# JEE Advanced (2025)

# PAPER 1

## MATHEMATICS

*General Instructions*:

**SECTION 1 (Maximum Marks: 12)**

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme:**
  *Full Marks* : +3, If **ONLY** the correct option is chosen;
  *Zero Marks* : 0, If none of the options is chosen (i.e., the question is unanswered);
  *Negative Marks* : −1, in all other cases.

**1.** Let $\mathbb{R}$ denote the set of all real numbers. Let $a_i, b_i \in \mathbb{R}$ for $i \in \{1, 2, 3\}$.

Define the functions $f: \mathbb{R} \to \mathbb{R}$, $g: \mathbb{R} \to \mathbb{R}$ and $h: \mathbb{R} \to \mathbb{R}$ by

$$f(x) = a_1 + 10x + a_2x^2 + a_3x^3 + x^4,$$
$$g(x) = b_1 + 3x + b_2x^2 + b_3x^3 + x^4,$$
$$h(x) = f(x + 1) - g(x + 2).$$

If $f(x) \neq g(x)$ for every $x \in \mathbb{R}$, then the coefficient of $x^3$ in $h(x)$ is:

**(A)** 8 **(B)** 2 **(C)** –4 **(D)** –6

**2.** Three students $S_1$, $S_2$ and $S_3$ are given a problem to solve. Consider the following events:

$U$ : At least one of $S_1$, $S_2$ and $S_3$ can solve the problem,

$V$ : $S_1$ can solve the problem, given that neither $S_2$ nor $S_3$ can solve the problem

$W$ : $S_2$ can solve the problem and $S_3$ cannot solve the problem

$T$ : $S_3$ can solve the problem.

For any event $E$, let $P(E)$ denote the probability of $E$. If

$$P(U) = \frac{1}{2}, \quad P(V) = \frac{1}{10} \quad \text{and} \quad P(W) = \frac{1}{12},$$

then $P(T)$ is equal to:

**(A)** $\frac{13}{36}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{19}{60}$ **(D)** $\frac{1}{4}$

**3.** Let $\mathbb{R}$ denote the set of all real numbers. Define the function $f: \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} 2 - 2x^2 - x^2 \sin\frac{1}{x} & \text{if } x \neq 0, \\ 2 & \text{if } x = 0. \end{cases}$$

Then which one of the following statements is TRUE?

**(A)** The function $f$ is **NOT** differentiable at $x = 0$

**(B)** There is a positive real number $\delta$, such that $f$ is a decreasing function on the interval $(0, \delta)$

**(C)** For any positive real number $\delta$, the function $f$ is **NOT** an increasing function on the interval $(-\delta, 0)$

**(D)** $x = 0$ is a point of local minima of $f$

**4.** Consider the matrix

$$P = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

Let the transpose of a matrix $X$ be denoted by $X^T$. Then the number of $3 \times 3$ invertible matrices $Q$ with integer entries, such that $Q^{-1} = Q^T$ and $PQ = QP$, is:

**(A)** 32 **(B)** 8 **(C)** 16 **(D)** 24

*General Instructions*:

**SECTION 2 (Maximum Marks: 12)**

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
  *Full Marks* : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  *Partial Marks* : +3 If all the four options are correct but **ONLY** three options are chosen;
  *Partial Marks* : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  *Partial Marks* : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  *Zero Marks* : 0 If none of the options is chosen (i.e., the question is unanswered);
  *Negative Marks* : −2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then
  choosing **ONLY** (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option(s) (i.e., the question is unanswered) will get 0 marks and
  choosing any other option(s) will get −2 marks.

**5.** Let $L_1$ be the line of intersection of the planes given by the equations

$2x + 3y + z = 4$ and $x + 2y + z = 5$.

Let $L_2$ be the line passing through the point $P(2, -1, 3)$ and parallel to $L_1$. Let $M$ denotes the plane given by the equation

$2x + y - 2z = 6$.

Suppose that the line $L_2$ meets the plane $M$ at the point $Q$. Let $R$ be the foot of the perpendicular drawn from $P$ to the plane $M$.

Then which of the following statements is (are) TRUE?

**(A)** The length of the line segment $PQ$ is $9\sqrt{3}$

**(B)** The length of the line segment $QR$ is 15

**(C)** The area of $\Delta PQR$ is $\frac{3}{2}\sqrt{234}$

**(D)** The acute angle between the line segments $PQ$ and $PR$ is $\cos^{-1}\left(\frac{1}{2\sqrt{3}}\right)$

**6.** Let $\mathbb{N}$ denote the set of all natural numbers, and $\mathbb{Z}$ denote the set of all integers. Consider the functions $f : \mathbb{N} \to \mathbb{Z}$ and $g : \mathbb{Z} \to \mathbb{N}$ defined by

$$f(n) = \begin{cases} (n+1)/2 & \text{if } n \text{ is odd,} \\ (4-n)/2 & \text{if } n \text{ is even,} \end{cases}$$

and

$$g(n) = \begin{cases} 3+2n & \text{if } n \geq 0, \\ -2n & \text{if } n < 0. \end{cases}$$

Define $(g \circ f)(n) = g(f(n))$ for all $n \in \mathbb{N}$, and $(f \circ g)(n) = f(g(n))$ for all $n \in \mathbb{Z}$.

Then which of the following statements is (are) TRUE?

**(A)** $g \circ f$ is **NOT** one–one and $g \circ f$ is **NOT** onto

**(B)** $f \circ g$ is **NOT** one–one but $f \circ g$ is onto

**(C)** $g$ is one–one and $g$ is onto

**(D)** $f$ is **NOT** one–one but $f$ is onto

**7.** Let $\mathbb{R}$ denote the set of all real numbers. Let $z_1 = 1 + 2i$ and $z_2 = 3i$ be two complex numbers, where $i = \sqrt{-1}$. Let $S = \{(x, y) \in \mathbb{R} \times \mathbb{R} : |x + iy - z_1| = 2|x + iy - z_2|\}$.

Then, which of the following statements is(are) TRUE?

**(A)** $S$ is a circle with centre $\left(-\frac{1}{3}, \frac{10}{3}\right)$

**(B)** $S$ is a circle with centre $\left(\frac{1}{3}, \frac{8}{3}\right)$

**(C)** $S$ is a circle with radius $\frac{\sqrt{2}}{3}$

**(D)** $S$ is a circle with radius $\frac{2\sqrt{2}}{3}$

***General Instructions***:

**SECTION 3 (Maximum Marks: 24)**

- This section contains **SIX (06)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal points, **truncate/round-off** the value of **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**

| | | |
|---|---|---|
| *Full Marks* | : +4 | **ONLY** if the correct numerical value is entered in the designated place; |
| *Zero Marks* | : 0 | In all other cases. |

**8.** Let the set of all relations $R$ on the set $\{a, b, c, d, e, f\}$, such that $R$ is reflexive and symmetric, and contains exactly 10 elements, be denoted by $\delta$.

Then the number of elements in $\delta$ is ________.

**9.** For any two points $M$ and $N$ in the $XY$-plane, let $\overrightarrow{MN}$ denote the vector from $M$ to $N$, and $\vec{0}$ denote the zero vector. Let $P$, $Q$ and $R$ be three distinct points in the $XY$-plane. Let $S$ be a point inside the triangle $\Delta PQR$ such that

$$\overrightarrow{SP} + 5\overrightarrow{SQ} + 6\overrightarrow{SR} = \vec{0}.$$

Let $E$ and $F$ be the mid-points of the sides $PR$ and $QR$, respectively. Then the value of

$$\frac{\text{length of the line segment } EF}{\text{length of the line segment } ES}$$

is ________.

**10.** Let $S$ be the set of all seven-digit numbers that can be formed using the digits 0, 1 and 2. For example, 2210222 is in $S$, but 0210222 is **NOT** in $S$.

Then the number of elements $x$ in $S$ such that at least one of the digits 0 and 1 appears exactly twice in $x$, is equal to ________.

**11.** Let $\alpha$ and $\beta$ be the real numbers such that

$$\lim_{x \to 0} \frac{1}{x^3}\left(\frac{\alpha}{2}\int_0^x \frac{1}{1-t^2}\,dt + \beta x \cos x\right) = 2.$$

Then the value of $\alpha + \beta$ is ________.

**12.** Let $\mathbb{R}$ denote the set of all real numbers. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $f(x) > 0$ for all $x \in \mathbb{R}$, and $f(x + y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$.

Let the real numbers $a_1, a_2, \ldots, a_{50}$ be in an arithmetic progression. If $f(a_{31}) = 64f(a_{25})$, and

$$\sum_{i=1}^{50} f(a_i) = 3(2^{25} + 1),$$

then the value of

$$\sum_{i=6}^{30} f(a_i)$$

is ________.

**13.** For all $x > 0$, let $y_1(x)$, $y_2(x)$, and $y_3(x)$ be the functions satisfying

$$\frac{dy_1}{dx} - (\sin x)^2 y_1 = 0, \quad y_1(1) = 5,$$

$$\frac{dy_2}{dx} - (\cos x)^2 y_2 = 0, \quad y_2(1) = \frac{1}{3},$$

$$\frac{dy_3}{dx} - \left(\frac{2 - x^3}{x^3}\right) y_3 = 0, \quad y_3(1) = \frac{3}{5e},$$

respectively. Then

$$\lim_{x \to 0+} \frac{y_1(x)y_2(x)y_3(x) + 2x}{e^{3x}\sin x}$$

is equal to ________.

*General Instructions*:

**SECTION 4 (Maximum Marks: 12)**

- This section contains **THREE (03) Matching Lists Sets.**
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II.**
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **FIVE** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated **according to the following marking scheme:**

*Full Marks* : +4 **ONLY** if the option corresponding to the correct combination is chosen;
*Zero Marks* : 0 If none of the options is chosen (i.e., the question is unanswered);
*Negative Marks* : –1 In all other cases.

**14.** Consider the following frequency distribution:

| Value | 4 | 5 | 8 | 9 | 6 | 12 | 11 |
|---|---|---|---|---|---|---|---|
| Frequency | 5 | $f_1$ | $f_2$ | 2 | 1 | 1 | 3 |

Suppose that the sum of the frequencies is 19 and the median of this frequency distribution is 6.

For the given frequency distribution, let $\alpha$ denote the mean deviation about the mean, $\beta$ denote the mean deviation about the median, and $\sigma^2$ denote the variance.

Match each entry in **List-I** to the correct entry in **List-II** and choose the correct option.

| List-I | List-II |
|---|---|
| (P) $7f_1 + 9f_2$ is equal to | (1) 146 |
| (Q) $19\alpha$ is equal to | (2) 47 |
| (R) $19\beta$ is equal to | (3) 48 |
| (S) $19\sigma^2$ is equal to | (4) 145 |
| | (5) 55 |

The correct option is:

**(A)** (P) → (5) (Q) → (3) (R) → (2) (S) → (4)
**(B)** (P) → (5) (Q) → (2) (R) → (3) (S) → (1)
**(C)** (P) → (5) (Q) → (3) (R) → (2) (S) → (1)
**(D)** (P) → (3) (Q) → (2) (R) → (5) (S) → (4)

**15.** Let $\mathbb{R}$ denote the set of all real numbers. For a real number $x$, let $[x]$ denote the greatest integer less than or equal to $x$. Let $n$ denote a natural number.

Match each entry in **List-I** to the correct entry in **List-II** and choose **the correct option.**

| List-I | List-II |
|---|---|
| (P) The minimum value of $n$ for which the function $f(x)=\left[\frac{10x^3-45x^2+60x+35}{n}\right]$ is continuous on the interval [1,2], is | (1) 8 |
| (Q) The minimum value of $n$ for which $g(x) = (2n^2 - 13n - 15)(x^3 + 3x)$, $x \in \mathbb{R}$, is an increasing function on $\mathbb{R}$, is | (2) 9 |
| (R) The smallest natural number $n$ which is greater than 5, such that $x = 3$ is a point of local minima of $h(x)=(x^2 - 9)^n(x^2 + 2x + 3)$, is | (3) 5 |
| (S) Number of $x_0 \in \mathbb{R}$ such that $l(x)=\sum_{k=0}^{4}\left(\sin\lvert x-k\rvert+\cos\left\lvert x-k+\frac{1}{2}\right\rvert\right)$, $x \in \mathbb{R}$, is **NOT** differentiable at $x_0$, is | (4) 6 |
| | (5) 10 |

The correct option is:

**(A)** (P) → (1) (Q) → (3) (R) → (2) (S) → (5)
**(B)** (P) → (2) (Q) → (1) (R) → (4) (S) → (3)
**(C)** (P) → (5) (Q) → (1) (R) → (4) (S) → (3)
**(D)** (P) → (2) (Q) → (3) (R) → (1) (S) → (5)

**16.** Let $\vec{w}=\hat{i}+\hat{j}-2\hat{k}$ and $\vec{u}$ and $\vec{v}$ be two vectors, such that $\vec{u}\times\vec{v}=\vec{w}$ and $\vec{v}\times\vec{w}=\vec{u}$. Let $\sigma, \beta, \gamma$ and $t$ be real numbers such that

$\vec{u}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$, $-t\alpha+\beta+\gamma=0$, $\alpha-t\beta+\gamma=0$, and $\alpha+\beta-t\gamma=0$.

Match each entry in **List-I** to the correct entry in **List-II**.

| List-I | List-II |
|---|---|
| (P) $\lvert\vec{v}\rvert^2$ is equals to | (1) 0 |
| (Q) If $\alpha=\sqrt{3}$, then $\gamma^2$ is equal to | (2) 1 |
| (R) If $\alpha=\sqrt{3}$, then $(\beta+\gamma^2)$ is equal to | (3) 2 |
| (S) If $\alpha=\sqrt{2}$, then $t + 3$ is equal to | (4) 3 |
| | (5) 5 |

The correct option is:

**(A)** (P) → (2) (Q) → (1) (R) → (4) (S) → (5)
**(B)** (P) → (2) (Q) → (4) (R) → (3) (S) → (5)
**(C)** (P) → (2) (Q) → (1) (R) → (4) (S) → (3)
**(D)** (P) → (5) (Q) → (4) (R) → (1) (S) → (3)

# Answer Key

| Q.No. | Answer key | Topic's name | Chapter's name |
|---|---|---|---|
| 1 | (C) | Function | Function |
| 2 | (A) | Conditional Probability | Probability |
| 3 | (C) | Miscellaneous | Application of Derivative |
| 4 | (C) | Product of Matrices | Matrix |
| 5 | (A, C) | Plane | 3D |
| 6 | (A, D) | Composite Function | Function |
| 7 | (A, D) | Circle | Complex Number |
| 8 | 105 | Types of Relation | Sets and Relation |
| 9 | 1.15 to 1.25 | Point and Triangle | Vector |
| 10 | 762 | Formation of Numbers | Permutation and Combination |
| 11 | 2.35 to 2.45 | Newton Lebinitz Rule | Definite iIntegral |
| 12 | 96 | Arithmetic Progression | Sequence and Series |
| 13 | 2 | First Order Differential Equation | Differential Equation |
| 14 | (C) | Statistics | Statistics |
| 15 | (B) | Miscellaneous | Application of Derivative |
| 16 | (A) | Vector Product | Vector |

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# ANSWERS WITH EXPLANATIONS

**1. Correct option is (C).**

$f(x) = a_1 + 10x + a_2x^2 + a_3x^3 + x^4,$

$g(x) = b_1 + 3x + b_2x^2 + b_3x^3 + x^4,$

$h(x) = f(x + 1) - g(x + 2)$

$f(x) - g(x) = (a_3 - b_3)x^3 + (a_2 - b_2)x^2 + 7x + a_2 - b_1$

$\therefore f(x) - g(x) \neq 0 \Rightarrow$ It should not meet $x - a \times b$ at any point $x$.

$\Rightarrow f(x) - g(x)$ must be an even degree $a_3 - b_3 = 0$.

Now $f(x + 1) = a_1 + 10(x + 1) + a_2(x + 1)^2 + a_3(x + 1)^3 + (x + 1)^4$

and $g(x + 2) = b_1 + 3(x + 2) + b_2(x + 2)^2 + b_3(x + 2)^3 + (x + 2)^4$

Coefficient of $x^3$ in $h(x)$

$\Rightarrow$ Coefficient $x^3$ in $\{a_3(x + 1)^3 + (x + 1)^4 - b_3(x + 2)^3 - (x+2)^4\}$

$\Rightarrow a_3 + 4 - b_3 - 4 \times 2$

$\Rightarrow -4 \ (\because a_3 - b_3 = 0)$

**2. Correct option is (A).**

Let $P(S_1) = x$

$P(S_2) = y$

$P(S_3) = z$

$\therefore P(U) = P(S_1 \cup S_2 \cup S_3) = \frac{1}{2}$

$\Rightarrow 1 - (1 - x)(1 - y)(1 - z) = \frac{1}{2}$

$\Rightarrow (1 - x)(1 - y)(1 - z) = \frac{1}{2}$ ... (1)

$\because P(V) = P\left(\frac{S_1}{(\overline{S_2} \cap \overline{S_3})}\right) = \frac{1}{10}$

$\Rightarrow \frac{P(S_1 \cap \overline{S_2} \cap \overline{S_3})}{P(\overline{S_2} \cap \overline{S_3})} = \frac{1}{10}$ $\left\{\because P(\overline{A \cup B}) = P(\overline{A} \cap \overline{B})\right.$

$\Rightarrow \frac{x(1-y)(1-z)}{(1-y)(1-z)} = \frac{1}{10}$

$\Rightarrow x = \frac{1}{10}$

and $P(S_2 \cap \overline{S_3}) = \frac{1}{12}$

$\Rightarrow y(1 - z) = \frac{1}{12}$ ...(2)

From Eq. (1) we get,

$\left(1 - \frac{1}{10}\right)(1-y)(1-z) = \frac{1}{2}\left\{x = \frac{1}{10}\right\}$

$\Rightarrow (1 - y)(1 - z) = \frac{5}{q}$ ... (3)

From Eqs (2) and (3), we get,

$\frac{y}{1-y} = \frac{1}{12} \times \frac{9}{5}$

$\Rightarrow 20y = 3 - 3y$

$\Rightarrow 20y + 3y = 3$

$\Rightarrow y = \frac{3}{23}$

From Eqs (2), we get,

$\frac{3}{23}(1 - z) = \frac{1}{12}$

$\Rightarrow 1 - z = \frac{23}{36}$

$\Rightarrow z = 1 - \frac{23}{36}$

$\Rightarrow z = \frac{13}{36}$

**3. Correct option is (C).**

$$f(x) = \begin{cases} 2 - 2x^2 - x^2 \sin\dfrac{1}{x} & \text{if } x \neq 0 \\ 2 & \text{if } x = 0 \end{cases}$$

$$\text{LHL} = \lim_{x \to 0^-} \left(2 - 2x^2 - \left(x^2 \sin\frac{1}{x}\right)\right) = 2 - 0 = 2$$

$$\text{RHL} = \lim_{x \to 0^+} \left(2 - 2x^2 - \left(x^2 \sin\frac{1}{x}\right)\right) = 2 - 0 = 2$$

LHL = RHL = $f(0)$.

$\therefore$ Function is continuous at $x = 0$.

$$\text{RHD} = f'(0^+) = \lim_{h \to 0} \frac{f(h) - f(0)}{h}$$

$$\lim_{h \to 0} \frac{2 - 2h^2 - h^2 \sin\left(\dfrac{1}{h}\right) - 2}{h} = 0$$

$$\text{LHD} = f'(0^-) \lim_{h \to 0} \frac{f(-h) - f(0)}{-h} = 0$$

$$= \lim_{h \to 0} \frac{2 - 2h^2 - h^2 \sin\left(\dfrac{1}{h}\right) - 2}{-h} = 0$$

$f'(0^+) = f'(0^-) = f(0)$ is differentiable at $x = 0$.

Now,

**For Option B:** There is a positive real number $\delta$, such that $f$ is a decreasing function on the interval $(0, \bar{\delta})$.

To check if $f$ is decreasing on $(0, \delta)$, we analyze the derivative $f'(x)$ for $x > 0$

Calculating $f'(x)$ for $x \neq 0$ :

$$f'(x) = -4x - \sin\left(\frac{1}{x}\right) + \frac{\cos\left(\dfrac{1}{x}\right)}{x^2}$$

The behavior of this derivative is complex due to the oscillatory nature of $\sin\left(\dfrac{1}{x}\right)$ and $\cos\left(\dfrac{1}{x}\right)$. Without further analysis, we cannot confirm if $f$ is decreasing.

So Option B is uncertain.

**For Option C:** For any positive real number $\delta$, the function $f$ is NOT an increasing function on the interval $(-\bar{\delta}, 0)$

For $x < 0$, we have $f(x) = 2 - 2x^2 - x^2 \sin\left(\dfrac{1}{x}\right)$. The derivative $f'(x)$ will determine if $f$ is increasing or not.

Calculating $f'(x)$ for $x < 0$ :

$$f'(x) = -4x - \sin\left(\frac{1}{x}\right) + \frac{\cos\left(\dfrac{1}{x}\right)}{x^2}$$

Due to oscillatory behavior of $\sin\left(\dfrac{1}{x}\right)$ and $\cos\left(\dfrac{1}{x}\right)$, $f'(x)$ can take both positive and negative values. Therefore, f is not guaranteed to be increasing on the interval $(-\delta, 0)$

So **Option C is correct.**

**4. Correct option is (C).**

$$P = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\text{Let, } Q = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$$

$$PQ = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} = \begin{bmatrix} 2a_1 & 2b_1 & 2c_1 \\ 2a_2 & 2b_2 & 2c_2 \\ 3a_3 & 3b_3 & 3c_3 \end{bmatrix}$$

$$QP = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 2a_1 & 2b_1 & 3c_1 \\ 2a_2 & 2b_2 & 3c_2 \\ 2a_3 & 2b_3 & 3c_3 \end{bmatrix}$$

$\therefore PQ = QP$

$$\Rightarrow \begin{pmatrix} 2a_1 & 2b_1 & 3c_1 \\ 2a_2 & 2b_2 & 2c_2 \\ 3a_3 & 2b_3 & 3c_3 \end{pmatrix} = \begin{pmatrix} 2a_1 & 2b_1 & 3c_1 \\ 2a_2 & 2b_2 & 3c_2 \\ 3a_3 & 2b_3 & 3c_3 \end{pmatrix}$$

$\Rightarrow c_1 = 0,\ c_2 = 0,\ a_3 = 0$ and $b_3 = 0$

$$\therefore Q = \begin{pmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ 0 & 0 & c_3 \end{pmatrix}$$

$\therefore Q^{-1} = Q^T \qquad I = QQ^T \qquad QQ^T = I$

$$\Rightarrow \begin{pmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ 0 & 0 & c_3 \end{pmatrix} \begin{pmatrix} a_1 & a_2 & 0 \\ b_1 & b_2 & 0 \\ 0 & 0 & c_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{bmatrix} a_1^2 + b_1^2 & a_1a_2 + b_1b_2 & 0 \\ a_1a_2 + b_1b_2 & a_2^2 + b_2^2 & 0 \\ 0 & 0 & c_3^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \quad {a^2}_1 + {b^2}_1 = 1$$

$${a^2}_2 + {b^2}_2 = 1$$

$${c^2}_3 = 1$$

$$a_1a_2 + b_1b_2 = 0$$

$a_1, a_2, b_1, b_2, c_3$ all are Integers.

$\Rightarrow {a^2}_1 = 1,\ \ {b^2}_1 = 0$ or ${a^2}_1 = 0, {b^2}_1 = 1$

${a^2}_2 = 1,\ \ {b^2}_2 = 0$ or ${a^2}_2 = 0, {b^2}_2 = 1,\ {c^2}_3 = 1$

$\Rightarrow\ a_1 = \pm 1,\ b_1 = 0$ or $a_1 = 0,\ b_1 = \pm 1$

$a_2 = \pm 1,\ b_2 = 0$ or $a_2 = 0,\ b_2 = \pm 1$

$c_3 = \pm 1$

If $\ b_1 = 0 \Rightarrow a_2 = 0 \qquad \{\because a_1b_2 + b_1b_2 = 0\}$

**Case I** $\Rightarrow a_1 = \pm 1,\ b_1 = 0,\ a_2 = 0,\ b_2 = \pm 1$

$c_3 = \pm 1$

Total 8 matrices possible

**Case II** $\Rightarrow a_1 = 0,\ b_1 = \pm 1,\ a_2 = \pm 1,\ b_2 = 0$

$c_3 = \pm 1$

Totally 8 matrices are possible.

$\therefore$ Totally 16 matrices are possible.

**5. Correct option is (A, C).**

$L_1 : 2x + 3y + z = 4$ ... (1)

$x + 2y + z = 5$ ... (2)

Let $z = 0 \Rightarrow 2x + 3y - 4 = 0$

$x + 2y - 5 = 0$

$$\Rightarrow \frac{x}{-15+8} = \frac{y}{-4+10} = \frac{1}{4-3}$$

$x = -7$ and $y = 6$

$L_1$ passes through $(-7, 6, 0)$.

Let dr's of $L_1$ are $a, b, c$

$$\therefore \quad 2a + 3b + c = 0$$

$$a + 2b + c = 0$$

$$\Rightarrow \frac{a}{3-2} = \frac{b}{1-2} = \frac{c}{4-3}$$

$\Rightarrow \quad \frac{a}{1} = \frac{b}{-1} = \frac{c}{1}$

$\therefore L_1 : \frac{x+7}{1} = \frac{y-6}{-1} = \frac{z}{1}$

$L_2 : \frac{x-2}{1} = \frac{y+1}{-1} = \frac{z-3}{1} = \lambda$. {$L_2$ passes through point $P(2, -1, 3)$ and parallel to $L_1$.}

Any point on $L_2$ is $Q$.

$Q = (\lambda+2, -\lambda-1, \lambda+3)$

Line $L_2$ meet plane at point $Q$,

$$2(\lambda+2)-\lambda-1-2(\lambda+3)=6$$

$\Rightarrow \quad -\lambda-3=6$

$\lambda=-9$

$\Rightarrow \quad Q = (-7, 8, -6)$

$2x + y - 2z - 6 = 0$

$\therefore$ Direction ratio of Line $PR$ is (2, 1, –2) equation of line $PR$ is

$$\frac{x-2}{2} = \frac{y+1}{1} = \frac{z-3}{-2} = \lambda$$

$\therefore$ Point $R = (2\lambda+2, \lambda-1, -2\lambda+3)$

R line on plane,

$2(2\lambda + 2) + \lambda - 1 - 2(-2\lambda + 3) - 6 = 0$

$\Rightarrow \quad 9\lambda - 9 = 0$

$\Rightarrow \quad \lambda = 1$

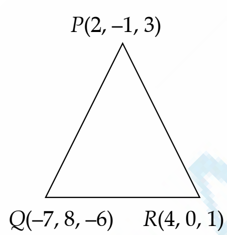
$\Rightarrow \quad R = R(4, 0, 1)$

$\therefore \quad PQ = \sqrt{81+81+81} = 9\sqrt{3}$

and $\quad QR = \sqrt{121+64+49} = \sqrt{234}$

$\overrightarrow{PQ} = -9\hat{i}+9\hat{j}-9\hat{k}$

$\overrightarrow{PQ} = 2\hat{i}+\hat{j}-2\hat{k}$




$\therefore \quad \overrightarrow{PQ}\cdot\overrightarrow{PR} = |\overrightarrow{PQ}||\overrightarrow{PR}|\cos\theta$

$-18 + 9 + 18 = 9\sqrt{3} \cdot 3 \cos\theta$

$\cos\theta = \frac{1}{3\sqrt{3}}$

Now, area of $\Delta PQR = \frac{1}{2}|\overrightarrow{PQ}\times\overrightarrow{PR}|$

$$\therefore \quad |\overrightarrow{PQ}\times\overrightarrow{PR}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -9 & 9 & -9 \\ 2 & 1 & -2 \end{vmatrix}$$

$$= \left[\hat{i}(-18+9)-\hat{j}(18+18)+\hat{k}(-9-18)\right]$$

$$= \left[-9\hat{i}-36\hat{j}-27\hat{k}\right]$$

$$\therefore \quad \text{Area} = \frac{1}{2}\times 9\sqrt{1+16+9}$$

$$= \frac{3}{2}\sqrt{234}$$

**6. Correct option is (A, D).**

$$f(n) = \begin{cases} (n+1)/2, & \text{if } n \text{ is odd,} \\ (4-n)/2, & \text{if } n \text{ is even.} \end{cases}$$

$f(1) = \frac{1+1}{2} = 1$

$f(2) = \frac{4-2}{2} = 1$

$\therefore f(n)$ is many to one function $f(n) \in \mathbb{N}$.

$\therefore f(n)$ is many one onto function.

$$g(n) = \begin{cases} 3+2n, & \text{if } n \geq 0, \\ -2n, & \text{if } n < 0. \end{cases}$$

$g(n) \in \{3, 5, 7, 9 ...\}$ $\quad n \geq 0$

$g(n) \in \{2, 4, 6, ...\}$ $\quad n > 0$

$\therefore$ g($n$) is one–one into function.

($\because$ g($n$) $\neq$ 1)

Now, for $f \circ g(n)$ domain of $f \circ g(n)$ is $\mathbb{Z}$

Now, $f \circ g(n) \in \{1, 2, 0, 3, -1, ...\}$

Hence, $f \circ g(n)$ is one–one onto.

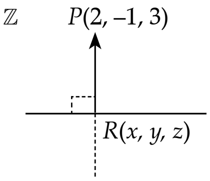
Now, for $g \circ f(n)$

Domain is $\mathbb{N}$

$g \circ f(n) \in \{5, 7, 9, ...,$ when $n$ = odd$\}$

$g \circ f(n) \in \{5, 3, 2, 4, ...,$ when $n$ = even$\}$

$g \circ f(n)$ is not one–one and $g \circ f$ is not onto.




**7. Correct option is (A, D).**

$|x + iy - z_1| = 2|x + iy - z_2|$

$z_1 = 1 + 2i$

$z_2 = 3i$

$|x + iy - 1 - 2i| = 2|x + iy - 3i|$

Squaring on both side we get,

$\Rightarrow \quad |x - 1) + i(y - 2)^2 = 4|x + i(y - 3)|^2$

$\Rightarrow \quad (x - 1)^2 + (y - 2)^2 = 4(x^2 + (y - 3)^2)$

$\Rightarrow \quad x^2 - 2x + 1 + y^2 - 4y + 4 = 4x^2 + 4y^2 - 24y + 36$

$\Rightarrow \quad 3x^2 + 3y^2 + 2x - 20y + 31 = 0$

$\Rightarrow x^2 + y^2 + \frac{2}{3}x - \frac{20}{3}y + \frac{31}{3} = 0$

Centre: $\left(-\frac{1}{3}, \frac{10}{3}\right)$

$\therefore$ Radius $= \sqrt{\frac{1}{9}+\frac{100}{9}-\frac{31}{3}} = \frac{2\sqrt{2}}{3}$

**8. Correct option is [105].**

$R$ must contain ($a$, $a$), ($b$, $b$), ($c$, $c$), ($d$, $d$) ($e$, $e$), ($f$, $f$)

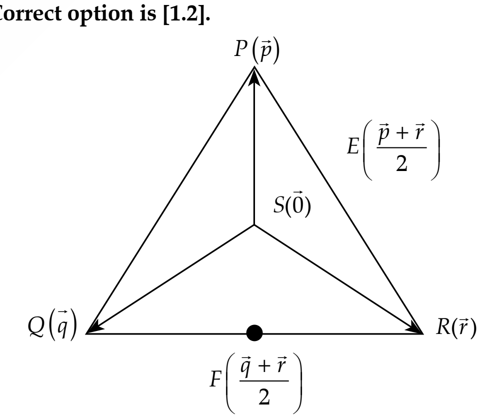
Out of 6 elements, 2 can be selected by ${}^6C_2 = \frac{6\times5}{2} = 15$ ways.

$R$ is symmetric, therefore, we can take only two elements from the following 15 pairs:

{($a$, $b$), ($a$, $c$), ($a$, $d$) ($a$, $e$) ($a$, $f$), ($b$, $c$), ($b$, $d$), ($b$, $e$), ($b$, $f$), ($c$, $d$), ($c$, $e$), ($c$, $f$), ($d$, $e$), ($d$, $f$), ($e$, $f$)}

Total ways $\Rightarrow {}^{15}C_2 = 105$

**9. Correct option is [1.2].**



Let $S = (0, 0)$

$\overrightarrow{SP} = \vec{p}$

$\overrightarrow{SQ} = \vec{q}$

$\overrightarrow{SR} = \vec{r}$

$\therefore\ \overline{SP} + 5\overline{SQ} + 6\overline{SR} = 0$

$\vec{p} + 5\vec{q} + 6\vec{r} = \vec{0}$

$\therefore\quad \vec{r} = -\left(\frac{\vec{p} + 5\vec{q}}{6}\right)$

$\therefore\ |\overline{EF}| = \frac{\left|\vec{q} + \vec{r} - (\vec{p} + \vec{r})\right|}{2} = \frac{|\vec{q} - \vec{p}|}{2}$

$\therefore\ |\overline{ES}| = \frac{|\vec{p} + \vec{r}|}{2}$

$= \frac{1}{2}\left|p\right| - \left(\frac{\vec{p} + 5\vec{q}}{6}\right) \qquad \left\{\therefore \vec{r} = -\frac{(\vec{p} + 5\vec{q})}{6}\right\}$

$= \frac{1}{12}\left|5(\vec{p} - \vec{q})\right| = \frac{5}{12}|\bar{p} - \bar{q}|$

$\therefore\ \frac{|\overline{EF}|}{|\overline{ES}|} = \frac{\frac{|\vec{q} - \vec{p}|}{2}}{\frac{5}{12}|\vec{p} - \vec{q}|} = \frac{5}{6} = 1.2$

**10. Correct option is (762).**

**Case I:** Seven–digit number contains exactly two Zeros.

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | | |

:

Zero must be here.

Total ways $\Rightarrow {}^6C_2\, 1 \cdot 2^5 = 480$

**Case II:** Number contains exactly two Ones.

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | | | | | | |

$\Rightarrow {}^6C_1 \cdot 2^5 = 192$

| | | | | | | |
|---|---|---|---|---|---|---|
| 2 | | | | | | |

$\Rightarrow {}^6C_2 \cdot 2^4 = 240 \Rightarrow$ Total $= 432$

**Case III:** Number contains 0, 0, 1, 1, 2, 2, 2.

total ways $= {}^6C_2 \times \frac{5!}{2!3!}$

$= 150$

Total possible numbers $= 480 + 432 - 150$

$= 762$

**11. Correct option is [2.4].**

$$\lim_{x\to 0}\frac{1}{x^3}\left(\frac{\alpha}{2}\int_0^x \frac{1}{1-t^2}dt + \beta x\cos x\right) = 2$$

$$\lim_{x\to 0}\frac{\alpha\int_0^x \frac{dt}{1-t^2} + 2\beta x\cos x}{x^3} = 4$$

Applying L'Hospital's rule,

$$\lim_{x\to 0}\frac{\frac{\alpha}{1-x^2} + 2\beta\cos x - \beta x\sin x}{3x^2} = 4$$

Limit will exist if $\alpha + 2\beta = 0$. ...(1)

Applying series expansion,

$$\lim_{x\to 0}\frac{\alpha\left(1-x^2\right)^{-1} - 2\beta\left(x^2 - \frac{x^4}{6}\right) + 2\beta\left(1 - \frac{x^2}{2}\right)}{3x^2} = 4$$

$$\frac{\alpha - 2\beta - \beta}{3} = 4$$

$\Rightarrow\quad \alpha - 3\beta = 12$ ...(2)

From Eqs (1) and (2), we get,

$-2\beta - 3\beta = 12$

$\beta = -12/5$

$\Rightarrow\quad \alpha = \frac{24}{5}$

$\Rightarrow\quad \alpha + \beta = \frac{24}{5} - \frac{12}{5} = 2.4$

**12. Correct option is [96].**

Given, $f(x) > 0,\ \forall\ x \in \mathbb{R}$

Noω $\quad f(x + y) = f(x)\,f(y)$

Let, $\quad a_1 = a$

Common difference of AP $= d$

$f(x + y) = f(x)\,f(y)$

Differentiating both sides by taking $y$ as a constant,

$f'(x + y) = f'(x)\,f(y)$

now put $\quad x = 0$

$f'(y) = f'(0)\,f(y)$

Let $\quad f'(0) = t,\ y = x$

$\Rightarrow\quad f'(x) = t\,f(x)$

$\Rightarrow\quad \frac{f'(x)}{f(x)} = t$

Integrating both sides, we get,

$\ln f(x) = xt + c$

$\Rightarrow\quad f(x) = e^{xt+c} = \lambda e^{tx}$

$\Rightarrow\quad f(a_{31}) = \lambda e^{ta_{31}}$ (Let $e^c = \lambda$)

$\Rightarrow\quad f(a_{25}) = \lambda e^{ta_{25}}$

$\therefore\quad f(a_{31}) = 64\, f(a_{25})$

$\therefore\quad \lambda\, e^{ta_{31}} = 64\lambda\, e^{ta_{25}}$

$\Rightarrow\quad et(e^{ta_{31} - ta_{25}}) = 64$

$\Rightarrow\quad e^{6td} = 64$

$\Rightarrow\quad e^{td} = 2$

$\therefore\quad f(x) = \lambda(2)^{\frac{x}{d}}$

$\therefore\quad \sum_{i=1}^{50} f(a_i) = 3\left(2^{25} + 1\right)$

$\Rightarrow \lambda\left\{2^{\frac{a_1}{d}} + \ldots + 2^{\frac{a_{50}}{d}}\right\} = 3\left(2^{25} + 1\right)$

$\Rightarrow \lambda\left\{2^{\frac{a}{d}} + \ldots + 2^{\frac{a+49d}{d}}\right\} = 3\left(2^{25} + 1\right)$

$\Rightarrow \lambda \cdot 2^{\frac{a}{d}\left\{1+2+\ldots+2^{49}\right\}} = 3\left(2^{25} + 1\right)$

$\Rightarrow \lambda \cdot 2^{\frac{a}{d}\left\{2^{50}-1\right\}} = 3\left\{2^{25} + 1\right\}$

$$\left\{\begin{array}{l}\therefore\ a + ar + \ldots + ar^{n-1} = a\frac{\left(1-r^n\right)}{1-r} \\ a^2 - b^2 = (a-b)(a+b)\end{array}\right\}$$

$$\lambda\ 2^{\frac{a}{d}\left\{2^{25}-1\right\}} = 3 \qquad ...(1)$$

$$\therefore \sum_{i=6}^{90} f(a_6)+f(a_7)+...+f(a_{30})$$

$$=\left(2^{\frac{a_6}{d}}+2^{\frac{a_7}{d}}+...+2^{\frac{a_{29}}{d}}+2^{(a_{30}/d)}\right)\cdot\lambda$$

$$=\lambda\cdot 2^{\frac{a}{d}\left\{2^5+...+2^{29}\right\}}$$

$$=\lambda\ 2^{\frac{a}{d}}\cdot 2^5\left(1+...+2^{24}\right)$$

$$= 32\lambda\ 2^{\frac{a}{d}\left(2^{25}-1\right)}$$

$= 32 \times 3$ (using Eq. (1))

$= 96$

**13. Correct option is [2].**

$$\frac{dy_1}{dx}-(\sin x)^2 y_1 = 0,\ y_1(1)=5 \qquad ...(1)$$

$$\frac{dy_2}{dx}-(\cos x)^2 y_2 = 0,\ y_2(1)=\frac{1}{3} \qquad ...(2)$$

$$\frac{dy_3}{dx}-\left(\frac{2-x^3}{x^3}\right)y_3 = 0,\ y_3(1)=\frac{5}{5e} \qquad ...(3)$$

Adding all three equations, we get,

$$\frac{dy_1}{y_1}+\frac{dy_2}{y_2}+\frac{dy_3}{y_3}=\left(\sin^2 x+\cos^2 x+\frac{2}{x^3}-1\right)dx$$

Integrating on both sides,

$$\Rightarrow \ln y_1 + \ln y_2 + \ln y_3 = \frac{1}{x^2}+c$$

$$\Rightarrow \qquad \ln(y_1 \cdot y_2 y_3) = -\frac{1}{x^2}+c$$

at $x = 1 \Rightarrow y_1(1) = 5$

$$y_2(1) = \frac{1}{3}$$

$$y_3(1) = \frac{3}{5e}$$

$$\Rightarrow \quad \ln\left(5\times\frac{1}{3}\times\frac{3}{5e}\right) = -1 + c$$

$$-1 = -1 + c$$

$$c = 0$$

$$\therefore y_1\, y_2\, y_3 = e^{(-1/x^2)}$$

$$\lim_{9\to 0}\left(\frac{y_1 . y_2 . y_3+2x}{e^{3x}\sin x}\right)=\lim_{x\to 0^+}\left(\frac{e^{-\frac{1}{x^2}}+2x}{e^{3x}.\frac{\sin x}{x}.x}\right)$$

$$\left\{\begin{array}{l}\therefore \lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right)=1\\ \lim\limits_{x\to 0} e^{3x}=1\end{array}\right\}$$

$$= \lim_{x\to 0}\left(\frac{e^{-\frac{1}{x^2}}}{x}+\frac{2x}{x}\right)$$

$$= \lim_{x\to 0}\left(\frac{1}{xe^{\frac{1}{x^2}}}+\frac{2x}{x}\right) \qquad \left\{\therefore \lim_{x\to 0}+\left(\frac{1}{x\cdot e\left(\frac{1}{x^2}\right)}\right)=0\right\}$$

$= 0+2=2$

**14. Correct option is [C] .**

| Value | 4 | 5 | 8 | 9 | 6 | 12 | 11 |
|---|---|---|---|---|---|---|---|
| Frequency | 5 | $f_1$ | $f_2$ | 2 | 1 | 1 | 3 |

$\therefore$ Sum of frequencies $= 19$

$$12 + f_1 + f_2 = 19$$

$$f_1 + f_2 = 7$$

$\therefore$ Median $= \dfrac{19+1}{2}$ observation

$= 10$th observation

| $x_i$ | $f_i$ | $c_f$ |
|---|---|---|
| 4 | 5 | 5 |
| 5 | $f_i$ | $5 + f_1$ |
| 6 | 1 | $6 + f_1$ |
| 8 | $f_2$ | $6 + f_1 + f_2$ |
| 9 | 2 | $8 + f_1 + f_2$ |
| 11 | 3 | $11 + f_1 + f_2$ |
| 12 | 1 | $12 + f_1 + f_2$ |

$\therefore \qquad 5 + f_1 + 1 = 10$

$f_1 = 4$

$\therefore \qquad f_1 + f_2 = 7$

$\therefore \qquad f_2 = 3$

$$\therefore\ \bar{x}=\frac{\sum x_i f_j}{\sum f_i}=\frac{20+20+60+24+18+33+12}{19}$$

$= 7$

| $x_i$ | $f_i$ | $f_i\,\lvert x_i-\bar{x}\rvert$ |
|---|---|---|
| 4 | 5 | +15 |
| 5 | 4 | +8 |
| 6 | 1 | +1 |
| 8 | 3 | 3 |
| 9 | 2 | 4 |
| 11 | 3 | 12 |
| 12 | 1 | 5 |

$$\sum f_i\,\lvert x_i-\bar{x}\rvert = 15+8+1+3+4+12+5$$

$= 48$

$$\Rightarrow \qquad \alpha=\frac{48}{19}$$

Let $M$ = Median

| $x_i$ | $f_i$ | $f_i\,\lvert x_i - M\rvert$ |
|---|---|---|
| 4 | 5 | 10 |
| 5 | 4 | 4 |
| 6 | 1 | 0 |
| 8 | 3 | 6 |
| 9 | 2 | 6 |
| 11 | 3 | 15 |
| 12 | 1 | 6 |

$$\therefore\ \sum f_i\,\lvert x_i - M\rvert = 47$$

$$\therefore \beta=\frac{47}{19}$$

$$\therefore\ \sigma^2=\frac{\sum f_i\left(x_i-\bar{x}\right)^2}{\sum f_i}$$

$$=\frac{5\times 9+4\times 4+1\times 1+3\times 1+2\times 4+3\times 16+1\times 25}{19}$$

$$\sigma^2=\frac{146}{19}$$

**15. Correct option is (B).**

Let $y = 10x^3 - 45x^2 + 60x + 35 x \in [1, 2]$

(differentiating both sides)

$$\therefore \quad \frac{dy}{dx} = 30x^2 - 90x + 60$$

$$= 30\left(x^2 - 3x + 2\right)$$

$$= 30\left(x-1\right)\left(x-2\right)$$

$$\therefore \quad \frac{dy}{dx} \leq 0 \;\; \forall\, x \in [1, 2].$$

$\therefore$ Function is decreasing, $\forall x \in [1, 2]$.

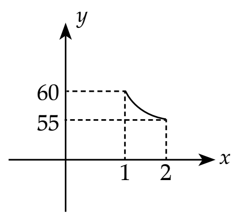
$\therefore \quad y(1) = 10 - 45 + 60 + 35 = 60$

$y(2) = 80 - 45 \times 4 + 120 + 35$

$= 80 - 180 + 120 + 35$

$= 55$

$\therefore \quad 55 \leq y \leq 60$




$$\Rightarrow \quad \frac{55}{9} \leq \frac{y}{9} \leq \frac{60}{9}$$

$\because \quad 6.1 \leq y \leq 6.6$

$[y] = 6 \Rightarrow$ minimum value of $n = 9$

(Q) $\quad g(x) = (2n^2 - 13n - 15)(x^3 + 3x)$

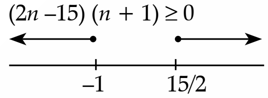
$g'(x) = (2n^2 - 13n - 15)(3x^2 + 3)$

$g(x)$ is an increasing function $\Rightarrow g'(x) \geq 0$

$\Rightarrow \quad 2n^2 - 3n - 15 \geq 0$

$2n^2 + 2n - 15n - 15 \geq 0$

$(2n - 15)(n + 1) \geq 0$




Minimum value of $n = 8$

(R) $\quad h(x) = (x^2 - 9)^n (x^2 + 2x + 3)$

$n > 5$

$h(x) = (x + 3)^n (x - 3)^n (x + 1)^2 + 2)$

$h(3) = 0$

$h(x)$ has local minima at $x = 3$

$h(3^-) > 0$ and $h(3^+) > 0$

$\Rightarrow n$ must be even

The minimum value of $n = 6$

(S) $\therefore \cos(-\theta) = \cos\theta$

$$\therefore \cos\left|x - k + \frac{1}{2}\right| = \cos\left(x - k + \frac{1}{2}\right)$$

$$l(x) = \sum_{k=0}^{4}\left(\sin|x-k| + \cos\left(x - k + \frac{1}{2}\right)\right)$$

$$\sum_{k=0}^{4}\sin|x-k| + \sum_{k=0}^{4}\cos\left(x - k + \frac{1}{2}\right)$$

$\downarrow$ $\qquad\qquad$ $\downarrow$

Not differentiable $\qquad$ Is always differentiable

Number of values of $x_0 = 5$

**16. Correct option is (A).**

$\vec{w}\cdot\vec{v} = 0$

$\vec{u}\cdot\vec{v} = 0$

$\vec{w}\cdot\vec{u} = 0$

$\therefore\ \vec{u}\times\vec{v} = \vec{w}$ and $\vec{v}\times\vec{w} = \vec{u}$

$$\vec{v}\times\vec{w} = \vec{u}$$

$$\Rightarrow \quad \left(\vec{v}\times\vec{w}\right)\times\vec{v} = \vec{u}\times\vec{v}$$

$$\Rightarrow \quad \left(\vec{v}\cdot\vec{v}\right)\vec{w} - \left(\vec{w}\cdot\vec{v}\right)\vec{v} = \vec{w}$$

$$\because \quad |\vec{v}| = 1$$

Now,

$$\vec{u}\times\vec{v} = \vec{w}$$

$$\Rightarrow \quad |\vec{u}\times\vec{v}| = |\vec{w}|$$

$$\Rightarrow \quad |\vec{u}|^2|\vec{v}|^2 = |\vec{w}|^2$$

$$|\vec{u}|^2 - 0 = 6$$

$$|\vec{u}|^2 = 6$$

$\therefore\ \vec{u} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$

$\alpha^2 + \beta^2 + \gamma^2 = 6$ ...(i)

Also, we have $\vec{U}\cdot\vec{W} = 0$

$\alpha + \beta - 2\gamma = 0$ ...(ii)

Now, from

$-ta + \beta + \gamma = 0$

$\alpha + t\beta + \gamma = 0$

$\alpha + \beta - t\gamma = 0$

**Case I:** When $\alpha = \beta = \gamma = 0$, trivial solution is not possible.

**Case II:** When non-trivial solution

$t = -1$ and $t = 2$

**Case I** : When $\quad t = -1$

Then $\quad \alpha + \beta + \gamma = 0$ ...(iii)

and $\quad \alpha + \beta - 2\gamma = 0$ ...(iv)

$\alpha^2 + \beta^2 + \gamma^2 = 6$ ...(v)

$3\gamma = 0$

$v = 0$

and $\quad \alpha = -\beta$

From Eq. (v), we get,

$\alpha_2 + \beta_2 + 0 = 6$

$a = \pm\sqrt{3}$

Now, when $t = 2$

$-2\alpha + \beta + \gamma = 0$ ...(vi)

and $\quad \alpha + \beta - 2\gamma = 0$ ...(vii)

$\alpha^2 + \beta^2 + \gamma^2 = 6$

Subtracting Eq. (vii) from Eq. (vi),

$-3\alpha + 3\gamma = 0$

$\alpha = \gamma = b$

Put in $\quad \alpha = \pm\sqrt{2}$

Now when $\alpha = \sqrt{2}$, We have, $t + 3 = 2 + 3 = 5$