Engineering Mathematics — GATE — Solved Papers 2025

COMPUTER SCIENCE & IT (CS-1)

1. g(.) is a function from A to B, f(.) is a function from B to C, and their composition defined as f(g(.)) is a mapping from A to C.

If f(.) and f(g(.)) are onto (subjective) functions, which ONE of the following is TRUE about the function g(.)?

(a) g(.) must be an onto (surjective) function.
(b) g(.) must be a one-to-one (injective) function.
(c) g(.) must be a bijective function, that is, both one-to-one and onto.
(d) g(.) is not required to be a one-to-one or onto function.

2. Consider the following recurrence relation:

$T(n) = 2T((n-1) + n2^n$ for $n > 0$, $T(0) = 1$

Which ONE of the following options is CORRECT?

(a) $T(n) = \Theta(n^2\, 2^n)$ **(b)** $T(n) = \Theta(n2^n)$
(c) $T(n) = \Theta(\log n)^2\, 2^n)$ **(d)** $T(n) = \Theta(4^n)$

3. Consider the given system of linear equations for variables x and y, where k is a real-valued constant. Which of the following option(s) is/are CORRECT?

$$x + ky = 1$$
$$kx + y = -1$$

(a) There is exactly one value of k for which the above system of equations has no solution.
(b) There exist an infinite number of values of k for which the system of equations has no solution.
(c) There exists exactly one value of k for which the system of equations has exactly one solution.
(d) There exists exactly one value of k for which the system of equations has an infinite number of solutions.

4. Consider the given function $f(x)$.

$$f(x) = \begin{cases} ax + b & \text{for } x < 1 \\ x^3 + x^2 + 1 & \text{for } x \geq 1 \end{cases}$$

If the function is differentiable everywhere, the value of b must be _____ . (Rounded off to one decimal place).

5. Let A be a 2×2 matrix as given:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Where are the eigenvalues of the matrix A^{13}?

(a) $1, -1$ **(b)** $2\sqrt{2}, -2\sqrt{2}$
(c) $4\sqrt{2}, -4\sqrt{2}$ **(d)** $64\sqrt{2}, -64\sqrt{2}$

6. $A = (0, 1, 2, 3, ...)$ is the set of non-negative integers. Let F be the set of functions from A to itself. For any two functions, $f_1, f_2 \in F$ we define

$$(f_1 \cdot f_2)\,(n) = f_1(n) + f_2(n)$$

for every number n in A. Which of the following is/are CORRECT about the mathematical structure $(F, \odot)$?

(a) $(F, \odot)$ is an Abelian group.
(b) $(F, \odot)$ is an Abelian monoid.
(c) $(F, \odot)$ is a non-Abelian group.
(d) $(F, \odot)$ is a non Abelian monoid.

7. Consider a probability distribution given by the density function $P(x)$.

$$P(x) = \begin{cases} Cx^2, & \text{for } 1 \leq x \leq 4 \\ 0, & \text{for } x < 1 \text{ or } x > 4 \end{cases}$$

The probability that x lies between 2 and 3, i.e., $P(2 \leq x \leq 3)$ is __________ . (Rounded off to three decimal places).

COMPUTER SCIENCE & IT (CS-2)

8. If $A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$, then which ONE of the following is A^8?

(a) $\begin{pmatrix} 25 & 0 \\ 0 & 25 \end{pmatrix}$ **(b)** $\begin{pmatrix} 125 & 0 \\ 0 & 125 \end{pmatrix}$

(c) $\begin{pmatrix} 625 & 0 \\ 0 & 625 \end{pmatrix}$ **(d)** $\begin{pmatrix} 3125 & 0 \\ 0 & 3125 \end{pmatrix}$

9. The value of x such that $x > 1$, satisfying the equation $\int_1^x t \ln t \, dt = \frac{1}{4}$ is

(a) $\sqrt{e}$ **(b)** e
(c) e^2 **(d)** $e - 1$

10. Let L, M, and N be non-singular matrices of order 3 satisfying the equations $L^2 = L^{-1}$, $M = L^8$ and $N = L^2$.

Which ONE of the following is the value of the determinant of $(M - N)$?

(a) 0 **(b)** 1 **(c)** 2 **(d)** 3

11. Let F be the set of all functions from $[1, ..., n]$ to $\{0, 1\}$. Define the binary relation $\preccurlyeq$ on F as follows: $\forall f.g \in F, f \preccurlyeq g$ if and only if $\forall x \in \{1, ..., n\}, f(x) \leq g(x)$, where $0 = 1$.
Which of the following statement(s) is/are TRUE?

(a) $\preccurlyeq$ is a symmetric relation
(b) $(F, \preccurlyeq)$ is a partial order
(c) $(F, \preccurlyeq)$ is a lattice
(d) $\preccurlyeq$ is an equivalence relation

12. Consider a system of linear equations $PX = Q$ where $P \in \mathbb{R}^{3\times3}$ and $Q \in \mathbb{R}^{3\times3}$. Suppose P has an LU decomposition, $P = LU$, where

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \text{ and } u = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

Which of the following statement(s) is/are TRUE?
(a) The system $PX = Q$ can be solved by first solving $LY = Q$ and then $UX = Y$.
(b) If P is invertible, then both L and U are invertible.
(c) If P is singular, then atleast one of the diagonal elements of U is zero.
(d) If P is symmetric, then both L and U are symmetric.

13. A quadratic polynomial $(x - \alpha)(x - \beta)$ over complex numbers is said to be square invariant if $(x - \alpha)(x - \beta) = (x - \alpha^2)(x - \beta^2)$. Suppose from the set of all square invariant quadratic polynomials we choose one at random.

The probability that the roots of the chosen polynomial are equal is (rounded off to one decimal place).

14. The unit interval (0, 1) is divided at a point chosen uniformly distributed over (0, 1) in R into two disjoint subintervals.
The expected length of the subinterval that contains 0.4 is ________ . (round off to two decimal places).

MECHANICAL ENGINEERING

15. Let A and B be real symmetric matrices of same size. Which one of the following options is correct?
(a) $A^T = A^{-1}$ (b) $AB = BA$
(c) $(AB)^T = B^TA^T$ (d) $A = A^{-1}$

16. For the differential equation given below, which one of the following options is correct?

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, 0 \le x \le 1, 0 \le y \le 1$$

(a) $u = e^{x+y}$ is a solution for all x and y
(b) $u = e^x \sin y$ is a solution for all x and y
(c) $u = \sin x \sin y$ is a solution for all x and y
(d) $u = \cos x \cos y$ is solution for all x and y

17. The divergence of the curl of a vector field
(a) the magnitude of this vector field
(b) the argument of this vector field
(c) the magnitude of the curl of this vector field
(d) Zero

18. If two unbiased coins are tossed, then what is the probability of having at least one head?
(a) 0.25 (b) 0.5
(c) 0.675 (d) 0.75

19. The values of a function f obtained for different values of x are shown in the table below.

x	0	0.25	0.5	0.75	1.0
$f(x)$	0.9	2.0	1.5	1.8	0.4

Using Simpson's one-third rule,

$$\int_0^1 f(x)\,dx \approx \_\_\_\_\_\_$$

[Round off to 2 decimal places]

20. In the closed interval [0, 3], the minimum value of the function f given below is $f(x) = 2x^3 - 9x^2 + 12x$
(a) 0 (b) 4 (c) 5 (d) 9

21. If C is the unit circle in the complex plane with its centre at the origin, then the value of n in the equation given below is __________ (rounded off to 1 decimal place).

$$\oint_C \frac{Z^3}{(Z^2+4)(Z^2-4)} dz = 2\pi i n$$

22. The directional derivative of the function f given below at the point (1, 0) in the direction of $\frac{1}{2}(\hat{i} + \sqrt{3}\hat{j})$ is _____ (Rounded off to 1 decimal place).

$$f(x, y) = x^2 + xy^2$$

23. Let y be the solution of the differential equation with the initial conditions given below. If $y(x = 2) = A \ln 2$, then the value of A is _________ (rounded off to 2 decimal places).

$$x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = 0$$

$$y(x = 1) = 0 \quad 3x\frac{dy}{dx}(x = 1) = 1$$

ELECTRICAL ENGINEERING

24. Consider the set S of points $(x, y) \in R^2$ which minimise the real valued function

$$f(x, y) = (x + y - 1)^2 + (x + y)^2$$

Which of the following statement is true about the set S?
(a) The number of elements in the set S is finite and more than one.
(b) The number of elements in the set S is infinite.
(c) The set S is empty.
(d) The number of elements in the set S is exactly one.

25. Let v_1 and v_2 be the two eigenvector corresponding to distinct eigenvalues of a 3×3 real symmetric matrix. Which one of the following statements is true?

(a) $v_1^T v_2 \neq 0$ **(b)** $v_1^T v_2 = 0$

(c) $v_1 + v_2 = 0$ **(d)** $v_1 - v_2 = 0$

26. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$ and $b = \begin{bmatrix} 1/3 \\ -1/3 \\ 0 \end{bmatrix}$, then the system of linear equations $AX = b$ has

(a) a unique solution

(b) infinitely many solutions

(c) a finite number of solutions

(d) no solution

27. Let $P = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and let I be the identify matrix.

Then P^2 is equal to

(a) $2P - I$ **(b)** P **(c)** I **(d)** $P + I$

28. Consider discrete random variable X and Y with probabilities as follows:

$P(X = 0 \text{ and } Y = 0) = \dfrac{1}{4}$

$P(X = 1 \text{ and } Y = 1) = \dfrac{1}{8}$

$P(X = 0 \text{ and } Y = 1) = \dfrac{1}{2}$

$P(X = 1 \text{ and } Y = 1) = \dfrac{1}{8}$

Given $X = 1$, the expected value of Y is

(a) $\dfrac{1}{4}$ **(b)** $\dfrac{1}{2}$ **(c)** $\dfrac{1}{8}$ **(d)** $\dfrac{1}{3}$

29. Consider ordinary differential equations given by $\dot{x}_1(t) = 2x_2(t), \dot{x}_2(t) = r(t)$ with initial conditions $x_1(0) = 1$ and $x_2(0) = 0$. If $r(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$, then $t = 1, x_1(t) =$ ________. (Round off to the nearest integer).

30. Let C be clockwise oriented closed curve in the complex plane defined by $|z| = 1$. Further, let $f(x) = jz$ be a complex function, where $j = \sqrt{-1}$. Then, $\oint_C f(z)\,dz =$ _______.

ELECTRONICS ENGINEERING

31. Consider the matrix A below:

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & 6 & 7 & 8 \\ 0 & 0 & \alpha & \beta \\ 0 & 0 & 0 & \gamma \end{bmatrix}$$

For which of the following combinations of α, β and γ, is the rank of A at least three?

(i) $\alpha = 0$ and $\beta = \gamma \neq 0$

(ii) $\alpha = \beta = \gamma = 0$

(iii) $\beta = \gamma = 0$ and $\alpha \neq 0$

(iv) $\alpha = \beta = \gamma \neq 0$

(a) Only (i), (iii), and (iv)

(b) Only (iv)

(c) Only (ii)

(d) Only (i) and (iii)

32. Consider the following series:

(i) $\sum_{n=1}^{\infty} \dfrac{1}{\sqrt{n}}$

(ii) $\sum_{n=1}^{\infty} \dfrac{1}{n(n+1)}$

(iii) $\sum_{n=1}^{\infty} \dfrac{1}{n!}$

(a) Only (ii) converges

(b) Only (ii) and (iii) converge

(c) Only (iii) converges

(d) All three converge

33. A pot contains two red balls and two blue balls. Two balls are drawn from this pot random without replacement.

What is the probability that the two balls drawn have different colours?

(a) $\dfrac{2}{3}$ **(b)** $\dfrac{1}{3}$

(c) $\dfrac{1}{2}$ **(d)** 1

34. Consider a continuous-time, real-valued signal $f(t)$ whose Fourier transform

$F(\omega) = \int_{-\infty}^{\infty} f(t) \exp(-j\omega t)\,dt$ exists.

Which one of the following statements is always TRUE?

(a) $|F(\omega)| \leq \int_{-\infty}^{\infty} |f(t)|\,dt$ **(b)** $|F(\omega)| > \int_{-\infty}^{\infty} |f(t)|\,dt$

(c) $|F(\omega)| \leq \int_{-\infty}^{\infty} f(t)\,dt$ **(d)** $|F(\omega)| \geq \int_{-\infty}^{\infty} f(t)\,dt$

35. Consider the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined as

$$f(x) = 2x^3 - 3x^2 - 12x + 1$$

Which of the following statements is/are correct?

(Here, $\mathbb{R}$ is the set of real numbers.)

(a) f has no global maximiser

(b) f has no global minimiser

(c) $x = -1$ is a local minimiser of f

(d) $x = 2$ is a local maximiser of f

36. The function $y(t)$ satisfies

$$t^2y''(t) - 2ty'(t) + 2y(t) = 0,$$

where $y'(t)$ and $y''(t)$ denote the first and second derivatives of $y(t)$, respectively.

Given $y'(0) = 1$ and $y'(1) = -1$, the maximum value of $y(t)$ over [0, 1] is _________ .

(rounded off to two decimal places).

37. The generator matrix of a (6, 3) binary linear block code is given by

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

The minimum Hamming distance d_{min} between codewords equals _________ (answer in integer).

38. Consider the polynomial $p(s) = s^5 + 7s^4 + 3s^3 - 33s^2 + 2s - 40$. Let (*L.I.R.*) be defined as follows:

L is the number of roots of $p(s)$ with negative real parts.

I is the number of roots of $p(s)$ that are purely imaginary.

R is the number of roots of $p(s)$ with positive real parts.

Which one of the following options is correct?

(a) $L = 2, I = 2$ and $R = 1$

(b) $L = 3, I = 2$ and $R = 0$

(c) $L = 1, I = 2$ and $R = 2$

(d) $L = 0, I = 4$ and $R = 1$

39. Consider a non-negative function $f(x)$ which is continuous and bounded over the interval [2, 8]. Let M and m denote, respectively, the maximum and the minimum values of $f(x)$ over the interval.

Among the combinations of α and β given below, choose the one(s) for which the inequality

$$\beta \le \int_2^\beta f(x)\,dx \le \alpha$$

is guaranteed to hold.

(a) β = 5 m, α = 7 M **(b)** β = 6 m, α = 5 M

(c) β = 7 m, α = 6 M **(d)** β = 7 m, α = 5 M

40. The random variable X takes values in {—1, 0, 1} with probabilities $P(X = -1) = P(X = 1)$ and α and $P(X = 0) = 1 - 2\alpha$, where $0 < \alpha < \frac{1}{2}$. Let $g(\alpha)$ denote the entropy of X (in bits), parameterised by α.

Which of the following statements is/are TRUE?

(a) $g(0.4) > g(0.3)$ **(b)** $g(0.3) > g(0.4)$

(c) $g(0.3) > g(0.25)$ **(d)** $g(0.25) > g(0.3)$

41. Two fair dice (with faces labelled 1, 2, 3, 4, 5, and 6) are rolled. Let the random variable X denote the sum of the outcomes obtained.

The expectation of X is _________ (rounded off to two decimal places).

CIVIL ENGINEERING (CE-1)

42. Suppose λ is an eigenvalue of matrix A and x is the corresponding eigenvector. Let x also be an eigenvector of the matrix B = A – 2I, where I is the identity matrix. Then, the eigenvalue of B corresponding to the eigenvector x is equal to

(a) λ **(b)** λ + 2 **(c)** 2λ **(d)** λ – 2

43. Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 3 \\ -2 & -3 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. For $Ax = b$ to be solvable, which one of the following options is the correct condition on b_1, b_2 and b_3:

(a) $b_1 + b_2 + b_3 = 1$ **(b)** $3b_1 + b_2 + 2b_3 = 0$

(c) $b_1 + 3b_2 + b_3 = 2$ **(d)** $b_1 + b_2 + b_3 = 2$

44. Which one of the following options is the correct Fourier series of the periodic function $f(x)$ described below:

$$f(x) = \begin{cases} 0 & \text{if } -2 < x < -1 \\ 2k & \text{if } -1 < x < 1;\ \text{period} = 4 \\ 0 & \text{if } -1 < x < 2 \end{cases}$$

(a) $f(x) = \frac{k}{2} + \frac{2k}{\pi}\left(\cos\frac{\pi}{2}x - \frac{1}{3}\cos\frac{3\pi}{2}x + \frac{1}{5}\cos\frac{5\pi}{2}x - +...\right)$

(b) $f(x) = \frac{k}{2} + \frac{2k}{\pi}\left(\sin\frac{\pi}{2}x - \frac{1}{3}\sin\frac{3\pi}{2}x + \frac{1}{5}\sin\frac{5\pi}{2}x - +...\right)$

(c) $f(x) = k + \frac{4k}{\pi}\left(\cos\frac{\pi}{2}x - \frac{1}{3}\cos\frac{3\pi}{2}x + \frac{1}{5}\cos\frac{5\pi}{2}x - +...\right)$

(d) $f(x) = k + \frac{4k}{\pi}\left(\sin\frac{\pi}{2}x - \frac{1}{3}\sin\frac{3\pi}{2}x + \frac{1}{5}\sin\frac{5\pi}{2}x - +...\right)$

45. X is the random variable that can take any one of the values, 0, 1, 7, 11 and 12. The probability mass function for X is

$P(X = 0) = 0.4$; $P(X = 1) = 0.3$; $P(X = 7) = 0.1$; $P(X = 11) = 0.1$; $P(X = 12) = 0.1$

Then, the variable of X is

(a) 20.81 **(b)** 28.40 **(c)** 31.70 **(d)** 10.89

46. Which of the following equations belong/belongs to the class of second-order, linear, homogeneous partial differential equations:

(a) $\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + xy$

(b) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

(c) $\frac{\partial u}{\partial t} = c\frac{\partial u}{\partial x}$

(d) $\left(\frac{\partial^2 u}{\partial t^2}\right)^2 = c^2\frac{\partial^2 u}{\partial x^2}$

47. The value of $\lim_{x\to\infty}\left(x-\sqrt{x^2+x}\right)$ is equal to

(a) –1 **(b)** –0.5 **(c)** –2 **(d)** 0

48. Let y be the solution of the initial value problem $y'' + 0.8y' + 0.16y = 0$ where $y(0) = 3$ and $y'(0) = 4.5$. Then, $y(1)$ is equal to ________ (rounded off to 1 decimal place).

49. The maximum value of the function $h(x) = -x^3 + 2x^2$ in the interval $[-1, 1.5]$ is equal to ________ . (rounded off to 1 decimal place).

50. Consider the differential equation given below. Using the Euler method with the step size(h) of 0.5, the value of y at $x = 1.0$ is equal to ________ (rounded off to 1 decimal place).

$$\frac{dy}{dx} + y + 2x - x^2; y(0) = 1 \quad (0 \le x < \infty)$$

CIVIL ENGINEERING (CE-2)

51. For the matrix [A] given below the transpose is _______ .

$$[A] = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 4 & 5 \\ 4 & 3 & 2 \end{bmatrix}$$

(a) $\begin{bmatrix} 2 & 1 & 4 \\ 3 & 4 & 3 \\ 4 & 5 & 2 \end{bmatrix}$ **(b)** $\begin{bmatrix} 4 & 3 & 2 \\ 5 & 4 & 1 \\ 2 & 3 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & 2 & 3 \\ 5 & 1 & 4 \\ 2 & 4 & 3 \end{bmatrix}$ **(d)** $\begin{bmatrix} 2 & 3 & 4 \\ 1 & 4 & 5 \\ 4 & 3 & 2 \end{bmatrix}$

52. Integration of $\ln(x)$ with x i.e., $\int \ln(x)dx$ = __________.

(a) $x{\cdot}\ln(x) - x$ + Constant
(b) $x - \ln(x)$ + Constant
(c) $x{\cdot}\ln(x) + x$ + Constant
(d) $\ln(x) - x$ + Constant

53. Consider a velocity vector, $\vec{V}$ in (x, y, z) coordinates given below. Pick one or more CORRECT statement(s) from the choices given below:

$$\vec{V} = u\hat{x} + v\hat{y}$$

(a) z-component of Curl of velocity;

$$\nabla \times \vec{V} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\vec{z}$$

(b) z-component of Curl of velocity;

$$\nabla \times \vec{V} = \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right)\vec{z}$$

(c) Divergence of velocity; $\nabla \cdot \vec{V} = \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$

(d) Divergence of velocity; $\nabla \cdot \vec{V} = \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$

54. Given that A and B are not null sets, which of the following statements regarding probability is/are CORRECT?

(a) $P(A \cap B) = P(A)\,P(B)$, if A and B are mutually exclusive.
(b) Conditional probability, $P(A \mid B) = 1$ if $B < A$.
(c) $P(A \cup B) = P(A) + P(B)$, if A and B are mutually exclusive.
(d) $P(A \cap B) = 0$, if A and B are independent.

55. The "order" of the following ordinary differential equation is ________ .

$$\frac{d^3y}{dx^3} + \left(\frac{d^2y}{dx^2}\right)^6 + \left(\frac{dy}{dx}\right)^4 + y = 0$$

56. Pick the CORRECT solution for the following differential equation $\frac{dy}{dx} = e^{x-y}$

(a) $y = \ln(e^x + \text{Constant})$
(b) $\ln(y) = x$ + Constant
(c) $\ln(y) = \ln(e^x)$ + Constant
(d) $y = x$ + Constant

57. Consider the function given below and pick one or more CORRECT statement(s) from the following choices.

$$f(x) = x^3 - \frac{15}{2}x^2 + 18x + 20$$

(a) $f(x)$ has a local minimum at $x = 3$
(b) $f(x)$ has a local maximum at $x = 3$
(c) $f(x)$ has local minimum at $x = 2$
(d) $f(x)$ has a local maximum at $x = 2$

58. Pick the CORRECT eigenvalue(s) of the matrix [A] from the following choices.

$$[A] = \begin{bmatrix} 6 & 8 \\ 4 & 2 \end{bmatrix}$$

(a) 10w **(b)** 4 **(c)** –2 **(d)** –10

59. Consider a discrete random variable X whose probabilities are given below. The standard deviation of the random variable is ________ (round off to one decimal place).

x_1	1	2	3	4
$P(X = x_i)$	0.3	0.1	0.3	0.3

PRODUCTION AND INDUSTRIAL ENGINEERING (PI)

60. In a work sampling, out of n observations, a worker was sitting idle in x observations. The standard deviation of the mean proportion of idle time is given by

(a) $\sqrt{\frac{x(n-x)}{n^2}}$ **(b)** $\sqrt{\frac{x(n-x)}{n^3}}$

(c) $\frac{x}{n}$ **(d)** $\sqrt{\frac{x^2}{n^3}}$

61. If $F(s)$ denotes the Laplace transform of some function $f(t)$, then the Laplace transform of $e^{bt} f(t)$, where b is a real constant, is

(a) $-F(s)$ **(b)** $F(b-s)$

(c) $F(s-b)$ **(d)** $F(s+b)$

62. The eigenvalues of the matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ are

(a) $1+\sqrt{-1}$ and $1-\sqrt{-1}$

(b) -1 and 1

(c) $-\sqrt{-1}$ and $\sqrt{-1}$

(d) $-1+\sqrt{-1}$ and $-1-\sqrt{-1}$

63. A bag contains 5 red, 7 green and 3 blue balls. Two balls are drawn at random from the bag one-by-one. The probability of the second drawn ball being red is

(a) $\frac{1}{3}$ **(b)** $\frac{2}{5}$

(c) $\frac{2}{3}$ **(d)** $\frac{1}{5}$

64. If **i**, **j** and **k** are the orthogonal unit vectors in Cartesian x-y-z coordinate system, the curl of the vector $-2y\mathbf{i} + x\mathbf{j}$ is

(a) $-\mathbf{k}$ **(b)** $3\mathbf{k}$

(c) $\mathbf{k}$ **(d)** $-3\mathbf{k}$

65. Which one of the following equations is a linear differential equation?

(a) $\left(\frac{dy}{dx}\right)^2 + 2x = y$ **(b)** $\frac{dy}{dx} + 2x = y^2$

(c) $x\frac{d^2y}{dx^2} + 2y\frac{dy}{dx} = 0$ **(d)** $x^3\frac{dy}{dx} + xy = x^2$

66. Which one of the following function is analytic, given $i = \sqrt{-1}$?

(a) $e^x(\cos y + i\sin y)$

(b) $e^x(-\cos y + i\sin y)$

(c) $e^x(\cos y - i\sin y)$

(d) $e^{-x}(\cos y + i\sin y)$

67. The solution of the linear differential equation $\frac{dy}{dx} + y = e^x$, when $y(0) = 0$, is

(a) $y = e^x - e^{-x}$

(b) $y = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$

(c) $y = e^x + e^{-x}$

(d) $y = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$

68. If **i**, **j** and **k** are the orthogonal unit vectors in Cartesian x-y-z coordinate system, the rate of the change of the function $f(x, y, z) = x^2 + 2y^2 + z$ at point (1, 1, 1) in the direction of $3\mathbf{i} + 4\mathbf{k}$ is _______ . (*Answer in integer*)

69. The value of the integral $\int_1^3 (x^2 - 2x)\,dx$ obtained by using Simpson's 1/3 rule with 4 subintervals is equal to $\frac{n}{3}$. The value of n is _______. (*Answer in integer*)

Answer Key

Q. No.	Answer	Topic Name	Chapter Name
1	(d)	Function	Relation & Function
2	(a)	Relation	Relation & Function
3	(a,d)	Equation	Algebra
4	-2	Function	Relation & Function
5	(d)	Matrix	Algebra
6	(b)	Function	Relation & Function

7	0.301	Probability Distribution	Probability
8	(c)	Matrix	Algebra
9	(a)	Integrals	Calculus
10	(a)	Matrix	Algebra
11	(b, c)	Relation	Relation & Function
12	(a, b, c)	Matrix	Algebra
13	0.5	Probability	Probability
14	0.75	Probability	Probability
15	(c)	Matrix	Algebra
16	(b)	Differential Equation	Calculus
17	(d)	Vectors	Vectors
18	(d)	Probability	Probability
19	1.63	Integrals	Calculus
20	(a)	Limits	Calculus
21	0	Integrals	Calculus
22	1	Vectors	Vectors
23	0.5	Differential Equation	Calculus
24	(b)	Function	Relation & Function
25	(b)	Vectors	Vectors
26	(b)	Matrix	Algebra
27	(a)	Matrix	Algebra
28	(b)	Probability	Probability
29	2	Differential Equation	Calculus
30	0	Complex Number	Algebra
31	(a)	Matrix	Algebra
32	(b)	Series	Algebra
33	(a)	Probability	Probability
34	(a)	Fourier Transform	Calculus
35	(a, b)	Function	Relation & Function
36	0.25	Differential Equation	Calculus
37	3	Matrix	Algebra
38	(a)	Equation	Algebra
39	(a)	Integrals	Calculus

40	(b, c)	Probability	Probability
41	7	Probability	Probability
42	(d)	Matrix	Algebra
43	(b)	Matrix	Algebra
44	(c)	Fourier transform	Calculus
45	(a)	Probability	Probability
46	(b)	Differential Equation	Calculus
47	(b)	Limits	Calculus
48	5.8	Differential Equation	Calculus
49	3	Limits	Calculus
50	2.6	Differential Equation	Calculus
51	(a)	Matrix	Matrix
52	(a)	Integrals	Calculus
53	(a, c)	3D vector	Vectors
54	(b, c)	Probability	Probability
55	3	Differential Equation	Calculus
56	(a)	Differential Equation	Calculus
57	(a, d)	Limits	Calculus
58	(a, c)	Matrix	Matrix
59	2.8	Random Variables	Probability
60	(b)	Statics	Algebra
61	(c)	Laplace Transformation	Calculus
62	(c)	Matrix	Linear Algebra
63	(a)	Probability	Probability
64	(b)	Curl	Calculus
65	(d)	Differential Equation	Ordinary Differential Equation
66	(a)	Analytic Function	Calculus
67	(d)	Differential Equation	Ordinary Differential Equation
68	2	Divergence and Curl	Calculus
69	2	Simpson Formula of integration	Numerical Analysis

COMPUTER SCIENCE & IT (CS-1)

1. Option (d) is correct.

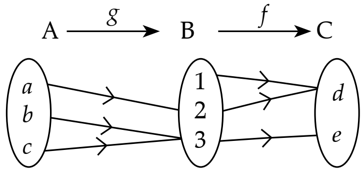
Given that, $g : A \rightarrow B$

and $f : B \rightarrow C$

Such that $f(g(\cdot)) : A \rightarrow C$

If $f(\cdot)$ and $f(g(\cdot))$ are onto function then $g(\cdot)$ need not be onto function.

For example: Let f is onto, fog is onto



$\therefore \quad f(1) = d, f(2) = d, f(3) = e$

$f(\cdot)$ is many to one and onto.

$$g(a) = 2, g(b) = 3, g(c) = 3$$

$g(\cdot)$ is many to one and into.

$$f(g(a)) = d$$
$$f(g(b)) = e$$
$$f(g(c)) = e$$

$\therefore f(g(1))$ is many to one and onto.

Hence, $f(\cdot)$ and $f(g(\cdot))$ are onto function but $g(\cdot)$ is need not be one-one or onto function.

2. Option (a) is correct.

$$T(n) = 2\,T(n-1) + n \,.\, 2^n \quad \text{...(i)}$$

replace n by $n-1$ we get,

$$T(n-1) = 2\,T(n-2) + (n-1) \,.\, 2^{n-1}$$

$$\therefore \quad T(n) = 2^2\,T(n-2) + (n-1) \,.\, 2^n + n.2^n \quad \text{...(ii)}$$

Replace n by $n-2$ in equation (i),

$$T(n-2) = 2\,T(n-3) + (n-2)2^{n-2} \quad \text{....(iii)}$$

From equations (ii) and (iii) we get,

$$T(n) = 2^3\,T(n-3) + 2^n\,[(n-2) + (n-2) + n]$$
$$= \Theta\,(2^n \,.\, n^2)$$

3. Option (a, d) are correct.

Given equations are $x + ky = 1$

and $\quad kx + y = -1$

We can solve both equation using matrix equation

$$AX = B$$

$$A = \begin{bmatrix} 1 & k \\ k & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$|A| = 1 - k^2$$

If $|A| \neq 0$, then given equation has unique solution

$\therefore k \neq \pm 1$, we get unique solution

If $\quad k = 1$ we get,

$$x + y = 1 \text{ and } x + y = -1$$

$\therefore$ We don't have any value of x and y which satisfy both equation together.

Hence, no solution for $k = 1$.

If $\quad k = -1$ we get,

$$x - y = 1$$
$$-x + y = -1$$

Above equation form two intersecting plane.

Therefore, we get infinite many solution for $k = -1$.

4. Correct answer is [–2].

If function is differentiable at $x = 1$

then it must be continuous at $x = 1$

If function is continuous at $x = 1$

then,

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$$

$$\lim_{x \to 1^-} (ax + b) = \lim_{x \to 1^+} (x^3 + x^2 + 1)$$

$$a + b = 3 \quad \text{....(i)}$$

If $f(x)$ is differentiable at $x = 1$

then,

$$\lim_{x \to 1^-} f'(x) = \lim_{x \to 1^+} f'(x)$$

$$\lim_{x \to 1^-} \frac{d}{dx}(ax + b) = \lim_{x \to 1^+} \frac{d}{dx}(x^3 + x^2 + 1)$$

$$a = \lim_{x \to 1^+} (3x^2 + 2x)$$

$$a = 5 \quad \text{...(ii)}$$

From equations (i) and (ii) we get

$$5 + b = 3$$
$$b = -2$$

5. Option (d) is correct.

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\text{Trace (A)} = 1 + (-1) = 0$$
$$|A| = -1 - 1 = -2$$

Characteristic equation,

$$\lambda^2 - \text{Trace (A)} + |A| = 0$$

$$\therefore \quad \lambda^2 - 2 = 0$$

$$\lambda = \pm\sqrt{2}$$

Eigenvalues of matrix A^n is given as λ^n.

$\therefore$ Eigenvalue of matrix $A^{13} = \left(\pm\sqrt{2}\right)^{13}$

$$= 64\sqrt{2}, \; -64\sqrt{2}$$

6. **Option (b) is correct.**

Given F be the set of all possible functions from A to A.

$$F = \{f_1, f_2, f_3, f_4 ....\}$$

Algebraic structure is defined as

$$(f_1 \odot f_2)(n) = f_1(n) + f_2(n)$$

Associative:

$$\begin{aligned}(f_1 \odot f_2) \odot f_3(n) &= (f_1 \odot f_2)(n) + f_3(n)\\ &= f_1(n) + f_2(n) + f_3(n)\\ &= f_1(n) + (f_2 \odot f_3)(n)\\ &= f_1 \odot (f_2 \odot f_3)\end{aligned}$$

$\therefore$ '$\odot$' is associative.

Identify:

$$f_1 \odot f_e = f_1 = f_e \odot f_1$$

Consider $(f_1 \odot f_e)(n) = f_1(n) + f_e(n)$

$= f_1(n) + 0 = f_1(n)$

Similarly, $(f_e \odot f_1)(n) = f_e(n) + f_1(n)$

$= 0 + f_1(n) = f_1(n)$

Therefore there exist an identity with respect to $\odot$.

Hence, it is monoid.

Inverse:

By definition

$$f_1 \odot f_2(n) = f_e(n) = (f_2 \odot f_1)(n)$$

$$f_1(n) + f_2(n) = 0 = f_2(n_1) + f_1(n)$$

$\Rightarrow$ $f_2(n) = -f_1(n)$ which is –ve integer

$\therefore$ Inverse does not exists.

Commutative:

$$\begin{aligned}f_1 \odot f_2(n) &= f_1(n) + f_2(n)\\ &= f_2(n_1) + f_1(n)\\ &= f_2 \odot f_1(n)\end{aligned}$$

$\therefore$ It is abelian.

Hence (F, $\odot$) is an abelian monoid.

7. **Correct answer is [0.301].**

Given,

$$P(x) = \begin{cases} Cx^2, & \text{for } 1 \le x \le 4 \\ 0, & \text{for } x < 1 \text{ or } x > 4 \end{cases}$$

Total probability = 1

$$\int_{-\infty}^{\infty} P(x)dx = 1$$

$$\int_{-\infty}^{1} 0dx + \int_{1}^{4} Cx^2dx + \int_{4}^{\infty} 0dx = 1$$

$$\Rightarrow \left[\frac{Cx^3}{3}\right]_1^4 = 1$$

$$\frac{C}{3}\{4^3 - 1^3\} = 1$$

$$\frac{63C}{3} = 1$$

$$\therefore C = \frac{1}{21}$$

$$\therefore P(2 < x \le 3) = \int_2^3 f(x)dx$$

$$= \frac{1}{21}\int_2^3 x^2 dx$$

$$= \frac{1}{21}\left(\frac{x^3}{3}\right)_2^3$$

$$= \frac{1}{21}\left[9 - \frac{8}{3}\right]$$

$$= \frac{19}{63} = 0.301$$

COMPUTER SCIENCE & IT (CS-2)

8. **Option (c) is correct.**

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$A^2 = 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore (A^2)^4 = \left(5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)^4$$

$$A^8 = 5^4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 625 & 0 \\ 0 & 625 \end{bmatrix}$$

9. **Option (a) is correct.**

$$\int_1^x t\ln t\,dt = \frac{1}{4}$$

Applying integration by parts,

$$\left[\frac{t^2}{2}\ln t\right]_1^x - \int_1^x \frac{t^2}{2}\cdot\frac{1}{t}dt = \frac{1}{4}$$

$$\frac{x^2}{2}\ln x - \left[\frac{t^2}{4}\right]_1^x = \frac{1}{4}$$

$$\frac{x^2}{2}\ln x - \frac{x^2}{4} + \frac{1}{4} = \frac{1}{4}$$

$$\frac{x^2}{2}\left[\ln x - \frac{1}{2}\right] = 0$$

$$\ln x = \frac{1}{2}$$
$$x = \sqrt{e}$$

10. Option (a) is correct.

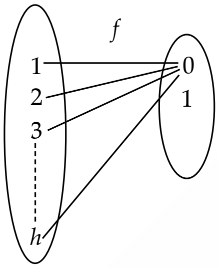
$$L^2 = L^{-1}$$
$$M = L^8$$
$$N = L^2$$
$$M = L^8 = L^2L^2L^2L^2$$
$$M = L^2L^2L^{-1}L^{-1}$$
$$= L^2\,L\,.\,L\,L^{-1}L^{-1} \qquad (LL^{-1} = I)$$
$$= L^2\,L\,I\,L^{-1} \qquad (LI = L)$$
$$= L^2\,LL^{-1}$$
$$= L^2\,I$$
$$M = L^2$$
$$M = N$$
$$|M - N| = 0$$

11. Options (b, c) are correct.

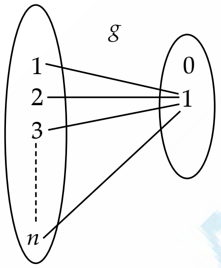
F : Set of all function from $\{1, 2, 3 \;..... \; n\}$ to $\{0, 1\}$ and R = "$\leq$"

$F \leq g$ iff $f(x) \leq g(x)$, $\forall x, \in \{1, 2, 3 \;......\; n\}$

Let,



$f(x) = 0$



$g(x) = 1$

Here $\quad f(x) \leq g(x)$

But $\quad g(x) \leq f(x)$

$\therefore$ It cannot be a symmetric.

$\Rightarrow$ It cannot be equivalence reaction.

But here, If $f(x) \leq g(x)$ and $g(x) \leq f(x)$

Then, $\quad f(x) = g(x)$

$(0 = 0)$

$(1 = 1)$

$\therefore$ It is antisymmetric.

Reflexive:

$\because \quad f(x) \leq f(x), \forall x, \in \{1, 2, ... \, n\}$

$0 \leq 0$

$1 \leq 0$

$\therefore$ It is reflexive.

Transitive:

Let, $\quad 0 \leq 1$

$1 \leq 0$

Let $0 \leq 0$ and $0 \leq 1$ then $0 \leq 1$

If $\quad f(x) \leq g(x)$ and $g(x) \leq h(x)$

Then $\quad f(x) \leq h(x)$

$\therefore$ It is transitive.

Hence it is poset.

We know that,

glb (f, g) wrt '$\leq$' is f

lub (f, g) wrt $\leq$ is g

$\therefore$ For every two function F $\exists$ a glb and lub also.

$\therefore$ $(f, \leq)$ is a lattice.

12. Option (a, b, c) is correct.

Given that,

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \text{ and } U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

and $\quad PX = Q, \quad P = LU$

$\therefore \quad LUX = Q$

Let $\quad Y = UX$

$\therefore \quad LY = Q$

Solving $LY = Q$ we get Y then

Sub in $UX = Y$ and save for X

$\therefore$ option (a) is correct.

Check for option (b)

Let P is invertible

$\therefore \quad P^{-1} = (LU)^{-1}$

$= U^{-1}\,L^{-1}$

$\Rightarrow$ U and L are invertible

$\therefore$ option (b) is true.

Let us check for option (c)

Let $\quad P$ is singular $= |P| = 0$

$\therefore \quad |LU| = 0$

$\Rightarrow \quad |L|\;|U| = 0$

$\therefore$ either $\quad |L| = 0$ or $|U| = 0$

$\therefore$ option (c) is true.

Let us check for option (d)

Let P is symmetric $= PT = P$

$$P = LU$$
$$P^T = (LU)^T$$
$$= U^T\,L^T$$

As, $\quad P = P^T \Rightarrow LU = U^T\,L^T$

If U and L are symmetric then $U^T = U$ and $L = L^T$

$\therefore \quad U^T\,L^T = UL \neq LU$

Hence, U and L need not be symmetric.

$\therefore$ option (d) is wrong.

13. Correct answer is [0.5].

Square invariant polynomial, if

$$(x - \alpha)\,(x - \beta) = (x - \alpha^2)\,(x - \beta^2)$$

On comparing both side we get,

$$\alpha + \beta = \alpha^2 + \beta^2 \qquad ...(i)$$

$$\alpha\beta = \alpha^2\beta^2 \qquad \text{...(ii)}$$

From equation (ii) we get,

$$\alpha\beta = 1 \quad \text{or} \quad \alpha\beta = 0$$

Case-I

If $\alpha\beta = 0$

(a) Let $\alpha = 0$, then from equation (i) we get, $\beta = \beta^2$

$\Rightarrow \quad \beta = 0, 1$

(b) Let $\beta = 0$, then from equation (i) we get, $\alpha = \alpha^2$

$\Rightarrow \quad \alpha = 0, 1$

(c) $\alpha = 0$ and $\beta = 0$

Hence, we get

$$(\alpha, \beta) = (0, 0) \text{ or } (1, 0) \text{ or } (0, 1)$$

For $\alpha = 0, \beta = 0$:

Polynomial will be $x^2 = x^2$

Which is square invariant

For $\alpha = 0, \beta = 1$,

Polynomial will be

$$x(x-1) = x(x-1)$$

$\therefore$ $x^2 - x$ is also a square invariant polynomial.

Similarly for $\alpha = 1, \beta = 0$ also.

Case (ii):

$$\alpha\beta = 1$$

$$\Rightarrow \quad \beta = \frac{1}{\alpha}$$

Sub in (i),

$$\Rightarrow \quad \alpha + \frac{1}{\alpha} = \alpha^2 + \frac{1}{\alpha^2}$$

$$\alpha - \alpha^2 = \frac{1}{\alpha^2} - \frac{1}{\alpha}$$

$$\alpha(1-\alpha) = \frac{\alpha - \alpha^2}{\alpha\alpha^2}$$

$$(\alpha - \alpha^2)\left[1 - \frac{1}{\alpha^3}\right] = 0$$

$$\therefore \quad 1 - \frac{1}{\alpha^3} = 0$$

$$\Rightarrow \quad \alpha^3 = 1$$

$$\therefore \quad \alpha^3 - 1 = 0$$

$$(\alpha - 1)(\alpha^2 + \alpha + 1) = 0$$

$$\Rightarrow \quad \alpha = 1, \omega, \omega^2$$

For $\alpha = \omega, \omega^2$ also

We get $x^2 + x + 1$ will be square invariant polynomial.

Hence, total number of square invariant polynomial $= 4$

Let E = Drawing a square invariant polynomial has equal roots

$$\therefore \quad n(E) = 2$$

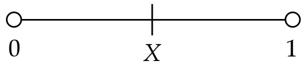
Hence, $\quad P(E) = \dfrac{n(E)}{n(S)} = \dfrac{2}{4} = \dfrac{1}{2} = 0.5$

14. Correct answer is [0.75].

Let X be a random variable that represents the point where the unit integral (0, 1) is divided into disjoints subintervals.

X is uniformly distributed over (0, 1).

Then, subintervals are $(0, X)$ and $(X, 1)$.



Condition is 0.4 lies interval,

So, if $X < 0.4$, 0.4 will be in $(X, 1)$

and if $X > 0.4$, 0.4 will be in $(0, X)$

Let $f(x)$ = maximum $(x, 1-x)$ $0 < x < 1$

Let Y = Expected length of interval containing 0.4

$$\therefore \quad Y = \int_0^1 f(x)\,dx$$

$$Y = \int_0^{\frac{1}{2}} (1-x)\,dx + \int_{\frac{1}{2}}^1 x\,dx$$

$$= \left(x - \frac{x^2}{2}\right)_0^{0.5} + \left(\frac{x^2}{2}\right)_{0.5}^1$$

$$= \left(\frac{1}{2} - \frac{1}{8}\right) + \left(\frac{1}{2} - \frac{1}{8}\right) = 0.75$$

MECHANICAL ENGINEERING

15. Option (c) is correct.

A is symmetric matrix $\Rightarrow A = A^T$

B is symmetric matrix $\Rightarrow B = B^T$

$$(AB)^T = B^T \cdot A^T = BA$$

16. Option (b) is correct.

Option (c) :

$$u = \sin x \sin y$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin x \sin y$$

$$\frac{\partial^2 u}{\partial y^2} = -\sin x \sin y$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -2 \sin x \sin y$$

Therefore, option (c) is wrong.

Option (b) :

$$u = e^x \sin y$$

$$\frac{\partial^2 u}{\partial x^2} = e^x \sin y$$

$$\frac{\partial^2 u}{\partial y^2} = -e^x \sin y$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Therefore, option (b) is correct.

Option (d) :

$$u = \cos x \cos y$$

$$\frac{\partial^2 u}{\partial x^2} = -\cos x \cos y$$
$$\frac{\partial^2 u}{\partial y^2} = -\cos x \cos y$$
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -2 \cos x \cos y$$

Therefore, option (d) is wrong.

Option (a) :

$$\frac{\partial^2 u}{\partial x^2} = e^x e^y$$
$$\frac{\partial^2 u}{\partial y^2} = e^x e^y$$
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 2 e^x e^y$$

Therefore, option (a) is wrong.

17. Option (d) is correct.

Let $\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$

Cut of $\vec{F}$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

$$\nabla \times \vec{F} = \hat{i}\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) - \hat{j}\left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z}\right) + \hat{k}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$$

Now div. (curl $\bar{F}$) = $\nabla . \ (\nabla \times \bar{F})$

$$= \frac{\partial}{\partial x}\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) - \frac{\partial}{\partial y}\left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z}\right) + \frac{\partial}{\partial z}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$$

$$= \frac{\partial^2 F_z}{\partial x \partial y} - \frac{\partial^2 F_y}{\partial x \partial z} - \frac{\partial^2 F_z}{\partial x \partial y} + \frac{\partial^2 F_x}{\partial y \partial z} + \frac{\partial^2 F_y}{\partial z \partial x} - \frac{\partial^2 F_x}{\partial y \partial z}$$

$= 0$

18. Option (d) is correct.

S = ∫HH, HT, TH, TT∫

Probability of at least one head = $\frac{3}{4}$

19. Correct answer is (1.63).

Simpson's $\frac{1}{3}$ rule of integration is given as,

$$\int_0^1 f(x)dx = \frac{h}{3}[(y_0 + y_L) + 4(y_1 + y_3 + y_5 + \ldots) + 2(y_2 + y_4 + y_6 \ldots)]$$

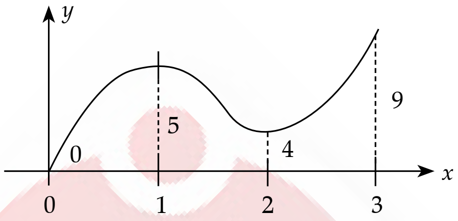
Here
$y_0 = 0.9$
$y_L = 0.4$
$y_1 = 2$
$y_2 = 1.5$
$y_3 = 1.8$

$y_i = 0 \ \forall \ i \geq y$ and $i \in N$

$$\int_0^1 f(x)\,dx = \frac{0.25}{3}[(0.9+0.4)+4(2+1.8)+2(1.5)]$$
$$= \frac{0.25}{3}[1.3 + 4 \times 3.8 + 3]$$
$$= \frac{19.5}{12}$$
$$= 1.625$$

20. Option (a) is correct.

$$f(x) = 2x^3 - 9x^2 + 12x$$
$$f'(x) = 6x^2 - 18x + 12$$



If $f'(x) = 0 \Rightarrow 6x^2 - 18x + 12 = 0$

$x^2 - 3x + 2 = 0$

$\therefore \quad x = 1, 2$

$f(0) = 0$

$f(1) = 2 - 9 + 12 = 5$

$f(2) = 2(2)^3 - 9(2)^2 + 2(12)$

$= 16 - 36 + 24 = 4$

$f(3) = 2(3)^3 - 9(3)^2 + 3(12)$

$= 54 - 81 + 36 = 9$

Therefore, minimum value is zero.

21. Correct answer is (0).

$$\oint_C \frac{z^3 dz}{(z^2+4)(z^2-4)} = 2\pi n$$

Poles $(z^2 + 4)(z^2 - 4) = 0$

$z^2 + 4 = 0 \Rightarrow z = \pm 2i$

$z^2 - 4 = 0 \Rightarrow z = \pm 2$

All the poles lies outside the closed curve $\Rightarrow n = 0$

22. Correct answer is (1).

$$f(x, y) = x^2 + xy^2$$

Let $\vec{a} = \frac{1}{2} + \frac{\sqrt{3}}{2}\hat{j}$

$$\nabla f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} = (2x + y^2)\hat{i} + (2xy)\hat{j}$$

∇f at $(1, 0) = (2+0)\hat{i} + 0$

$= 2\hat{i}$

Directional derivative $= \nabla f \cdot \frac{\vec{a}}{|\vec{a}|}$

$$= (2\hat{i}) \cdot \frac{\left(\frac{\hat{i}}{2} + \frac{\sqrt{3}}{2}\hat{j}\right)}{\sqrt{\frac{1}{4} + \frac{3}{4}}}$$

$$= 2\times\frac{1}{2}=1$$

23. Correct answer is [0.5]

$$x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+y = 0$$

Let $\quad x = e^t,\ \frac{d}{dx}=D$

$[D(D-1) + 3D + 1]y = 0$

$(D^2 - D + 3D + 1)y = 0$

$(D^2 + 2D + 1)y = 0$

$(D + 1)^2 y = 0$

$\therefore$ Auxiliary equation $\Rightarrow (m + 1)^2 = 0$

$m = -1, -1$

$\therefore$ general solution will be,

$$y = (c_1 + c_2 t)\, e^{-t}$$

$$= (c_1 + c_2 \ln x)e^{-\ln x}$$

$$y = \frac{c_1+c_2\ln x}{x}$$

at $x = 1, y = 0$

$$\therefore \quad 0 = \frac{c_1+c_2\ln 1}{1}$$

$$0 = c_1 + 0$$

$$c_1 = 0 \qquad \text{...(i)}$$

$$y = \frac{c_1+c_2\ln 1}{1}$$

$$\frac{dy}{dx} = \frac{-c_1}{x_2}+c_2\left(\frac{x\cdot\frac{1}{x}-\ln x}{x^2}\right)$$

$$\frac{dy}{dx} = \frac{-c_1+c_2-c_2\ln x}{x^2}$$

at $x = 1$, $\frac{dy}{dx} = 1$

$$1 = \frac{-c_1+c_2-c_1\ln 1}{1}$$

as $c_1 = 0$ from equation (i)

we get, $c_2 = 1$

Hence, $\quad y = \frac{\ln x}{x}$

At $x = 2$, $y = A\ln 2$

$$\therefore \quad A\ln 2 = \frac{\ln 2}{x}$$

$$\Rightarrow \quad A = \frac{1}{2}$$

ELECTRICAL ENGINEERING

24. Option (b) is correct.

$f(x, y) = (x + y - 1)^2 + (x + y)^2$

$= x^2 + y^2 - 2x - 2y + 2xy + 1 + x^2 + y^2 + 2xy$

$= 2x^2 + 2y^2 + 4xy - 2x - 2y + 1$

We can find stationary points by partial differentiation.

$$\frac{\partial f}{\partial x} = 4x + 4y - 2$$

$$\frac{\partial f}{\partial y} = 4y + 4x - 2$$

On solving $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$ we get, $2x + 2y = 1$

$$r = f_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)=4$$

$$t = f_{yy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right)=4$$

$$S = f_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)=4$$

$$rt - S^2 = 16 - 16 = 0$$

$\therefore$ We have infinite stationary points which lie along $x + y = \frac{1}{2}$

$\therefore$ It is minimum at infinite points.

$f(x, y) = (x + y - 1)^2 + (x + y)^2$

$f(x, y)$ is sum of two perfect square number, therefore it will be minimum when both are minimum at same time.

Let $x + y = u$

$$\therefore \quad f(x, y) = (u - 1)^2 + u^2$$

$$= 2u^2 - 2u + 1$$

$$= 2\left(u-\frac{1}{2}\right)^2+\frac{1}{2}$$

$\Rightarrow f(x, y)$ is minimum when $u - \frac{1}{2} = 0$

$$\therefore \quad x + y = \frac{1}{2}$$

$\therefore$ Function is minimum at infinite points.

25. Option (b) is correct.

Eigenvectors of symmetric matrix of 3×3.

Corresponding to distinct λ are orthogonal to each other.

$$\therefore \quad V_1^T V_2 = 0$$

26. Option (b) is correct.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

$$b = \begin{bmatrix} 1/3 \\ -1/3 \\ 0 \end{bmatrix}$$

$$A\,X = b$$

Let $\quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ -\frac{1}{3} \\ 0 \end{bmatrix} \quad \text{...(i)}$$

From equation (i) we get,

$$\begin{bmatrix} x+y+z \\ -x-y-z \\ 0+y-z \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ -\frac{1}{3} \\ 0 \end{bmatrix} \quad \text{...(ii)}$$

From equation (ii), we get,

$$x + y + x = \frac{1}{3} \quad \text{(iii)}$$

$$-x-y-z = -\frac{1}{3} \quad \text{(iv)}$$

$$y - z = 0 \quad \text{...(v)}$$

Let $z = \lambda \quad \Rightarrow y = \lambda$

From equation (iii) we get,

$$x + 2\lambda = \frac{1}{3}$$

$$x = \frac{1}{3} - 2\lambda$$

$\therefore$ System of linear equations has infinite solutions as, $\frac{1}{3} - 2\lambda, \lambda, \lambda$ are the values of x, y and z respectively.

27. Option (a) is correct.

$$P = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4-1 & 2 & 0 \\ -2 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4-1 & 2 & 0 \\ -2 & 0-1 & 0 \\ 0 & 0 & 2-1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 0 \\ -2 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$= 2P - I$$

$$|P - \lambda I| = \begin{bmatrix} 2-\lambda & 1 & 0 \\ -1 & 0-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix}$$

$(1 - \lambda)(-2\lambda + \lambda^2 + 1) = 0$

By Cayley-Hamilton theorem,

Replace λ by P we get,

$$= (I - P)(-2P + P^2 + I) = 0$$

$$= -2P + P^2 + I = 0$$

$$\therefore \quad P^2 = 2P - \mathrm{I}$$

28. Option (b) is correct.

Given,

$$P(X = 0 \text{ and } Y = 0) = \frac{1}{4}$$

$$P(X = 1 \text{ and } Y = 1) = \frac{1}{8}$$

$$P(X = 0 \text{ and } Y = 1) = \frac{1}{2}$$

$$P(X = 1 \text{ and } Y = 0) = \frac{1}{8}$$

$$E\left[\frac{Y}{K=1}\right] = \sum_j Y_j P\left(\frac{Y/y_j}{X=1}\right)$$

$$= 0 + 1 \times \frac{p(x=1 \text{ and } y=1)}{p(x=1)}$$

$$= \frac{\frac{1}{8}}{\frac{1}{8}+\frac{1}{8}} = \frac{1}{2}$$

29. Correct answer is [2].

Given, $x_1(0) = 1$ and $x_2(0) = 0$

$x_1(t) = 2x_2(t)$, $x_2(t) = r(t)$ ...(i)

Equation (i) can be written as,

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t)$$

Let $$A = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$$

$$SI - A = \begin{bmatrix} S & -2 \\ 0 & S \end{bmatrix}$$

$$(SI - A)^{-1} = \frac{1}{S^2}\begin{bmatrix} S & 2 \\ 0 & S \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{S} & \frac{2}{S^2} \\ 0 & \frac{1}{S} \end{bmatrix}$$

$\therefore$ State transition matrix $= \phi(t)$

$$\phi(t) = L^{-1}\left\{[sI - A]^{-1}\right\} = L^{-1}\left\{\begin{bmatrix} \frac{1}{S} & \frac{2}{S^2} \\ 0 & \frac{1}{S} \end{bmatrix}\right\}$$

from the question, $r(t) = u(t)$

$$\phi(t) = \begin{bmatrix} u(t) & 2tu(t) \\ 0 & u(t) \end{bmatrix}$$

$$x(t) = \phi(t)\cdot x(0) + L^{-1}\{[sI - A]^{-1} B\cdot u(s)\}$$

$$[sI - A]^{-1} B\cdot u(s) = \begin{bmatrix} \frac{1}{s} & \frac{2}{s^2} \\ 0 & \frac{1}{s} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\frac{1}{s} = \begin{bmatrix} \frac{2}{s^2} \\ \frac{1}{s} \end{bmatrix}\frac{1}{s} = \begin{bmatrix} \frac{2}{s^3} \\ \frac{1}{s^2} \end{bmatrix}$$

$$\therefore \quad L^{-1}\begin{bmatrix} \frac{2}{s^3} \\ \frac{1}{s^2} \end{bmatrix} = \begin{bmatrix} t^2u(t) \\ tu(t) \end{bmatrix} = \begin{bmatrix} t^2 \\ t \end{bmatrix}u(t)$$

$$\therefore \quad x(t) = \begin{bmatrix} 1 & 2t \\ 0 & 1 \end{bmatrix}u(t)\cdot\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} + \begin{bmatrix} t^2 \\ t \end{bmatrix}u(t)$$

Given, $x_1(0) = 1, x(0) = 0$

$$x(t) = \begin{bmatrix} 1 & 2t \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}u(t) + \begin{bmatrix} t^2 \\ t \end{bmatrix}u(t)$$

$$x(t) = \begin{bmatrix} 1+0 \\ 0 \end{bmatrix}u(t) + \begin{bmatrix} t^2 \\ t \end{bmatrix}u(t)$$

$$x(t) = \begin{bmatrix} 1+t^2 \\ t \end{bmatrix}u(t)$$

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$\therefore \quad x_1(t) = (1 + t^2)\, u(t)$

At $\quad t = 1,$

$$x_1(1) = (1 + 1^2)\, u(1) = 2$$

30. Correct answer is [0].

$$f(z) = iz$$
$$z = x + iy$$
$$f(z) = i(x + iy) = ix - y$$

Let, $f(z) = u + iv$

$$u = -y, v = x$$

$$\frac{\partial u}{\partial x} = 0, \frac{\partial u}{\partial y} = -1$$

$$\frac{\partial v}{\partial x} = 1, \frac{\partial v}{\partial y} = 0$$

It satisfies Cauchy's Reimann equations,

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

and $$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Hence, $f(z) = iz$ is analytic function.

$$\therefore \quad \oint_C iz\,dz = 0$$

ELECTRONICS ENGINEERING

31. Option (a) is correct.

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & 6 & 7 & 8 \\ 0 & 0 & \alpha & \beta \\ 0 & 0 & 0 & \gamma \end{bmatrix}$$

If rank of matrix A is atleast three. Then atleast three elements of principle diagonal.

$\therefore$ Atleast one of α, β and γ should be non-zero.

(i) $\alpha = 0$ and $\beta = \gamma \neq 0 \rightarrow$ Correct

(ii) $\alpha = \beta = \gamma = 0 \rightarrow$ Wrong

(iii) $\beta = \gamma = 0$ and $\alpha \neq 0 \rightarrow$ Correct

(iv) $\alpha = \beta = \gamma \neq 0 \rightarrow$ Correct.

32. Option (b) is correct.

$$\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty}\frac{1}{n^p}$$

Using P series test : $P = \frac{1}{2} < 1$

$\therefore$ It is divergent series.

or $\sqrt{n} < n \quad \forall \quad n > 0$

$$\therefore \frac{1}{\sqrt{n}} > \frac{1}{n}$$

$$\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}} > \sum_{n=1}^{\infty}\frac{1}{n}$$

Now, $\sum_{n=1}^{\infty}\frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots$ upto ∞

It's Harmonic sequence and it is divergent series, therefore $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ is also divergent.

(ii) $$\sum_{n=1}^{\infty}U_n = \sum_{n=1}^{\infty}\frac{1}{n(n+1)}$$

$$= \sum_{n=1}^{\infty}\frac{n+1-n}{n(n+1)}$$

$$= \sum_{n=1}^{\infty}\left\{\frac{n+1}{n(n+1)} - \frac{n}{n(n+1)}\right\}$$

$$\sum_{n=1}^{\infty}U_n = \sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$U_n = \frac{1}{n} - \frac{1}{n+1}$$

$$U_1 = 1 - \frac{1}{2}$$

$$U_2 = \frac{1}{2} - \frac{1}{3}$$

$$U_3 = \frac{1}{3} - \frac{1}{4}$$

$$U_n = \frac{1}{n} - \frac{1}{n+1}$$

$$\therefore U_1 + U_2 + \ldots U_n = 1 - \frac{1}{n+1}$$

Hence, $$\sum_{n=1}^{\infty} U_n = \lim_{n\to\infty}\left(1 - \frac{1}{n+1}\right)$$
$$= 1$$

$\therefore$ It is convergent.

(iii) $\sum_{n=1}^{\infty} \frac{1}{n!}$, Let $U_n = \frac{1}{n!}$

By ratio test:

$$\lim_{n\to\infty} \frac{U_n}{U_{n+1}}$$
$$= \lim_{n\to\infty} \frac{1/n!}{1/(n+1)!}$$
$$= \lim_{n\to\infty} (n+1) \to \infty > 1$$

$\therefore$ It is Convergent.

33. Option (a) is correct.

$$\begin{matrix} 2R \\ 2B \end{matrix} \underline{draw}\ 1R \,\&\, 1B$$

Let E = Draw one red and one blue ball from pot.

$$P(E) = \frac{n(E)}{n(S)}$$
$$= \frac{{}^2C_1 \cdot {}^2C_1}{{}^4C_2}$$
$$= \frac{2\times 2}{6}$$
$$= \frac{2}{3}$$

34. Option (a) is correct.

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

$$|F(\omega)| = \left|\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt\right|$$

$$|F(\omega)| \le \int_{-\infty}^{\infty} \left|f(t)e^{-j\omega t}\right|dt$$

$$\le \int_{-\infty}^{\infty} |f(t)|\cdot\left|e^{-j\omega t}\right|dt$$

$$|F(\omega)| \le \int_{-\infty}^{\infty} |f(t)|dt$$

35. Option (a, b) are correct.

$$f(x) = 2x^3 - 3x^2 - 12x + 1$$
$$f'(x) = 6x^2 - 6x - 12$$

To get critical point we have to solve $f'(x) = 0$.

$$6x^2 - 6x - 12 = 0$$
$$6(x^2 - x - 2) = 0$$
$$6(x-2)(x+1) = 0$$
$$x = 2$$
$$x = -1$$
$$f''(x) = 12x - 6$$
$$f''(2) = 12 \times 2 - 6$$
$$= 18 > 0$$

$\therefore$ $x = 2$ is point of local minima

$$f''(-1) = 12 \times (-1) - 6$$
$$= -18 < 0$$

$\therefore$ $x = -1$ is point of local maxima

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} (2x^3 - 3x^2 - 12x + 1)$$
$$= \infty$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} (2x^3 - 3x^2 - 12x + 1)$$
$$= -\infty$$

Function is unbounded above and below,
So, function has neither global maxima nor global minima.

36. Correct answer is [0.25].

$$t^2y''(t) - 2ty'(t) + 2y(t) = 0 \quad \ldots\text{(i)}$$

Cauchy's Euler differential equation,

$$x = e^t$$
$$t = e^u$$
$$\ln t = u$$

$\therefore$ From equation (i) we get,

$$D(D-1)y - 2Dy + 2y = 0$$
$$(D^2 - D - 2D + 2)y = 0$$
$$(D^2 - 3D + 2)y = 0$$

$\therefore$ Auxilary equation is,

$m^2 - 3m + 2 = 0$

$m = 1, 2$

$$y = C_1e^u + C_2e^{2u}$$
$$y = C_1t + C_2t^2$$
$$\frac{dy}{dt} = C_1 + 2tC_2$$

at $t = 0$, $\frac{dy}{dt} = 1$

$\therefore$ $1 = C_1 + 0$

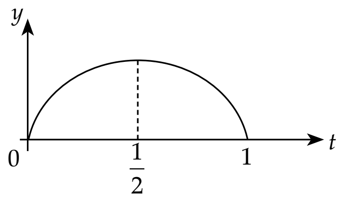
$C_1 = 1$

at $t = 1$, $\frac{dy}{dt} = -1$

$\therefore$ $-1 = 1 + 2C_2$

$C_2 = -1$

$\Rightarrow$ $y = t - t^2$



$$\frac{dy}{dt} = 1 - 2t$$

If $$\frac{dy}{dt} = 0 \Rightarrow t = \frac{1}{2}$$

$$\frac{d^2y}{dt^2} = -2 < 0$$

$\therefore t = \frac{1}{2}$ is point of local maxima.

$$y\left(\frac{1}{2}\right) = \frac{1}{2} - \left(\frac{1}{2}\right)^2 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

37. Correct answer is [3].

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}_{\substack{3\times 6 \\ K\times n}}$$

$K = 3, n = 6$

Method 1:

Since, $K = 3, 2^K = 8$ distinct message blocks possible i.e., 000 to 111.

Find corresponding codewords $[c] = [d] \,.\, [G]$.

Find Hamming weight of each of the codewords.

Smallest possible Hamming weight of non zero codeword equals to d_{min}.

$\therefore \quad d_{min} = 3.$

Method 2:

Minimum number of columns of G that sum to zero equals to d_{min}.

Sum of 1st, 2nd and 6th columns (or) 4th, 5th and 6th columns (or) 2nd, 3rd and 5th columns equals to zero.

Minimum number of columns of G matrix that sum to zero equals to 3.

$\therefore \quad d_{min} = 3.$

38. Option (a) is correct.

$p(s) = s^5 + 7s^4 + 3s^3 - 33s^2 + 2s - 40$

$p(i) = i^5 + 7(i)^4 + 3(i)^3 - 33(i)^2 + 2i - 40$

$p(i) = i + 7 - 3i + 33 + 2i - 40$

$= 0 \qquad (i^2 = -1, i^4 = 1, i^3 = -i)$

$\therefore$ 'i' is a root of $p(s) = 0$

$p(i) = (-i)^5 + 7(-i)^4 + 3(-i)^3 - 33(-i)^2 - 2i - 40$

$= -i + 7 + 3i - 33 - 2i - 40$

$= 0$

$\therefore$ '$-i$' is a root of $p(s) = 0$

$p(s) = s^5 + 7s^4 + 3s^3 - 33s^2 + 2s - 40$

$= (s^2 + 1)(s^3 + 7s^2 + 2s - 40)$

$= (s^2 + 1)(s - 2)(s^2 + 9s + 20)$

$\therefore$ roots of $p(s) = 0$,

$s = \pm i, 2, -4, -5$

Type of root	Roots	Count
Purely Imaginary	$+i$	2
Positive Real Part	2	1
Negative Real Part	$-4, -5$	2

39. Option (a) is correct.

$f(x) \geq 0$ given in $[2, 8] = [a, b]$

We know that,

$$m(b-a) \leq \int_a^b f(x)dx \leq M(b-a) \qquad \text{...(i)}$$

m = minimum value of $f(x)$

$\forall\, x \in$ [a, b]

m = maximum value of $f(x)$

$\forall\, x \in [a, b]$

from equation (i) we get

$$m(8-2) \leq \int_2^8 f(x)dx \leq M(8-2)$$

$$6m \leq \int_2^8 f(x)dx \leq 6M$$

$\because \quad f(x) \geq 0$

$\Rightarrow m$ and $M \geq 0$

$$\beta \leq 6m \leq \int_2^8 f(x)dx \leq 6M \leq \alpha$$

$\therefore \beta = 5m, \alpha = 7M$

40. Option (b, c) are correct.

$X \in \{-1, 0, 1\}$

$P(X = -1) = P(X = 1) = \alpha$

$P(X = 0) = 1 - 2\alpha$ where $0 < \alpha < \frac{1}{2}$

Entropy of $X = g(\alpha)$

$\therefore g(\alpha) = -\{\alpha \log_2 \alpha + \alpha \log_2 \alpha + (1 - 2\alpha) \log_2 (1 - 2\alpha)\}$

$g(\alpha) = -\{2\alpha \log_2 \alpha + (1 - 2\alpha) \log_2 (1 - 2\alpha)\}$

$g(0.25) = -\{0.5 \log_2 0.25 + 0.5 \log_2 0.5\} = 1.5$

$g(0.3) = -\{0.6 \log_2 0.3 + 0.4 \log_2 0.4\} = 1.57$

$g(0.4) = -\{0.8 \log_2 0.4 + 0.2 \log_2 0.2\} = 1.52$

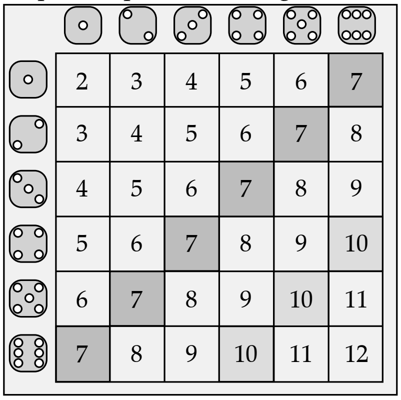
$g(0.3) > g(0.4) > g(0.25)$

$\therefore g(0.3) > g(0.4)$

and $g(0.3) > g(0.25)$

41. Correct answer is [7].

Sample space of pair of dice is given as,



x	2	3	4	5	6	7	8	9	10	11	12
$P(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$$= 2\times\frac{1}{36}+3\times\frac{2}{36}+4\times\frac{3}{36}+5\times\frac{4}{36}+6\times\frac{5}{36}+7\times\frac{6}{36}$$

$$+8\times\frac{5}{36}+9\times\frac{4}{36}+10\times\frac{3}{36}+11\times\frac{2}{36}+12\times\frac{1}{36}$$

$$E(x) = \frac{252}{36}$$

$$= 7$$

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42. Option (d) is correct.

Given eigen value of matrix A is λ.

$$\therefore \quad |A-\lambda I| = 0$$

$$B = A-2I$$

$$B-\lambda I = A-\lambda I-2I$$

$$B-\lambda I+2I = A-\lambda I$$

$$|B-\lambda I+2I| = |A-\lambda I|$$

$$\therefore |B-(\lambda-2)I| = 0$$

$\therefore$ eigenvalue of matrix B is $\lambda-2$.

43. Option (b) is correct.

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 3 \\ -2 & -3 \end{bmatrix}$$

$$b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$Ax = b$$

$$[A : B] = \begin{bmatrix} 1 & 1 & b_1 \\ 1 & 3 & b_2 \\ -2 & -3 & b_3 \end{bmatrix}$$

Using row operation we can covert matrix into echelon form.

$$\begin{bmatrix} 1 & 1 & b_1 \\ 1 & 3 & b_2 \\ -2 & -3 & b_3 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

and $R_3 \to R_3 + 2R_1$

$$= \begin{bmatrix} 1 & 1 & b_1 \\ 0 & 2 & b_2-b_1 \\ 0 & -1 & b_3+2b_1 \end{bmatrix}$$

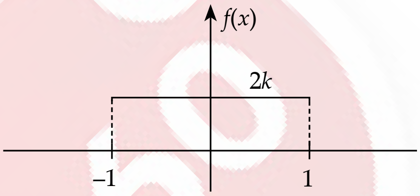
$$R_3 \to R_3 + \frac{R_2}{2}$$

$$= \begin{bmatrix} 1 & 1 & b_1 \\ 0 & 2 & b_2-b_1 \\ 0 & 0 & \frac{3b_1+b_2+2b_3}{2} \end{bmatrix}$$

$$\Rightarrow 3b_1 + b_2 + 2b_3 = 0$$

44. Option (c) is correct.

$$f(x) = \begin{cases} 0 & \text{if } -2<x<-1 \\ 2k & \text{if } -1<x<1;\ \text{period} = 4 \\ 0 & \text{if } 1<x<2 \end{cases}$$



$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)dx$$

$$= \frac{1}{2}\int_{-2}^{2} f(x)dx$$

$$= \frac{1}{2}\times 2\int_{0}^{2} f(x)dx \qquad (\because f(x) \text{ is even function})$$

$$= \int_{0}^{1} f(x)dx + \int_{1}^{2} f(x)dx$$

$$= \int_{0}^{1} 2k\,dx + 0$$

$$= 2k$$

$f(x)$ is even function, therefore $b_n = 0$

$L = 2$

$$a_n = \frac{1}{L}\int_{-L}^{L} \cos\left(\frac{n\pi x}{2}\right) f(x)dx$$

$$= \frac{1}{2}\int_{-2}^{2} \cos\left(\frac{n\pi x}{2}\right)\cdot f(x)dx$$

$$= \frac{1}{2}\int_{-2}^{-1} 0\,dx + \frac{1}{2}\int_{-1}^{1} \cos\left(\frac{n\pi x}{2}\right)2k dx + \frac{1}{2}\int_{1}^{2} 0\,dx$$

$$= K\int_{-1}^{1} \cos\frac{n\pi x}{2}\,dx$$

$$= 2K\int_0^1 \cos\left(\frac{n\pi x}{2}\right)dx$$

$$a_n = 2K\left(\frac{\sin\frac{n\pi x}{2}}{\frac{n\pi}{2}}\right)_0^1$$

$$a_n = \frac{4k}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

Now, $$f(x) = \frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{n\pi x}{L}\right)$$

$$= \frac{2k}{2}+\sum_{n=1}^{\infty}a_n\cos\left(\frac{n\pi x}{L}\right)$$

$$= k+\frac{4k}{\pi}\cos\frac{\pi x}{2}+\frac{4k}{2\pi}\times 0+\frac{4k}{3\pi}(-1)\cos\left(\frac{3\pi x}{2}\right)+ 0+\frac{4k}{5\pi}\times\cos\frac{5\pi x}{2}+\ldots$$

$$f(x) = k+\frac{4k}{\pi}\left[\cos\left(\frac{\pi x}{2}\right)-\frac{1}{3}\cos\frac{3\pi k}{2}+\frac{1}{5}\cos\frac{5\pi x}{2}-\ldots\right]$$

45. Option (a) is correct.

$P(X = 0) = 0.4$; $P(X = 1) = 0.3$; $P(X = 7) = 0.1$;
$P(X = 11) = 0.1$; $P(X = 12) = 0.1$

X	0	1	7	11	12
$P(X)$	0.4	0.3	0.1	0.1	0.1

$E(X) = \sum Xp(X)$
$E(X) = 0 \times 0.4 + 1 \times 0.3 + 7 \times 0.1 + 11 \times 0.1 + 12 \times 0.1 = 3.3$
$E(X^2) = \sum X^2p(X)$
$E(X^2) = 0^2 \times 0.4 + 1^2 \times 0.3 + 7^2 \times 0.1 + 11^2 \times 0.1 + 12^2 \times 0.1$
$= 0 + 0.3 + 4.9 + 12.1 + 14.4$
$= 31.7$
$\text{Var}(X) = E(X^2) - (E(X))^2$
$= 31.7 - (3.3)^2$
$= 31.7 - 10.89$
$= 20.81$

46. Option (b) is correct.

Option(a)

$\frac{\partial^2 u}{\partial t^2}=c^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+xy$ is not linear homogenous equation.

Option (b) is second order linear homogenous equation.

Option (c) is first order equation.

Option (d) is not linear equation.

47. Option (b) is correct.

$$= \lim_{x\to\infty}\left(x-\sqrt{x^2+x}\right) \qquad (\infty-\infty) \text{ form}$$

We can multiply and divide by $x+\sqrt{x^2+x}$.

$$= \lim_{x\to\infty}\left(x-\sqrt{x^2+x}\right)\times\frac{\left(x+\sqrt{x^2+x}\right)}{\left(x+\sqrt{x^2+x}\right)}$$

$$= \lim_{x\to\infty}\frac{x^2-(x^2+x)}{x+\left(\sqrt{x^2+x}\right)}$$

$$= \lim_{x\to\infty}\frac{-x}{x+\sqrt{x^2+x}}$$

Let $x = \frac{1}{t}$

as $x\to\infty \Rightarrow t\to 0$

$$= \lim_{t\to 0}\frac{-\frac{1}{t}}{\frac{1}{t}+\sqrt{\frac{t}{t^2}+\frac{1}{t}}}$$

$$= \lim_{t\to 0}\frac{-1}{1+\sqrt{1+t}}$$

$$= \frac{-1}{1+\sqrt{1}}$$

$$= -\frac{1}{2}$$

48. Correct answer is [5.8].

$y'' + 0.8y' + 0.16y = 0$
$(D^2 + 0.8D + 0.16)y = 0$

Auxiliary equation is,

$m^2 + 0.8m + 0.16 = 0$
$(m + 0.4)^2 = 0$
$m = -0.4, -0.4$

Roots are identical therefore,

$y = (c_1 + c_2x)e^{-0.4x}$

Replacing, $x = 0$, $y = 3$

$3 = (c_1 + 0)e^0$
$c_1 = 3$
$y' = (c_1 + c_2x)e^{-0.4x} \times (-0.4) + e^{0.4x} \times c_2$

Replacing $x = 0$, $y' = 4.5$

$4.5 = (3 + 0)\, e^0 \times (-0.4) + c_2$
$4.5 = -1.2 + c_2$
$c_2 = 5.7$

Now, $x = 1$

$y = (3 + 5.7)e^{-0.04}$
$= 5.83$
$= 5.8$

49. Correct answer is [3].

$f(x) = -x^3 + 2x^2$
$f'(x) = -3x^2 + 4x$

To get critical point, $f'(x) = 0$

$-3x^2 + 4x = 0$

$$x = 0, \frac{4}{3}$$

$f''(x) = -6x + 4$

$$f''(x)|_{x=0} = -6 \times 0 + 4 = 4 > 0$$

$$\Rightarrow x = 0 \text{ point of minima}$$

$$f''(x)\Big|_{x=\frac{4}{3}} = -6\times\frac{4}{3}+4 = -4 < 0$$

$$\Rightarrow x = \frac{4}{3} \text{ is point of maxima}$$

$$f(x) = -x^3 + 2x^2$$

$$f(0) = 0$$

$$f(-1) = -(-1)^3 + 2(-1)^2$$

$$= 3$$

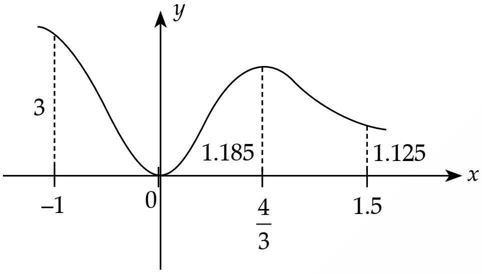
$$f\left(\frac{4}{3}\right) = -\left(\frac{4}{3}\right)^3+2\left(\frac{4}{3}\right)^2$$

$$= \frac{-64}{27}+\frac{32}{9}=1.185$$

$$f(1.5) = -(1.5)^3 + 2(1.5)^2$$

$$= 1.125$$

$\therefore$ Maxima value is 3.



CIVIL ENGINEERING (CE-2)

50. Correct answer is [2.6].

$$\frac{dy}{dx} = y + 2x - x^2$$

$$f(x, y) = \frac{dy}{dx}$$

$$f(x, y) = y + 2x - x^2$$

$$y(0) = 1$$

$$x_0 = 0, y_0 = 1$$

$$x_1 = 0.5$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$= y_0 + h(y_0, + 2x_0 - x_0^2)$$

$$= 1 + 0.5(1 + 0) = 1.5$$

$$x_2 = 1$$

$$y_2 = y_1 + h(f(x_1, y_1))$$

$$= y_1 + h(y_1, + 2x_1 - x_1^2)$$

$$= 1.5 + 0.5(1.5 + 2 \times 0.5 - 0.5^2)$$

$$y_2 = 2.625$$

51. Option (a) is correct.

$$[A] = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 4 & 5 \\ 4 & 3 & 2 \end{bmatrix}$$

To find transpose of a matrix, we have to interchange Row and Column.

$$A^T = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 4 & 3 \\ 4 & 5 & 2 \end{bmatrix}$$

52. Option (a) is correct.

$$I = \int \ln x \, dx$$

We have to apply integration by parts.

$$\int f(x)\, g(x)\, dx = f(x) \int g(x)\, dx - \int (f'(x) \int g(x)\, d(x)\, dx$$

$$\therefore \quad I = \int 1 \cdot \ln x \, dx$$

$$f(x) = \ln x, g(x) = 1$$

$$f'(x) = \frac{1}{x}, \int g(x)\, dx = x$$

$$I = x \ln x - \int \frac{1}{x} \cdot x \, dx$$

$$I = x \ln x - \int dx$$

$$I = x \ln x - x + c$$

c = Integration constant

53. Options (a, c) are correct.

$$\vec{V} = u\hat{x} + v\hat{y} = u\hat{i} + v\hat{j}$$

$$\text{Div } \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

$$\nabla.\vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

$$\text{curl } \vec{V} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & 0 \end{vmatrix}$$

$$\nabla \times \vec{V} = \hat{i}\left(0-\frac{\partial v}{\partial z}\right)-\hat{j}\left(0-\frac{\partial u}{\partial z}\right)+\hat{k}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$$

$$z - \text{component of curl } \vec{V} = \left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)\hat{k}$$

54. Option (b, c) is correct.

A and B are null set

$P(A \cap B) = P(A) \,.\, P(B)$ of A and B are independent, so option (a) is wrong.

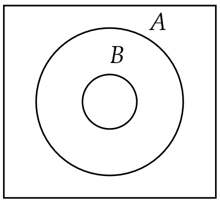
If B is subset of A, $(B \subset A)$

then, $P(A \cap B) = P(B)$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(B)}{P(B)}$$

$$P(A/B) = 1$$



So, option (b) is correct.

If A and B are mutually exclusive, then

$$P(A \cap B) = 0$$

$$P(A \cap B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B)$$

Hence, option (c) is correct.

If A and B are independent then,

$P(A \cap B) = P(A) \cdot P(B)$

So, option (d) is wrong.

55. Correct answer is [3].

$$\frac{d^3y}{dx^3} + \left(\frac{d^2y}{dx^2}\right)^6 + \left(\frac{dy}{dx}\right)^4 + y = 0$$

Order of differential equation is the highest order derivative term and power of the highest order term and power of the highest order derivative term is degree.

$\therefore$ Order = 3

Degree = 1

56. Option (a) is correct.

$$\frac{dy}{dx} = e^{x-y}$$

$$\frac{dy}{dx} = e^x \cdot e^{-y}$$

$$e^y dy = e^x dx$$

$$\int e^y dy = \int e^x dx$$

$$e^y = e^x + c$$

taking ln of both side

$$\ln(e^y) = \ln(e^x + c)$$

$$y = \ln(e^x + c)$$

C is integration constant.

57. Options (a, d) are correct.

$$f(x) = x^3 - \frac{15}{2}x^2 + 18x + 20$$

$$f'(x) = 3x^2 - 15x + 18$$

To get critical points, we have to solve

$$f'(x) = 0$$

$$\therefore\ 3x^2 - 15x + 18 = 0$$

$$x = 2, 3$$

$$f''(x) = 6x - 15$$

$$f''(2) = 6 \times 2 - 15 = -3 < 0$$

$\therefore$ $x = 2$ is point of local maxima

$$f''(3) = 6 \times 3 - 15 = 3 > 0$$

$x = 3$ is point of local minima

58. Options (a, c) are correct.

$$A = \begin{bmatrix} 6 & 8 \\ 4 & 2 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 6 & 8 \\ 4 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 6 & 8 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 6-\lambda & 8 \\ 4 & 2-\lambda \end{bmatrix}$$

To get eigenvalues, we have to solve

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 6-\lambda & 8 \\ 4 & 2-\lambda \end{vmatrix} = 0$$

$$(6-\lambda)(2-\lambda) - 32 = 0$$

$$\lambda^2 - 8\lambda - 20 = 0$$

$$\lambda^2 - 10\lambda + 2\lambda - 20 = 0$$

$$\lambda(\lambda - 10) + 2(\lambda - 10) = 0$$

$$\lambda = -2, 10$$

59. Correct answer is [2.8].

x_i	1	2	3	4
$P(X = x_i)$	0.3	0.1	0.3	0.3

$$E(x) = \sum x\, P(x) = 1 \times 0.3 + 2 \times 0.1 + 4 \times 0.3 + 8 \times 0.3 = 4.1$$

$$E(x^2) = \sum x^2\, P(x) = 1^2 \times 0.3 + 2^2 \times 0.1 + 4^2 \times 0.3 + 8^2 \times 0.3 = 24.7$$

$$V(x) = E(x^2) - (E(x))^2 = (24.7) - (4.1)^2 = 7.89$$

Standard deviation, $\sigma = \sqrt{7.89} = 2.808$

PRODUCTION AND INDUSTRIAL ENGINEERING (PI)

60. Option (b) is correct.

Let p = proportion of idle time $= \frac{x}{n}$

$$\therefore\quad \text{Standard deviation} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{x}{n}\left(1 - \frac{x}{n}\right) \cdot \frac{1}{n}} = \sqrt{\frac{x(n-x)}{n^3}}$$

61. Option (c) is correct.

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt = F(s) \quad \text{...(i)}$$

$$L\{e^{bt} f(t)\} = \int_0^\infty e^{-st} e^{bt} f(t)\, dt = \int_0^\infty e^{-(s-b)t} f(t)\, dt$$

Let, $s - b = \lambda$

$$\therefore\quad L\{e^{bt} f(t)\} = \int_0^\infty e^{-\lambda t} f(t)\, dt$$

$$= F(\lambda)$$

$$\Rightarrow \quad L\{e^{bt} f(t)\} = F(s-b)$$

62. Option (c) is correct.

Let $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -\lambda & -1 \\ 1 & -\lambda \end{bmatrix}$$

$$|A - \lambda I| = \begin{bmatrix} -\lambda & -1 \\ 1 & -\lambda \end{bmatrix}$$

If, $|A - \lambda I| = 0$

$$\Rightarrow \begin{bmatrix} -\lambda & -1 \\ 1 & -\lambda \end{bmatrix} = 0$$

$$\lambda^2 + 1 = 0$$

$$\lambda = \pm\sqrt{-1}$$

$\therefore$ eigen values $= \sqrt{-1}, -\sqrt{-1}$

63. Option (a) is correct.

5 Red
7 Green
3 Blue

P (first ball Red and second ball Red) $= \dfrac{5}{15} \times \dfrac{4}{14}$

$$= \frac{2}{21}$$

P (first ball Green and second ball Red) $= \dfrac{7}{15} \times \dfrac{5}{14}$

$$= \frac{1}{6}$$

P (first ball blue and second ball Red $= \dfrac{3}{15} \times \dfrac{5}{14}$

$$= \frac{1}{14}$$

Hence, required probability $= \dfrac{2}{21} + \dfrac{1}{6} + \dfrac{1}{14}$

$$= \frac{4+7+3}{42}$$

$$= \frac{14}{42}$$

$$= \frac{1}{3}$$

64. Option (b) is correct.

Curl of vector $= \nabla \times F$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

$$\nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ -2y & x & 0 \end{vmatrix}$$

$$= \hat{i}\left(0 - \frac{\partial x}{\partial z}\right) - \hat{j}\left(0 - \frac{\partial(-2y)}{\partial z}\right)$$

$$+\hat{k}\left(\frac{\partial x}{\partial x} - \frac{\partial}{\partial y}(-2y)\right)$$

$$= \hat{k}(1+2)$$

$$= 3\hat{k}$$

65. Option (d) is correct.

Linear differential equation is,

$$\frac{dy}{dx} + y \cdot P(x) = Q(x)$$

(a) $\left(\dfrac{dy}{dx}\right) + 2x = y \rightarrow$ 2^{nd} degree differential equation, hence, non-linear.

(b) $\dfrac{dy}{dx} + 2x = y^2 \rightarrow$ Quadratic with respect to y, hence, non-linear.

(c) $x\dfrac{d^2y}{dx^2} + 2y\dfrac{dy}{dx} = 0 \rightarrow y$ and $\dfrac{dy}{dx}$ are in product form, hence, non-linear.

(d) $x^3 \dfrac{dy}{dx} + xy = x^2$

$$\frac{dy}{dx} + y\left(\frac{1}{x^2}\right) = \frac{1}{x}$$

here, $P(x) = \dfrac{1}{x^2}$, $Q(x) = \dfrac{1}{x}$

Hence, it's a linear differential equation.

66. Option (a) is correct.

If $f(z) = u + iv$ is analytic.

then $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$ and $\dfrac{\partial u}{\partial y} = \dfrac{-\partial v}{\partial x}$

(a) $e^x(\cos y + i \sin y)$

$u = e^x \cos y$ and $v = e^x \sin y$

$$\frac{\partial u}{vx} = e^x \cos y$$

$$\frac{\partial u}{\partial y} = -e^x \sin y$$

$$\frac{\partial v}{\partial x} = e^x \sin y$$

$$\frac{\partial v}{\partial y} = e^x \cos y$$

$\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$ and $\dfrac{\partial u}{\partial y} = \dfrac{-\partial v}{\partial x}$

Hence, analytic.

(b) $e^x(-\cos y + i\sin y)$

$\therefore\ u = -e^x \cos y, v = e^x \text{ in } y$

$$\frac{\partial u}{\partial x} = -e^x \cos y$$

$$\frac{\partial u}{\partial y} = e^x \sin y$$

$$\frac{\partial v}{\partial x} = e^x \sin y$$

$$\frac{\partial v}{\partial y} = e^x \cos y$$

$$\frac{\partial u}{\partial y} \neq \frac{\partial v}{\partial y}$$

Hence, not analytic.

(c) $e^x(\cos y - i\sin y)$

$\therefore\ u = e^x \cos y, v = -e^x \sin y$

$$\frac{\partial u}{\partial x} = e^x \cos y$$

$$\frac{\partial u}{\partial y} = -e^x \sin y$$

$$\frac{\partial v}{\partial x} = -e^x \sin y$$

$$\frac{\partial v}{\partial y} = -e^x \cos y$$

$$\frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y}$$

Hence, not analytic.

(d) $e^{-x}(\cos y + i\sin y)$

$\therefore\ u = e^{-x} \cos y, v = e^{-x} \sin y$

$$\frac{\partial u}{\partial x} = -e^{-x} \cos y$$

$$\frac{\partial u}{\partial y} = -e^{-x} \sin y$$

$$\frac{\partial v}{\partial x} = -e^{-x} \sin y$$

$$\frac{\partial v}{\partial y} = e^{-x} \cos y$$

$$\frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y}$$

Hence, not analytic.

67. Option (d) is correct.

$$\frac{dy}{dx} + y = e^x$$

$$\text{I.F.} = e^{\int 1.dx} = e^x$$

$$\therefore\ y.e^x = \int e^x \cdot e^x\, dx$$

$$y\,e^x = \frac{e^{2x}}{2} + c$$

$$y(0) = 0$$

$$\therefore\ 0 = \frac{1}{2} + c$$

$$c = -\frac{1}{2}$$

$$\Rightarrow\ y\,e^x = \frac{e^{2x}}{2} - \frac{1}{2}$$

$$\therefore\ y = \frac{e^x - e^{-x}}{2}$$

68. Correct answer is [2].

$$f(x, y, z) = x^2 + 2y^2 + z$$

$$\nabla f(x, y, z) = 2x\hat{i} + 4y\hat{j} + \hat{k}$$

$$\nabla f(x, y, z) \text{ at } (1, 1, 1) = 2\hat{i} + 4\hat{j} + \hat{k}$$

$$= 3\hat{i} + 4\hat{k}$$

$$\therefore\ \hat{n} = \frac{3\hat{i} + 4\hat{k}}{\sqrt{9+16}} = \frac{3\hat{i} + 4\hat{k}}{5}$$

$$\hat{n} \cdot \nabla f(x, y, z) = \left(\frac{3\hat{i} + 4\hat{k}}{5}\right) \cdot \left(2\hat{i} + 4\hat{j} + \hat{k}\right) = \frac{6+4}{5} = 2$$

69. Correct answer is [2].

Simpson's $\frac{1}{3}$ rule,

$$\int_a^b f(x)\,dx = \frac{h}{3}\{f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\}$$

$$h = \frac{b-a}{4}$$

$$\therefore\ f(x) = x^2 - 2x$$

$$h = \frac{3-1}{4} = \frac{1}{2}$$

$$f(x_0) = (1^2 - 2) = -1$$

$$f(x_1) = ((1.5)^2 - 2 \times 1.5) = 2.25 - 3 = -0.75$$

$$f(x_2) = (2^2 - 2 \times 2) = 0$$

$$f(x_3) = ((2.5)^2 - 2 \times 2.5) = 6.25 - 5 = 1.25$$

$$f(x_4) = (3^2 - 2 \times 3) = 9 - 6 = 3$$

$$\int_1^3 (x^2 - 2x)\,dx = \frac{1}{6}\{-1 + 4(-0.75) + 0 + 4(1.25) + 3\}$$

$$= \frac{1}{6}\{-1 - 3 + 5 + 3\} = \frac{4}{6} = \frac{2}{3}$$

$$\therefore\ n = 2$$