^{1}$$
$$= 8 - \frac{8}{3} = \frac{24}{3} - \frac{8}{3}$$
$$= \frac{16}{3} \text{ sq. units}$$

36. $\int e^x \left(\frac{2x+1}{2\sqrt{x}}\right)dx =$

(1) $\frac{1}{2\sqrt{x}}e^x + C$ **(2)** $-e^x\sqrt{x} + C$

(3) $-\frac{1}{2\sqrt{x}}e^x + C$ **(4)** $e^x\sqrt{x} + C$

Ans. Option (4) is correct.

Explanation:

Let $I = \int \frac{(2x+1)e^x}{2\sqrt{x}}dx$

$$= \frac{1}{2}\int \frac{(2x+1)e^x}{\sqrt{x}}dx$$
$$= \int e^x\left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right)dx$$
$$= \sqrt{x}e^x + C$$

$\left[\text{Since, } \int e^x(f(x) + f'(x))dx = e^x f(x) + C\right]$

37. If $f(x)$, defined by $f(x) = \begin{cases} kx+1 & \text{if } x \leq \pi \\ \cos x & \text{if } x > \pi \end{cases}$ is continuous at $x = \pi$, then the value of k is :

(1) 0 **(2)** π **(3)** $\frac{2}{\pi}$ **(4)** $-\frac{2}{\pi}$

Ans. Option (4) is correct.

Explanation:

1. Left-hand limit:
$$\lim_{x\to\pi^-} f(x) = k\pi + 1$$
2. Right-hand limit:
$$\lim_{x\to\pi^+} f(x) = \cos\pi$$
Since $(\cos\pi = -1)$,
$$\lim_{x\to\pi^+} f(x) = -1$$
3. Continuity at $(x = \pi)$:
For the function to be continuous at $(x = \pi)$:
$$\lim_{x\to\pi^-} f(x) = \lim_{x\to\pi^+} f(x) = f(\pi)$$

Thus, $k\pi + 1 = -1$
Solving for k: $k\pi + 1 = -1$
$$k\pi = -2$$
$$k = -\frac{2}{\pi}$$

38. If $P = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ and $Q = [2\ \ -4\ \ 1]$ are two matrices, then $(PQ)'$ will be:

(1) $\begin{bmatrix} 4 & 5 & 7 \\ -3 & -3 & 0 \\ 0 & -3 & -2 \end{bmatrix}$ **(2)** $\begin{bmatrix} -2 & 4 & 2 \\ 4 & -8 & -4 \\ -1 & 2 & 1 \end{bmatrix}$

(3) $\begin{bmatrix} 5 & 5 & 2 \\ 7 & 6 & 7 \\ -9 & -7 & 0 \end{bmatrix}$ **(4)** $\begin{bmatrix} -2 & 4 & 8 \\ 7 & 5 & 7 \\ -8 & -2 & 6 \end{bmatrix}$

Ans. Option (2) is correct.

Explanation:
$$PQ = \begin{bmatrix} -1\times 2 & -1\times -4 & -1\times 1 \\ 2\times 2 & 2\times -4 & 2\times 1 \\ 1\times 2 & 1\times -4 & 1\times 1 \end{bmatrix}$$
Simplify the elements:
$$PQ = \begin{bmatrix} -2 & 4 & -1 \\ 4 & -8 & 2 \\ 2 & -4 & 1 \end{bmatrix}$$
$$(PQ)^T = \begin{bmatrix} -2 & 4 & 2 \\ 4 & -8 & -4 \\ -1 & 2 & 1 \end{bmatrix}$$

39. $\Delta = \begin{vmatrix} 1 & \cos x & 1 \\ -\cos x & 1 & \cos x \\ -1 & -\cos x & 1 \end{vmatrix}$

(A) $\Delta = 2(1 - \cos^2 x)$
(B) $\Delta = 2(2 - \sin^2 x)$
(C) Minimum value of Δ is 2
(D) Minimum value of Δ is 4

Choose the **correct** answer from the options given below:

(1) (A), (C) and (D) only **(2)** (A), (B) and (C) only
(3) (A), (B), (C) and (D) **(4)** (B), (C) and (D) only

Ans. Option (4) is correct.

Explanation:

By using cofactor expansion method along the first row:
$$\Delta = 1\cdot\begin{vmatrix} 1 & \cos x \\ -\cos x & 1 \end{vmatrix} - \cos x\cdot\begin{vmatrix} -\cos x & \cos x \\ -1 & 1 \end{vmatrix} + 1\cdot\begin{vmatrix} -\cos x & 1 \\ -1 & -\cos x \end{vmatrix}$$

By Computing the 2×2 determinants
$$\begin{vmatrix} 1 & \cos x \\ -\cos x & 1 \end{vmatrix} = (1)(1) - (\cos x)(-\cos x) = 1 + \cos^2 x$$
$$\begin{vmatrix} -\cos x & \cos x \\ -1 & 1 \end{vmatrix} = (-\cos x)(1) - (\cos x)(-1) = -\cos x + \cos x = 0$$
$$\begin{vmatrix} -\cos x & 1 \\ -1 & -\cos x \end{vmatrix} = (-\cos x)(-\cos x) - (1)(-1) = \cos^2 x + 1$$

By Substituting the 2×2 determinants
$$\Delta = 1\cdot(1 + \cos^2 x) - \cos x\cdot 0 + 1\cdot(\cos^2 x + 1)$$
$$\Delta = 2 + 2\cos^2 x$$

We can simplify this further using the trigonometric identity $(\cos^2 x = 1 - \sin^2 x)$:
$$\Delta = 2 + 2(1 - \sin^2 x) = 2 + 2 - 2\sin^2 x$$
$$= 4 - 2\sin^2 x$$
$$\Delta = 2(2 - \sin^2 x)$$

(C) Minimum value of (Δ) is (2):
When $(\sin^2 x = 1)$,
$\Delta = 4 - 2 \times 1 = 2$

(D) Maximum value of (Δ) is (4):
When $(\sin^2 x = 0)$,
$\Delta = 4 - 2 \times 0 = 4$

40. $f(x) = \sin x + \frac{1}{2}\cos 2x$ in $\left[0, \frac{\pi}{2}\right]$

(A) $f'(x) = \cos x - \sin 2x$
(B) The critical points of the function are $x = \frac{x}{6}$ and $x = \frac{\pi}{2}$
(C) The minimum value of the function is 2
(D) The maximum value of the function is $\frac{3}{4}$

Choose the **correct** answer from the options given below:

(1) (A), (B) and (D) only **(2)** (A), (B) and (C) only
(3) (A), (B), (C) and (D) **(4)** (B), (C) and (D) only

Ans. Option (1) is correct.

Explanation:

By analysing each element

Statement (A): $f(x) = \sin x + \frac{1}{2}\cos 2x$

$$f'(x) = \cos x - \sin 2x$$

So, statement (A) is true.

Statement (B):

To find critical points, we set

$$f'(x) = 0:$$

$$\cos x - \sin 2x = 0$$

This equation determines the critical points.

Solving this equation in the interval $\left(\left[0, \frac{\pi}{2}\right]\right)$

For $\left(x = \frac{\pi}{6}\right)$:

$$\cos\frac{\pi}{6} - \sin 2\cdot\frac{\pi}{6} = \frac{\sqrt{3}}{2} - \frac{1}{2}$$

$$= \frac{\sqrt{3}-1}{2} \approx 0.366$$

For $\left(x = \frac{\pi}{2}\right)$:

$$\cos\frac{\pi}{2} - \sin 2.\frac{\pi}{2} = 0 - 1 = -1$$

Therefore, $\left(x = \frac{\pi}{6}\right)$ and $\left(x = \frac{\pi}{2}\right)$ are critical points.

Statement (D):

From the evaluations above:

$$f\left(\frac{\pi}{6}\right) = \frac{3}{4}$$

$$f(0) = \sin 0 + \frac{1}{2}\cos 0$$

$$= 0 + \frac{1}{2}.1 = \frac{1}{2}$$

The maximum value of $f(x)$ on $\left(\left[0, \frac{\pi}{2}\right]\right)$ is $\left(\frac{3}{4}\right)$.

41. The direction cosines of the line which is perpendicular to the lines with direction ratios 1, – 2, – 2 and 0, 2, 1 are:

(1) $\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$ **(2)** $-\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$

(3) $\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}$ **(4)** $\frac{2}{3}, \frac{1}{3}, \frac{2}{3}$

Ans. Option (1) is correct.

Explanation:

l, m, n be the direction of cosines of the needed line.

It is perpendicular to the line who's proportional to 1, – 2, – 2 and 0, 2, 1.

$$l - 2m - 2n = 0$$

$$0l + 2m + n = 0$$

By solving these equations

$$\frac{l}{2} = \frac{m}{-1} = \frac{n}{2}$$

Since, $l = \frac{a}{|d|}, m = \frac{b}{|d|},$

$$n = \frac{c}{|d|}$$

where, $|d| = \sqrt{a^2 + b^2 + c^2}$

So, direction cosines are:

$$\frac{2}{\sqrt{2^2+(-1)^2+2^2}}, \frac{-1}{\sqrt{2^2+(-1)^2+2^2}},$$

$$\frac{2}{\sqrt{2^2+(-1)^2+2^2}} = \frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$$

42. Let X denote the number of hours you play during a randomly selected day. The probability that X can take values x has the following form, where c is some constant.

$$P(X = x) = \begin{cases} 0.1 & , \text{ if } x = 0 \\ cx & , \text{ if } x = 1 \text{ or } x = 2 \\ c(5-x) & , \text{ if } x = 3 \text{ or } x = 4 \\ 0 & , \text{ otherwise} \end{cases}$$

Match **List-I** with **List-II**:

List-I		List-II	
A.	c	**I.**	0.75
B.	$P(X \le 2)$	**II.**	0.3
C.	$P(X = 2)$	**III.**	0.55
D.	$P(X \ge 2)$	**IV.**	0.15

Choose the **correct** answer from the options given below:

(1) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(2) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(3) (A)-(I), (B)-(II), (C)-(IV), (D)-(III)

(4) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Ans. Option (2) is correct.

Explanation:

$\Sigma P(X = x) = 1$

$P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X > 4) = 1$

$\Rightarrow 0.1 + c + 2c + c(5 - 3) + c(5 - 4) + 0 = 1$

$\Rightarrow 0.1 + c + 2c + 2c + c = 1$

$C = 0.15$

(B) $P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$= 0.1 + k + 2k = 0.1 + 3k$

$= 0.1 + 3 \times 0.15$

$\therefore P(X \le 2) = 0.55$

(C) $P(X = 2) = 2k = 2 \times 0.15$

$\therefore P(X = 2) = 0.3$

(D) $P(X \ge 2) = P(X = 2) + P(X = 3) + P(X = 4)$

$P(X \ge 2) = 2k + 2k + k = 5k = 5 \times 0.15$

$\therefore P(X \ge 2) = 0.75$

43. If $\sin y = x \sin (a + y)$, then $\frac{dy}{dx}$ is:

(1) $\frac{\sin^2 a}{\sin(a+y)}$ **(2)** $\frac{\sin(a+y)}{\sin^2 a}$

(3) $\frac{\sin(a+y)}{\sin a}$ **(4)** $\frac{\sin^2(a+y)}{\sin a}$

Ans. Option (4) is correct.

Explanation:

From the question

$$\sin y = \sin(a + y)$$

$$x = \frac{\sin y}{\sin(a+y)}$$

$$\left[\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u\frac{d}{dx}(v)-(v)\frac{d}{dx}(u)}{v^2}\right]$$

$$\frac{dx}{dy} = \frac{\sin(a+y)\frac{d}{dy}\sin y - \sin y\frac{d}{dy}\sin(a+y)}{\sin^2(a+y)}$$

$$= \frac{\sin[(a+y)-y]}{\sin^2(a+y)}$$

$$= \frac{\sin y}{\sin^2(a+y)}$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin y}$$

44. The unit vector perpendicular to each of the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$, where $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ and $b = \hat{i}+2\hat{j}+3\hat{k}$, is:

(1) $\frac{1}{\sqrt{6}}\hat{i}+\frac{2}{\sqrt{6}}\hat{j}+\frac{1}{\sqrt{6}}\hat{k}$ **(2)** $-\frac{1}{\sqrt{6}}\hat{i}+\frac{1}{\sqrt{6}}\hat{j}-\frac{1}{\sqrt{6}}\hat{k}$

(3) $-\frac{1}{\sqrt{6}}\hat{i}+\frac{2}{\sqrt{6}}\hat{j}+\frac{2}{\sqrt{6}}\hat{k}$ **(4)** $-\frac{1}{\sqrt{6}}\hat{i}+\frac{2}{\sqrt{6}}\hat{j}-\frac{1}{\sqrt{6}}\hat{k}$

Ans. Option (4) is correct.

Explanation:

$$\vec{c} = (\vec{a}+\vec{b})\times(\vec{a}-\vec{b})$$

$$\vec{c} = \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{pmatrix}$$

$$= [-6-(-4)] - \hat{j}[(-4)-0] + \hat{k}[(-2)-0]$$

$$= -2\hat{i}+4\hat{j}-2\hat{k}$$

$$\vec{c} = \frac{1}{\text{magnitude of } \vec{c}}\times\vec{c}$$

Unit of vector

$$= \frac{1}{\sqrt{(-2)^2+4^2+(-2)^2}}\times(-2\hat{i}+4\hat{j}-2\hat{k})$$

$$= \frac{-1}{\sqrt{6}}\hat{i}+\frac{2}{\sqrt{6}}\hat{j}-\frac{1}{\sqrt{6}}\hat{k}$$

45. The distance between the lines

$\vec{r} = \hat{i}-2\hat{j}+3\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$ and

$\vec{r} = 3\hat{i}-2\hat{j}+1\hat{k}+\mu(4\hat{i}+6\hat{j}+12\hat{k})$ is:

(1) $\frac{\sqrt{28}}{7}$ **(2)** $\frac{\sqrt{199}}{7}$ **(3)** $\frac{\sqrt{328}}{7}$ **(4)** $\frac{\sqrt{421}}{7}$

Ans. Option (3) is correct.

Explanation:

Line I: $\vec{r} = \hat{i}-2\hat{j}+3\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$

Line II: $\vec{r} = 3\hat{i}-2\hat{j}+1\hat{k}+\mu(4\hat{i}+6\hat{j}+12\hat{k})$

$$\vec{r} = 3\hat{i}-2\hat{j}+1\hat{k}+2\mu(2\hat{i}+3\hat{j}+6\hat{k})$$

$$\vec{r} = 3\hat{i}-2\hat{j}+1\hat{k}+\mu'(2\hat{i}+3\hat{j}+6\hat{k})$$

Here, $\vec{a_1} = \hat{i}-2\hat{j}+\hat{k}$

$$\vec{a_2} = 3\hat{i}-2\hat{j}+1\hat{k}$$

Since, lines are parallel therefore they have common normal

$$\vec{b} = 2\hat{i}+3\hat{j}+6\hat{k}$$

$$|\vec{b}| = \sqrt{2^2+3^2+6^2}$$

$$|\vec{b}| = \sqrt{49} = 7$$

The shortest distance between lines is

$$d = \frac{|(\vec{a_2}-\vec{a_1})\times\vec{b}|}{|\vec{b}|}$$

$(\vec{a_2}-\vec{a_1}) =$

$(3\hat{i}-2\hat{j}+1\hat{k})-(\hat{i}-2\hat{j}+3\hat{k})$

$$(\vec{a_2}-\vec{a_1}) = 2\hat{i}+0\hat{j}-2\hat{k}$$

$$(\vec{a_2}-\vec{a_1})\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & -2 \\ 2 & 3 & 6 \end{vmatrix}$$

$$= (0+6)\hat{i}-(12+4)\hat{j}+(6-0)\hat{k}$$

$$= 6\hat{i}-16\hat{j}+6\hat{k}$$

Now, $|(\vec{a_2}-\vec{a_1})\times\vec{b}| = \sqrt{(6)^2+(-16)^2+(6)^2}$

$\Rightarrow$ $|(\vec{a_2}-\vec{a_1})\times\vec{b}| = \sqrt{328}$

Putting these values in the expression,

$$d = \frac{|(\vec{a_2}-\vec{a_1})\times\vec{b}|}{|\vec{b}|}$$

$$d = \frac{\sqrt{328}}{7} \text{ units}$$

46. If $f(x) = 2\left(\tan^{-1}(e^x)-\frac{\pi}{4}\right)$, then $f(x)$ is:

(1) Even and is strictly increasing in $(0, \infty)$
(2) Even and is strictly decreasing in $(0, \infty)$
(3) Odd and is strictly increasing in $(-\infty, \infty)$
(4) Odd and is strictly decreasing in $(-\infty, \infty)$

Ans. Option (3) is correct.

Explanation:

Since $(\tan^{-1}(e^{-x}))$ is odd, $\left(2\left(\tan^{-1}(e^{-x})-\frac{\pi}{4}\right)\right)$ will be an odd function because:

$$f(-x) = 2\left(-\tan^{-1}(e^{x})-\frac{\pi}{4}\right)$$

$$= -2\left(\tan^{-1}(e^x) - \frac{\pi}{4}\right)$$

$$= -f(x)$$

Therefore, $(f(x)$ is an odd function.

It is strictly increasing in $(-\infty, \infty)$.

47. For the differential equation $(x \log_e x)dy = (\log_e x - y)dx$

(A) Degree of the given differential equation is 1.

(B) It is a homogeneous differential equation.

(C) Solution is $2y \log_e x + A = (\log_e x)^2$, where A is an arbitrary constant

(D) Solution is $2y \log_e x + A = \log_e (\log_e x)$, where A is an arbitrary constant

Choose the **correct** answer from the options given below:

(1) (A), and (C) only **(2)** (A), (B) and (C) only

(3) (A), (B), and (D) only **(4)** (A) and (D) only

Ans. Option (1) is correct.

Explanation:

(A) Degree of the given differential equation is 1: True. The equation is $((x \log_e x)dy = (\log_e x - y)dx)$, which involves first-order differentials only.

(C) We have, $(x \log_e x)dy = (\log_e x - y)dx$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x\log_e x} = \frac{1}{x}$$

On comparing with $\frac{dy}{dx}$, $Py = Q$, we get

$$P = \frac{1}{x\log_e x}$$

and
$$Q = \frac{1}{x}$$

$$\text{I.F.} = e^{\int P dx}$$

$$= e^{\int \frac{1}{x\log_e x}dx}$$

$$= e^{\log_e(\log_e x)}$$

$$= \log_e x$$

Solution is:

$$y\,\text{I.F.} = \int Q \times \text{I.F.}\, dx + c$$

$$\Rightarrow y\log_e x = \int\left(\frac{1}{x}\times\log_e x\right)dx + c$$

$$\Rightarrow y\log_e x = \frac{(\log_e x)^2}{2} + c$$

$$\Rightarrow 2y\log_e x = (\log_e x)^2 + 2c$$

$$\Rightarrow 2y\log_e x - 2c = (\log_e x)^2$$

$$\Rightarrow 2y\log_e x + A = (\log_e x)^2$$

where $-2c = A$

48. There are two bags. Bag-1 contains 4 white and 6 black balls and Bag-2 contains 5 white and 5 black balls. A die is rolled, if it shows a number divisible by 3, a ball is drawn from Bag-1, else a ball is drawn from Bag-2. If the ball drawn is not black in colour, the probability that it was not drawn from Bag-2 is:

(1) $\frac{4}{9}$ **(2)** $\frac{3}{8}$ **(3)** $\frac{2}{7}$ **(4)** $\frac{4}{19}$

Ans. Option (3) is correct.

Explanation:

Determine the probability of drawing from each bag

Probability of rolling a number divisible by 3 (i.e., 3 or 6) on a die is

$$\frac{2}{6} = \frac{1}{3}.$$

Probability of rolling a number not divisible by 3 (i.e., 1, 2 , 4 and 5) on a die is

$$\frac{4}{6} = \frac{2}{3}.$$

Calculate the probability of drawing a non-black ball from each bag

From Bag-1 (4 white, 6 black):

$$\text{P(non-black|Bag-1)} = \frac{4}{10} = \frac{2}{5}$$

From Bag-2 (5 white, 5 black):

$$\text{P(non-black|Bag-2)} = \frac{5}{10} = \frac{1}{2}$$

Calculate the total probability of drawing a non-black ball

P(non-black) = P(non-black|Bag-1) × P(Bag-1) + P(non-black|Bag-2) × P(Bag-2)

$$\text{P(non-black)} = \left(\frac{2}{5}\times\frac{1}{3}\right)+\left(\frac{1}{2}\times\frac{2}{3}\right)$$

$$= \frac{2}{15}+\frac{1}{3} = \frac{7}{15}$$

Use Bayes' theorem to find the probability that the ball was not drawn from Bag-2 given it is not black

P(not bag-2|non-black)

$$= \frac{\text{P(non-black | not bag-2)} \times \text{P(not bag-2)}}{\text{P(non-black)}}$$

$$\text{P(non-black|not bag-2)} = \text{P(non-black|Bag-1)} = \frac{2}{5}$$

$$\text{P(not bag-2)} = \text{P(Bag}-1) = \frac{1}{3}$$

$$\text{P(not bag-2|non-black)} = \frac{\left(\frac{2}{5}\times\frac{1}{3}\right)}{\frac{7}{15}} = \frac{2}{7}$$

49. Which of the following ***cannot*** be the direction ratios of the straight line $\frac{x-3}{2} = \frac{2-y}{3} = \frac{z+4}{-1}$?

(1) 2, –3, –1 **(2)** –2, 3, 1 **(3)** 2, 3, –1 **(4)** 6, –9, –3

Ans. Option (3) is correct.

Explanation:

Given line is : $\frac{(x-3)}{2} = \frac{(y-3)}{(-3)} = \frac{(z+4)}{(-1)}$

We need to check which of the given options does not match the direction ratios.

Option (1): (2, –3, –1) — This matches the direction ratios.

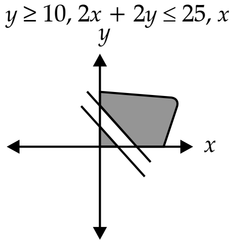
Option (2): (–2, 3, 1) — This is the negative of the direction ratios, which is still valid.

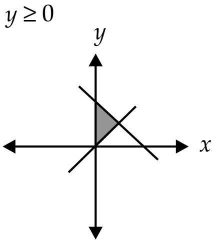
Option (3): (2, 3, –1) This does not match the direction ratios.

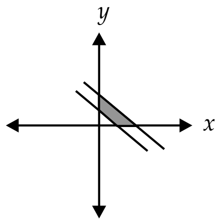
Option (4): (6, –9, –3) This is a multiple of the direction ratios (multiplied by 3), which is still valid.

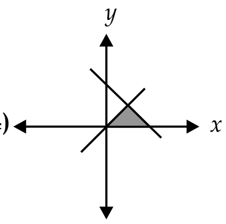
50. Which one of the following represents the correct feasible region determined by the following constraints of an LPP?

$x + y \geq 10, 2x + 2y \leq 25, x \geq 0, y \geq 0$

(1) 

(2) 

(3) 

(4) 

Ans. Option (3) is correct.

Explanation:

The feasible region is the area in the first quadrant ($x \geq 0, y \geq 0$) bounded by the lines $x + y = 10$ and $x + y = 12.5$.

This region is a trapezoid with vertices at (0, 10), (10, 0), (12.5, 0), and (0, 12.5).

Section - B2

Applied Mathematics

51. The least non-negative remainder when 3^{51} is divided by 7 is:

(1) 2 **(2)** 3 **(3)** 6 **(4)** 5

Ans. Option (3) is correct.

Explanation:

$$3^{51} = 3^{6.8+3} = (3^6)^8.3^3$$

Since $(3^6 \equiv 1 \pmod 7)$, we have:

$$(3^6)^8 \equiv 1^8 \equiv 1 \pmod 7$$

Therefore, $3^{51} \equiv 3^3 \pmod 7$

Now we calculate (3^3) modulo 7:

$$3^3 = 27$$

$$27 \div 7 = \text{quotient } 3 \text{ remainder } 6$$

The least non-negative remainder when (3^{51}) is divided by 7 is: 6

52. If $\begin{bmatrix} 5x+8 & 7 \\ y+3 & 10x+12 \end{bmatrix} = \begin{bmatrix} 2 & 3y+1 \\ 5 & 0 \end{bmatrix}$ then the value of $5x + 3y$ is equal to:

(1) -1 **(2)** 8 **(3)** 2 **(4)** 0

Ans. Option (4) is correct.

Explanation:

By finding equations one by one:

$$5x + 8 = 2$$

$$x = -\frac{6}{5}$$

$$7 = 3y + 1$$

$$y = 2$$

$$y + 3 = 5$$

$$5 = 5$$

This equation confirms value of

$$y = 2.$$

$$10x + 12 = 0$$

$$x = -\frac{6}{5}$$

This confirms the value of $\left(x = -\frac{6}{5}\right)$.

Finally, we calculate $5x + 3y$:

$$5x + 3y = 5\left(-\frac{6}{5}\right) + 3(2)$$

$$5x + 3y = 0$$

53. There are 6 cards numbered 1 to 6, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two cards drawn. Then $P(X > 3)$ is:

(1) $\frac{14}{15}$ **(2)** $\frac{1}{15}$ **(3)** $\frac{11}{12}$ **(4)** $\frac{1}{12}$

Ans. Option (1) is correct.

Explanation:

All the possible pairs of cards and their sums:

(1, 2) with sum 3
(1, 3) with sum 4
(1, 4) with sum 5
(1, 5) with sum 6
(1, 6) with sum 7
(2, 3) with sum 5
(2, 4) with sum 6
(2, 5) with sum 7
(2, 6) with sum 8
(3, 4) with sum 7
(3, 5) with sum 8
(3, 6) with sum 9
(4, 5) with sum 9
(4, 6) with sum 10
(5, 6) with sum 11

There are a total of ($^6C_2 = 15$) possible pairs.

Next, we count the pairs where the sum X is greater than 3:

Pairs with sums: (1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 5), (4, 6), (5, 6)

All pairs except (1, 2) result in a sum greater than 3.

Since there are 14 pairs that result in $X > 3$, the probability is:

$$P(X > 3) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{14}{15}$$

54. Which of the following are components of a time series?

(A) Irregular component

(B) Cyclical component

(C) Chronological Component

(D) Trend Component

Choose the **correct** answer from the options given below:

(1) (A), (B) and (D) only **(2)** (A), (B) and (C) only

(3) (A), (B), (C) and (D) **(4)** (B), (C) and (D) only

Ans. Option (1) is correct.

Explanation:

The Chronological Component (C) is not typically considered a standard component of time series analysis.

55. The following data is from a simple random sample:

15, 23, x, 37, 19, 32

If the point estimate of the population mean is 23, then the value of x is:

(1) 12 **(2)** 30 **(3)** 21 **(4)** 24

Ans. Option (1) is correct.

Explanation:

The formula for the sample mean $(\bar{x})$ is: $\left[\bar{x} = \frac{\sum x_i}{n}\right]$

$$n = 6.$$

sample mean is 23:

$$\bar{x} = 23$$

Substitute the given values into the formula:

$$23 = \frac{15 + 23 + x + 37 + 19 + 32}{6}$$

$$x = 12$$

56. For an investment, if the nominal rate of interest is 10% compounded half yearly, then the effective rate of interest is:

(1) 10.25% **(2)** 11.25% **(3)** 10.125% **(4)** 11.025%

Ans. Option (1) is correct.

Explanation:

The formula for the effective annual rate (EAR):

$$\text{EAR} = \left(1 + \frac{r}{n}\right)^n - 1$$

$$r = 10\%$$

$$n = 2 \text{ (half-yearly compounding)}$$

Substitute the values into the formula:

$$\text{EAR} = \left(1 + \frac{0.10}{2}\right)^2 - 1$$

$$\text{EAR} = 0.1025$$

Convert the decimal to a percentage:

$$\text{EAR} = 10.25\%$$

57. A mixture contains apple juice and water in the ratio 10 : x. When 36 litres of the mixture and 9 litres of water are mixed, the ratio of apple juice and water becomes 5 : 4. The value of x is:

(1) 4 **(2)** 4.4 **(3)** 5 **(4)** 8

Ans. Option (2) is correct.

Explanation:

The amount of apple juice and water in the 36 litres taken out as follows:

Apple juice: $\left(\frac{10}{10+x} \times 36\right)$ litres

Water: $\left(\frac{x}{10+x} \times 36\right)$ litres

When 9 litres of water is added, the new amounts of apple juice and water are:

Apple juice: $\left(\frac{10}{10+x} \times 36\right)$ litres

Water: $\left(\frac{x}{10+x} \times 36 + 9\right)$ litres

According to the problem, the new ratio of apple juice to water is $5:4$. Thus, we can write:

$$\frac{\frac{10 \times 36}{10+x}}{\frac{x \times 36}{10+x} + 9} = \frac{5}{4}$$

$$4 \times \frac{360}{10+x} = 5 \times \left(\frac{36x}{10+x} + 9\right)$$

$$\frac{1440}{10+x} = \frac{180x}{10+x} + 45$$

Multiply through out by 10 + x to clear the denominator:

$$1440 = 180x + 45(10 + x)$$
$$1440 = 180x + 450 + 45x$$
$$1440 = 225x + 450$$

Subtract 450 from both sides:

$$990 = 225x$$
$$x = 4.4$$

58. For $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, if X and Y are square matrices of order 2 such that $XY = X$ and $YX = Y$, then $(Y^2 + 2Y)$ equals to:

(1) $2Y$ **(2)** $I + 3X$ **(3)** $I + 3Y$ **(4)** $3Y$

Ans. Option (4) is correct.

Explanation:

$$\begin{aligned} Y^2 + 2Y &= YY+2Y \\ &= (YX)(YX) + 2Y \\ &= Y(XY)X + 2Y \\ &= Y(X)X + 2Y \\ &= (YX)X + 2Y \\ &= YX + 2Y \\ &= Y + 2Y = 3Y \end{aligned}$$

59. A coin is tossed K times. If the probability of getting 3 heads is equal to the probability of getting 7 heads, then the probability of getting 8 tails is:

(1) $\frac{5}{512}$ **(2)** $\frac{45}{2^{21}}$ **(3)** $\frac{45}{1024}$ **(4)** $\frac{210}{2^{21}}$

Ans. Option (3) is correct.

Explanation:

Conditions:

A coin is tossed K times.

The probability of getting 3 heads is equal to the probability of getting 7 heads.

P(3 heads) = P(7 heads)

By Binomial probability formula:

$$P(k \text{ heads}) = \binom{K}{k}\left(\frac{1}{2}\right)^K$$

Therefore, $$\binom{K}{3}\left(\frac{1}{2}\right)^K = \binom{K}{7}\left(\frac{1}{2}\right)^K$$

$$\binom{K}{3} = \binom{K}{7}$$

$$\left[\text{Since, } \binom{K}{k} = \binom{K}{K-k}\right]$$

$$\binom{K}{K-3} = \binom{K}{7}$$

$$K-3 = 7$$

$\Rightarrow$ $$K = 10$$

Probability of getting 8 tails:

Getting 8 tails in 10 tosses means getting 10 – 8 = 2 heads.

$$P(2 \text{ heads}) = \binom{10}{2}\left(\frac{1}{2}\right)^{10}$$

Since, $$\binom{10}{2} = \frac{10\times 9}{2\times 1} = 45$$

$$P(2 \text{ heads}) = 45\left(\frac{1}{2}\right)^{10}$$

$$= 45\left(\frac{1}{1024}\right) = \frac{45}{1024}$$

60. If 95% confidence interval for the population mean was reported to be 160 to 170 and $\sigma = 25$, then size of the sample used in this study is:

(Given $Z_{0.025} = 1.96$)

(1) 96 **(2)** 125 **(3)** 54 **(4)** 81

Ans. Option (1) is correct.

Explanation:

By central limit theorem

Confidence Interval= $\bar{x} \pm Z_{\frac{\alpha}{2}}\left(\frac{\sigma}{\sqrt{n}}\right)$

The width of the confidence interval is:

$$170 - 160 = 10$$

The margin of error E is half the width of the confidence interval:

$$E = \frac{10}{2} = 5$$

Using the margin of error formula:

$$E = Z_{\frac{\alpha}{2}}\left(\frac{\sigma}{\sqrt{n}}\right)$$

solve for (n): $$5 = 1.96\left(\frac{25}{\sqrt{n}}\right)$$

Solving for ($\sqrt{n}$): $$\sqrt{n} = 1.96\left(\frac{25}{5}\right)$$

$$= 1.96 \times 5 = 9.8$$

Squaring both sides to solve for n:

$$n = 9.8^2 = 96.04$$

$$n = 96$$

61. Two pipes A and B together can fill a tank in 40 minutes. Pipe A is twice as fast as pipe B. Pipe A alone can fill the tank in :

(1) 1 h **(2)** 2 h **(3)** 80 min **(4)** 20 min

Ans. Option (1) is correct.

Explanation:

Let Pipe B fills the tank alone in t minutes.

Since Pipe A is twice as fast as Pipe B the time it takes for Pipe A to fill the tank alone is $\left(\frac{t}{2}\right)$ min.

The combined rate of both pipes A and B together is:

$$\frac{1}{t} + \frac{2}{t} = \frac{3}{t}$$

Together they can fill the tank in 40 min so

$\frac{1}{40}$ th part of tank per min

Equating the two rates:

$$\frac{3}{t} = \frac{1}{40}$$

Solving for t:

$$t = 3 \times 40 = 120 \text{ min}$$

So, Pipe B alone can fill the tank in 120 min.

Pipe A, being twice as fast as Pipe B, can fill the tank in:

$$\frac{120}{2} = 60 \text{ min}$$

$$= 1 \text{ h}$$

62. An even number is the determinant of

(A) $\begin{bmatrix} 1 & -1 \\ -1 & 5 \end{bmatrix}$ **(B)** $\begin{bmatrix} 13 & -1 \\ -1 & 15 \end{bmatrix}$

(C) $\begin{bmatrix} 16 & -1 \\ -11 & 15 \end{bmatrix}$ **(D)** $\begin{bmatrix} 6 & -12 \\ 11 & 15 \end{bmatrix}$

Choose the **correct** answer from the options given below:

(1) (A), (B) and (D) only **(2) (A), (B) and (C) only**
(3) (A), (B), (C) and (D) **(4) (B), (C) and (D) only**

Ans. Option (1) is correct.

Explanation:

Matrix (A): $\begin{vmatrix} 1 & -1 \\ -1 & 5 \end{vmatrix} = (1 \times 5) - (-1 \times -1)$

$= 5 - 1 = 4$

4 is an even number.

Matrix (B): $\begin{vmatrix} 13 & -1 \\ -1 & 15 \end{vmatrix} = (13 \times 15) - (-1 \times -1)$

$= 195 - 1 = 194$

194 is an even number.

Matrix (C):

$\begin{vmatrix} 16 & -1 \\ -11 & 15 \end{vmatrix} = (16 \times 15) - (-1 \times -11)$

$= 240 - 11 = 229$

229 is an odd number.

Matrix (D):

$\begin{vmatrix} 6 & -12 \\ 11 & 15 \end{vmatrix} = (6 \times 15) - (-12 \times 11)$

$= 90 + 132 = 222$

222 is an even number.

63. Match **List-I** with **List-II**:

List-I Function		**List-II** Derivative w.r.t. x	
A.	$\frac{5^x}{\log_e 5}$	**I.**	$5^x(\log_e 5)^2$
B.	$\log_e 5$	**II.**	$5^x \log_e 5$
C.	$5^x \log_e 5$	**III.**	5^x
D.	5^x	**IV.**	0

Choose the correct answer from the options given below :

(1) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)
(2) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)
(3) (A)-(I), (B)-(II), (C)-(IV), (D)-(III)
(4) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Ans. Option (4) is correct.

Explanation:

Let's match each function in List-I with its corresponding derivative in List-II.

(A) Using the chain rule

$$\frac{d}{dx}\left(\frac{5^x}{\log_e 5}\right) = \frac{d}{dx}\left(5^x \frac{1}{\log_e 5}\right)$$

$$\frac{1}{\log_e 5} 5^x \log_e 5 = 5^x$$

(B) $\frac{d}{dx}(\log_e 5) = 0$

(C) Using the product

$$\frac{d}{dx}(5^x \log_e 5) = 5^x(\log_e 5)^2$$

(D) (5^x)

$$\frac{d}{dx}(5^x) = 5^x \log_e 5$$

64. A random variable X has the following probability distribution:

X	1	2	3	4	5	6	7
$P(X)$	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Match the options of **List-I** with **List-II**:

List-I		**List-II**	
A.	k	**I.**	$\frac{7}{10}$
B.	$P(X < 3)$	**II.**	$\frac{53}{100}$
C.	$P(X > 2)$	**III.**	$\frac{1}{10}$
D.	$P(2 < X < 7)$	**IV.**	$\frac{3}{10}$

Choose the **correct** answer from the options given below:

(1) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)
(2) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)
(3) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)
(4) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Ans. Option (4) is correct.

Explanation:

Summing these probabilities

$k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$10k^2 + 9k - 1 = 0$

Solving this quadratic equation for k

$$k = \frac{1}{10}$$

(B) $P(X < 3) = P(X = 1) + P(X = 2)$

$= k + 2k = 3k$

(C) $P(X > 2) = P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6) + P(X = 7)$

$= 2k + 3k + k^2 + 2k^2 + 7k^2 + k$

$= 6k + 10k^2$

$= \frac{7}{10}$

(D) $P(2 < X < 7) = P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6)$

$= 2k + 3k + k^2 + 2k^2$

$= 5k + 3k^2$

$= \frac{53}{100}$

65. For which one of the following purposes is CAGR (Compounded Annual Growth Rate) ***not*** used?

(1) To calculate and communicate the average growth of a single investment

(2) To understand and analyse the donations received by a non-government organisation

(3) To demonstrate and compare the performance of investment advisors

(4) To compare the historical returns of stocks with a savings account

Ans. Option (1) is correct.

> ***Explanation:***
>
> To calculate and communicate the average growth of a single investment

66. A flower vase costs ₹36,000. With an annual depreciation of ₹2,000, its cost will be ₹6,000 in .... years.

(1) 10 **(2)** 15 **(3)** 17 **(4)** 6

Ans. Option (2) is correct.

> ***Explanation:***
>
> By the following equation:
> Initial Cost – (Annual Depreciation × Number of Years) = Final Cost
> Let n be the number of years.
>
> $$36{,}000 - (2{,}000 \times n) = 6{,}000$$
>
> Solving for n:
>
> $$36{,}000 - 6{,}000 = 2{,}000 \times n$$
> $$n = 15$$

67. Arun's speed of swimming in still water is 5 km/h. He swims between two points in a river and returns back to the same starting point. He took 20 minutes more to cover the distance upstream than downstream. If the speed of the stream is 2 km/h, then the distance between the two points is:

(1) 3 km **(2)** 1.5 km

(3) 1.75 km **(4)** 1 km

Ans. Option (3) is correct.

> ***Explanation:***
>
> Speed downstream = 5 + 2 = 7 km/h (since he's aided by the stream).
>
> Time downstream $\left(t_{\text{down}} = \frac{d}{7}\right)$ hours.
>
> Speed upstream $= 5 - 2 = 3$ km/ h (since he's working against the stream).
>
> Time upstream $\left(t_{\text{up}} = \frac{d}{3}\right)$ hours.
>
> $\left(t_{\text{up}} = t_{\text{down}} + \frac{20}{60}\right)$ hours (converting 20 min to h)
>
> $$\frac{d}{3} = \frac{d}{7} + \frac{1}{3}$$
>
> Multiply through by 21 (to eliminate fractions):
>
> $$7d = 3d + 7$$
> $$d = \frac{7}{4} = 1.75 \text{ km}$$

68. If $e^y = x^x$, then which of the following is true?

(1) $y\frac{d^2y}{dx^2} = 1$ **(2)** $\frac{d^2y}{dx^2} - y = 0$

(3) $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$ **(4)** $y\frac{d^2y}{dx^2} - \frac{dy}{dx} + 1 = 0$

Ans. Option (4) is correct.

> ***Explanation:***
>
> We have, $e^y = x^x$
>
> Taking log both sides, we get
>
> $$y \log e = x \log x$$
> $$y = x \log x$$
>
> Now, $\frac{dy}{dx} = x \times \frac{1}{x} + \log x$
>
> $$\frac{dy}{dx} = 1 + \log x$$
>
> and $\frac{d^2y}{dx^2} = \frac{1}{x}$
>
> $$\therefore \quad y\frac{d^2y}{dx^2} - \frac{dy}{dx} + 1 = y \times \frac{1}{x} - (1 + \log x) + 1$$
> $$= x \log x \frac{1}{x} - 1 - \log x + 1$$
> $$= 0$$
>
> Hence, option 4 is correct.

69. The probability of a shooter hitting a target is $\frac{3}{4}$.

How many minimum number of times must he fire so that the probability of hitting the target at least once is more than 90%?

(1) 1 **(2)** 2 **(3)** 3 **(4)** 4

Ans. Option (2) is correct.

> ***Explanation:***
>
> Let X = No. of times the shooter hited the target
>
> X has bionomial distribution
>
> $$P(X = x) = {}^{n}C_x q^{n-x} p^x$$
> $$p = \frac{3}{4}$$
> $$q = 1 - p = \frac{1}{4}$$
> $$P(X = x) = {}^{n}C_x \left(\frac{1}{4}\right)^{n-x} \left(\frac{3^x}{4}\right)$$
> $$P(X \geq 1) > 90\%$$
> $$1 - P(X = 0) > 90\%$$
> $$1 - {}^{n}C_0 \left(\frac{1}{4}\right)^n \left(\frac{3}{4}\right)^0 > 0.90$$
> $$1 - \left(\frac{1}{4}\right)^n > 0.90$$
>
> He must fire at least 2 times.

70. Match **List-I** with **List-II**:

	List-I		List-II
A.	Distribution of a sample leads to becoming a normal distribution	**I.**	Central Limit Theorem
B.	Some subset of the entire population	**II.**	Hypothesis
C.	Population mean	**III.**	Sample
D.	Some assumptions about the population	**IV.**	Parameter

Choose the **correct** answer from the options given below :

(1) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(2) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)

(3) (A)-(I), (B)-(II), (C)-(IV), (D)-(III)

(4) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Ans. Option (2) is correct.

Explanation:

The Central Limit Theorem states that regardless of the shape of the data distribution, the sampling distribution of the sampling means approaches a normal distribution as the sample size increases.

71. Ms. Sheela creates a fund of ₹ 1,00,000 for providing scholarships to needy children. The scholarship is provided in the beginning of the year. This fund earns an interest of r % per annum. If the scholarship amount is taken as ₹ 8,000, then r =

(1) $8\frac{1}{2}\%$ **(2)** $8\frac{16}{23}\%$ **(3)** $8\frac{17}{25}\%$ **(4)** $8\frac{2}{5}\%$

Ans. Option (2) is correct.

Explanation:

$$P = R + \frac{R}{i}$$

$$100000 = 8000 + \frac{8000}{r} \times 100$$

$$r = \frac{200}{23}$$

$$r = 8\frac{16}{23}$$

72. A person wants to invest an amount of ₹ 75,000. He has two options A and B yielding 8% and 9% return respectively on the invested amount. He plans to invest at least ₹ 15,000 in Plan A and at least ₹ 25,000 in Plan B. Also he wants that his investment in Plan A is less than or equal to his investment in Plan B. Which of the following options describes the given LPP to maximize the return (where x and y are investments in Plan A and Plan B respectively)?

(1) maximize $Z = 0.08x + 0.09y$
$x \geq 15000$
$y \geq 25000$
$x + y \geq 75000$
$x \leq y$
$x, y \geq 0$

(2) maximize $Z = 0.08x + 0.09y$
$x \geq 15000$
$y \leq 25000$
$x + y \geq 75000$
$x \leq y$
$x, y \geq 0$

(3) maximize $Z = 0.08x + 0.09y$
$x \geq 15000$
$y \geq 25000$
$x + y \leq 75000$
$x \geq y$
$x, y \geq 0$

(4) maximize $Z = 0.08x + 0.09y$
$x \geq 15000$
$y \geq 25000$
$x + y \leq 75000$
$x \leq y$
$x, y \geq 0$

Ans. Option (4) is correct.

Explanation:

To Maximize the return, which is the total interest earned:

$$\text{Interest} = 0.08x + 0.09y$$

The LPP to maximize the return $0.08x + 0.09y$ subject to the constraints is:

Maximize:

$$Z = 0.08x + 0.09y$$
$$x + y \leq 75000$$
$$x \geq 15000$$
$$y \geq 25000$$
$$x \leq y$$
$$x, y \geq 0$$

73. In a 700 m race, Amit reaches the finish point in 20 seconds and Rahul reaches in 25 seconds. Amit beats Rahul by a distance of:

(1) 120 m **(2)** 150 m **(3)** 140 m **(4)** 100 m

Ans. Option (3) is correct.

Explanation:

Amit's speed:

$$\text{Speed of Amit} = \frac{\text{Distance}}{\text{Time}} = \frac{700\,m}{20\,s}$$
$$= 35 \text{ m/s}$$

Rahul's speed:

$$\text{Speed of Rahul} = \frac{\text{Distance}}{\text{Time}} = \frac{700\,m}{25\,s}$$
$$= 28 \text{ m/s}$$

The distance Rahul covers in the 20 s, it takes Amit to finish the race:

Distance covered by Rahul in 20 s

= Speed of Rahul × Time

= 28 m/s × 20 s = 560 m

Distance Amit beats Rahul

= 700 m – 560 m = 140 m

74. For the given five values 12, 15, 18, 24, 36; the three-year moving averages are:

(1) 15, 25, 21 **(2)** 15, 27, 19

(3) 15, 19, 26 **(4)** 15, 19, 30

Ans. Option (3) is correct.

Explanation:

The three-year moving average

Lets find first year

$$\frac{12+15+18}{3} = 15$$

$$\frac{15+18+24}{3} = \frac{57}{3} = 19$$

$$\frac{18+24+36}{3} = \frac{78}{3} = 26$$

75. A property dealer wishes to buy different houses given in the table below with some down payments and balance in EMI for 25 years. Bank charges 6% per annum compounded monthly.

$$\left(\text{Given } \frac{(1.005)^{300} \times 0.005}{(1.005)^{300} - 1} = 0.0064\right)$$

Property type	Price of the property (in ₹)	Down Payment (in ₹)
P	45,00,000	5,00,000
Q	55,00,000	5,00,000
R	65,00,000	10,00,000
S	75,00,000	15,00,000

Match **List-I** with **List-II**:

List-I Property Type		List-II EMI amount (in ₹)	
A.	P	**I.**	25,600
B.	Q	**II.**	38,400
C.	R	**III.**	32,000
D.	S	**IV.**	35,200

Choose the **correct** answer from the options given below :

(1) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(2) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)

(3) (A)-(I), (B)-(II), (C)-(IV), (D)-(III)

(4) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Ans. Option (2) is correct.

Explanation:

$$n = 25 \times 12 = 300$$

$$i = \frac{6}{12}\% = 0.05$$

$$\text{EMI} = \frac{p \times i \times (1+i)^n}{(1+i)^{n-1}}$$

$$= \frac{p \times 0.005 \times (1.005)^{300}}{1.005^{300} - 1}$$

EMI amount

$$\begin{aligned} P &= (4500000 - 500000) \times 0.064 \\ &= 400000 \times 0.064 \\ &= 25{,}600 \\ Q &= (5500000 - 500000) \times 0.064 \\ &= 500000 \times 0.064 \\ &= 32{,}000 \\ R &= (65{,}00{,}000 - 10{,}00{,}00) \times 0.064 \\ &= 5500000 \times 0.064 \\ &= 35{,}200 \\ S &= (75{,}00{,}000 - 15{,}00{,}00) \times 0.064 \\ &= 600000 \times 0.064 \\ &= 38{,}400 \end{aligned}$$

76. The corner points of the feasible region for an L.P.P. are (0, 10), (5, 5), (5, 15) and (0, 30). If the objective function is $Z = \alpha x + \beta y$, $\alpha, \beta > 0$, the condition on α and β so that maximum of Z occurs at corner points (5, 5) and (0, 20) is:

(1) $\alpha = 5\beta$ **(2)** $5\alpha = \beta$

(3) $\alpha = 3\beta$ **(4)** $4\alpha = 5\beta$

Ans. Option (3) is correct.

Explanation:

Given function $Z = \alpha x + \beta y$

$$Z(5, 5) = 5\alpha + 5\beta$$

$$Z(0, 20) = 20\beta$$

$$5\alpha + 5\beta = 20\beta$$

$$\alpha = 3\beta$$

77. The solution set of the inequality $|3x| \geq |6 - 3x|$ is:

(1) $(-\infty, 1]$ **(2)** $[1, \infty)$

(3) $(-\infty, 1) \cup (1, \infty)$ **(4)** $(-\infty, -1) \cup (-1, \infty)$

Ans. Option (2) is correct.

Explanation:

To solve the inequality

$$|3x| \geq |6 - 3x|$$

Squaring on both sides

$$x^2 \geq (2 - x)^2$$

$$x^2 \geq 4 + x^2 - 4x$$

$$4x \geq 4$$

$$x \geq 1$$

78. If the matrix $\begin{bmatrix} 0 & -1 & 3x \\ 1 & y & -5 \\ -6 & 5 & 0 \end{bmatrix}$ is skew-symmetric, then the value of $5x - y$ is:

(1) 12 **(2)** 15 **(3)** 10 **(4)** 14

Ans. Option (3) is correct.

Explanation:

Since, given matrix is skew-symmetric, then

$$a_{13} = -a_{31}$$

i.e., $3x = -(-6)$

$$3x = 6$$

$$x = 2$$

In case of skew-symmetric matrix all diagonal elements are 0.

i.e., $y = 0$

Substituting in $5x - y$

$$5(2) - 0 = 10$$

79. A company is selling a certain commodity 'x'. The demand function for the commodity is linear. The company can sell 2000 units when the price is ₹ 8 per unit and it can sell 3000 units when the price is ₹ 4 per unit. The Marginal revenue at $x = 5$ is:

(1) ₹ 79.98 **(2)** ₹ 15.96

(3) ₹ 16.04 **(4)** ₹ 80.02

Ans. Option (2) is correct.

Explanation:

Linear function of demand is

$$P = ax + b$$

For first condition $\quad 8 = 2000a + b \quad ...1$

For Second Condition

$$4 = 3000a + b \quad ...2$$

Solving 1 and 2

$$a = -\frac{1}{250}$$

Substituting in 1

$$b = 16$$

$$P = -\frac{1}{250}x + 16$$

Revenue function $R(x) = p \times x$

$$-\frac{1}{250}x^2 + 16x$$

$$\text{Marginal Revenue} = \frac{dR(x)}{dx} = \frac{-2}{250}x + 16$$

Substitute $x = 5$ in above equation

$= 15.96.$

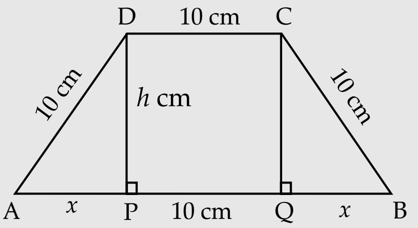
80. If the lengths of the three sides of a trapezium other than the base are 10 cm each, then the maximum area of the trapezium is:

(1) 100 cm^2 **(2)** $25\sqrt{3} \text{ cm}^2$

(3) $75\sqrt{3} \text{ cm}^2$ **(4)** $100\sqrt{3} \text{ cm}^2$

Ans. Option (3) is correct.

Explanation:



By Pythagoras theorem *BQX*

$x^2 + h^2 = 10^2$

$$A = \frac{1}{2} \times h \times (a + b)$$

$$A = \frac{1}{2} \times \sqrt{100 - x^2}\,(10 + 10 + 2x)$$

Squaring on both sides

$$A^2 = \frac{1}{4}(100 - x^2)(20 + 2x)^2$$

$$= \frac{4}{4}(10 - x)(10 + x)(10 + x)^2$$

$$= (10 - x)\,(10 + x)^3$$

$$\frac{d(A^2)}{dx} = -1(10 + x)^3 + (10 - x)3(10 + x)^2$$

$$= (10 + x)^2(-4x + 20) = 0$$

$$x = 5$$

$$A = \frac{1}{2}\sqrt{100 - 25}(20 + 2 \times 5)$$

$$A = 75\sqrt{3} \text{ cm}^2$$

81. Three defective bulbs are mixed with 8 good ones. If three bulbs are drawn one by one with replacement, the probabilities of getting exactly 1 defective, more than 2 defective, no defective and more than 1 defective respectively are:

(1) $\frac{27}{1331}, \frac{576}{1331}, \frac{243}{1331}$ and $\frac{512}{1331}$

(2) $\frac{27}{1331}, \frac{243}{1331}, \frac{576}{1331}$ and $\frac{512}{1331}$

(3) $\frac{576}{1331}, \frac{27}{1331}, \frac{512}{1331}$ and $\frac{243}{1331}$

(4) $\frac{243}{1331}, \frac{576}{1331}, \frac{512}{1331}$ and $\frac{27}{1331}$

Ans. Option (3) is correct.

Explanation:

Probability of drawing a defective bulb

$$P(D) = \frac{3}{11}$$

Probability of drawing a good bulb

$$P(G) = \frac{8}{11}$$

1. Exactly 1 defective bulb:

$$P(\text{Exactly 1 defective}) = \binom{3}{1}\left(\frac{3}{11}\right)\left(\frac{8}{11}\right)^2$$

$$= 3 \cdot \frac{3}{11} \cdot \left(\frac{8}{11}\right)^2$$

Calculating this:

$$P(\text{Exactly 1 defective}) = 3 \cdot \frac{3}{11} \cdot \frac{64}{121}$$

$$= 3 \cdot \frac{192}{1331} = \frac{576}{1331}$$

2. More than 2 defective bulbs:

This scenario means drawing exactly 3 defective bulbs (since 3 is the maximum we can draw). The probability is:

P(More than 2 defective) = P(Exactly 3 defective)

$$= \left(\frac{3}{11}\right)^3 = \frac{27}{1331}$$

3. No defective bulbs:

This scenario involves drawing 3 good bulbs. The probability is:

$$P(\text{No defective}) = \left(\frac{8}{11}\right)^3 = \frac{512}{1331}$$

4. More than 1 defective bulb:

Probability of exactly 2 defective bulbs:

$$P(\text{Exactly 2 defective}) = \binom{3}{2}\left(\frac{3}{11}\right)^2\left(\frac{8}{11}\right)$$

$$= 3 \cdot \left(\frac{3}{11}\right)^2 \cdot \frac{8}{11}$$

$$= 3 \cdot \frac{9}{121} \cdot \frac{8}{11}$$

$$= 3 \cdot \frac{72}{1331} = \frac{216}{1331}$$

Probability of exactly 3 defective bulbs:

$$\text{P(Exactly 3 defective)} = \left(\frac{3}{11}\right)^3 = \frac{27}{1331}$$

Summing these probabilities gives:

P(More than 1 defective)

$$= \frac{216}{1331} + \frac{27}{1331} = \frac{243}{1331}$$

82. If $A = \begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix}, X = \begin{bmatrix} n \\ 1 \end{bmatrix}, B = \begin{bmatrix} 8 \\ 11 \end{bmatrix}$

and $AX = B$, then the value of n will be:

(1) 0 **(2)** 1
(3) 2 **(4)** not defined

Ans. Option (3) is correct.

Explanation:

$$A^{-1} = \frac{adj A}{|A|}$$

$$|A| = -10$$

$$A^{-1} = -\frac{1}{10}\begin{pmatrix} 3 & -4 \\ -4 & 2 \end{pmatrix}$$

$$X = A^{-1}B$$

$$= -\frac{1}{10}\begin{pmatrix} 3 & -4 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} 8 \\ 11 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

83. The equation of the tangent to the curve $x^{\frac{5}{2}} + y^{\frac{5}{2}}$ = 33 at the point (1, 4) is:

(1) $x + 8y - 33 = 0$ **(2)** $12x + y - 8 = 0$
(3) $x + 8y - 12 = 0$ **(4)** $x + 12y - 8 = 0$

Ans. Option (1) is correct.

Explanation:

$$\frac{5}{2}x^{\frac{3}{2}} + \frac{5}{2}y^{\frac{3}{2}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x^{\frac{3}{2}}}{y^{\frac{3}{2}}} = -\frac{1}{8}$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{1}{8}(x-1)$$

$$x + 8y - 33 = 0$$

84. A random variable X has the following probability distribution:

X	– 2	– 1	0	1	2
$P(X)$	0.2	0.1	0.3	0.2	0.2

The variance of X will be :

(1) 0.1 **(2)** 1.42 **(3)** 1.89 **(4)** 2.54

Ans. Option (3) is correct.

Explanation:

$$E(X) = X \times p(X)$$

$$E(X) = -0.4 - 0.1 + 0 + 0.2 + 0.4$$

$$= 0.1$$

$$E(X^2) = X^2 \times p(X)$$

$$= 0.8 + 0.1 + 0 + 0.2 + 0.8$$

$$= 1.9$$

$$\text{var}(X) = E(X^2) - (E(X))^2$$

$$= 1.9 - 0.1^2$$

$$= 1.89$$

85. A Multinational company creates a sinking fund by setting a sum of ₹ 12,000 annually for 10 years to pay off a bond issue of ₹ 72,000. If the fund accumulates at 5% per annum compound interest, then the surplus after paying for bond is:

(Use $(1.05)^{10} \approx 1.6$)

(1) ₹ 78,900 **(2)** ₹ 68,500
(3) ₹ 72,000 **(4)** ₹ 1,44,000

Ans. Option (3) is correct.

Explanation:

$$A = \frac{R(1+i)^n - 1}{i}$$

$$= 12,000\frac{(1+0.05)^{10} - 1}{0.05}$$

$$= 1,44,000$$

Need value of after paying bond surplus

$$= 1,44,000 - 72,000$$

$$= 72,000$$

❑❑❑