JEE Advanced

(2023)

PAPER 1

ANSWERS WITH EX

Mathe

1. Correct options are (A, C and D).

Given $S = (0, 1) \cup (1, 2) \cup (3, 4)$

$T = \{0, 1, 2, 3\}$

$\because$ For function $S \rightarrow T$, set S (domai
elements but set T (codomain) has

$\therefore$ There are infinite functior
is impossible to make a fu
increasing from S to T.

$\therefore$ All functions mu

$\therefore$ Option (A) is

and option (B

Accordin

contin

Tot

ible)

$= -x - 3$

is (–3, 0).

(Chord of contact for ellipse)

$= -2$

$A_1 = (-2, 1)$

$A_4 = (-2, -1)$

quation of $A_2 A_3$

$T = 0$ (Chord of contact for parabola)

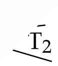
$$y(0) = 12\left(\frac{x-3}{2}\right)$$

$\Rightarrow$ $x = 3$

$\Rightarrow$ $A_2 = (3, 6)$ and $A_3 = (3, -6)$

$\therefore$ Area of quadrilateral $A_1 A_2 A_3 A_4$

$$= \frac{1}{2} \times (2 + 12) \times 5 = 35 \text{ sq. units}$$



Equation of tange

$y = mx \pm \sqrt{\ldots + 3}$ ...(1)

Equation of tangent for parabola

$$y = mx + \frac{a}{m}$$

$\Rightarrow$ $y = mx + \dfrac{3}{m}$ ...(2)

3. Correct options are (B, C and D).

Given $f : [0, 1] \rightarrow [0, 1]$

$$f(x) = \frac{x^3}{3} - x^2 + \frac{5}{9}x + \frac{17}{36}$$

$$f'(x) = \frac{3x^2}{3} - 2x + \frac{5}{9}$$

OSWAAL 360

$$f'(x) = 0$$

$$9x^2 - 18x + 5 = 0$$

$$\Rightarrow 9x^2 - 15x - 3x + 5 = 0$$

$$\Rightarrow 3x(3x - 5) - 1(3x - 5) = 0$$

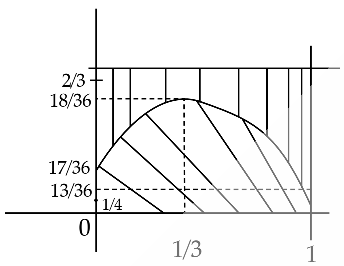
$$\Rightarrow (3x - 5)(3x - 1) = 0$$

$$\Rightarrow \quad x = \frac{1}{3} \text{ or } \frac{5}{3}$$

$$f''(x) = 2x - 2$$

$$f''\left(\frac{1}{3}\right) = \frac{2}{3} - 2 < 0 \text{ point of maxima}$$

Graph of $f(x)$



$$\text{Area}_{\text{red}} = \int_0^1 f(x)dx$$

$$= \left[\frac{x^4}{12} - \frac{x^3}{3} + \frac{5x^2}{18} + \frac{17x}{36}\right]_0^1$$

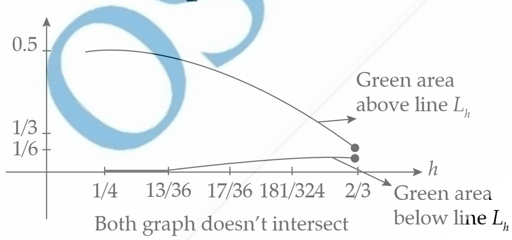
$$= \frac{1}{12} - \frac{1}{3} + \frac{5}{18} + \frac{17}{36}$$

$$= \frac{3 - 12 + 10 + 17}{36}$$

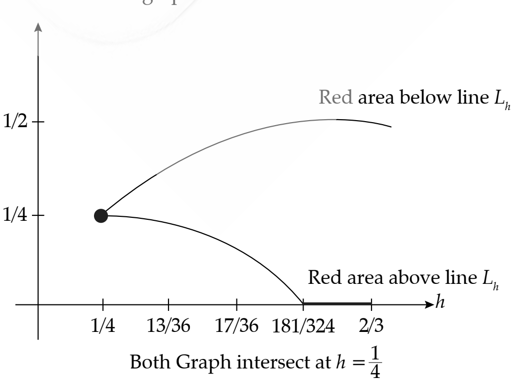
$$= \frac{18}{36} = \frac{1}{2} = 0.5$$

$$\therefore \quad (\text{Area})_{\text{green}} = 1 - \frac{1}{2} = 0.5$$

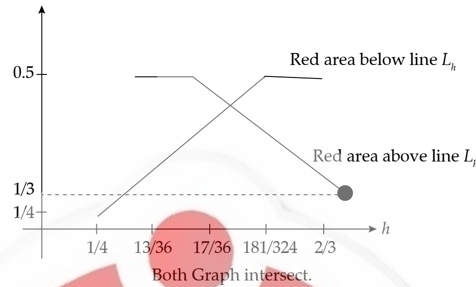
(A)



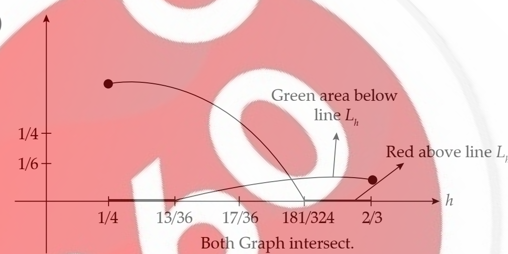
(B)



(C)



(D)



4. Correct option is (C).

$f : (0, 1) \to R,$

$$f(x) = \sqrt{n}, \ x \in \left[\frac{1}{n+1}, \frac{1}{n}\right), \ n \in N$$

$g : (0, 1) \to R$ where

$$\int_{x^2}^{x} \sqrt{\frac{1-t}{t}}\,dt < g(x) < 2\sqrt{x}, \ x \in (0, 1)$$

Now (According to the question)

$$\lim_{x \to \infty} f(x) \cdot g(x)$$

$$\Rightarrow \quad \text{Put} \quad x = \frac{1}{n}$$

$$\lim_{n \to \infty} f\left(\frac{1}{n}\right) g\left(\frac{1}{n}\right)$$

$$\lim_{n \to \infty} \sqrt{n-1} \int_{\frac{1}{n^2}}^{\frac{1}{n}} \sqrt{\frac{1-t}{t}}\,dt \le \lim_{n \to \infty} f\left(\frac{1}{n}\right) g\left(\frac{1}{n}\right)$$

$$\le \lim_{n \to \infty} \sqrt{n} - 1 \frac{2}{\sqrt{n}}$$

$$\Rightarrow \quad \lim_{n \to \infty} \frac{\int_{\frac{1}{n^2}}^{\frac{1}{n}} \sqrt{\frac{1-t}{t}}\,dt}{\frac{1}{\sqrt{n-1}}} \le \lim_{n \to \infty} f\left(\frac{1}{n}\right) g\left(\frac{1}{n}\right) \le 2$$

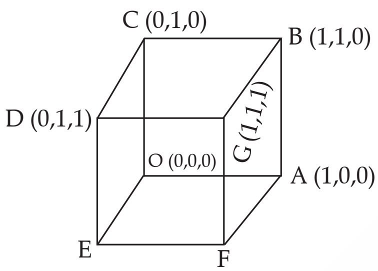
$$\Rightarrow \quad \frac{\lim\limits_{n \to \infty} \frac{-1}{n^2}\sqrt{n-1} + \frac{2}{n^3}\sqrt{n^2 - 1}}{\frac{1}{2(n-1)^{\frac{3}{2}}}}$$

$$\le \lim_{n \to \infty} f\left(\frac{1}{n}\right) g\left(\frac{1}{n}\right) \le 2$$

$$\therefore \quad \lim_{n\to\infty} \frac{2(n-1)^2}{n^2} - \frac{4(n-1)^{\frac{3}{2}}\sqrt{n^2-1}}{n^3} = 2$$

$$\therefore \lim_{n\to\infty} f\left(\frac{1}{n}\right)g\left(\frac{1}{n}\right) = 2 \text{ (Using Sandwich Theorem)}$$

5. Correct option is (A).



$$\overrightarrow{OG} = \hat{i} + \hat{j} + \hat{k} = \hat{b}_1$$

$$\overrightarrow{AC} = -\hat{i} + \hat{j} = \hat{b}_2$$

Equation of line OG

$$\Rightarrow \quad \frac{x}{1} = \frac{y}{1} = \frac{z}{1}$$

Equation of line AC

$$\Rightarrow \quad \frac{x-1}{-1} = \frac{y}{1} = \frac{z}{0}$$

$$\text{S.D.} = \frac{|(\bar{a}_2 - \bar{a}_1)\cdot(\bar{b}_1 \times \bar{b}_2)|}{|\bar{b}_1 \times \bar{b}_2|}$$

$$\bar{a}_2 - \bar{a}_1 = -\hat{i}$$

$$\bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ -1 & 1 & 0 \end{vmatrix}$$

$$= \hat{i}(-1) - \hat{j}(+1) + \hat{k}(1+1)$$

$$= -\hat{i} - \hat{j} + 2\hat{k}$$

$$\text{S.D.} = \frac{|(-\hat{i}).(-\hat{i} - \hat{j} + 2\hat{k})|}{|-\hat{i}||-\hat{i} - \hat{j} + 2\hat{k}|}$$

$$= \frac{1}{1\sqrt{1+1+4}} = \frac{1}{\sqrt{6}}$$

6. Correct option is (B).

$$\frac{x^2}{8} + \frac{y^2}{20} < 1 \text{ and } y^2 < 5x$$

Let $\dfrac{x^2}{8} + \dfrac{y^2}{20} = 1$ ...(1)

and $y^2 = 5x$ ...(2)

On solving (1) and (2), we get

$$\frac{x^2}{8} + \frac{5x}{20} = 1$$

$$\frac{x^2}{8} + \frac{x}{4} = 1$$

$$x^2 + 2x = 8$$

$$\Rightarrow \quad x^2 + 2x - 8 = 0$$

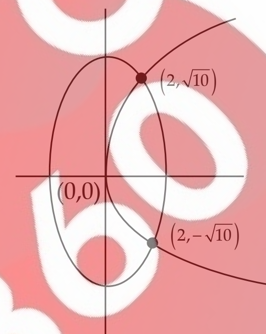
$$\Rightarrow \quad (x+4)(x-2) = 0$$

$$\Rightarrow \quad x = -4, 2$$

$$\Rightarrow \quad x = 2 \text{ (}-4\text{ is not possible)}$$

$$\Rightarrow \quad y^2 = 10$$

$$y = \pm\sqrt{10}$$



$X = \{(1, 1), (1, 0), (1, -1), (1, 2), (1, -2), (2, 1), (2 -1), (2, 3), (2, -3), (2, -2), (2, 2), (2, 0)\}$

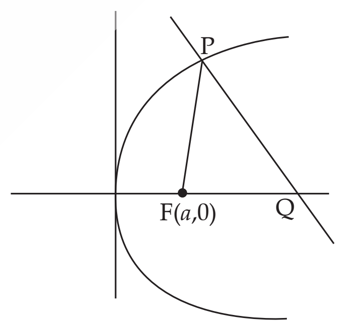
$$n(s) = {}^{12}C_3$$

A is even of selecting 3 points for which area of Δ is positive integer.

$$n(A) = 4 \times 7 + 9 \times 5 = 73$$

$$P(A) = \frac{73}{{}^{12}C_3} = \frac{73}{220}$$

7. Correct option is (A).



$$y^2 = 4ax$$

Equation of normal

$y = mx - 2am - am^3$

Point of contact

$P(am^2, -2am)$

and Point $Q\ (2a + am^2, 0)$

$$\text{Area of } \Delta\, PFQ = \frac{1}{2} \times |a + am^2|\ |-2am|$$

$$120 = a^2(1 + m^2)m \qquad \text{...(1)}$$

$$a = 2, m = 3$$

Satisfies the equation (1), hence (2, 3) will be the correct answer.

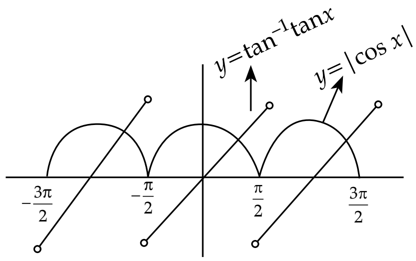
8. Correct answer is [3].

$$\sqrt{1 + \cos 2x} = \sqrt{2}\tan^{-1}(\tan x)$$

$\Rightarrow \sqrt{2\cos^2 x} = \sqrt{2}\ \tan^{-1} \tan x$

$\Rightarrow \sqrt{2}\ |\cos x| = \sqrt{2} \tan^{-1} \tan x$

$\Rightarrow \quad |\cos x| = \tan^{-1} \tan x$

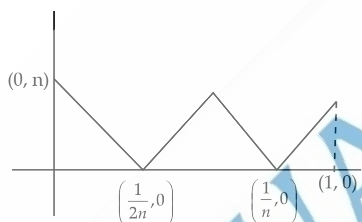


Number of solution = 3.

9. Correct option is [8].

$f: [0, 1] \rightarrow R$

$$f(x) = \begin{cases} n(1 - 2nx) & 0 \le x \le \frac{1}{2n} \\ 2n(2nx - 1) & \frac{1}{2n} \le x \le \frac{3}{4n} \\ 4n(1 - nx) & \frac{3}{4n} \le x \le \frac{1}{n} \\ \frac{n}{n-1}(nx - 1) & \frac{1}{n} \le x \le 1 \end{cases}$$



$$\text{Area} = \frac{1}{2} \times \frac{1}{2n} \times n + \frac{1}{2} \times \frac{1}{2n} \times n + \frac{1}{2} \times \left(1 - \frac{1}{n}\right) \times n$$

$$4 = \frac{1}{4} + \frac{1}{4} + \frac{n-1}{2}$$

$$4 = \frac{1}{2} + \frac{n-1}{2}$$

$$4 = \frac{n}{2}$$

$n = 8$

10. Correct answer is [1219].

$$S = 77 + 757 + 7557 + \ldots \overbrace{755 \ldots 57}^{98 \text{ times}}$$

$$S = 70 + 700 + 7000 + \ldots \overbrace{70000 \ldots 00}^{99 \text{ times}}$$

$$+ \underbrace{(50 + 550 + 5550 + \ldots.)}_{98 \text{times}}$$

Let T_r be the general term.

$$T_r = 7 \times 10^{r-1} + 5(10 + 100 + \ldots 10^{r-2}) + 7 \; r \ge 2$$

$$= 7 \times 10^{r-1} + 5\left[\frac{10(1 - 10^{r-2})}{1 - 10}\right] + 7$$

$$= 7 \times 10^{r-1} + \frac{50}{9}(10^{r-2} - 1) + 7$$

$$= 7 \times 10^{r-1} + \frac{50}{9}(10^{r-2}) - \frac{50}{9} + 7$$

$$= 7 \times 10^{r-1} + \frac{50}{9}10^{r-2} + \frac{13}{9}$$

$$S = \sum_{r=2}^{100} T_r = \sum_{r=2}^{100} 7 \times 10^{r-1} + \frac{50}{90} \times 10^{r-2} + \frac{13}{9}$$

$$= \frac{70}{9}(10^{99} - 1) + \frac{50}{81}(10^{99} - 1) \times 13 \times 11$$

$$\text{RHS} = \frac{\overbrace{7555 \ldots 57}^{99 \text{times}} + m}{n}$$

$$\frac{7 \times 10^{100} + \frac{50}{9}(10^{99}) + \frac{13}{9} + m}{n}$$

Now,

$$\frac{70}{9}(10^{99} - 1) + \frac{50}{81}(10^{99} - 1)\ 13 \times 11$$

$$= \frac{\frac{70}{9}10^{100} + \frac{50}{9} \times 10^{99} + \frac{13}{9} + m}{n}$$

$$= \frac{7}{n} + 10100 + \frac{50}{9n}1099 + \frac{13}{9n} + \frac{m}{n}$$

By Comparison,

$9 = n$ or $81 = 9n \Rightarrow n = 9$

$\therefore$ Put $n = 9$

$13 \times 11 \times 9^2 - 50 = 13 + 9m$

$m = 1210$

$\therefore \quad m + n = 1219$

11. Correct answer is [281].

$$A = \left\{\frac{1967 + 1686 i \sin\theta}{7 - 3i\cos\theta}, \theta \in R\right\}$$

$\because$ A contains exactly one positive integer n.

Now simplifying

$$Z = \frac{1967 + 4686 i \cos\theta}{7 - 3i\cos\theta}$$

$$= 281\frac{(7 + 6i\sin\theta)}{7 - 3i\cos\theta} \times \frac{7 + 3i\cos\theta}{7 + 3i\cos\theta}$$

$$= 281\frac{(49 - 9\sin 2\theta)}{49 + 9\cos^2\theta} + \frac{281\,(3)(2\sin\theta + \cos\theta)}{49 + 9\cos^2\theta}i$$

$$= 281\left(\frac{49 - 9\sin 2\theta}{49 + 9\cos^2\theta}\right) + 562\left(\frac{2\sin\theta + \cos\theta}{49 + 9\cos^2\theta}\right)i$$

For positive integer $Im(z) = 0$

We get, $2\sin\theta + \cos\theta = 0$

$$\tan\theta = \frac{-1}{2}$$

$$\Rightarrow \quad \cos^2\theta = \frac{4}{5}$$

$$\Rightarrow \quad \sin 2\theta = \frac{2+\tan\theta}{1+\tan^2\theta}$$

$$= \frac{-1}{1+\frac{1}{4}} = \frac{-4}{5}$$

$$\therefore \quad Z = 281\frac{\left(49 - 9\left(\frac{-4}{5}\right)\right)}{49 + 9\left(\frac{4}{5}\right)}$$

$$= 281$$

$$\therefore \quad n = 281$$

12. Correct answer is [45].

$P : \sqrt{3}x + 2y + 3z = 16$

$$S = \left\{\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k} : \alpha^2 + \beta^2 + \gamma^2 = 1, \; d_p = \frac{7}{2}\right\}$$

$\because |\vec{u} - \vec{v}| = |\vec{v} - \vec{w}| = |\vec{w} - \vec{u}|$ ...(1)

$\vec{u}, \vec{v}, \vec{w}$ are elements of set S and in set S magnitude of vector is 1

$\therefore$ $\vec{u}, \vec{v}, \vec{w}$ are unit vectors and by equation (1) we can system $\vec{u}, \vec{v}, \vec{w}$ are equally inclined and vertices of equilateral triangle also lying on a circle which is intersection of sphere $|\vec{r}| = 1$

Distance from origin to P,

$$d = \frac{|-16|}{\sqrt{3+4+9}} = \frac{16}{4} = 4$$

$\therefore$ Plane containing $\hat{u}, \hat{v}, \hat{w}$ are at a distance $4 - \frac{7}{2} = \frac{1}{2}$ from origin and Parallel to $\sqrt{3}x + 2y + 3z = 16$.

$\therefore$ Equation of the plane is

$$\sqrt{3}x + 2y + 3z = \gamma$$

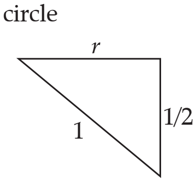
$$\therefore \quad \frac{1}{2} = \left|\frac{\gamma}{4}\right|$$

$$\Rightarrow \quad \gamma = \pm 2$$

$$\sqrt{3}x + 2y + 3z = 2$$

Equation of sphere $x^2 + y^2 + z^2 = 1$

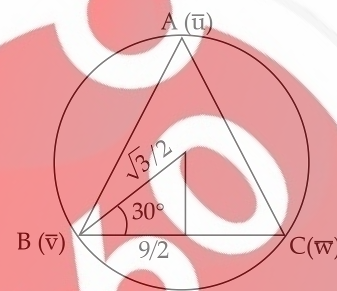
$\therefore$ Radius or circle



$$r = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$$

then $\quad \frac{a}{2} = \frac{\sqrt{3}}{2}\cos 30°$

$$a = \sqrt{3} \times \frac{\sqrt{3}}{2} = \frac{3}{2}$$



$\therefore$ Area or triangle

$$= \frac{\sqrt{3}}{2}a^2 = \frac{\sqrt{3}}{2} \times \frac{9}{4} = \frac{9\sqrt{3}}{16}$$

$\therefore$ Velocity of Parallelepiped

$$= 2 \times \frac{1}{2} \times \frac{9\sqrt{3}}{16}$$

$$V = \frac{9\sqrt{3}}{16}$$

$$\therefore \quad \frac{80V}{\sqrt{3}} = \frac{80}{\sqrt{3}} \times \frac{9\sqrt{3}}{16} = 45$$

13. Correct answer is [3].

General term of $\left(ax^2 + \frac{70}{27bx}\right)^4$

$$T_{r+1} = {}^4C_r\,(ax^2)^{4-r}\left(\frac{70}{27bx}\right)^r$$

$$= {}^4C_r\,a^{4-r}\,\frac{70^r}{(27b)^r}\,(x^{8-3r})$$

For Coefficient of x^5

$$8 - 3r = 5$$

$$r = 1$$

$$\therefore \quad \text{Coefficient} = {}^4C_1\,a^3 \cdot \frac{70}{27b}$$

$$= \frac{280}{27}\frac{a^3}{2}$$

General term of $\left(ax - \frac{1}{bx^2}\right)^7$ is

$$T_{r+1} = {}^7C_r\,(ax)^{7-r}\left(\frac{-1}{bx^2}\right)^r$$

$$= {}^7C_r\,a^{7-r}\left(-\frac{1}{b}\right)^r x^{7-3r}$$

For Coefficient of x^{-5}

$$7 - 3r = -5$$
$$r = 4$$

$\therefore$ Coefficient $= {}^7C_4\, a^3 \times \frac{1}{b^4}$

$\therefore$ According to the question,

$$\frac{280}{27}\frac{a^3}{b} = \frac{35 \times a^3}{b^4}$$

$$\Rightarrow \quad b^3 = \frac{27}{8}$$

$$\Rightarrow \quad b = \frac{3}{2}$$

$$\therefore \quad 2b = 3$$

14. Correct option is (A).

Given $x + 2y + z = 7$

$$x + \alpha z = 11$$
$$2x - 3y + \beta z = \gamma$$

Using Cramer's rule

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & \alpha \\ 2 & -3 & \beta \end{vmatrix}$$

$$= 1(3\alpha) - 2(\beta - 2\alpha) + 1(-3)$$
$$= 3\alpha - 2\beta + 4\alpha - 3$$
$$= 7\alpha - 2\beta - 3$$

$$\Delta_x = \begin{vmatrix} 7 & 2 & 1 \\ 11 & 0 & \alpha \\ \gamma & -3 & \beta \end{vmatrix}$$

$$= 7(3\alpha) - 2(11\beta - \gamma\alpha) + 1(-33)$$
$$= 21\alpha - 22\beta + 22\gamma - 33$$

$$\Delta y = \begin{vmatrix} 1 & 7 & 1 \\ 1 & 11 & \alpha \\ 2 & \gamma & \beta \end{vmatrix}$$

$$= 1(11\beta - \alpha\gamma) - 7(\beta - 2\alpha) + 1(\gamma - 22)$$
$$= 11\beta - \alpha\gamma - 7\beta + 14\alpha + \gamma - 22$$
$$= 14\alpha + 4\beta + \gamma - \alpha\gamma - 22$$

$$\Delta z = \begin{vmatrix} 1 & 2 & 7 \\ 1 & 0 & 11 \\ 2 & -3 & \gamma \end{vmatrix}$$

$$= 1(33) - 2(\gamma - 22) + 7(-3)$$
$$= 33 - 2\gamma + 44 - 21$$
$$= -2\gamma + 56$$

For unique solution $\Delta \neq 0$

For infinite solution

$$\Delta = \Delta x = \Delta y = \Delta z = 0$$

For no solution $\Delta = 0$ and atleast one in $\Delta x, \Delta y, \Delta z$ is non zero.

$$\Delta = 0$$

$$\Rightarrow \quad \beta = \frac{1}{2}(7\alpha - 3)$$

(P) $\quad \beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma = 28$

then $\quad \Delta = 0, \Delta x = \Delta y = \Delta z = 0$

$\therefore$ Infinite solution

(Q) $\quad \beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma \neq 28$

$\therefore \quad \Delta = 0$ and $\Delta_2 \neq 0$

$\Rightarrow$ No solution

(R) $\beta \neq \frac{1}{2}(7\alpha - 3),\ \alpha = 1, \gamma \neq 28$

$\Rightarrow \quad \Delta \neq 0 =$ unique solution

(S) $\quad \beta \neq \frac{1}{2}(7\alpha - 3),\ \alpha = 1, \gamma = 28$

$\therefore \quad \Delta \neq 0, \quad \Delta = 4 - 2\beta$

$$\Delta x = 44 - 22\beta$$
$$\Delta y = 4\beta - 8$$
$$\Delta z = 0$$

$\therefore \quad x = 11, y = -2, z = 0$ is the solution.

15. Correct option is (A).

x_1	f_i	f_ix_i	$f_i\lvert x_i - \bar{x}\rvert$	$f_i\lvert x_i - N\rvert$
3	5	15	15	10
4	4	16	8	4
5	4	20	4	0
8	2	16	4	6
10	2	20	8	10
11	3	33	15	18
	$\Sigma f_i = 20$	$\Sigma f_ix_i = 120$	sum = 54	sum = 48

(P) $\quad$ Mean $= \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{120}{20} = 6$

(Q) $\quad$ Median $= \frac{(10^{th} + 11^{th})\text{observation}}{2}$

$$= \frac{5+5}{2} = 5$$

(both observation are same)

(R) $\quad$ Mean deviation

$$= \frac{\Sigma f_i \mid x_i - \bar{x} \mid}{\Sigma f_i} = \frac{54}{20}$$

$$= 2.7$$

(S) $\quad$ Mean deviation about median

$$= \frac{\Sigma f_i \mid x_i - M \mid}{\Sigma f_i} = \frac{48}{20}$$

$$= 2.4$$

16. Correct option is (B).

$l_1 : \quad \vec{r} = \lambda(\hat{i} + \hat{j} + \hat{k})$

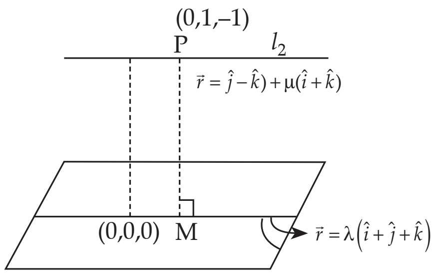
$l_2 : \quad \vec{r} = \hat{j} - \hat{k} + \mu(\hat{i} + \hat{k})$

For plane

$d(H)$ = Smallest possible distance between the points of l_2 and Plane.

$d(H_0)$ = Maximum value of $d(H)$

For $d(H_0)$



l_2 is Parallel to plane containing l_1

Equation of plane

$a(x) + by + cz = 0$

$a(x) + by + cz = 0$ → $\vec{n} \perp l_1$, $\vec{n} \perp l_2$

$\therefore \quad a + b + c = 0 \qquad ...(1)$

$a + c = 0 \qquad ...(2)$

By (1) and (2) $a = -c, b = 0$

$\therefore$ Equation of plane $x - z = 0$

(P) $d(H_0) = PM = \left|\dfrac{0-(-1)}{\sqrt{1+1}}\right| = \dfrac{1}{\sqrt{2}}$

(Q) Distance from (0, 1, 2)

$= \left|\dfrac{0-2}{\sqrt{2}}\right| = \sqrt{2}$

(R) Distance from origin (0, 0, 0)

$= \left|\dfrac{0}{\sqrt{2}}\right| = 0$

(S) Point of Intersection,

$x - z = 0 \qquad ...(1)$

and $x = 1, y = z \qquad ...(2)$

$\therefore \quad x = 1 = z = y$

$\therefore$ Point of intersection (1, 1, 1)

Distance from origin

$= \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

17. Correct option is (B).

$|Z|^3 + 2Z^2 + 4\bar{Z} - 8 = 0$

let $Z = x + iy$

$|Z| = \sqrt{x^2 + y^2}$

$\bar{Z} = x - iy$

$Z^2 = x^2 - y^2 + 2ixy$

$\therefore \quad |Z|^3 + 2Z^2 + 4\bar{Z} - 8 = 0$

$(x^2 + y^2)^{3/2} + 2(x^2 - y^2) + 4ixy + 4x - 4iy - 8 = 0$

$\therefore \quad (x^2 + y^2)^{3/2} + 2(x^2 - y^2) + 4x - 8 = 0 \qquad ...(1)$

and $2xy - 4y = 0$

$\Rightarrow \quad y = 0$ or $x = 1$

At $x = 1$

$(1 + y^2)^{3/2} + 2 - 2y^2 + 4 - 8 = 0$

$\Rightarrow (1 + y^2)^{3/2} - 2y^2 - 2 = 0$

$\Rightarrow (1 + y^2)^{3/2} - 2(1 + y^2) = 0$

$(1 + y^2)\left(\sqrt{1 + y^2} - 2\right) = 0$

then $1 + y^2 = 0$ (wich is not possible)

or $1 + y^2 = 4$

$\Rightarrow \quad y^2 = 3$

$\therefore \quad x = 1$ and $y^2 = 3$

(P) $|Z|^2 = x^2 + y^2 = 1 + 3 = 4$

(Q) $|Z - \bar{Z}|^2 = |2Im(z)|^2$

$= (2y)^2 = 4y^2 = 12$

(R) $|Z|^2 + |Z + \bar{Z}|^2 = 4 + |2x|^2$

$= 4 + 4(1) = 8$

(S) $|Z + 1|^2 = |x + iy + 1|^2$

$= (x + 1)^2 + y^2$

$= 4 + 3 = 7.$