CUET (UG) Exam Paper 2025

National Testing Agency

MATHEMATICS/APPLIED MATHEMATICS

(Solved)

[This includes Questions pertaining to Domain Specific Subject only]

Time Allowed: 60 Minutes **Maximum Marks: 250**

General Instructions:

(i) *This paper consists of 85 MCQs.*
(ii) *Mathematics Section A consist of 15 questions (Q. 1 to Q. 15), all are compulsory.*
(iii) *Mathematics Section B1 Core consist of 35 questions (Q. 16 to Q. 50), all are compulsory.*
(iv) *Mathematics Section B2 Applied consist of 35 questions (Q. 51 to Q. 85), all are compulsory.*
(v) *Correct answer or the most appropriate answer: Five marks (+5).*
(vi) *Any incorrect option marked will be given minus One mark (–1).*
(vii) *Unanswered/Marked for Review will be given No mark (0).*
(viii) *If more than one option is found to be correct then. Five marks (+5) will be awarded to only those who have marked any of the correct options.*
(ix) *If all options are found to be correct then Five marks (+5) will be awarded to all those who have attempted the question.*
(x) *If none of the options is found correct or a Question is found to be wrong or a Question is dropped then all candidates who have appeared will be given five marks (+5).*
(xi) *Calculator / any electronic gadgets are not permitted.*

Section : Mathematics Section A

1. Value of $\int \frac{2}{(x-3)\sqrt{x+1}}dx$ is: (Here ....) (Here C is an arbitrary constant.)

(1) $\log\left|\frac{x-3}{x+1}\right|+C$

(2) $\log\left|\frac{\sqrt{x-1}-2}{\sqrt{x+1}}\right|+C$

(3) $\frac{1}{2}\log\left|\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right|+C$

(4) $\log\left|\frac{\sqrt{x+1}-2}{\sqrt{x+1}+2}\right|+C$

Ans. Option (4) is correct.

Explanation: We have, $\int \frac{2}{(x-3)\sqrt{x+1}}dx$

Let, $x + 1 = t^2 \Rightarrow x = t^2 - 1$

Then, $dx = 2t\,dt$

Also, $x - 3 = t^2 - 1 - 3 = t^2 - 4$

Now rewrite the integral as

$$\int \frac{2}{(x-3)\sqrt{x+1}}dx = \int \frac{2}{(t^2-4)t}.2t\,dt$$

$$= \int \frac{4t}{(t^2-4)t}dt = \int \frac{4}{t^2-4}dt$$

$$= \int \frac{4}{(t-2)(t+2)}dt$$

$$\int \frac{4}{t^2-4}dt = \frac{4}{2\times 2}\log\left|\frac{t-2}{t+2}\right|$$

$$= \log\left|\frac{t-2}{t+2}\right|+C$$

Substituting $t = \sqrt{x+1}$, we get $\log\left|\frac{\sqrt{x+1}-2}{\sqrt{x+1}+2}\right|+C$

2. The maximum value of $\sin x.\cos x$ is:

(1) 1 **(2)** $\frac{1}{2}$

(3) $\frac{1}{4}$ **(4)** $\sqrt{2}$

Ans. Option (2) is correct.

Explanation: We know that,

$$\sin x.\cos x = \frac{1}{2}\sin 2x$$

Maximum value of $\sin 2x$:

$\sin 2x \in [-1, 1] \Rightarrow \frac{1}{2}\sin 2x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Thus, maximum value of $\sin x.\cos x$ is $\frac{1}{2}$.

3. The general solution of the differential equation $\frac{dy}{dx} = e^{ax+by}$ is: (Here C is an arbitrary constant.)

(1) $be^{-by} + ae^{ax} = C$ **(2)** $-be^{-by} + ae^{ax} = C$

(3) $ae^{-by} + be^{ax} = C$ **(4)** $e^{ax} + e^{by} = C$

Ans. Option (3) is correct.

Explanation: We are given the differential equation:

$$\frac{dy}{dx} = e^{ax+by}$$

$$\frac{dy}{dx} = e^{ax+by} = e^{ax}.e^{by}$$

$$\int e^{-by}dy = \int e^{ax}dx$$

On integrating both sides, we get

$$\Rightarrow \quad \frac{-1}{b}e^{-by} = \frac{1}{a}e^{ax} + C_1$$

$$\Rightarrow \quad -ae^{-by} - be^{ax} = abC_1$$

[Multiplying both sides by ab]

$$\Rightarrow \quad ae^{-by} + be^{ax} = -abC_1$$

$$\Rightarrow \quad ae^{-by} + be^{ax} = C \quad [\text{Here, } -abC_1 = C.]$$

4. The difference of two different skew-symmetric matrices is:

(1) Null matrix
(2) Identity matrix
(3) Symmetric matrix
(4) Skew-symmetric matrix

Ans. Option (4) is correct.

Explanation: A skew-symmetric matrix A satisfies:

$$A^T = -A$$

Let A and B be two skew-symmetric matrices. That means:

$$A^T = -A \text{ and } B^T = -B$$

Now consider their difference:

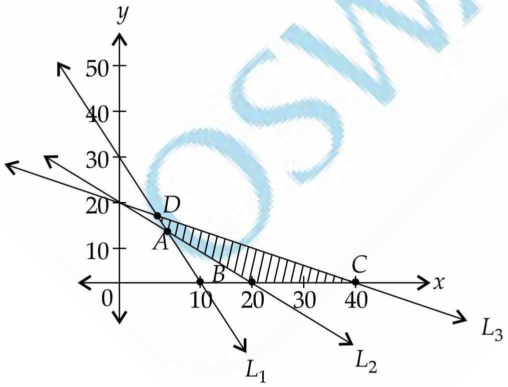
$$(A - B)^T = A^T - B^T = (-A) - (-B)$$
$$= -A + B = -(A - B)$$

This shows:

$$(A - B)^T = -(A - B)$$

So, $A - B$ is also skew-symmetric.

5. Which one of the following inequalities is redundant for the shaded feasible region ($ABCDA$) shown below?



(1) $x + 2y \le 40$ **(2)** $3x + y \ge 30$
(3) $4x + 3y \le 60$ **(4)** $x \ge 0, y \ge 0$

Ans. Option (3) is correct.

Explanation: From the graph, we observe that the shaded region is enclosed by lines L_1, L_2, L_3, the x-axis, and the y-axis.

Line L_1 passes through (10, 0) and (0, 30).
Equation $3x + y = 30 \rightarrow$ inequality is $3x + y \ge 30$.
Line L_2 passes through (10, 0) and (0, 30).
Equation $x + 2y = 40 \rightarrow$ inequality is $x + 2y \le 40$.
Line L_3 passes through (40, 0) and (0, 20).
Equation $4x + 3y = 60 \rightarrow$ inequality is $4x + 3y \le 60$.
Also, the feasible region is bounded by the x-axis and y-axis, hence:
$x \ge 0, y \ge 0$
Now,
Option 1: $x + 2y \le 40 \rightarrow$ forms boundary $L_3 \rightarrow$ essential.
Option 2: $3x + y \ge 30 \rightarrow$ forms boundary $L_1 \rightarrow$ essential.
Option 3: $4x + 3y \le 60$, this line passes through:
● When $x = 0, y = 20$.
● When $y = 0, x = 15$, this line lies above the shaded region.
This is not part of boundary $ABCDA$.
This inequality doesn't affect the region. Hence, $4x + 3y \le 60$ is redundant.
Option 4: $x \ge 0, y \ge 0 \rightarrow$ bounds the region in first quadrant $\rightarrow$ essential.

6. If A and B are square matrices of the same order 3, such that $\det(A) = 3$ and $AB = 3I$, where I is an identity matrix of order 3. Then the value of $\det(B)$ is:

(1) 3 **(2)** 27
(3) 9 **(4)** 1

Ans. Option (3) is correct.

Explanation: We know from properties of determinants that: $\det(AB) = \det(A).\det(B)$
Given, $AB = 3I$, therefore

$$\det(AB) = \det(3I)$$

Also, if I is the identity matrix of order n, then:

$$\det(kI) = k^n.\det(I) = k^n$$

[Since, $\det(I) = 1$.]

So, $\det(3I) = 3^3 = 27$

Now,

$$\det(AB) = \det(A).\det(B)$$
$$\Rightarrow \quad 27 = 3.\det(B)$$
$$\Rightarrow \quad \det(B) = \frac{27}{3} = 9$$

7. If $e^x + e^y = e^{x+y}$, then $\frac{dy}{dx} =$

(1) e^{x-y} **(2)** e^{y-x}
(3) $-e^{y-x}$ **(4)** e^{x+y}

Ans. Option (3) is correct.

Explanation: Given,

$$e^x + e^y = e^{x+y}$$

Dividing both sides by e^{x+y},

$$\frac{e^x}{e^{x+y}} + \frac{e^y}{e^{x+y}} = 1 \Rightarrow e^{-y} + e^{-x} = 1$$

Differentiating both sides with respect to x,

$$-e^{-y}.\frac{dy}{dx} - e^{-x} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{e^{-x}}{e^{-y}} = -e^{y-x}$$

8. If $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -3 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 \\ 4 & -5 \\ 1 & 2 \end{bmatrix}$, then which of the following statements are TRUE?

(A) AB is defined

(B) Both AB and BA are defined and $AB = I$, where I is an identity matrix of order 2

(C) BA is defined

(D) Both AB and BA are defined and $AB = BA$

Choose the correct answer from the options given below:

(1) (A), (B) and (C) only **(2)** (B), (C) and (D) only

(3) (A) and (C) only **(4)** (A), (C) and (D) only

Ans. Option (3) is correct.

Explanation: We are given two matrices:

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 1 & 5 \end{bmatrix}_{(2\times3)} \text{ and } B = \begin{bmatrix} -2 & 3 \\ 4 & -5 \\ 1 & 2 \end{bmatrix}_{(3\times2)}$$

(A) Matrix multiplication is defined if number of columns of A = number of rows of B.

A is 2×3

B is 3×2

So, AB is defined and will be a 2×2 matrix.

Thus, (A) is true.

(B) AB is defined (from above).

Here, BA is also defined (as columns of B = rows of A).

Also, order of BA is 3×3.

$$\text{Now, } AB = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 1 & 5 \end{bmatrix}_{(2\times3)} \begin{bmatrix} -2 & 3 \\ 4 & -5 \\ 1 & 2 \end{bmatrix}_{(3\times2)}$$

$$= \begin{bmatrix} -5 & 17 \\ 1 & 17 \end{bmatrix} \neq I$$

Thus, (B) is false.

(C) BA is defined (from above).

Thus, (C) is true.

(D) AB and BA both are defined.

AB is 2×2, BA is $3 \times 3 \Rightarrow$ different sizes, so can't be equal.

Thus, **(D)** is false.

9. The area (in sq. units) bounded by the parabola $y^2 = 4ax$, its latus rectum and the x-axis in the first quadrant is:

(1) $\frac{1}{3}a^2$ **(2)** $\frac{2}{3}a^2$

(3) $\frac{4}{3}a^2$ **(4)** $\frac{8}{3}a^2$

Ans. Option (3) is correct.

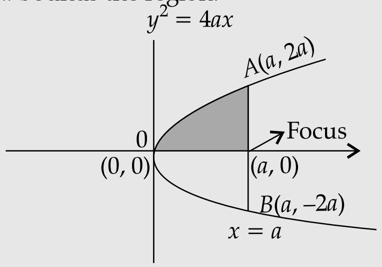
Explanation: For the parabola $y^2 = 4ax$, opens to the right.

The latus rectum is the line $x = a$, and its length is $4a$.

The latus rectum cuts the parabola at $x = a$

$\Rightarrow y^2 = 4a^2 \Rightarrow y = \pm 2a$

So, in the first quadrant, the latus rectum runs from $(a, 0)$ to $(a, 2a)$ and the parabola from $x = 0$ to $x = a$ bounds the region.



Area under the curve $y^2 = 4ax$ from $x = 0$ to $x = a$, in first quadrant, bounded above by the upper half of the parabola.

So, the area is: $A = \int_{x=0}^{a} y\,dx = \int_{x=0}^{a} \sqrt{4ax}\,dx$

$$= \int_{x=0}^{a} 2\sqrt{ax}\,dx$$

$$A = 2\sqrt{a}\int_0^a \sqrt{x}\,dx = 2\sqrt{a}\left[\frac{2}{3}x^{3/2}\right]_0^a$$

$$= 2\sqrt{a}.\frac{2}{3}a^{3/2} = \frac{4}{3}a^2 \text{ sq. units}$$

10. Match **List-I** with **List-II.**

List-I	List-II
Differential equation	**Order and degree of differential equation**
(A) $\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 = e^{\frac{dy}{dx}} + 1$	**(I)** Order = 1, Degree = 2
(B) $\left(\frac{d^2y}{dx^2}\right)^2 + 4\left(\frac{dy}{dx}\right)^3 = e^y - 1$	**(II)** Order = 2, Degree = 1
(C) $3\left(\frac{dy}{dx}\right) + 4y + e^y = \frac{dx}{dy}$	**(III)** Order = 2, Degree = 2
(D) $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right) = \left(e^y + \frac{dy}{dx}\right)^2$	**(IV)** Order = 2, Degree = Not defined

Choose the correct answer from the options given below:

(1) (A)–(I), (B)–(II), (C)–(III), (D)–(IV)
(2) (A)–(II), (B)–(III), (C)–(IV), (D)–(I)
(3) (A)–(IV), (B)–(III), (C)–(I), (D)–(II)
(4) (A)–(IV), (B)–(III), (C)–(II), (D)–(I)

Ans. Option (3) is correct.

***Explanation*:** We know that:
The order of a differential equation is the highest order of the derivative present.
The degree of a differential equation is the power of the highest-order derivative, provided the equation is a polynomial in derivatives.

(A): $\left(\frac{d^2y}{dx^2}\right)+2\left(\frac{dy}{dx}\right)^2 = e^{\frac{dy}{dx}}+1.$

The highest derivative is $\frac{d^2y}{dx^2}$, so the order is 2.

The term $e^{\frac{dy}{dx}}$ makes the equation non-polynomial in derivatives, so the degree is not defined.
This matches **(IV)**: Order = 2, Degree = Not defined.

(B): $\left(\frac{d^2y}{dx^2}\right)^2+4\left(\frac{dy}{dx}\right)^3 = e^y-1.$

The highest derivative is $\frac{d^2y}{dx^2}$, so the order is 2.

The power of the highest derivative is 2, so the degree is 2.
This matches **(III)**: Order = 2, Degree = 2.

(C): $3\left(\frac{dy}{dx}\right)+4y+e^y = \frac{dx}{dy}.$

Rewrite as $3\left(\frac{dy}{dx}\right)+4y+e^y = \frac{1}{\left(\frac{dy}{dx}\right)}$

Multiply by $\frac{dy}{dx}$: $3\left(\frac{dy}{dx}\right)^2+4y\left(\frac{dy}{dx}\right)+e^y\left(\frac{dy}{dx}\right) = 1$

The highest derivative is $\frac{dy}{dx}$, so the order is 1.

The power of the highest derivative is 2, so the degree is 2.
This matches **(I)**: Order = 1, Degree = 2.

(D): $\left(\frac{d^2y}{dx^2}\right)+3\left(\frac{dy}{dx}\right) = \left(e^y+\frac{dy}{dx}\right)^2$

The highest derivative is $\frac{d^2y}{dx^2}$, so the order is 2.

The power of the highest derivative is 1, so the degree is 1.
This matches **(II)**: Order = 2, Degree = 1.

11. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then the value of A^{20} is:

(1) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ **(2)** $\begin{bmatrix} 20 & 20 \\ 0 & 20 \end{bmatrix}$

(3) $\begin{bmatrix} 1 & 20 \\ 0 & 1 \end{bmatrix}$ **(4)** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Ans. Option (3) is correct.

***Explanation*:** First, compute the first few powers of A to observe any emerging pattern:

$A^1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

$A^2 = A \times A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

$A^3 = A^2 \times A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$

$A^4 = A^3 \times A = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$

From the pattern observed, we can generalise:

$$A^n = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$$

So, $$A^{20} = \begin{pmatrix} 1 & 20 \\ 0 & 1 \end{pmatrix}$$

12. Function $f(x) = x^3 - 3x + 3$ is

(A) Increasing in the interval $(-1, 1)$
(B) Increasing in the interval $(1, \infty)$
(C) Decreasing in the interval $(-1, 1)$
(D) Increasing in the interval $(-\infty, -1) \cup (\infty, 1)$

Choose the correct answer from the options given below:

(1) (A), (C) and (D) only
(2) (B), (C) and (D) only
(3) (A) and (B) only
(4) (A), (B) and (D) only

Ans. Option (2) is correct.

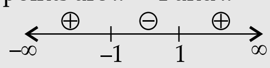
***Explanation*:** A function is increasing when its first derivative is positive ($f'(x) > 0$).
A function is decreasing when its first derivative is negative ($f'(x) < 0$).
The derivative of $f(x) = x^3 - 3x + 3$ is

$$f'(x) = 3x^2 - 3.$$

For critical points, put $f'(x) = 0$.

or, $$3(x^2 - 1) = 0.$$
$$3(x-1)(x+1) = 0.$$

The critical points are $x = 1$ and $x = -1$.



Test sign of $f'(x)$ in each interval:

Interval	Test value	$f'(x) = 3x^2 - 3$	Sign of $f'(x)$	Behaviour of $f(x)$
$(-\infty, -1)$	$x = -2$	$3(4-3) = 9$	Positive	Increasing
$(-1, 1)$	$x = 0$	-3	Negative	Decreasing
$(1, \infty)$	$x = 2$	$3(4) - 3 = 9$	Positive	Increasing

Thus,

(A) Increasing in $(-1, 1)$ is false.

(B) Increasing in $(1, \infty)$ is true.

(C) Decreasing in $(-1, 1)$ is true.

(D) Increasing in $(-\infty, -1) \cup (1, \infty)$ is true.

13. Let X denote the number of hours a person uses a mobile and the probability distribution of X is as

X	0	1	2	3	4
$P(X)$	0.1	K	$2K$	$2K$	K

Then the value of K is

(1) 0.15 **(2)** 0.20

(3) 0.12 **(4)** 0.16

Ans. Option (1) is correct.

***Explanation*:** Since, the sum of all probabilities must be 1:

$$0.1 + K + 2K + 2K + K = 1$$
$$\Rightarrow \quad 0.1 + 6K = 1$$
$$\Rightarrow \quad 6K = 1 - 0.1 = 0.9$$
$$\Rightarrow \quad K = \frac{0.9}{6} = 0.15$$

14. The value of $\int_0^1 xe^x \, dx$ is:

(1) $\frac{1}{e}$ **(2)** e

(3) 1 **(4)** -1

Ans. Option (3) is correct.

***Explanation*:** Given, $\int_0^1 xe^x dx$

Here,
$$\int xe^x dx = xe^x - \int e^x dx$$
$$= xe^x - e^x + C$$
(Integration by parts)

Now, apply limits from 0 to 1:
$$\int_0^1 xe^x dx = [xe^x - e^x]_0^1$$
$$= (e - e) - (0 - 1) = 1$$

15. Consider the Linear Programming Problem

Maximise $z = x + y$

subject to the constraints $x - y \le -1$, $x \ge y$, $x \ge 0$, $y \ge 0$.

Then, which one of the following is TRUE?

(1) Maximum $z = 3$ at $(1, 2)$

(2) There is no solution

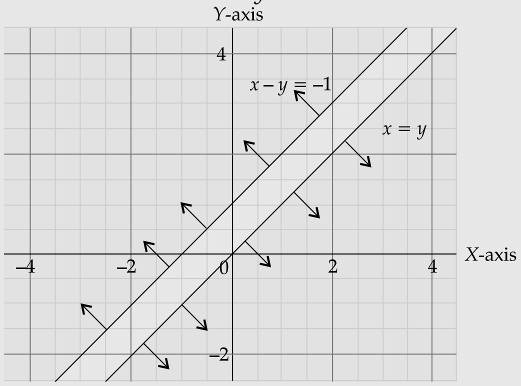
(3) Maximum $z = 15$ at $(7, 8)$

(4) Maximum $z = 10$ at all points on the line $x - y = -1$

Ans. Option (2) is correct.

***Explanation*:** Given, objective function Max. $z = x + y$, subject to constraints:

$$x - y \le -1$$
$$x \ge y$$
$$x \ge 0$$
$$y \ge 0$$



From the above graph, we can see there is no feasible region.

Therefore, the objective function $z = x + y$ has no maximum value under these constraints.

Section : Mathematics Section B1 Core

16. Match **List-I** with **List-II.**

Let A and B be any two events.

List-I	List-II
(A) $P(A')$	**(I)** $\frac{P(A \cap B)}{P(A)}$; $P(A) \neq 0$
(B) $P(\phi)$	**(II)** $\frac{P(A \cap B)}{P(B)}$; $P(B) \neq 0$
(C) $P(A\|B)$	**(III)** $1 - P(A)$
(D) $P(B\|A)$	**(IV)** 0

Choose the correct answer from the options given below:

(1) (A)–(III), (B)–(IV), (C)–(I), (D)–(II)

(2) (A)–(IV), (B)–(III), (C)–(I), (D)–(II)

(3) (A)–(III), (B)–(IV), (C)–(II), (D)–(I)

(4) (A)–(IV), (B)–(III), (C)–(II), (D)–(I)

Ans. Option (3) is correct.

***Explanation*: (A)** $P(A')$

The probability of the complement of A is $P(A') = 1 - P(A)$.

Matches with **(III)**.

(B) $P(\phi)$

The probability of an impossible event (empty set) is 0.

Matches with **(IV)**.

(C) $P(A|B)$

The conditional probability of A given B is $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$ (where $P(B) \neq 0$).

Matches with **(II)**.

(D) $P(B|A)$

The conditional probability of B given A is $P(B|A) = \dfrac{P(A \cap B)}{P(A)}$ (where $P(A) \neq 0$).

Matches with **(I)**.

17. If C_{ij} represents the cofactor of element a_{ij} of the matrix $A = \begin{vmatrix} 2 & -1 & 3 \\ 1 & 2 & 0 \\ 4 & 1 & 5 \end{vmatrix}$ then the value of

$C_{23} + C_{31} - C_{22}$ is

(1) 14 **(2)** –4

(3) –10 **(4)** 8

Ans. Option (3) is correct.

***Explanation*:** Cofactor of C_{23}:

$$(-1)^{2+3}\begin{vmatrix} 2 & -1 \\ 4 & 1 \end{vmatrix} = -[2(1) - (-1)(4)]$$

$$= -[2 + 4] = -6$$

Cofactor of C_{31}:

$$(-1)^{3+1}\begin{vmatrix} -1 & 3 \\ 2 & 0 \end{vmatrix} = [(-1)(0) - (3)(2)]$$

$$= -6$$

Cofactor of C_{22}:

$$(-1)^{2+2}\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = [2(5) - 3(4)]$$

$$= 10 - 12 = -2$$

Now, $C_{23} + C_{31} - C_{22} = -6 - 6 + 2 = -10$

18. Match **List-I** with **List-II**.

List-I	List-II
(A) $\cos^{-1} x + \cos^{-1}(-x)$	**(I)** $\dfrac{\pi}{3}$
(B) $\operatorname{cosec}^{-1}(-x) + \sec^{-1}(-x)$	**(II)** $-\dfrac{\pi}{3}$
(C) $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$	**(III)** π
(D) $\tan^{-1}\left(\tan\dfrac{4\pi}{3}\right)$	**(IV)** $\dfrac{\pi}{2}$

Choose the correct answer from the options given below:

(1) (A)–(IV), (B)–(III), (C)–(II), (D)–(I)

(2) (A)–(III), (B)–(IV), (C)–(II), (D)–(I)

(3) (A)–(III), (B)–(II), (C)–(IV), (D)–(I)

(4) (A)–(I), (B)–(II), (C)–(IV), (D)–(III)

Ans. Option (2) is correct.

***Explanation*: (A)** $\cos^{-1} x + \cos^{-1}(-x) = \cos^{-1} x + (\pi - \cos^{-1} x) = \pi$.

[By Property: $\cos^{-1}(-x) = \pi - \cos^{-1} x$ for $x \in [-1, 1]$]

Matches with (III).

(B) $\operatorname{cosec}^{-1}(-x) + \sec^{-1}(-x) = -\operatorname{cosec}^{-1} x + \pi - \sec^{-1} x$.

[By Property: $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$(for $x \geq 1$ or $x \leq -1$).]

We know that, $\operatorname{cosec}^{-1} x + \sec^{-1} x = \dfrac{\pi}{2}$ for $|x| \geq 1$.

So,

$$-\operatorname{cosec}^{-1} x + \pi - \sec^{-1} x = -\left(\frac{\pi}{2} - \sec^{-1} x\right) + \pi - \sec^{-1} x$$

$$= -\frac{\pi}{2} + \sec^{-1} x + \pi - \sec^{-1} x = \frac{\pi}{2}$$

Matches with (IV).

(C) $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

Since, $\tan^{-1}\sqrt{3} = \dfrac{\pi}{3}$

and $\sec^{-1}(-2) = \pi - \sec^{-1} 2$

$$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Therefore, $\dfrac{\pi}{3} - \dfrac{2\pi}{3} = -\dfrac{\pi}{3}$

Matches with (II).

(D) $$\tan^{-1}\left(\tan\frac{4\pi}{3}\right) = \tan^{-1}\left(\tan\left(\pi + \frac{\pi}{3}\right)\right)$$

$$= \tan^{-1}\left(\tan\frac{\pi}{3}\right) = \frac{\pi}{3}$$

Since $\dfrac{\pi}{3} \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

Matches with (I).

19. If $\begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$, then the product of all values of a is:

(1) 21 **(2)** –21

(3) 10 **(4)** –4

Ans. Option (2) is correct.

***Explanation*:** Given, $\begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$

$$\begin{aligned}
\Rightarrow \quad 1(2a^2 + 4) + 2(4a) + 5(8) &= 86 \\
\Rightarrow \quad 2a^2 + 4 + 8a + 40 &= 86 \\
\Rightarrow \quad 2a^2 + 8a + 44 &= 86 \\
\Rightarrow \quad 2a^2 + 8a - 42 &= 0 \\
\Rightarrow \quad a^2 + 4a - 21 &= 0 \\
\Rightarrow \quad (a + 7)(a - 3) &= 0 \Rightarrow a = -7, 3
\end{aligned}$$

Now, product of all $a = -7 \times 3 = -21$

20. The relation R in $\mathbb{R}$ (set of real numbers) is defined by $R = \{(a, b): a \leq b^3)\}$, then R is

(1) Reflexive relation

(2) Transitive but not symmetric

(3) Reflexive and transitive but not symmetric

(4) Neither reflexive nor symmetric nor transitive

Ans. Option (4) is correct.

***Explanation*:** Given: $R = \{(a, b) \mid a \leq b^3\}$ on $\mathbb{R}$

Not Reflexive because

Let $a = 0.5 \Rightarrow 0.5 \nleq (0.5)^3 = 0.125$

Not Symmetric because

$(0, 2) \in$ R but $(2, 0) \notin$ R

Not Transitive because

if $a = 3, b = \frac{3}{2}, c = \frac{4}{3}$

$b^3 = 3.375 \Rightarrow a = 2 \leq 3.375$, so $a \leq b^3$

$c^3 = 2.37 \Rightarrow b = 1.5 \leq 2.37$, so $b \leq c^3$

Now, $3 \nleq 2.37$, so $a \nleq c^3$

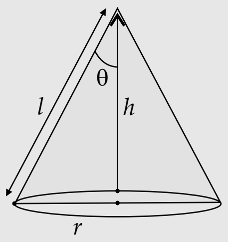
21. The semi-vertical angle of a right circular cone of maximum volume of a given slant height is

(1) $\tan^{-1} 2$ **(2)** $\sin^{-1}\left(\frac{1}{3}\right)$

(3) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ **(4)** $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Ans. Option (3) is correct.

***Explanation*:** Volume of cone: $V = \frac{1}{3}\pi r^2 h$



Using geometry:

$r = l \sin\theta, h = l \cos\theta$

$$\therefore \quad V = \frac{1}{3}\pi(l \sin\theta)^2 (l \cos\theta)$$
$$= \frac{1}{3}\pi l^3 \sin^2\theta \cos\theta$$

$$\text{Maximise } V(\theta) = \sin^2\theta \cos\theta$$

Let $f(\theta) = \sin^2\theta \cos\theta$

Use the product rule:

$f'(\theta) = 2 \sin\theta \cos\theta . \cos\theta + \sin^2\theta . (-\sin\theta)$

$f'(\theta) = 2 \sin\theta \cos^2\theta - \sin^3\theta$

Put $f'(\theta) = 0$:

$$2 \sin\theta \cos^2\theta - \sin^3\theta = 0$$
$$\Rightarrow \quad \sin\theta(2 \cos^2\theta - \sin^2\theta) = 0$$

So, either

$\sin\theta = 0 \Rightarrow \theta = 0 \rightarrow$ not in open interval $\left(0, \frac{\pi}{2}\right)$

so discard.

$$\begin{aligned}
\text{or,} \quad 2\cos^2\theta &= \sin^2\theta \\
2\cos^2\theta &= 1 - \cos^2\theta \\
\Rightarrow \quad 3\cos^2\theta &= 1 \\
\Rightarrow \quad \cos\theta &= \frac{1}{\sqrt{3}} \\
\text{So,} \quad \theta &= \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)
\end{aligned}$$

22. If a matrix P is both symmetric and skew-symmetric, then

(1) P is a diagonal matrix

(2) P is a square matrix

(3) P is a zero matrix

(4) P is an identity matrix

Ans. Option (3) is correct.

***Explanation*:** A matrix P is symmetric if $P^T = P$

A matrix P is skew-symmetric if $P^T = -P$

If both hold:

$$\begin{aligned}
P = P^T &= -P \\
\Rightarrow \quad P &= -P \\
\Rightarrow \quad 2P &= 0 \\
\Rightarrow \quad P &= 0
\end{aligned}$$

Thus, the only matrix that is both symmetric and skew-symmetric is the zero matrix.

23. A letter is known to have come either from KOLKATA or TATANAGAR. On the envelope, just two consecutive letters TA are visible. The probability that letter has come from TATANAGAR is:

(1) $\frac{2}{5}$ **(2)** $\frac{3}{5}$

(3) $\frac{1}{4}$ **(4)** $\frac{2}{3}$

Ans. Option (2) is correct.

Explanation: The problem states that a letter is known to have come either from KOLKATA or TATANAGAR, and on the envelope, the two consecutive letters 'TA' are visible.

Occurrences of 'TA' in each city name:

KOLKATA contains 'TA' once.

TATANAGAR contains 'TA' twice.

We need to find P(TATANAGAR | TA).

Here, P(TA | KOLKATA) = $\frac{1}{6}$

P(TA | TATANAGAR) = $\frac{2}{8} = \frac{1}{4}$

Apply Bayes' Theorem (assuming equal priors $P(K) = P(T) = \frac{1}{2}$)

$$P(T \mid TA) = \frac{\frac{1}{4}\cdot\frac{1}{2}}{\frac{1}{4}\cdot\frac{1}{2}+\frac{1}{6}\cdot\frac{1}{2}}$$

$$= \frac{\frac{1}{8}}{\frac{1}{8}+\frac{1}{12}} = \frac{\frac{1}{8}}{\frac{5}{24}} = \frac{3}{5}$$

24. The shortest distance between lines $\frac{-x-3}{4} = \frac{y-6}{3} = \frac{z}{2}$ and $\frac{-x-2}{4} = \frac{y}{1} = \frac{z-7}{1}$ is:

(1) $\frac{1}{9}$ units (2) 81 units

(3) 9 units (4) 7 units

Ans. Option (3) is correct.

***Explanation*: Line 1:** $\frac{-x-3}{4} = \frac{y-6}{3} = \frac{z}{2}$.

Line 2: $\frac{-x-2}{4} = \frac{y}{1} = \frac{z-7}{1}$.

The shortest distance d between two skew lines with position vectors $\vec{a_1}, \vec{a_2}$ and direction vectors $\vec{b_1}, \vec{b_2}$ is given by $d = \frac{|(\vec{a_2}-\vec{a_1}).(\vec{b_1}\times\vec{b_2})|}{|\vec{b_1}\times\vec{b_2}|}$.

From Line 1:

Position vector $\vec{a_1} = \langle -3, 6, 0\rangle$.

Direction vector $\vec{b_1} = \langle -4, 3, 2\rangle$.

From Line 2:

Position vector $\vec{a_2} = \langle -2, 0, 7\rangle$.

Direction vector $\vec{b_2} = \langle -4, 1, 1\rangle$.

Now,

$$\vec{a_2}-\vec{a_1} = \langle -2-(-3), 0-6, 7-0\rangle$$

$$\vec{a_2}-\vec{a_1} = \langle 1, -6, 7\rangle$$

$$\vec{b_1}\times\vec{b_2} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ -4 & 3 & 2 \\ -4 & 1 & 1 \end{vmatrix}$$

$= \boldsymbol{i}(3 \times 1 - 2 \times 1) - \boldsymbol{j}(-4 \times 1 - 2 \times (-4))$
$\quad + \boldsymbol{k}(-4 \times 1 - 3 \times (-4))$

$= \boldsymbol{i}(1) - \boldsymbol{j}(4) + \boldsymbol{k}(8)$

$= \langle 1, -4, 8\rangle$

$$|\vec{b_1}\times\vec{b_2}| = \sqrt{1^2+(-4)^2+8^2}$$

$$= \sqrt{1+16+64}$$

$$= \sqrt{81} = 9$$

$$(\vec{a_2}-\vec{a_1}).(\vec{b_1}\times\vec{b_2}) = \langle 1, -6, 7\rangle.\langle 1, -4, 8\rangle$$

$$= (1)(1) + (-6)(-4) + (7)(8)$$

$$= 1 + 24 + 56 = 81$$

Now, the shortest distance,

$$d = \frac{|81|}{9}$$

$$d = 9 \text{ units}$$

25. If $\vec{a} = \hat{i}+\hat{k}$, $\vec{b} = \hat{j}-\hat{k}$ and $\vec{c} = \hat{i}+\hat{j}+\hat{k}$ such that $\vec{r}\times\vec{b} = \vec{c}\times\vec{b}$ and $\vec{r}.\vec{a} = 0$, then $\vec{r}$ is:

(1) $\hat{i}+3\hat{j}-\hat{k}$ (2) $\hat{i}-3\hat{j}+\hat{k}$

(3) $3\hat{i}+\hat{j}-\hat{k}$ (4) $\hat{i}-\hat{j}+3\hat{k}$

Ans. Option (1) is correct.

Explanation: Given, $\vec{r}\times\vec{b} = \vec{c}\times\vec{b}$.

This implies $(\vec{r}-\vec{c})\times\vec{b} = \vec{0}$

Therefore, $\vec{r}-\vec{c}$ is parallel to $\vec{b}$

So, $\vec{r}-\vec{c} = \lambda\vec{b}$ for some scalar λ.

Thus, $\vec{r} = \vec{c}+\lambda\vec{b}$

Substituting the given vectors into the expression for $\vec{r}$

$$\vec{r} = (\hat{i}+\hat{j}+\hat{k})+\lambda(\hat{j}-\hat{k})$$

$$\vec{r} = \hat{i}+(1+\lambda)\hat{j}+(1-\lambda)\hat{k}$$

Given, $\vec{r}.\vec{a} = 0$.

Substituting $\vec{r}$ and $\vec{a}$: $(\hat{i}+(1+\lambda)\hat{j}+(1-\lambda)\hat{k}).(\hat{i}+\hat{k})$ = 0.

$$(1)(1) + (1+\lambda)(0) + (1-\lambda)(1) = 0$$
$$1 + 0 + 1 - \lambda = 0$$
$$2 - \lambda = 0$$
$$\lambda = 2$$

Substituting the value of λ back into the expression for $\vec{r}$,

$$\vec{r} = \hat{i}+(1+2)\hat{j}+(1-2)\hat{k}$$
$$\vec{r} = \hat{i}+3\hat{j}-\hat{k}$$

26. Solution of the differential equation $\frac{dy}{dx} = \sqrt{1+x^2+y^2+x^2y^2}$ is: (Here C is an arbitrary constant.)

(1) $\log\left|\frac{y+\sqrt{1+y^2}}{x+\sqrt{1+x^2}}\right| = C$

(2) $\log\left|\frac{y+\sqrt{1+y^2}}{\sqrt{x+\sqrt{1+x^2}}}\right| = C$

(3) $\log\left|\frac{y+\sqrt{1+y^2}}{\sqrt{x+\sqrt{1+x^2}}}\right| = \frac{x}{2}\sqrt{1+x^2}+C$

(4) $\log\left|\frac{y+\sqrt{1+y^2}}{x+\sqrt{1+x^2}}\right| = \frac{x}{2}\sqrt{1+x^2}+C$

Ans. Option (3) is correct.

Explanation: We have, $\frac{dy}{dx} = \sqrt{1+x^2+y^2+x^2y^2}$

or, $$\frac{dy}{dx} = \sqrt{(1+x^2)(1+y^2)}$$

$$\frac{dy}{\sqrt{1+y^2}} = \sqrt{1+x^2}dx$$

Integrating both sides, we get,

$$\int\frac{dy}{\sqrt{1+y^2}} = \int\sqrt{1+x^2}dx$$

$$\log\left|y+\sqrt{1+y^2}\right| = \frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+C$$

$$\left[\text{Since, } \int\sqrt{a^2+x^2}dx = \frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\log\left|x+\sqrt{a^2+x^2}\right|\right]$$

$$\log\left|y+\sqrt{1+y^2}\right|-\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right| = \frac{x}{2}\sqrt{1+x^2}+C$$

$$\log\left|y+\sqrt{1+y^2}\right|-\log\left|(x+\sqrt{1+x^2})^{\frac{1}{2}}\right| = \frac{x}{2}\sqrt{1+x^2}+C$$

$$\log\left|\frac{y+\sqrt{1+y^2}}{\sqrt{x+\sqrt{1+x^2}}}\right| = \frac{x}{2}\sqrt{1+x^2}+C$$

27. If matrix $A = \begin{bmatrix} p & -3 \\ -4 & p \end{bmatrix}$ and $|A^3| = 64$, then the value of p is:

(1) ± 1 **(2)** ± 2

(3) ± 3 **(4)** ± 4

Ans. Option (4) is correct.

Explanation: Given:

$A = \begin{bmatrix} p & -3 \\ -4 & p \end{bmatrix}$, and $|A^3| = 64$

Using property of determinant of powers, we have

$$|A^3| = |A|^3$$
$$\Rightarrow \quad |A|^3 = 64$$
$$\Rightarrow \quad |A| = \sqrt[3]{64} = \pm 4$$

Also, $$|A| = p.p-(-3)(-4) = p^2-12$$

Therefore, $p^2 - 12 = \pm 4$

Case 1: $p^2 - 12 = 4$
$$p^2 = 16$$
$$\Rightarrow \quad p = \pm 4$$

Case 2: $p^2 - 12 = -4$
$$p^2 = 8$$
$$\Rightarrow \quad p = \pm\sqrt{8} = \pm 2\sqrt{2} \notin \text{options}$$

28. Distance of the point (2, 4, –1) from the line $\frac{10+2x}{2} = \frac{y+3}{4} = \frac{6-z}{9}$ is

(1) $\sqrt{14}$ units **(2)** 13 units

(3) 7 units **(4)** $\sqrt{7}$ units

Ans. Option (3) is correct.

***Explanation*:** The given line is

$$\frac{10+2x}{2} = \frac{y+3}{4} = \frac{6-z}{9}$$

The standard symmetric form is

$$\frac{x-(-5)}{1} = \frac{y-(-3)}{4} = \frac{z-6}{-9}$$

A point on the line is $A = (-5, -3, 6)$.

The direction vector of the line is $\vec{d} = \langle 1, 4, -9 \rangle$.

Let $L = (-5 + t, -3 + 4t, 6 - 9t)$ be a general point on the line.

(direction ratio) of the $\vec{PL}$

$\vec{PL} = \langle (-5 + t) - 2, (-3 + 4t) - 4, (6 - 9t) - (-1) \rangle = \langle t - 7, 4t - 7, 7 - 9t \rangle$.

Since $\vec{PL}$ is perpendicular to the line, $\vec{PL}.\vec{d} = 0$.

$$(t-7)(1) + (4t-7)(4) + (7-9t)(-9) = 0$$
$$t - 7 + 16t - 28 - 63 + 81t = 0$$
$$98t - 98 = 0$$
$$t = 1$$

Substituting $t = 1$ into the coordinates of L:

$$L = (-5 + 1, -3 + 4(1), 6 - 9(1))$$
$$= (-4, 1, -3)$$

The distance PL

$$= \sqrt{(-4-2)^2 + (1-4)^2 + (-3-(-1))^2}$$
$$= \sqrt{(-6)^2 + (-3)^2 + (-2)^2}$$
$$= \sqrt{36+9+4}$$
$$= \sqrt{49}$$
$$= 7 \text{ units}$$

29. If A and B are independent events, then which of the following statements are TRUE?

(A) $P(A \cap B) = P(A).P(B)$

(B) $P(A \cap B) = P(A) - P(B)$

(C) $P(A \cup B) = P(A) + P(B) - P(A).P(B)$

(D) $P(A \cup B) = P(A).P(B|A)$

Choose the correct answer from the options given below:

(1) (A), (B) and (C) only **(2)** (B) and (C) only

(3) (C) and (D) only **(4)** (A), (C) and (D) only

Ans. Option (4) is correct.

***Explanation*:** For independent events A and B:

1. (A) $P(A \cap B) = P(A).P(B)$

True (Definition of independence).

2. (B) $P(A \cap B) = P(A) - P(B)$

False (This is incorrect; it doesn't represent independence).

3. (C) $P(A \cup B) = P(A) + P(B) - P(A).P(B)$

True (Since $P(A \cap B) = P(A)P(B)$, the general addition rule simplifies to this).

4. (D) $P(A \cap B) = P(A).P(B|A)$

True (By definition of conditional probability, and since $P(B|A) = P(B)$ when independent).

30. A function $f: \mathbb{R} \to \{x \in \mathbb{R}: -1 < x < 1\}$ is defined as $f(x) = \frac{x}{1+|x|}$, then f is:

(1) neither one-one nor onto

(2) one-one only

(3) onto only

(4) both one-one and onto

Ans. Option (4) is correct.

***Explanation*:** Given, function $f: \mathbb{R} \to \{x \in \mathbb{R} : -1 < x < 1\}$, where $f(x) = \frac{x}{1+|x|}$

1. One-One (Injective)

For $x \geq 0$, $f(x) = \frac{x}{1+x}$ is strictly increasing $\left(\text{derivative } f'(x) = \frac{1}{(1+x)^2} > 0\right)$.

For $x < 0$, $f(x) = \frac{x}{1-x}$ is strictly increasing $\left(\text{derivative } f'(x) = \frac{1}{(1-x)^2} > 0\right)$.

Since $f(x) \geq 0$ for $x \geq 0$ and $f(x) < 0$ for $x < 0$, no two distinct inputs give the same output.

$\Rightarrow f$ is injective.

2. Onto (Surjective)

$$y = \frac{X}{1+|x|}$$

If $y \geq 0$, $x = \frac{y}{1-y}$ (since $x \geq 0$).

If $y < 0$, $x = \frac{y}{1+y}$ (since $x < 0$).

Every $y \in (-1, 1)$ has a corresponding x.

$\Rightarrow f$ is surjective.

So, f is both one-one and onto.

31. Bag A contains 2 unbiased and 3 biased coins whereas Bag B contains 3 unbiased and 2 biased coins. A bag is selected at random and 2 coins are taken out simultaneously. The probability, that both coins are unbiased is:

(1) $\frac{1}{5}$ **(2)** $\frac{1}{10}$

(3) $\frac{2}{5}$ **(4)** $\frac{3}{10}$

Ans. Option (1) is correct.

Explanation: There are two bags, A and B, and one is selected at random.

Probability of selecting Bag $A(P(A)) = \frac{1}{2}$

Probability of selecting Bag $B(P(B)) = \frac{1}{2}$

Step 2: Find the probability of drawing 2 unbiased coins from each bag.

For Bag A:

Contents: 2 unbiased and 3 biased coins

(Total = 5 coins).

Probability of drawing 2 unbiased coins

$(P(2U|A))$: $\frac{^2C_2}{^5C_2} = \frac{1}{10}$

For Bag B:

Contents: 3 unbiased and 2 biased coins

(Total = 5 coins).

Probability of drawing 2 unbiased coins

$(P(2U|B))$: $\frac{^3C_2}{^5C_2} = \frac{3}{10}$

By the Law of Total Probability, the overall probability $P(2U)$ is the weighted sum of the probabilities from each bag:

$P(2U) = P(A) \times P(2U|A) + P(B) \times P(2U|B)$

$$= \frac{1}{2}\times\frac{1}{10}+\frac{1}{2}\times\frac{3}{10} = \frac{1}{20}+\frac{3}{20} = \frac{4}{20} = \frac{1}{5}$$

32. If the line $\frac{-x+1}{3}=\frac{-y-2}{-2k}=\frac{z+3}{2}$ and $\frac{-1+x}{3k} = \frac{-1+y}{1} = \frac{-z+6}{5}$ are perpendicular, then the value of k is:

(1) $\frac{10}{7}$ **(2)** $-\frac{10}{7}$

(3) $\frac{1}{2}$ **(4)** $\frac{4}{3}$

Ans. Option (2) is correct.

Explanation: **Line 1:** $\frac{-x+1}{3} = \frac{-y-2}{-2k} = \frac{z+3}{2}$

$\Rightarrow$ Direction ratios (DRs) = $\langle -3, 2k, 2\rangle$

Line 2: $\frac{-1+x}{3k} = \frac{-1+y}{1} = \frac{-z+6}{5}$

$\Rightarrow$ Direction ratios = $\langle 3k, 1, -5\rangle$

We know that, the two vectors are perpendicular if their dot product is 0.

Let, $\vec{d_1} = \langle -3, 2k, 2\rangle$, $\vec{d_2} = \langle 3k, 1, -5\rangle$

$$\vec{d_1}.\vec{d_2} = -3(3k) + 2k(1) + 2(-5)$$
$$= -9k + 2k - 10$$
$$-7k - 10 = 0$$
$$\Rightarrow \quad k = -\frac{10}{7}$$

33. $\int \tan^{-1}\sqrt{x}\,dx$ equals to: (Here C is an arbitrary constant.)

(1) $(x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$

(2) $x\tan^{-1}\sqrt{x} - \sqrt{x} + C$

(3) $\sqrt{x} - x\tan^{-1}\sqrt{x} + C$

(4) $\sqrt{x} - (x+1)\tan^{-1}\sqrt{x} + C$

Ans. Option (1) is correct.

Explanation: Let $I = \int \tan^{-1}\sqrt{x}\,dx$

Put $\sqrt{x} = t$

$$\frac{1}{2\sqrt{x}}dx = dt$$
$$dx = 2\sqrt{x}\,dt$$
$$\Rightarrow \quad dx = 2t\,dt$$
$$\therefore \quad I = \int 2t\tan^{-1}t\,dt$$
$$I = 2\left[\frac{t^2}{2}\tan^{-1}t - \frac{1}{2}\int\frac{t^2}{1+t^2}dt\right]$$
$$I = 2\left[\frac{t^2}{2}\tan^{1}t - \frac{1}{2}\int\left[\frac{1+t^2}{1+t^2} - \frac{1}{1+t^2}\right]dt\right]$$

$I = [t^2\tan^{-1}t - t + \tan^{-1}t] + C$

$I = (x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$

34. The minimum value of the function $f(x) = x^3 + (10 - x)^3$ occurs at:

(1) $x = 0$ **(2)** $x = 3$

(3) $x = 5$ **(4)** $x = \frac{10}{3}$

Ans. Option (3) is correct.

Explanation: Given, $f(x) = x^3 + (10 - x)^3$

$$f'(x) = \frac{d}{dx}[x^3 + (10-x)^3]$$
$$f'(x) = 3x^2 + 3(10-x)^2(-1)$$
$$f'(x) = 3x^2 - 3(10-x)^2$$

Put the first derivative to zero and solve for x.

$$3x^2 - 3(10-x)^2 = 0$$
$$x^2 = (10-x)^2$$
$$x = \pm(10-x)$$

Case 1: $x = 10 - x$

$\Rightarrow$ $2x = 10$

$\Rightarrow$ $x = 5$

Case 2: $x = -(10-x)$

$\Rightarrow$ $x = -10 + x$

$\Rightarrow$ $0 = -10$, which is not possible.

Now,

$$f''(x) = \frac{d}{dx}[3x^2 - 3(10-x)^2]$$

$$f''(x) = 6x - 6(10-x)(-1)$$
$$f''(x) = 6x + 6(10-x)$$
$$f''(x) = 6x + 60 - 6x$$
$$f''(x) = 60$$

Since $f''(x) = 60 > 0$ for all x, the critical point $x = 5$ corresponds to a local minimum.

35. The area (in sq. units) of the region bounded by the curves $3y^2 = ax$, $y = a$, $a > 0$ and y-axis is:

(1) a **(2)** $3a$

(3) $2a^2$ **(4)** a^2

Ans. Option (4) is correct.

Explanation: Given curves:

$$3y^2 = ax$$

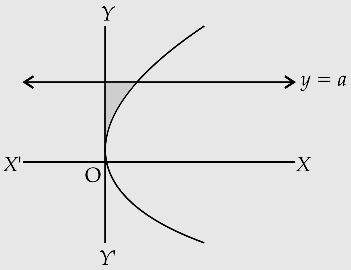
$\Rightarrow$ Parabola $y = a$

$\Rightarrow$ Horizontal line y-axis

$\Rightarrow$ i.e., $x = 0$

Also, $a > 0$

From $3y^2 = ax$, $x = \dfrac{3y^2}{a}$



$$\text{Area} = \int_{y=0}^{a} x\,dy = \int_0^a \frac{3y^2}{a}dy$$

$$= \frac{3}{a}\int_0^a y^2 dx = \frac{3}{a}\left[\frac{y^3}{3}\right]_0^a$$

$$= \frac{3}{a}\cdot\frac{a^3}{3} = a^2 \text{ sq. units}$$

36. If $2f(x) + f\left(\frac{1}{x}\right) = x^2 + 1$, then $\int f(x)dx$ is: (Here C is an arbitrary constant.)

(1) $\frac{1}{3}\left(\frac{2}{3}x^3 + \frac{1}{x} + x\right) + C$ **(2)** $\frac{2}{3}x^3 + \frac{1}{x} - x + C$

(3) $\frac{x^3}{3} + x + C$ **(4)** $\frac{1}{3}\left(\frac{2}{3}x^3 - x\right) + C$

Ans. Option (1) is correct.

Explanation: Given, $2f(x) + f\left(\frac{1}{x}\right) = x^2 + 1$ ...(i)

Replace x with $\frac{1}{x}$ in Eq. (i):

$$2f\left(\frac{1}{x}\right) + f(x) = \frac{1}{x^2} + 1 \quad \text{...(ii)}$$

Solving Eqs (i) and (ii),

Multiply Eq. (i) by 2:

$$4f(x) + 2f\left(\frac{1}{x}\right) = 2x^2 + 2$$

Now, subtract ing Eq. (ii):

$$\left(4f(x) + 2f\left(\frac{1}{x}\right)\right) - \left(f(x) + 2f\left(\frac{1}{x}\right)\right)$$

$$= (2x^2 + 2) - \left(\frac{1}{x^2} + 1\right)$$

$$3f(x) = 2x^2 + 2 - \frac{1}{x^2} - 1$$

$$= 2x^2 - \frac{1}{x^2} + 1$$

$$\Rightarrow \quad f(x) = \frac{1}{3}\left(2x^2 - \frac{1}{x^2} + 1\right)$$

Now, $\int f(x)dx = \int \frac{1}{3}\left(2x^2 - \frac{1}{x^2} + 1\right)dx$

$$= \frac{1}{3}\left(\int 2x^2\,dx - \int \frac{1}{x^2}dx + \int 1\,dx\right)$$

$$= \frac{1}{3}\left(\frac{2x^3}{3} + \frac{1}{x} + x\right) + C$$

37. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the value of $A^2 - 5A + 6I$ is:

(1) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ **(2)** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(3) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ **(4)** $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

Ans. Option (1) is correct.

Explanation: First, calculate A^2:

$$A^2 = A.A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (3)(3)+(1)(-1) & (3)(1)+(1)(2) \\ (-1)(3)+(2)(-1) & (-1)(1)+(2)(2) \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$5A = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$$

$$6I = 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

Now, $A^2 - 5A + 6I$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 8-15+6 & 5-5+0 \\ -5-(-5)+0 & 3-10+6 \end{bmatrix}$$

$$= \begin{bmatrix} -1+6 & 0+0 \\ 0+0 & -1+6 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

38. Match **List-I** with **List-II.**

List-I	List-II
Differential equation	**Integrating factor**
(A) $y\,dx + (x - y^3)dy = 0$	**(I)** e^{-x}
(B) $x\dfrac{dy}{dx} + y = x^2$	**(II)** $\dfrac{1}{x}$
(C) $\dfrac{dy}{dx} - y = e^x$	**(III)** y
(D) $x\,dy - y\,dx = x^3dx$	**(IV)** x

Choose the correct answer from the options given below:

(1) (A)–(IV), (B)–(III), (C)–(II), (D)–(I)
(2) (A)–(IV), (B)–(II), (C)–(I), (D)–(III)
(3) (A)–(III), (B)–(IV), (C)–(I), (D)–(II)
(4) (A)–(III), (B)–(II), (C)–(I), (D)–(IV)

Ans. Option (3) is correct.

***Explanation*:** **(A)** $y\,dx + (x - y^3)dy = 0$

Write it as: $y\dfrac{dx}{dy} + (x - y^3) = 0$

$$\Rightarrow \quad \frac{dx}{dy} + \frac{x}{y} = y^2$$

This is a linear differential equation in x.
Standard form:

$$\frac{dx}{dy} + P(y)x = Q(y)$$

$$\Rightarrow \quad P(y) = \frac{1}{y}$$

$$\Rightarrow \quad \text{IF} = e^{\int \frac{1}{y}dx} = e^{\ln y} = y$$

Match: **(A)** $\rightarrow$ **(III)**

(B) $$x\frac{dy}{dx} + y = x^2$$

Rewriting: $\dfrac{dy}{dx} + \dfrac{y}{x} = x$

his is a linear DE in y.

$$P(x) = \frac{1}{x}$$

$$\Rightarrow \quad \text{IF} = e^{\int \frac{1}{x}dx} = x$$

Match: **(B)** $\rightarrow$ **(IV)**

(C) $$\frac{dy}{dx} - y = e^x$$

Linear DE in y, with:

$$P(x) = -1$$

$$\Rightarrow \quad \text{IF} = e^{\int -1dx} = e^{-x}$$

Match: **(C)** $\rightarrow$ **(I)**

(D) $$x dy - y dx = x^3dx$$

Rewriting: $x\dfrac{dy}{dx} - y = x^3$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{y}{x} = x^2$$

Linear in y, with:

$$P(x) = -\frac{1}{x}$$

$$\Rightarrow \quad IF = e^{\int -\frac{1}{x}dx} = \frac{1}{x}$$

Match: **(D)** $\rightarrow$ **(II)**

39. Match **List-I** with **List-II.**

List-I	List-II
Mathematical statement	**Value**
(A) $\hat{i}.(\hat{j} \times \hat{k})$	**(I)** $-\hat{k}$
(B) $\hat{j}.(\hat{i} \times \hat{k})$	**(II)** 1
(C) $\hat{i} \times (\hat{j} \times \hat{k})$	**(III)** -1
(D) $\hat{j} \times \hat{i}$	**(IV)** $\vec{0}$

Choose the correct answer from the options given below:

(1) (A)–(II), (B)–(III), (C)–(IV), (D)–(I)
(2) (A)–(III), (B)–(IV), (C)–(I), (D)–(II)
(3) (A)–(IV), (B)–(III), (C)–(I), (D)–(II)
(4) (A)–(I), (B)–(II), (C)–(III), (D)–(IV)

Ans. Option (1) is correct.

***Explanation*:** **(A)** $\hat{i}.(\hat{j} \times \hat{k}) = \hat{i}.\hat{i} = 1$

Match **(A)** $\rightarrow$ **(II)**

(B) $\hat{j}.(\hat{i} \times \hat{k})$

$$\hat{i} \times \hat{k} = -\hat{j},$$

so: $$\hat{j}.(-\hat{j}) = -1$$

Match **(B)** $\rightarrow$ **(III)**

(C) $\hat{i} \times (\hat{j} \times \hat{k}) = \hat{i} \times (\hat{i}) = \vec{0}$

Match **(C)** $\rightarrow$ **(IV)**

(D) $\hat{j} \times \hat{i}$

$$\hat{i} \times \hat{j} = \hat{k},$$

so: $$\hat{j} \times \hat{i} = -\hat{k}$$

Match **(D)** $\rightarrow$ **(I)**

40. If $A = \begin{bmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}$, then the value of det (adj(2A)) is:

(1) 100 **(2)** 320

(3) 400 **(4)** 1600

Ans. Option (4) is correct.

Explanation: We know that:
det(adj(A)) = $[\det(A)]^{n-1}$, where A is an $n \times n$ matrix.
$\det(kA) = k^n.\det(A)$ for A of order n.
Here, A is a 3×3 matrix $\Rightarrow n = 3$.

$$\text{Now, } A = \begin{bmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}$$

$$\therefore \quad |A| = 2\begin{vmatrix} 2 & -1 \\ -5 & 0 \end{vmatrix} - (-1)\begin{vmatrix} 0 & -1 \\ 3 & 0 \end{vmatrix} + (-2)\begin{vmatrix} 0 & 2 \\ 3 & -5 \end{vmatrix}$$

$$= 2(2 \times 0 - (-1)(-5)) + 1(0 \times 0 - (-1)(3)) + (-2)(0 \times (-5) - 2 \times 3)$$
$$= 2(-5) + 1(3) + (-2)(-6)$$
$$= -10 + 3 + 12 = 5$$

Now, $\det(\text{adj}(2A)) = [\det(2A)]^{n-1}$
$= [\det(2A)]^2$

But, $\det(2A) = 2^3 \times \det(A)$
$= 8 \times 5 = 40$

So, $\det(\text{adj}(2A)) = 40^2 = 1600$

41. In a linear programming problem, the constraints on decision variables x and y are $y - 2x \leq 0$, $y \geq 0$, $0 \leq x \leq 5$. The feasible region of the above problem:

(1) is bounded in the first quadrant

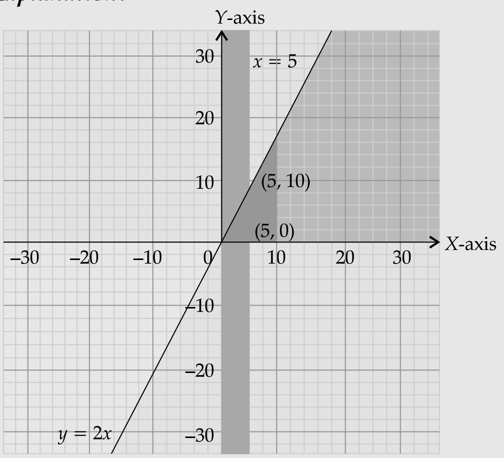
(2) is unbounded in the first quadrant

(3) is unbounded in first and second quadrants

(4) does not exist

Ans. Option (1) is correct.

Explanation:



From the above graph, we observe that the feasible region exists.
It lies entirely in the first quadrant.
And it is bounded (since x and y both have upper and lower limits).

42. If $\vec{a} = 3\hat{i} - 6\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{j} + \lambda\hat{k}$ are such that $\vec{a} \,||\, \vec{b}$, then $3\lambda + 2 =$

(1) 0 **(2)** 2

(3) 4 **(4)** 6

Ans. Option (3) is correct.

Explanation: Given vectors $\vec{A} = 3\hat{i} - 6\hat{j} + \hat{k}$ and $\vec{B} = 2\hat{i} - 4\hat{j} + \lambda\hat{k}$ are parallel.

The components of the vectors are:

For $\vec{A} : a_1 = 3, b_1 = -6, c_1 = 1$

For $\vec{B} : a_2 = 2, b_2 = -4, c_2 = \lambda$

Two vectors are parallel if the ratios of their corresponding components are equal:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

This gives us the following equations:

$$\frac{3}{2} = \frac{-6}{-4} = \frac{1}{\lambda}$$

or $$\frac{3}{2} = \frac{3}{2} = \frac{1}{\lambda}$$

Thus, $$\lambda = \frac{2}{3}$$

Now, $$3\lambda + 2 = 3\left(\frac{2}{3}\right) + 2 = 2 + 2 = 4$$

43. The corner points of the bounded feasible region determined by the system of linear inequalities are (0, 0), (2, 4), (0, 5) and (4, 0). If the maximum value of $Z = ax + by$, where $a, b > 0$ occurs at both (2, 4) and (4, 0), then

(1) $3a = b$ **(2)** $a = b$

(3) $a = 2b$ **(4)** $2a = b$

Ans. Option (3) is correct.

Explanation: Z at (2, 4):
Substitute $x = 2$ and $y = 4$ into $Z = ax + by$.
$Z_{(2,4)} = a(2) + b(4) = 2a + 4b$
Z at (4, 0):
Substitute $x = 4$ and $y = 0$ into $Z = ax + by$.
$Z_{(4,0)} = a(4) + b(0) = 4a$
Since the maximum value occurs at both points, put $Z_{(2,4)} = Z_{(4,0)}$

$\Rightarrow \quad 2a + 4b = 4a$

$\Rightarrow \quad 4b = 2a$

$\Rightarrow \quad a = 2b$

44. If $|\vec{a}| = 1$, $|\vec{b}| = 2$, $|2\vec{a} + \vec{b}| = 2\sqrt{3}$ then $|\vec{a} - \vec{b}|$ is:

(1) $\sqrt{3}$ **(2)** $\sqrt{5}$

(3) $\frac{2}{\sqrt{3}}$ **(4)** $\frac{2}{\sqrt{5}}$

Ans. Option (1) is correct.

Explanation: Given,

$$|\vec{a}| = 1, |\vec{b}| = 2, |2\vec{a}+\vec{b}| = 2\sqrt{3}$$

$$|2\vec{a}+\vec{b}|^2 = (2\sqrt{3})^2 = 4\times 3 = 12$$

$$(2\vec{a}+\vec{b})\cdot(2\vec{a}+\vec{b}) = 4\vec{a}\cdot\vec{a}+4\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{b}$$

$$12 = 4(1)^2+4\vec{a}\cdot\vec{b}+(2)^2$$

$$12 = 4+4\vec{a}\cdot\vec{b}+4$$

$$12 = 8+4\vec{a}\cdot\vec{b}$$

$$4\vec{a}\cdot\vec{b} = 12-8 = 4$$

$$\vec{a}\cdot\vec{b} = \frac{4}{4} = 1$$

Now, $|\vec{a}-\vec{b}|^2 = (\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b})$

$$= \vec{a}\cdot\vec{a}-2\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{b}$$

$$= |\vec{a}|^2-2\vec{a}\cdot\vec{b}+|\vec{b}|^2$$

$$= (1)^2-2(1)+(2)^2$$

$$= 1-2+4 = 3$$

$$|\vec{a}-\vec{b}| = \sqrt{3}$$

45. The value of k for which the function

$$f(x) = \begin{cases}\dfrac{1-\cos 8x}{16x^2}, & \text{if } x\neq 0\\ k & \text{if } x = 0\end{cases}$$ is continuous at $x = 0$ is:

(1) 0 **(2)** 2

(3) –2 **(4)** 1

Ans. Option (2) is correct.

Explanation: Given,

$$f(x) = \begin{cases}\dfrac{1-\cos 8x}{16x^2}, & \text{if } x\neq 0\\ k & \text{if } x = 0\end{cases}$$

The limit is: $\lim\limits_{x\to 0}\dfrac{1-\cos 8x}{16x^2}$

Using trigonometric identity:

$$1-\cos(\theta) = 2\sin^2\left(\frac{\theta}{2}\right)$$

Applying this: $= \lim\limits_{x\to 0}\dfrac{2\sin^2(4x)}{16x^2}$

$$= \lim_{x\to 0}\frac{\sin^2(4x)}{8x^2}$$

Now use: $\dfrac{\sin(4x)}{4x}\to 1$ as $x\to 0$

$$= \lim_{x\to 0}\left(\frac{\sin(4x)}{4x}\right)^2\cdot\frac{16}{8}$$

$$= 1^2 . 2 = 2$$

$\lim\limits_{x\to 0} f(x) = k$ $\lim\limits_{x\to 0} f(x) = 2 \Rightarrow k = 2$

46. The function $f(x) = \sin 3x$, $x \in \left[0, \frac{\pi}{2}\right]$

(A) is increasing on $\left[0, \frac{\pi}{6}\right]$

(B) is decreasing on $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$

(C) is increasing on $\left[0, \frac{\pi}{2}\right]$

(D) is decreasing on $\left[0, \frac{\pi}{2}\right]$

Choose the correct answer from the options given below:

(1) (A), (B) and (C) only **(2)** (A) and (B) only

(3) (B), (C) and (D) only **(4)** (D) and (A) only

Ans. Option (2) is correct.

Explanation: Given, $f(x) = \sin 3x$, $x\in\left[0,\frac{\pi}{2}\right]$

Increasing when $f'(x) > 0$:

$3\cos 3x > 0 \Rightarrow \cos 3x > 0$

On $\left[0,\frac{\pi}{2}\right]$, $3x$ ranges from 0 to $\frac{3\pi}{2}$.

$\cos 3x > 0$ when:

$$3x\in\left[0,\frac{\pi}{2}\right)\Rightarrow x\in\left[0,\frac{\pi}{6}\right)$$

So, $f(x)$ is increasing on $\left[0,\frac{\pi}{6}\right]$.

Decreasing when $f'(x) < 0$:

$3\cos 3x < 0 \Rightarrow \cos 3x < 0$.

On $\left[0,\frac{\pi}{2}\right]$, $\cos 3x < 0$ when:

$$3x\in\left(\frac{\pi}{2},\frac{3\pi}{2}\right]\Rightarrow x\in\left(\frac{\pi}{6},\frac{\pi}{2}\right]$$

So, $f(x)$ is decreasing on $\left[\frac{\pi}{6},\frac{\pi}{2}\right]$.

Now,

(A) $f(x)$ is increasing on $\left[0,\frac{\pi}{6}\right]$: True (as shown above).

(B) $f(x)$ is decreasing on $\left[\frac{\pi}{6},\frac{\pi}{2}\right]$: True (as shown above).

(C) $f(x)$ is increasing on $\left[0,\frac{\pi}{2}\right]$: False (it decreases after $\frac{\pi}{6}$).

(D) $f(x)$ is decreasing on $\left[0,\frac{\pi}{2}\right]$: False (it increases first).

47. Match **List-I** with **List-II.**

List-I	List-II
(A) $\int_{-a}^{a} f(x)dx = 0$	**(I)** 0
(B) $\int_{0}^{2a} f(x)dx = 2\int_{0}^{a} f(x)dx$	**(II)** 1
(C) $\int_{-\pi}^{\pi} \cos x\, dx$	**(III)** f is an odd function
(D) $\int_{-1}^{1} x^{101}dx + 1$	**(IV)** $f(2a - x) = f(x)$

Choose the correct answer from the options given below:

(1) (A)–(III), (B)–(IV), (C)–(I), (D)–(II)
(2) (A)–(III), (B)–(IV), (C)–(II), (D)–(I)
(3) (A)–(IV), (B)–(III), (C)–(II), (D)–(I)
(4) (A)–(IV), (B)–(III), (C)–(I), (D)–(II)

Ans. Option (1) is correct.

***Explanation*: (A)** $\int_{-a}^{a} f(x)dx = 0$

This is true if $f(x)$ is an odd function (since the integral of an odd function over symmetric limits $[-a, a]$ is zero).
Match: **(A) → (III)**

(B) $\int_{0}^{2a} f(x)dx = 2\int_{0}^{a} f(x)dx$

This holds if $f(x)$ satisfies $f(2a - x) = f(x)$ (i.e., $f(x)$ is symmetric about $x = a$).
Match: **(B) → (IV)**

(C) $\int_{-\pi}^{\pi} \cos x\, dx$

The integral of $\cos x$ over $[-\pi, \pi]$ is:

$$\int_{-\pi}^{\pi} \cos x\, dx = 2\int_{0}^{\pi} \cos x\, dx = 2[\sin x]_0^{\pi}$$

$$= 2(0 - 0) = 0$$

Match: **(C) → (I)**
(D) $\int_{-1}^{1} x^{101}dx + 1$

Since x^{101} is an odd function, $\int_{-1}^{1} x^{101}dx = 0$.

Thus: $\int_{-1}^{1} x^{101}dx + 1 = 0 + 1 = 1$

Match: **(D) → (II)**

48. Area of region bounded by the curves $x = y^3$, $x = 0$ between $y = -1$ and $y = 2$ is:

(1) $\frac{15}{4}$ sq. units **(2)** $\frac{17}{4}$ sq. units
(3) $\frac{19}{4}$ sq. units **(4)** $\frac{21}{4}$ sq. units

Ans. Option (2) is correct.

***Explanation*:** The curve $x = y^3$ lies to the right of the y-axis (which is $x = 0$) for $y > 0$, and to the left of the y-axis for $y < 0$.
So, the area between the curves $x = y^3$ and $x = 0$ from $y = -1$ to $y = 2$ is:

$$A = \int_{-1}^{2} |y^3| dy = \int_{-1}^{0} (-y^3)dy + \int_{0}^{2} y^3 dy$$

$$= \left[-\frac{y^4}{4}\right]_{-1}^{0} + \left[\frac{y^4}{4}\right]_{0}^{2}$$

$$= \left\{\left(-\frac{0^4}{4}\right) - \left(-\frac{(-1)^4}{4}\right)\right\} + \left\{\left(\frac{2^4}{4}\right) - \left(\frac{0^4}{4}\right)\right\}$$

$$= \left\{0 - \left(-\frac{1}{4}\right)\right\} + \left\{\left(\frac{16}{4}\right) - 0\right\}$$

$$= \frac{1}{4} + 4 = \frac{17}{4} \text{ sq. units}$$

49. If $y^{1/m} + y^{-1/m} = 2x$, then the value of $(x^2 - 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is:

(1) 0 **(2)** my
(3) m^2y **(4)** m^2

Ans. Option (3) is correct.

***Explanation*:** Given, $y^{\frac{1}{m}} + y^{\frac{-1}{m}} = 2x$

Differentiating the left-hand side using the chain rule:

$$\frac{d}{dx}(y^{\frac{1}{m}}) + \frac{d}{dx}(y^{-\frac{1}{m}}) = \frac{d}{dx}(2x)$$

$$\frac{1}{m}y^{\frac{1}{m}-1}\frac{dy}{dx} - \frac{1}{m}y^{-\frac{1}{m}-1}\frac{dy}{dx} = 2$$

$$\frac{1}{my}y^{\frac{1}{m}}\frac{dy}{dx} - \frac{1}{my}y^{-\frac{1}{m}}\frac{dy}{dx} = 2$$

$$\frac{dy}{dx}\left(\frac{1}{my}y^{\frac{1}{m}} - \frac{1}{my}y^{-\frac{1}{m}}\right) = 2$$

$$\frac{dy}{dx}(y^{\frac{1}{m}} - y^{-\frac{1}{m}}) = 2my$$

Squaring both sides:

$$\left(\frac{dy}{dx}\right)^2 (y^{\frac{1}{m}} - y^{-\frac{1}{m}})^2 = (2my)^2$$

$$\left(\frac{dy}{dx}\right)^2 (y^{\frac{2}{m}} + y^{-\frac{2}{m}} - 2) = 4m^2y^2 \quad \ldots\text{(i)}$$

Since, given $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$.

On squaring both sides, we get.

$$(y^{\frac{1}{m}} + y^{-\frac{1}{m}})^2 = (2x)^2$$

$$y^{\frac{2}{m}} + y^{-\frac{2}{m}} + 2 = 4x^2$$

$$y^{\frac{2}{m}} + y^{-\frac{2}{m}} = 4x^2 - 2$$

Substituting this into the squared Eq. (i), we get,

$$\left(\frac{dy}{dx}\right)^2 (4x^2 - 2 - 2) = 4m^2 y^2$$

$$\left(\frac{dy}{dx}\right)^2 (x^2 - 1) = m^2 y^2$$

Differentiating again w.r.t. x, we get

$$2\left(\frac{dy}{dx}\right)\frac{d^2x}{dy^2}(x^2-1) + 2x\left(\frac{dy}{dx}\right)^2 = 2m^2 y\left(\frac{dy}{dx}\right)$$

$$(x^2-1)\frac{d^2x}{dy^2} + x\left(\frac{dy}{dx}\right) = m^2 y$$

50. If $x = e^{\cos 2t}$, $y = e^{\sin 2t}$, then $\frac{dy}{dx}$ equals to

(1) $\frac{y \log_e x}{x \log_e y}$ **(2)** $\frac{x \log_e x}{y \log_e y}$

(3) $-\frac{y \log_e x}{x \log_e y}$ **(4)** $\frac{-x \log_e x}{y \log_e y}$

Ans. Option (3) is correct.

Explanation: Given that: $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$

Differentiating both the parametric functions w.r.t. t,

$$x = e^{\cos 2t}$$

$$\frac{dx}{dt} = e^{\cos 2t}\cdot\frac{d}{dt}(\cos 2t)$$

$$= e^{\cos 2t}(-\sin 2t)\cdot\frac{d}{dt}(2t)$$

$$= -e^{\cos 2t}.\sin 2t.2$$

$$= -2e^{\cos 2t}.\sin 2t$$

and $y = e^{\sin 2t}$

$$\frac{dy}{dt} = e^{\sin 2t}\cdot\frac{d}{dt}(\sin 2t)$$

$$= e^{\sin 2t}\cdot\cos 2t\cdot\frac{d}{dt}(2t)$$

$$= e^{\sin 2t}.\cos 2t.2$$

$$= 2e^{\sin 2t}.\cos 2t$$

$$\therefore \quad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{2e^{\sin 2t}\cdot\cos 2t}{-2e^{\cos 2t}\cdot\sin 2t}$$

$$= \frac{e^{\sin 2t}\cdot\cos 2t}{-e^{\cos 2t}\cdot\sin 2t}$$

$$= \frac{y\cos 2t}{-x\sin 2t}$$

Now, $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$

On taking log on both sides of each equation, we get.

$\cos 2t = \log_e x$ and $\sin 2t = \log_e y$

Thus, $\frac{dy}{dx} = -\frac{y\log_e x}{x\log_e y}$

Section : Mathematics Section B2 Applied

51. If the corner points of the bounded feasible region for a Linear Programming Problem (LPP) are $A(0, 2)$, $B(3, 0)$, $C(2, 3)$ and $D(3, 1)$, then the maximum value of the objective function $Z = 4x + 2y$ occurs at

(1) (0, 2) only

(2) the mid-point of the line segment joining the points (2, 3) and (3, 1) only

(3) (2, 3) and (3, 1) only

(4) every point on the line segment joining the points (2, 3) and (3, 1)

Ans. Option (4) is correct.

Explanation: Z at each corner point

At $A(0, 2)$: $Z = 4(0) + 2(2) = 0 + 4 = 4$

At $B(3, 0)$: $Z = 4(3) + 2(0) = 12 + 0 = 12$

At $C(2, 3)$: $Z = 4(2) + 2(3) = 8 + 6 = 14$

At $D(3, 1)$: $Z = 4(3) + 2(1) = 12 + 2 = 14$

The maximum value of Z is 14, achieved at both $C(2, 3)$ and $D(3, 1)$.

So, maximum value occurs at every point on the line segment joining C and D.

52. The standard deviation of the number of tails in three tosses of a coin is:

(1) $\frac{3}{4}$ **(2)** $\frac{3}{2}$

(3) $\frac{3}{\sqrt{2}}$ **(4)** $\frac{\sqrt{3}}{2}$

Ans. Option (4) is correct.

Explanation: Let X be the number of tails in 3 tosses of a fair coin.

Each toss is a Bernoulli trial with probability $p = 0.5$ (tail or head).

The total number of tails in 3 tosses is a binomial random variable.

So, $X \sim$ Binomial ($n = 3, p = 0.5$)

For a binomial variable:

$$\sigma = \sqrt{np(1-p)}$$

$$= \sqrt{3\cdot\frac{1}{2}\cdot\frac{1}{2}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

53. The value of a depreciable asset at the end of its useful life is called_______.

(1) Nominal Value **(2)** Book value

(3) Marginal value **(4)** Scrap value

Ans. Option (4) is correct.

Explanation: Scrap value (also known as salvage value or residual value) refers to the estimated amount that an asset is worth at the end of its useful life. It is the value for which the asset can be sold or disposed of after depreciation.

54. Three bad eggs are mixed with 7 good ones. If two eggs are drawn one by one without replacement, then the probability distribution of the number (X) of bad eggs drawn is:

(1)

X	0	1	2
$P(X)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

(2)

X	0	1	2
$P(X)$	$\frac{15}{61}$	$\frac{20}{61}$	$\frac{26}{61}$

(3)

X	0	1	2
$P(X)$	$\frac{7}{15}$	$\frac{7}{15}$	$\frac{1}{15}$

(4)

X	0	1	2
$P(X)$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{5}{8}$

Ans. Option (3) is correct.

***Explanation*:** We are given,

3 bad eggs, 7 good eggs $\rightarrow$ total $=10$ eggs.

Two eggs are drawn one by one without replacement.

Let X be the number of bad eggs drawn.

We are to find the probability distribution of $X = 0, 1, 2$.

Case 1: $X = 0$ (no bad egg is drawn)

Both eggs drawn are good.

Number of ways to choose 2 good eggs:

$$^7C_2 = 21$$

Total number of ways to choose any 2 eggs:

$$^{10}C_2 = 45$$

$$P(X = 0) = \frac{21}{45} = \frac{7}{15}$$

Case 2: $X = 1$ (exactly one bad egg drawn)

This can happen in two ways:

First bad, second good

First good, second bad

Choose 1 bad and 1 good: $^3C_1 \times {}^7C_1 = 3.7 = 21$

$$\text{P}(X = 1) = \frac{21}{45} = \frac{7}{15}$$

Case 3: $X = 2$ (both bad)

$$P(X = 2) = \frac{^3C_2}{^{10}C_2} = \frac{3}{45} = \frac{1}{15}$$

So, distribution table:

X	0	1	2
$P(X)$	$\frac{7}{15}$	$\frac{7}{15}$	$\frac{1}{15}$

55. Which of the following statements are correct?

(A) If A is a square matrix, then $|A^2| = |A|^2$.

(B) If A and B are square matrices of the same order, then $\det(AB) = \det(A) + \det(B)$.

(C) If A is a square matrix of order 3 and $|A| = 2$, then the value of $|-3A|$ is 54.

(D) If the matrix $\begin{bmatrix} 5-x & x-1 \\ 3 & 5 \end{bmatrix}$ is singular, then the value of x is $\frac{7}{2}$.

Choose the correct answer from the options given below:

(1) (A), (B) and (D) only **(2)** (A), (B) and (C) only

(3) (A) and (D) only **(4)** (B), (C) and (D) only

Ans. Option (3) is correct.

***Explanation*:** **(A)** True.

For any square matrix A,

$$|A^2| = |A.A| = |A|.|A| = |A|^2.$$

(Determinant of a product is the product of determinants.)

(B) False.

The correct property is $\det(AB) = \det(A).\det(B)$, not addition.

(C) False.

For an $n \times n$ matrix $A, |kA| = k^n|A|$.

Here, $n = 3, k = -3$:

$$|-3A| = (-3)^3.|A| = -27.2 = -54.$$

But the statement says $|-3A| = 54$, which is incorrect in magnitude and sign.

(D) True.

A singular matrix has determinant zero:

$$\begin{vmatrix} 5-x & x-1 \\ 3 & 5 \end{vmatrix} = 0$$

$$(5-x)(5) - (x-1)(3) = 0$$

$$25 - 5x - 3x + 3 = 0$$

$$28 - 8x = 0$$

$$x = \frac{28}{8} = \frac{7}{2}$$

56. An automobile dealer wishes to buy four luxury cars of different brands given in the table below with some down payment and balance in equal monthly instalments (EMI) for 10 years. The bank charges 9% interest per annum compounded monthly.

$$\left(\text{Given } \frac{0.0075 \times (1.0075)^{120}}{(1.0075)^{120} - 1} = 0.01266\right)$$

Luxury Car	**Price of the Car (in ₹)**	**Down payment (in ₹)**
P	25,00,000	5,00,000
Q	35,00,000	12,00,000
R	45,00,000	15,00,000
S	42,00,000	15,00,000

Match **List-I** with **List-II**.

List-I	List-II
Luxury car	**EMI (in ₹)**
(A) P	**(I)** 34,182
(B) Q	**(II)** 37,980
(C) R	**(III)** 29,118
(D) S	**(IV)** 25,320

Choose the correct answer from the options given below.

(1) (A)–(I), (B)–(II), (C)–(III), (D)–(IV)
(2) (A)–(II), (B)–(I), (C)–(III), (D)–(IV)
(3) (A)–(IV), (B)–(III), (C)–(II), (D)–(I)
(4) (A)–(III), (B)–(IV), (C)–(I), (D)–(I)

Ans. Option (3) is correct.

Explanation: Given that

$$\frac{0.0075 \times (1.0075)^{120}}{(1.0075)^{120} - 1} = 0.01266,$$

$$\text{EMI} = \text{Loan Amount} \times 0.01266$$

The loan amount for each car:
Loan amount = Price – Down payment
EMI:

$$\text{EMI} = \text{Loan Amount} \times 0.01266$$

(A) Car P:
Price: ₹ 25,00,000
Down payment: ₹ 5,00,000
Loan amount: 25, 00, 000 – 5,00,000 = ₹ 20,00,000
EMI: 20,00,000 × 0.01266 = ₹ 25,320
Match: **(A)–(IV)**
(B) Car Q:
Price: ₹ 35,00,000
Down payment: ₹ 12,00,000
Loan amount: 35,00,000 – 12,00,000 = ₹ 23,00,000
EMI: 23,00,000 × 0.01266 = ₹ 29,118
Match: **(B)–(III)**
(C) Car R:
Price: ₹ 45,00,000
Down payment: ₹ 15,00,000
Loan amount: 45, 00, 000 – 15,00,000 = ₹ 30,00,000
EMI: 30,00,000 × 0.01266 = ₹ 37,980
Match: **(C)–(II)**
(D) Car S:
Price: ₹ 42,00,000
Down payment: ₹ 15,00,000
Loan amount: 42,00,000 – 15,00,000 = ₹ 27,00,000
EMI: 27,00,000 × 0.01266 = ₹ 34,182
Match: **(D)–(I)**

57. If X is a normal variate with mean 16 and standard deviation 4, then the value of standard normal variate Z corresponding to $X = 17$ is:

(1) 1.25 **(2)** –1.25
(3) –0.25 **(4)** 0.25

Ans. Option (4) is correct.

Explanation: Formula for the Z-score:

$$Z = \frac{X - \mu}{\sigma}$$

Given: $X = 17,\ \mu = 16, \sigma = 4$

So, $$Z = \frac{17-16}{4} = \frac{1}{4} = 0.25$$

58. If a revenue function is given by $R(x) = 2027x - 1013x^2 - 675x^3$, then the marginal revenue function (MR) is:

(1) $\text{MR} = 2027 - 1013x - 675x^2$
(2) $\text{MR} = 2026 - 4050x$
(3) $\text{MR} = 2027 - 2026x - 2025x^2$
(4) $\text{MR} = 1013x - 2025x^2$

Ans. Option (3) is correct.

Explanation: Given,

$$R(x) = 2027x - 1013x^2 - 675x^3$$

Differentiate $R(x)$ with respect to x:

$$\text{MR} = \frac{dR}{dx} = \frac{d}{dx}(2027x - 1013x^2 - 675x^3)$$

$$= 2027 - 2 \times 1013x - 3 \times 675x^2$$

$$= 2027 - 2026x - 2025x^2$$

59. A furniture trader deals in only two items, chairs and tables. He has ₹ 50,000 to invest and a space to store atmost 35 items. A chair costs him ₹ 1000 and a table costs him ₹ 2000. The trader earns a profit of ₹ 150 and ₹ 250 on a chair and a table, respectively. Choose the correct option among following that describes the given linear programming problem (LPP) to maximise the profit (where x and y are the number of chairs and tables that trader buys and sells)?

(1) Maximise $Z = 150x + 250y$,
Subjected to constants,
$x + y \le 35, x + 2y \ge 50, x \ge 0, y \ge 0$

(2) Maximise $Z = 150x + 250y$,
Subjected to constants,
$x + y \le 35, x + 2y \le 50, x \ge 0, y \ge 0$

(3) Maximise $Z = 150x + 250y$,
Subjected to constants,
$x + y \ge 35, 2x + y \le 50, x \ge 0, y \ge 0$

(4) Maximise $Z = 150x + 250y$,
Subjected to constants,
$x + y \ge 35, 2x + y \ge 50x, x \ge 0, y \ge 0$

Ans. Option (2) is correct.

Explanation: We are given a Linear Programming Problem (LPP) for a trader dealing in:
Chairs (cost ₹ 1000, profit ₹ 150)
Tables (cost ₹ 2000, profit ₹ 250)
Let,

$$x = \text{number of chairs}$$
$$y = \text{number of tables}$$

The total profit is:

Maximise Z = $150x + 250y$

(i) Investment constraint:

The trader has ₹ 50,000 to invest.

₹ 1000 per chair → total ₹ $1000x$

₹ 2000 per table → total ₹ $2000y$

So, $1000x + 2000y \le 50000 \Rightarrow x + 2y \le 50$

(ii) Storage space constraint:

He can store at most 35 items: $x + y \le 35$

(iii) Non-negativity: $x \ge 0, y \ge 0$

Thus LPP is:

$$\text{Maximise } Z = 150x + 250y$$

Subject to:
$$x + y \le 35$$
$$x + 2y \le 50$$

$x \ge 0, y \ge 0$

60. The area of the region bounded by the curves $y = x^2 + 2$ and x-axis, between $x = 0$ and $x = 3$ in the first quadrant is:

(1) $\frac{16}{3}$ sq. units **(2)** 15 sq. units

(3) $\frac{21}{2}$ sq. units **(4)** 12 sq. units

Ans. Option (2) is correct.

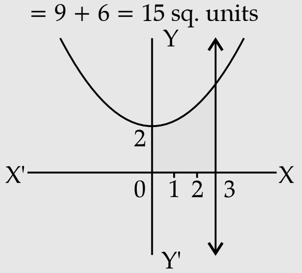
***Explanation*:** The area (A) of the region bounded by the curve $y = x^2 + 2$, the x-axis, and the vertical lines $x = 0$ and $x = 3$ in the first quadrant is given by:

$$A = \int_0^3 (x^2 + 2)\,dx = \left[\frac{x^3}{3} + 2x\right]_0^3$$

$$= \left(\frac{3^3}{3} + 2 \times 3\right) - \left(\frac{0^3}{3} + 2 \times 0\right)$$

$$= \left(\frac{27}{3} + 6\right) - 0$$

$$= 9 + 6 = 15 \text{ sq. units}$$



61. Which of the following are types of 'statistical inferences'?

(A) Point estimation **(B)** Interval estimation

(C) Marginal estimation **(D)** Hypothesis testing

Choose the correct answer from the options given below:

(1) (A), (B) and (D) only **(2)** (A), (B) and (C) only

(3) (A), (B), (C) and (D) **(4)** (C) and (D) only

Ans. Option (1) is correct.

***Explanation*:** Statistical inference involves drawing conclusions about a population based on sample data. The main types of statistical inferences are:

(A) Point estimation: Estimating an unknown population parameter with a single value (e.g., sample mean as an estimate of the population mean).

(B) Interval estimation: Estimating an unknown population parameter within a range (e.g., confidence intervals).

(D) Hypothesis testing: Making decisions or testing claims about a population parameter using sample data.

(C) Marginal estimation: This is not a standard type of statistical inference.

62. Pipe A can fill the tank 3 times faster than pipe B. If both pipes A and B running together can fill the tank in 15 minutes, then the time taken by B alone to fill the tank is:

(1) 42 min **(2)** 72 min

(3) 60 min **(4)** 48 min

Ans. Option (3) is correct.

***Explanation*:** Let the time taken by Pipe B alone to fill the tank be t min.

Since Pipe A is 3 times faster, Pipe A takes $\frac{t}{3}$ minutes to fill the tank alone.

$$\text{Pipe } B\text{'s rate} = \frac{1}{t} \text{ (tank per minute).}$$

$$\text{Pipe } A\text{'s rate} = \frac{1}{t/3} = \frac{3}{t} \text{ (tank per minute).}$$

Combined rate ($A + B$) = $\frac{3}{t} + \frac{1}{t} = \frac{4}{t}$.

Together, they fill the tank in 15 min., so their combined rate is also $\frac{1}{15}$ (tank per min).

So, $$\frac{4}{t} = \frac{1}{15}$$

$$t = 4 \times 15 = 60 \text{ min}$$

63. For the given five values, 15, 24, 18, 33, 42, the three-year moving averages are

(1) 19, 22, 23 **(2)** 19, 25, 31

(3) 19, 30, 31 **(4)** 19, 25, 33

Ans. Option (2) is correct.

***Explanation*:** First average:

$$\frac{15 + 24 + 18}{3} = \frac{57}{3} = 19$$

Second average:

$$\frac{24 + 18 + 33}{3} = \frac{75}{3} = 25$$

Third average:

$$\frac{18 + 33 + 42}{3} = \frac{93}{3} = 31$$

64. A steamer can row at the speed of 16 km/h in still water. If the river is flowing at 8 km/h and it takes 12 h for a round trip, then the distance between the two places is:

(1) 48 km **(2)** 72 km

(3) 96 km **(4)** 54 km

Ans. Option (2) is correct.

***Explanation*:** Given:

Speed of the steamer in still water (v) = 16 km/h

Speed of the river current (c) = 8 km/h

Time taken for a round trip = 12 h

Here,

Downstream speed (with the current) = $v + c$

= 16 + 8 = 24 km/h

Upstream speed (against the current) = $v - c$

= 16 – 8 = 8 km/h

Round trip time: Time downstream + Time upstream = Total time.

Let the distance between the two places be d km.

Time downstream:

$$t_1 = \frac{d}{\text{Downstream speed}} = \frac{d}{24}\text{h}$$

Time upstream:

$$t_2 = \frac{d}{\text{Upstream speed}} = \frac{d}{8}\text{h}$$

Total round trip time:

$$t_1 + t_2 = 12\text{ h}$$

$$\frac{d}{24} + \frac{d}{8} = 12$$

$$\frac{d}{24} + \frac{3d}{24} = 12\left(\text{since } \frac{d}{8} = \frac{3d}{24}\right)$$

$$\frac{4d}{24} = 12$$

$$\frac{d}{6} = 12$$

$$d = 72\text{ km}$$

65. If the matrix $M = \begin{bmatrix} 0 & -1 & 3\alpha \\ 1 & \beta & -5 \\ -6 & 5 & 0 \end{bmatrix}$ is skew-symmetric, then

(1) $\alpha = 2, \beta = 1$ **(2)** $\alpha = 2, \beta = -1$

(3) $\alpha = 2, \beta = 0$ **(4)** $\alpha = 1, \beta = 0$

Ans. Option (3) is correct.

***Explanation*:** We are given the matrix:

$$M = \begin{bmatrix} 0 & -1 & 3\alpha \\ 1 & \beta & -5 \\ -6 & 5 & 0 \end{bmatrix}$$

A matrix M is skew-symmetric if: $M^T = -M$

That is, for all i, j, $M_{ij} = -M_{ji}$

Also, all diagonal elements of a skew-symmetric matrix must be 0.

So, $\beta = 0$.

Entry $M_{12} = -1$, so $M_{21} = -(-1) = 1$

Entry $M_{13} = 3\alpha$, so $M_{31} = -6$

But from the matrix:

$$M_{13} = 3\alpha$$

$$M_{31} = -6$$

So, $-6 = -3\alpha \Rightarrow \alpha = 2$

66. If the slope of the tangent to the curve $y = f(x)$ at any point (x, y) is $\frac{2x}{y^2}$ and the curve passes through the point $\left(\frac{1}{\sqrt{3}}, 1\right)$, then equation of curve is:

(1) $y^2 = 3x^3$ **(z2)** $y^3 = 3x^2$

(3) $y^4 = 3x^2$ **(4)** $y = 3x^2$

Ans. Option (2) is correct.

***Explanation*:** Given,

Slope of the tangent (derivative): $\frac{dy}{dx} = \frac{2x}{y^2}$

Curve passes through the point: $\left(\frac{1}{\sqrt{3}}, 1\right)$

Now, $$\frac{dy}{dx} = \frac{2x}{y^2}$$

This is a separable differential equation.

$$\int y^2 dy = \int 2x dx$$

$$\frac{y^3}{3} = x^2 + C$$

where C is the constant of integration.

Substitute the point $\left(\frac{1}{\sqrt{3}}, 1\right)$ into the equation to solve for C:

$$\frac{(1)^3}{3} = \left(\frac{1}{\sqrt{3}}\right)^2 + C$$

$$\frac{1}{3} = \frac{1}{3} + C = 0$$

Substitute $C = 0$ back into the general solution:

$$\frac{y^3}{3} = x^2$$

or, $$y^3 = 3x^2$$

67. Mr. Mittal invested ₹ 20,000 in a mutual fund in the year 2019. The value of the mutual fund increased to ₹ 32,000 in the year 2024. The compound annual growth rate of his investment is:

[Given $(1.6)^{1/5} = 1.098$]

(1) 8.5% **(2)** 10%

(3) 9.8% **(4)** 9.1%

Ans. Option (3) is correct.

Explanation: Given,

Initial investment P = ₹ 20,000

Final value A = ₹ 32,000

Time period n = 5 years

CAGR Formula: CAGR $= \left(\frac{A}{P}\right)^{1/n} - 1$

$$\text{CAGR} = \left(\frac{32000}{20000}\right)^{\frac{1}{5}} - 1 = (1.6)^{\frac{1}{5}} - 1$$

$$= 1.098 - 1 = 0.098 = 9.8\%$$

68. If A is a non-singular matrix of order 3 such that $|\text{adj}(A)| = 121$, then $|AA^T|$ is equal to:

(1) 121 **(2)** 21

(3) 63 **(4)** 11

Ans. Option (1) is correct.

Explanation: For any $n \times n$ non-singular matrix A:

$$|\text{adj}(A)| = |A|^{n-1}$$

Here, $n = 3$, so:

$$|\text{adj}(A)| = |A|^2 = 121 \Rightarrow |A| = \sqrt{121} = 11 \text{ (since } |A| > 0)$$

For any square matrix A:

$$|AA^T| = |A|.|A^T| = |A|^2$$

(because $|A^T| = |A|$)

Thus, $|AA^T| = |A|^2 = 11^2 = 121$

69. If $y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots \ldots \ldots}}}$, then

(1) $(2y+1)\frac{dy}{dx} + 3\sqrt{x} = 0$ **(2)** $3\frac{dy}{dx} + 3y^2 = 0$

(3) $3x\frac{dy}{dx} + (2y+1) = 0$ **(4)** $(2y-1)\frac{dy}{dx} - 1 = 0$

Ans. Option (4) is correct.

Explanation: We have, $y = \sqrt{x + \sqrt{x + \sqrt{x + \ldots\ldots\ldots}}}$

Since the expression under the root repeats identically, we can write:

$$y = \sqrt{x + y}$$

Now square both sides:

$$y^2 = x + y$$

$$\Rightarrow \quad y^2 - y - x = 0 \quad \ldots(i)$$

Differentiate Eq. (i):

$$\frac{d}{dx}(y^2 - y - x) = 0$$

$$\Rightarrow \quad 2y \cdot \frac{dy}{dx} - \frac{dy}{dx} - 1 = 0$$

$$\Rightarrow \quad (2y - 1) \cdot \frac{dy}{dx} - 1 = 0$$

70. If the system of equations $2x + 3y = 10$, $x + ky = 4$ has a unique solution, then

(1) $k = \frac{3}{2}$ **(2)** $k \neq \frac{3}{2}$

(3) $k \neq 0$ **(4)** $k \neq \frac{1}{2}$

Ans. Option (2) is correct.

Explanation: Given system of equations:

$$2x + 3y = 10$$
$$x + ky = 4$$

The system can be represented as:

$$\begin{bmatrix} 2 & 3 \\ 1 & k \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 4 \end{bmatrix}$$

For the system to have a unique solution, the determinant of the coefficient matrix must be non-zero:

$$\begin{vmatrix} 2 & 3 \\ 1 & k \end{vmatrix} \neq 0$$

$$2k - 3 \neq 0 \Rightarrow 2k \neq 3 \Rightarrow k \neq \frac{3}{2}$$

71. The following data shows the percentage of urban Indian households who have a high speed 5G internet connection:

Year (x)	2020	2021	2022	2023	2024
Urban house hold (y)	9%	18%	21%	29%	38%

If a straight line trend by the method of least square for the above data is $y = 23 + 6.9(x - 2022)$, then the forecast the year 2025 is:

(1) 54.6% **(2)** 43.7%

(3) 47.6% **(4)** 34.8%

Ans. Option (2) is correct.

Explanation: We are given:

The least squares linear trend:

$$y = 23 + 6.9(x - 2022)$$

Put $x = 2025$,

$$y = 23 + 6.9(2025 - 2022)$$
$$= 23 + 6.9 \times 3$$
$$= 23 + 20.7 = 43.7\%$$

72. For the function $f(x) = x^{1/x}$, $x > 0$, which of the following are correct?

(A) $x = 0$ is the only point where extremum may occur.

(B) The given function is maximum at $x = e$.

(C) The function has no extreme value for $x > 0$.

(D) The maximum value of the function $f(x)$ is $e^{1/e}$.

Choose the correct answer from the options given below:

(1) (A), (B) and (D) only **(2)** (A) and (C) only

(3) (C) and (D) only **(4)** (B) and (D) only

Ans. Option (4) is correct.

Explanation: To analyse the function $f(x) = x^{1/x}$ for $x > 0$, well determine its critical points and behaviour:

First, take the natural logarithm of $f(x)$:

$$\ln f(x) = \frac{1}{x}\ln x$$

Differentiate implicitly with respect to x:

$$\frac{f'(x)}{f(x)} = \frac{1}{x^2} - \frac{\ln x}{x^2}$$

$$f'(x) = x^{1/x}\left(\frac{1-\ln x}{x^2}\right)$$

For Critical Points, put $f'(x) = 0$:

$$x^{1/x}\left(\frac{1-\ln x}{x^2}\right) = 0$$

Since $x^{1/x} > 0$ and $x^2 > 0$, the critical point occurs when:

$1 - \ln x = 0 \Rightarrow \ln x = 1 \Rightarrow x = e$

For $x < e$: $\ln x < 1$, so $f'(x) > 0$ (function is increasing).

For $x > e$: $\ln x > 1$, so $f'(x) < 0$ (function is decreasing).

Thus, $x = e$ is a maximum point.

Substitute $x = e$ into $f(x)$:

$$f(e) = e^{1/e}$$

Now,

(A) Incorrect. The only critical point is at $x = e$, not $x = 0$.

(B) Correct. The function attains its maximum at $x = e$.

(C) Incorrect. The function has a maximum at $x = e$.

(D) Correct. The maximum value is $e^{1/e}$.

73. Which of the following are examples of irregular trends in a time series?

(A) Decrease in production due to a sudden strike.
(B) The rise in prices before festivals.
(C) Unusual rise in income of the printing press due to the announcement of an election.
(D) Fall in crop yield due to floods.

Choose the correct answer from the options given below:

(1) (A), (B) and (D) only **(2)** (A), (C) and (D) only
(3) (B), (C) and (D) only **(4)** (B) and (C) only

Ans. Option (2) is correct.

Explanation: **(A)** Decrease in production due to a sudden strike.

Irregular—sudden, unplanned event affecting output.

(B) The rise in prices before festivals.

Seasonal—this occurs regularly around festival times.

(C) Unusual rise in income of the printing press due to the announcement of an election.

Irregular—elections are not regularly timed or predicted by the model.

(D) Fall in crop yield due to floods.

Irregular—floods are natural disasters and not regular or seasonal.

74. In what ratio a grocery shopkeeper mix two varieties of pulses worth ₹ 85 per kg and ₹ 100 per kg, respectively, so as to get a mixture worth ₹ 92 per kg?

(1) 8:7 **(2)** 5:6
(3) 3:7 **(4)** 11:10

Ans. Option (1) is correct.

Explanation: Given,

Price of first variety (A) = ₹ 85 per kg

Price of second variety (B) = ₹ 100 per kg

Desired mixture price (M) = ₹ 92 per kg

The alligation rule states,

$$\text{Ratio} = \frac{\text{Price of } B - M}{M - \text{Price of } A} = \frac{100-92}{92-85} = \frac{8}{7}$$

The ratio $\frac{8}{7}$ means that for every 8 kg of the ₹ 85/kg pulses, we need to mix 7 kg of the ₹ 100/kg pulses to achieve the desired mixture price of ₹ 92/kg.

75. Match **List-I** with **List-II**.

List-I	**List-II**
(Inequality)	**(Solution set)**
(A) $2x - 3 < x + 2 \leq 3x + 5, x \in \mathbb{R}$	**(I)** $x \in (-1, \infty)$
(B) $\lvert 2x + 3\rvert < 7, x \in \mathbb{R}$	**(II)** $x \in (-\infty, 120]$
(C) $\frac{1}{2}\left(\frac{3}{5}x+4\right) \geq \frac{1}{3}(x-6)$, $x \in \mathbb{R}$	**(III)** $x \in (-5, 2)$
(D) $\frac{\lvert x+1\rvert}{x+1} > 0,\ x \in \mathbb{R} - \{1\}$	**(IV)** $x \in \left[-\frac{3}{2}, 5\right)$

Choose the correct answer from the options given below:

(1) (A)–(II), (B)–(I), (C)–(III), (D)–(IV)
(2) (A)–(I), (B)–(II), (C)–(III), (D)–(IV)
(3) (A)–(III), (B)–(IV), (C)–(II), (D)–(I)
(4) (A)–(IV), (B)–(III), (C)–(II), (D)–(I)

Ans. Option (4) is correct.

Explanation: **(A)** $2x - 3 < x + 2 \leq 3x + 5$

Break it into two parts:

$2x - 3 < x + 2 \Rightarrow x < 5$

$x + 2 \leq 3x + 5 \Rightarrow -3 \leq 2x \Rightarrow x \geq -\frac{3}{2}$

Take intersection: $x \in \left[-\frac{3}{2}, 5\right)$

Matches with **(IV)**

(B) $\lvert 2x + 3\rvert < 7$

Break absolute inequality:

$-7 < 2x + 3 < 7 \Rightarrow -10 < 2x < 4$

$\Rightarrow -5 < x < 2$ $x \in (-5, 2)$

Matches with **(III)**

(C) $\frac{1}{2}\left(\frac{3}{5}x+4\right) \geq \frac{1}{3}(x-6)$

Simplify both sides:

LHS:

$$\frac{1}{2}\cdot\left(\frac{3}{5}x+4\right) = \frac{3}{10}x+2$$

So, $$\frac{3}{10}x+2 \geq \frac{1}{3}x-2$$

Multiply both sides by 30:

$9x + 60 \geq 10x - 60 \Rightarrow 120 \geq x \Rightarrow x \leq 120$

$x \in (-\infty, 120]$

Matches with **(II)**

(D) $\frac{|x+1|}{x+1} > 0$, with $x \neq -1$

This expression simplifies as:

$$\frac{|x+1|}{x+1} = \begin{cases} 1, & \text{if } x+1>0 \Rightarrow x>-1 \\ -1, & \text{if } x+1<0 \Rightarrow x<-1 \\ \text{undefined}, & \text{if } x=-1 \end{cases}$$

We want $\frac{|x+1|}{x+1} > 0 \Rightarrow$ numerator and denominator have same sign, so:

$x + 1 > 0 \Rightarrow x > -1$

Exclude the point $x = -1$, so:

$x \in (-1, \infty)$

Matches with **(I)**

76. If 95% confidence interval for the population mean was reported to be 140 to 150 and $\sigma = 25$, then size of the sample used in this study is:

(Given: $Z_{0.025} = 1.96$)

(1) 120 **(2)** 81

(3) 96 **(4)** 112

Ans. Option (3) is correct.

Explanation: Given,

Confidence Interval (CI): 140 to 150

Population Standard Deviation (σ): 25

Critical Z-value ($Z_{0.025}$): 1.96 (since 95% CI corresponds to $\alpha = 0.05$, and $Z_{\alpha/2} = Z_{0.025} = 1.96$)

The CI is symmetric around the sample mean $\bar{X}$, so:

$\bar{X} - E = 140$ and $\bar{X} + E = 150$

Subtracting the two equations:

$(\bar{X} + E) - (\bar{X} - E) = 150 - 140$

$2E = 10 \Rightarrow E = 5$

So, the Margin of Error (E) is 5.

The formula for the margin of error when σ is known is:

$$E = Z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}$$

$$5 = 1.96\cdot\frac{25}{\sqrt{n}}$$

$$5 = 1.96\cdot\frac{25}{\sqrt{n}}$$

$$\frac{5}{1.96} = \frac{25}{\sqrt{n}}$$

$$\sqrt{n} = \frac{25\times1.96}{5}$$

$$\sqrt{n} = 5 \times 1.96 = 9.8$$

$$n = (9.8)^2 = 96.04$$

Since the sample size must be a whole number, we round it to the nearest integer:

$$n = 96$$

77. The unit digit of 12^{12} is:

(1) 4 **(2)** 2

(3) 0 **(4)** 6

Ans. Option (4) is correct.

Explanation: Observe the units digit of powers of 2

$2^1 = 2$ (unit digit = 2)

$2^2 = 4$ (unit digit = 4)

$2^3 = 8$ (unit digit = 8)

$2^4 = 16$ (unit digit = 6)

$2^5 = 32$ (unit digit = 2)

Since the cycle repeats every 4 powers, we find the remainder when 12 is divided by 4:

$12 \div 4 = 3$ with a remainder of 0.

A remainder of 0 means the exponent corresponds to the last number in the cycle, which is 6. Thus, the unit digit of 12^{12} is the same as the unit digit of $2^4=16$, which is 6.

78. Consider the following hypothesis test:

H_0: $\mu = 18$

H_1: $\mu \neq 18$

If a sample of 48 provided a sample mean $\bar{x} = 17$ and a sample standard deviation $\sigma = 4.5$, then the value of the t-test statistic is:

(1) 2.14 **(2)** –1.54

(3) 0.84 **(4)** 1.988

Ans. Option (2) is correct.

Explanation: Given,

Null hypothesis (H_0): $\mu = 18$

Alternative hypothesis (H_1): $\mu \neq 18$ (two-tailed test)

Sample size (n): 48

Sample mean ($\bar{x}$): 17

Sample standard deviation (s): 4.5

The t-test statistic for the given hypothesis test:

$$t = \frac{\bar{x}-\mu_0}{\frac{s}{\sqrt{n}}}$$

$$= \frac{\frac{17-18}{4.5}}{\sqrt{48}}$$

$$= -\frac{\frac{1}{4.5}}{6.9282} \quad [\because \sqrt{n} = \sqrt{48} \approx 6.9282]$$

$$t = \frac{-1}{0.6495} \approx -1.54$$

79. The value of the definite integral $I = \int_0^2 x\sqrt{2-x}\,dx$ is:

(1) $\frac{16}{15\sqrt{2}}$ **(2)** $\frac{16\sqrt{3}}{15}$

(3) $\frac{16\sqrt{2}}{15}$ **(4)** $\frac{5\sqrt{2}}{7}$

Ans. Option (3) is correct.

Explanation: $I = \int_0^2 x\sqrt{2-x}\,dx$

Let $u = 2-x \Rightarrow du = -dx \Rightarrow dx = -du$

When $x = 0, u = 2$

When $x = 2, u = 0$

Rewriting the integral in terms of u :

$$I = \int_{u=2}^{u=0} (2-u)\sqrt{u}(-du)$$

$$I = \int_0^2 (2-u)\sqrt{u}\,du$$

$$I = \int_0^2 (2u^{1/2} - u^{3/2})du$$

$$I = 2\left[\frac{u^{3/2}}{3/2}\right]_0^2 - \left[\frac{u^{5/2}}{5/2}\right]_0^2$$

$$I = 2\left(\frac{2}{3}u^{3/2}\right)\Bigg|_0^2 - \left(\frac{2}{5}u^{5/2}\right)\Bigg|_0^2$$

$$I = \frac{4}{3}\left(2^{3/2}-0\right) - \frac{2}{5}\left(2^{5/2}-0\right)$$

$$I = \frac{4}{3}\left(2\sqrt{2}\right) - \frac{2}{5}\left(4\sqrt{2}\right)$$

$$= \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5} = \frac{16\sqrt{2}}{15}$$

80. A person has taken a loan of ₹ 40,000 for 3 months from a lender who has deducted ₹2,000 as interest at the time of lending. Then the effective rate of interest charged per annum by lender is (given $(1.0526)^4 = 1.2275$):

(1) 10.50% **(2)** 21%

(3) 22.75% **(4)** 16.75%

Ans. Option (3) is correct.

Explanation: Given,

Loan amount sanctioned: ₹ 40,000

Interest deducted upfront: ₹ 2,000

So, amount actually received = ₹ 40,000 – ₹ 2,000 = ₹ 38,000

He pays back ₹ 40,000 at the end of 3 months, but received ₹ 38,000.

So in 3 months, he pays:

Interest = $\frac{2000}{38000}$ =0.0526 = 5.26%

This is quarterly effective interest rate.

Effective Annual Rate (EAR) = $(1 + r)^4 - 1$

$= (1.0526)^4 - 1$

$= 1.2275 - 1$

$= 0.2275 = 22.75\%$

81. Which of the following is NOT correct about 'Sinking Fund'?

(1) It is set up for particular upcoming expense.

(2) A fixed amount at regular intervals is deposited in it.

(3) It can be used only for the purpose it was created.

(4) It is a long-term fund which can be closed at any time.

Ans. Option (4) is correct.

Explanation: **Statement 1 is correct**—the sinking funds are typically established to accumulate money over time for a known future expense (like debt repayment, equipment replacement, etc.).

Statement 2 is correct—the essence of a sinking fund is periodic contributions, usually fixed, to meet a future obligation.

Statement 3 is correct—for accounting and discipline, a sinking fund is earmarked and should only be used for its intended purpose.

Statement 4 is incorrect—a sinking fund has a specific purpose (e.g., repaying debt) and cannot be closed prematurely.

82. In a 700 m race, the ratio of speeds of two participants A and B is 5:6. If A has a start of 150 m, then the distance which A wins the race is:

(1) 80 m **(2)** 40 m

(3) 75 m **(4)** 100 m

Ans. Option (2) is correct.

Explanation: Given,

Length of race = 700 m

Speed ratio of A to B = 5:6

A has a head start of 150 m, so A runs only 550 m.

Let time taken by both be equal (since they finish at same time)

Let speed of $A = 5x$, speed of $B = 6x$

Time taken by A to run 550 m $= \frac{550}{5x} = \frac{110}{x}$

In the same time, distance run by B

$= $ speed $\times$ time

$= 6x \times \frac{110}{x} = 660$ m

So, B runs 660 m while A finishes the race (due to head start), but race is 700 m.

Distance B still had left to run $= 700 - 660 = 40$ m

83. Let $A = \begin{bmatrix} 0 & 2\alpha+1 \\ 1 & \beta \end{bmatrix}$ and $B = [b_{ij}]$ a skew-symmetric matrix of order 2 such that $b_{12} = 1$. If $AB = I_2$ where I_2 is identity matrix of order 2, then

(1) $\alpha + \beta = 1$ **(2)** $\beta - \alpha = 1$

(3) $\alpha + \beta = -2$ **(4)** $\alpha\beta = 1$

Ans. Option (2) is correct.

Explanation: We are given:

Matrix $A = \begin{bmatrix} 0 & 2\alpha+1 \\ 1 & \beta \end{bmatrix}$

Matrix $B = [b_{ij}]$ is a skew-symmetric matrix of order 2

$b_{12} = 1$

$AB = I_2$, where $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

A skew-symmetric matrix satisfies: $B^T = -B$

So, $B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(since $b_{12} = 1, b_{21} = -1, b_{11} = b_{22} = 0$)

Now,

$$AB = \begin{bmatrix} 0\cdot 0 + (2\alpha+1)(-1) & 0\cdot 1 + (2\alpha+1)(0) \\ 1\cdot 0 + \beta(-1) & 1\cdot 1 + \beta(0) \end{bmatrix}$$

$$= \begin{bmatrix} -(2\alpha+1) & 0 \\ -\beta & 1 \end{bmatrix}$$

Also, given

$$AB = I_2$$

$$\begin{bmatrix} -(2\alpha+1) & 0 \\ -\beta & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Comparing the entries, we get,

$-(2\alpha + 1) = 1 \Rightarrow 2\alpha + 1 = -1 \Rightarrow \alpha = -1$

$-\beta = 0 \Rightarrow \beta = 0$

So, $\alpha = -1, \beta = 0 \Rightarrow \alpha + \beta = -1$

Now,

$\beta - \alpha = 0 - (-1) = 1$ is the correct option.

84. What sum of money is needed to invest now, so as to get ₹ 5000 at the beginning of every month forever, if the money is worth 6% per annum compounded monthly?

(1) ₹ 10,05,000 **(2)** ₹ 15,00,000

(3) ₹ 10,50,000 **(4)** ₹ 12,50,000

Ans. Option (1) is correct.

Explanation: Given,

Monthly payment (PMT): ₹ 5,000

Annual interest rate (r): 6% or 0.06

$i = \frac{6\%}{12} = 0.5\% = 0.005$ (monthly interest rate)

For payments at the beginning of each period (perpetuity due), the present value (PV) is:

$$PV = \frac{PMT}{i} \times (1+i)$$

$$= \frac{5000}{0.005} \times (1 + 0.005)$$

$= 5000 \times 200 \times 1.005$

$= 10,00,000 \times 1.005$

$=$ ₹ 10,05,000

85. In 5 trials of binomial distribution, the probability of 3 successes is 4 times the probability of 2 successes. The probability of success in each trial is:

(1) $\frac{1}{5}$ **(2)** $\frac{4}{5}$

(3) $\frac{3}{4}$ **(4)** $\frac{2}{3}$

Ans. Option (2) is correct.

Explanation: Given,

A binomial distribution with $n = 5$ trials.

Let p be the probability of success, and $q = 1 - p$ be the probability of failure.

It is given that: $P(X = 3) = 4.P(X = 2)$

By binomial probability formula:

$P(X = r) = nC_r\, p^r q^{n-r}$

$P(X = 3) = {}^5C_3 p^3 q^2 = 10p^3q^2$

$P(X = 2) = {}^5C_2 p^2 q^3 = 10p^2q^3$

Given, $10p^3q^2 = 4.10p^2q^3$

$\Rightarrow p^3 q^2 = 4p^2 q^3$

$\Rightarrow p = 4q$

Now use $q = 1 - p$:

$p = 4(1 - p) \Rightarrow p = 4 - 4p \Rightarrow 5p = 4 \Rightarrow p = \frac{4}{5}$

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