# JEE Advanced (2025)

# PAPER 2

## MATHEMATICS

*General Instructions*:

**SECTION 1 (Maximum Marks: 12)**

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme:**
  *Full Marks* : +3 If **ONLY** the correct option is chosen;
  *Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);
  *Negative Marks* : –1 in all other cases.

**1.** Let $x_0$ be the real number such that $e^{x_0} + x_0 = 0$. For a given real number $\alpha$, define

$$g(x) = \frac{3xe^2 + 3x - \alpha e^x - \alpha x}{3(e^x + 1)}$$

for all real numbers $x$.

Then which one of the following statements is TRUE?

**(A)** For $\alpha = 2$, $\lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 0$

**(B)** For $\alpha = 2$, $\lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 1$

**(C)** For $\alpha = 3$, $\lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = 0$

**(D)** For $\alpha = 3$, $\lim_{x \to x_0} \left| \frac{g(x) + e^{x_0}}{x - x_0} \right| = \frac{2}{3}$

**2.** Let $\mathbb{R}$ denote the set of all real numbers. Then the area of the region

$$\left\{(x, y) \in \mathbb{R} \times \mathbb{R} : x > 0, y > \frac{1}{x}, 5x - 4y - 1 > 0, 4x + 4y - 17 < 0\right\}$$ is:

**(A)** $\frac{17}{16} - \log_e 4$ **(B)** $\frac{33}{8} - \log_e 4$

**(C)** $\frac{57}{8} - \log_e 4$ **(D)** $\frac{17}{2} - \log_e 4$

**3.** The total number of real solutions of the equation

$$\theta = \tan^{-1}(2\tan\theta) - \frac{1}{2}\sin^{-1}\left(\frac{6\tan\theta}{9 + \tan^2\theta}\right)$$ is

(Here, the inverse trigonometric functions $\sin^{-1} x$ and $\tan^{-1} x$ assume values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, respectively).

**(A)** 1 **(B)** 2 **(C)** 3 **(D)** 5

**4.** Let $S$ denote the locus of the point of intersection of the pair of lines

$$4x - 3y = 12\alpha$$
$$4\alpha x + 3\alpha y = 12$$

where $\alpha$ varies over the set of non-zero real numbers. Let $T$ be the tangent to $S$ passing through the points $(p, 0)$ and $(0, q)$, $q > 0$, and parallel to the line $4x - \frac{3}{\sqrt{2}}y = 0$. Then the value $pq$ is:

**(A)** $-6\sqrt{2}$ **(B)** $-3\sqrt{2}$ **(C)** $-9\sqrt{2}$ **(D)** $-12\sqrt{2}$

*General Instructions*:

**SECTION 2 (Maximum Marks: 16)**

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY OR MORE THAN ONE** of these four options is (are) the correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme:**
  *Full Marks* : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  *Partial Marks* : +3 If all the four options are correct but **ONLY** three options are chosen;
  *Partial Marks* : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  *Partial Marks* : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  *Zero Marks* : 0 If none of the options is chosen (i.e., the question is unanswered);
  *Negative Marks* : –2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e., the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –2 marks.

5. Let $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $P = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$. Let $Q = \begin{pmatrix} x & y \\ z & 4 \end{pmatrix}$ for some non-zero real numbers $x$, $y$ and $z$, for which there is $2 \times 2$ matrix $R$ with all entries being non-zero real numbers, such that $QR = RP$.
Then which of the following statements is (are) TRUE?
**(A)** The determinant of $Q - 2I$ is zero
**(B)** The determinant of $Q - 6I$ is 12
**(C)** The determinant of $Q - 3I$ is 15
**(D)** $yz = 2$

6. Let $S$ denote the locus of the mid-points of those chords of the parabola $y^2 = x$, such that the area of the region enclosed between the parabola and the chord is 4/3. Let $R$ denote the region lying in the first quadrant, enclosed by the parabola $y^2 = x$, the curve $S$, and the lines $x = 1$ and $x = 4$.
Then which of the following statements is (are) TRUE?
**(A)** $(4, \sqrt{3}) \in S$
**(B)** $(5, \sqrt{2}) \in S$
**(C)** Area of $R$ is $\frac{14}{3} - 2\sqrt{3}$
**(D)** Area of $R$ is $\frac{14}{3} - \sqrt{3}$

7. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two distinct points on the ellipse
$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$
such that $y_1 > 0$, and $y_2 > 0$. Let $C$ denote the circle $x^2 + y^2 = 9$, and $M$ be the point (3, 0).
Suppose the line $x = x_1$ intersects $C$ at $R$, and the line $x = x_2$ intersects $C$ at $S$, such that the $y$-coordinates of $R$ and $S$ are positive. Let $\angle ROM = \frac{\pi}{6}$ and $\angle SOM = \frac{\pi}{3}$, where $O$ denotes the origin (0, 0). Let $|XY|$ denote the length of the line segment $XY$.
Then which of the following statements is (are) TRUE?
**(A)** The equation of the line joining $P$ and $Q$ is $2x + 3y = 3(1+\sqrt{3})$
**(B)** The equation of the line joining $P$ and $Q$ is $2x + y = 3(1+\sqrt{3})$
**(C)** If $N_2 = (x_2, 0)$, then $3|N_2Q| = 2|N_2S|$
**(D)** If $N_1 = (x_1, 0)$, then $9|N_1P| = 4|N_1R|$

8. Let $\mathbb{R}$ denote the set of all real numbers. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by
$$f(x) = \begin{cases} \dfrac{6x + \sin x}{2x + \sin x} & \text{if } x \neq 0, \\ \dfrac{7}{3} & \text{if } x = 0 \end{cases}$$
Then which of the following statements is (are) TRUE?
**(A)** The point $x = 0$ is a point of local maxima of $f$
**(B)** The point $x = 0$ is a point of local minima of $f$
**(C)** Number of points of local maxima of $f$ in the interval $[\pi, 6\pi]$ is 3
**(D)** Number of points of local minima of $f$ in the interval $[2\pi, 4\pi]$ is 1

***General Instructions***:

**SECTION 3 (Maximum Marks: 32)**

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**

*Full Marks* : +4 If **ONLY** the correct numerical value is entered in the designated place,
*Zero Marks* : 0 In all other cases.

9. Let $y(x)$ be the solution of the differential equation
$$x^2 \frac{dy}{dx} + xy = x^2 + y^2, \quad x > \frac{1}{e}, \text{ satisfying } y(1) = 0.$$
Then the value of $2\frac{(y(e))^2}{y(e^2)}$ is:

10. Let $a_0, a_1, \ldots, a_{23}$ be real numbers such that
$$\left(1 + \frac{2}{5}x\right)^{23} = \sum_{i=0}^{23} a_i x^i$$
for every real number $x$. Let $a_r$ be the largest among the numbers $a_j$ for $0 \le j \le 23$.
Then the value of $r$ is ................. .

11. A factory has a total of three manufacturing units, $M_1$, $M_2$, and $M_3$, which produce bulbs independent of each other. The units $M_1$, $M_2$ and $M_3$ produce bulbs in the proportions of 2:2:1, respectively. It is known that 20% of the bulbs produced in the factory are defective. It is also known that, of all the bulbs produced by $M_1$, 15% are defective. Suppose that, if a randomly chosen bulb produced in the factory is found to be defective, the probability that it was produced by $M_2$ is $\frac{2}{5}$.
If a bulb is chosen randomly from the bulbs produced by $M_3$, then the probability that it is defective is ................. .

12. Consider the vectors
$$\vec{x} = \hat{i} + 2\hat{j} + 3\hat{k}, \quad \vec{y} = 2\hat{i} + 3\hat{j} + \hat{k}, \quad \text{and} \quad \vec{z} = 3\hat{i} + \hat{j} + 2\hat{k}.$$
For two distinct positive real numbers $\alpha$ and $\beta$, define
$$\vec{X} = \alpha\vec{x} + \beta\vec{y} - \vec{z}, \quad \vec{Y} = \alpha\vec{y} + \beta\vec{z} - \vec{x}, \quad \text{and} \quad \vec{Z} = \alpha\vec{z} + \beta\vec{x} - \vec{y}.$$
If the vectors added from source lie in a plane, then the value of $\alpha + \beta - 3$ is ................. .

13. For a non-zero complex number $z$, let $\arg(z)$ denote the principal argument of $z$, with $-\pi < \arg(z) \le \pi$. Let $\omega$ be the cube root of unity for which $0 < \arg(\omega) < \pi$. Let
$$\alpha = \arg\left(\sum_{n=1}^{2025} (-\omega)^n\right).$$
Then the value of $\frac{3\alpha}{\pi}$ is ................. .

**14.** Let $\mathbb{R}$ denote the set of all real numbers. Let $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to (0, 4)$ be functions defined by

$$f(x) = \log_e(x^2 + 2x + 4), \text{ and } g(x) = \frac{4}{1+e^{-2x}}$$

Define the composite function $f \circ g^{-1}$ by $(f \circ g^{-1})(x) = f(g^{-1}(x))$, where $g^{-1}$ is the inverse of the function $g$.

Then the value of the derivative of the composite function $f \circ g^{-1}$ at $x = 2$ is ________ .

**15.** Let

$$\alpha = \frac{1}{\sin 60^\circ \sin 61^\circ} + \frac{1}{\sin 62^\circ \sin 63^\circ} + \ldots + \frac{1}{\sin 118^\circ \sin 119^\circ}.$$

Then the value of $\left(\frac{\operatorname{cosec} 1^\circ}{\alpha}\right)^2$ is ________ .

**16.** If

$$\alpha = \int_{1/2}^{2} \frac{\tan^{-1} x}{2x^2 - 3x + 2} dx,$$

then the value of $\sqrt{7} \tan\left(\frac{2\alpha\sqrt{7}}{\pi}\right)$ is ________ .

(Here, the inverse trigonometric function $\tan^{-1} x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.)

# Answer Key

| Q.No. | Answer key | Topic name | Chapter name |
|---|---|---|---|
| 1 | (C) | Basic limit | Limit continuity and differentiability |
| 2 | (B) | Area | Area |
| 3 | (C) | Basic identities | Inverse trigonometric function |
| 4 | (A) | Tangent | Hyperbola |
| 5 | (A, B) | Product of matrices | Matrix |
| 6 | (A, C) | Area between chord and parabola | Area |
| 7 | (A, C) | Chord equation | Ellipse |
| 8 | (B, C, D) | Local maxima or minima | Application of derivative |
| 9 | [0.7 to 0.8] | First-order differential equation | Differential equation |
| 10 | 6 | Numerically greatest term | Binomial theorem |
| 11 | [0.27 to 0.33] | Total probability | Probability |
| 12 | –2 | Co-planar vector | Vector |
| 13 | –2 | Argument | Complex number |
| 14 | 0.2 to 0.3 | Derivative of inverse function | Differentiation |
| 15 | 3 | Basic identities | Trigonometric ratio and identities |
| 16 | 21 | Properties of definite integration | Definite integration |

# ANSWERS WITH EXPLANATIONS

**1. Correct option is (C).**

$$g(x) = \frac{3xe^x + 3x - \alpha e^x - \alpha x}{3(e^x + 1)} \text{ and}$$

$e^{x_0} + x_0 = 0$

$$\Rightarrow \quad g(x) = \frac{3x(e^x+1) - \alpha(e^x + x)}{3(e^x+1)}$$

$$\Rightarrow \quad g(x) = x - \frac{\alpha(e^x + x)}{3(e^x+1)}$$

Now, $\lim\limits_{x \to x_0} \frac{g(x) + e^{x_0}}{x - x_0}$

Let $x - x_0 = h$

$x \to x_0 \Rightarrow h \to 0$

$$\lim_{h \to 0} \frac{g(h + x_0) + e^{x_0}}{h}$$

$$= \lim_{h \to 0} \frac{h + x_0 - \frac{\alpha(e^{h+x_0} + h + x_0)}{3(e^{h+x_0}+1)} + e^{x_0}}{h}$$

$$= \lim_{h \to 0} \left(1 - \frac{\alpha(e^{h+x_0} + h + x_0)}{3h(e^{h+x_0}+1)}\right)$$

$$= \lim_{h \to 0} \left(1 - \frac{\alpha}{3(e^{x_0+h}+1)}\left\{\frac{e^{x_0}(e^h - 1)}{h} + 1\right\}\right)$$

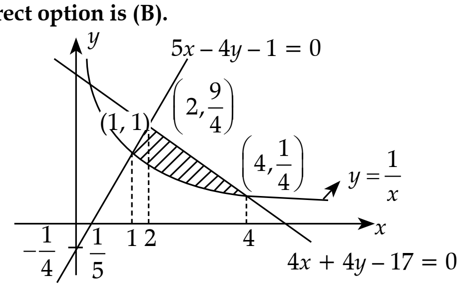
$$= 1 - \frac{\alpha}{3(e^{x_0}+1)}(e^{x_0}+1)$$

$$= 1 - \frac{\alpha}{3}$$

For $\alpha = 2$, the required limit is $1 - \frac{2}{3} = \frac{1}{3}$.

For $\alpha = 3$, the required limit is $1 - \frac{3}{3} = 0$.

**2. Correct option is (B).**




$\therefore$ $xy = 1$ ...(i)

$5x - 4y = 1$ ...(ii)

$$4x + 4y = 17 \quad ...(iii)$$

Solving Eqs (i) and (ii), we get,

$$5x - \frac{4}{x} = 1$$

$$\Rightarrow \quad 5x^2 - 4 = x$$

$$\Rightarrow \quad 5x^2 - x - 4 = 0$$

$$\therefore x = 1$$

Solving Eqs (i) and (iii), we get,

$$4x + \frac{4}{x} = 17$$

$$\Rightarrow \quad 4x^2 + 4 = 17x$$

$$\Rightarrow \quad 4x^2 - 17x + 4 = 0$$

$$\Rightarrow \quad x = 4, \frac{1}{4}$$

Solving Eqs (ii) and (iii), we get,

$$9x = 18$$

$$x = 2$$

$$\text{Required Area} = \frac{1}{2} \times \frac{13}{4} \times 1 + \frac{1}{2} \times \frac{10}{4} \times 2 - \int_1^4 \frac{1}{x}\,dx$$

$$= \frac{33}{8} - \ln 4$$

**3. Correct option is (C).**

$$\theta = \tan^{-1}(2\tan\theta) - \frac{1}{2}\sin^{-1}\left(\frac{6\tan\theta}{9+\tan^2\theta}\right)$$

$$\Rightarrow \theta = \tan^{-1}(2\tan\theta) - \frac{1}{2}\sin^{-1}\left(\frac{2\left(\frac{\tan\theta}{3}\right)}{1+\left(\frac{\tan\theta}{3}\right)^2}\right)$$

$$\Rightarrow \theta = \tan^{-1}(2\tan\theta) - \frac{1}{2}\sin^{-1}(\sin 2\alpha)$$

$$\left\{\text{Let } \tan\alpha = \frac{\tan\theta}{3}, \sin 2A = \frac{2\tan A}{1+\tan^2 A}\right\}$$

$$\Rightarrow \theta = \tan^{-1}(2\tan\theta) - \frac{1}{2} \times 2\alpha$$

$$\Rightarrow \theta + \alpha = \tan^{-1}(2\tan\theta)$$

$$\Rightarrow \tan(\theta + \alpha) = 2\tan\theta$$

$$\Rightarrow \frac{\tan\theta + \tan\alpha}{1 - \tan\theta\tan\alpha} = 2\tan\theta$$

$$\Rightarrow \frac{\tan\theta + \frac{\tan\theta}{3}}{1 - \tan\theta \cdot \frac{\tan\theta}{3}} = 2\tan\theta$$

$$\Rightarrow \frac{4\tan\theta}{3} = 2\tan\theta\left(1 - \frac{\tan^2\theta}{3}\right)$$

$$\Rightarrow 4\tan\theta = 6\tan\theta - 2\tan^3\theta$$

$$\Rightarrow 2\tan^3\theta - 2\tan\theta = 0$$

$$\Rightarrow \tan\theta(\tan^2\theta - 1) = 0$$

$$\Rightarrow \tan\theta = 0, 1, -1$$

$$\Rightarrow \theta = 0, \frac{\pi}{4}, \frac{-\pi}{4}$$

**4. Correct option is (A).**

$$4x - 3y = 12\alpha \quad ...(i)$$

$$4\alpha x + 3\alpha y = 12 \quad ...(ii)$$

From Eqs (i) and (ii), we get,

$$(4x - 3y)(4\alpha x + 3\alpha y) = 12\alpha \cdot 12$$

$$16x^2 - 9y^2 = 144$$

$$\Rightarrow \quad \frac{x^2}{9} - \frac{y^2}{16} = 1$$

$T$ is parallel to $4x - \frac{3y}{\sqrt{2}} = 0$

$$\Rightarrow m = \frac{4}{3/\sqrt{2}} = \frac{4\sqrt{2}}{3}$$

$\therefore$ Equation of tangent,

$$y = mx + \sqrt{a^2m^2 - b^2}$$

$$\Rightarrow y = \frac{4\sqrt{2}}{3}x + \sqrt{9 + \frac{16\times 2}{9} - 16}$$

$$\Rightarrow y = \frac{4\sqrt{2}}{3}x + 4$$

It passes through $(p, 0)$.

$$\Rightarrow 0 = \frac{4\sqrt{2}}{3}p + 4$$

$$p = \frac{-3}{\sqrt{2}}$$

It also pass through $(0, q)$.

$$\Rightarrow \quad q = 0 + 4$$

$$q = 4$$

$$\therefore \quad p \cdot q = \frac{-3}{\sqrt{2}} \times 4 = -6\sqrt{2}$$

**5. Correct options are (A, B).**

Given, $QR = RP$

Subtracting $2R$ on both sides,

$$QR - 2R = RP - 2R$$

$$\Rightarrow \quad (Q - 2I)R = R(P - 2I) \quad ...(i)$$

$$\because \quad P = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\Rightarrow \quad P - 2I = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \quad P - 2I = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$|P - 2I| = 0$$

$\therefore$ From Eq. (i), we get,

$$|Q - 2I||R| = |R||P - 2I|$$

$$|Q - 2I||R| = |R| \times 0$$

$$|Q - 2I| = 0$$

Now,

$$QR = RP$$

Subtracting $6R$ on both side,

$$QR - 6R = RP - 6R$$

$$\Rightarrow \quad (Q - 6I)R = R(P - 6I) \quad ...(ii)$$

$$\Rightarrow \quad P - 6I = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 0 \\ 0 & -3 \end{bmatrix}$$

$$\therefore \quad |P - 6I| = 12$$

From Eq. (ii), we get,

$$|Q - 6I||R| = |R||P - 6I|$$

$$|Q - 6I| = 12$$

Now,

$$QR = RP$$

Subtracting $3R$ on both sides,

$$QR - 3R = RP - 3R$$

$$(Q - 3I)R = R(P - 3I) \quad ...(iii)$$

$$P - 3I = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$P-3I=\begin{bmatrix}-1 & 0\\ 0 & 0\end{bmatrix}$$

$$|P-3I|=0$$

From Eq. (iii), we get,

$$|Q-3I|\,|R|=|R|\,|P-3I|$$

$$|Q-3I|\,|R|=|R|\times 0$$

$$|Q-3I|=0$$

**6. Correct options are (A, C).**

$$y^2 = x$$

Let point $p = (h, k)$

$$T : ky-\frac{x+h}{2}$$

$$S_1 = k^2-h$$

Equation of chord with point $p$ as mid-point $\Rightarrow T = S_1$

$$ky-\left(\frac{x+h}{2}\right)=k^2-h$$

$\Rightarrow x-2ky + 2k^2-h = 0$ ...(i)

Solving Eq. (i) with $y^2 = x$

$\Rightarrow y^2-2ky + 2k^2-h = 0$

Let the roots be $y_1$ and $y_2$.

$\therefore\quad y_1 + y_2 = 2k$

$\&\quad y_1y_2 = 2k^2-h$

$\Rightarrow\quad y_1^2+y_2^2 = 4k^2-2(2k^2-h)$

$= 2h$

Now, $\left|\int_{y_1}^{y_2}(2ky-2k^2+h-y^2)dy\right|=\frac{4}{3}$

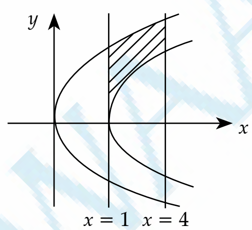
$$\left(ky^2-2k^2y+hy-\frac{y^3}{3}\right)\Bigg|_{y_1}^{y_2}=\frac{4}{3}$$

$$k(y_2^2-y_1^2)-2k^2(y_2-y_1)+h(y_2-y_1)-\frac{y_2^3-y_1^3}{3}=\frac{4}{3}$$

$$(y_2-y_1)\left\{3k(y_1+y_2)-6k^2+3h-(y_1^2+y_2^2+y_1y_2)\right\}=4$$

$$(y_2-y_1)\ \{6k^2-6k^2 + 3h-(2k^2 + h)\} = 4$$

$$(y_2-y_1)\ (2h-2k^2) = 4$$




$$\sqrt{4h-4k^2}(2h-2k^2)=4$$

$\therefore\quad h-k^2 = 1$

Now, $h = x, k = y$

$y^2 = x-1$

(A) $(4,\sqrt{3})$ lies on $S$ is correct.

(B) $(5,\sqrt{2})$ lies on $S$ is wrong.

Area of region $R = \int_1^4(\sqrt{x}-\sqrt{x-1})\,dx$

$$=\frac{2}{3}\left\{x^{3/2}-(x-1)^{3/2}\right\}_1^4$$

$$=\frac{2}{3}\left\{8-3\sqrt{3}-1\right\}=\frac{2}{3}(7-3\sqrt{3})=\frac{14}{3}-2\sqrt{3}$$

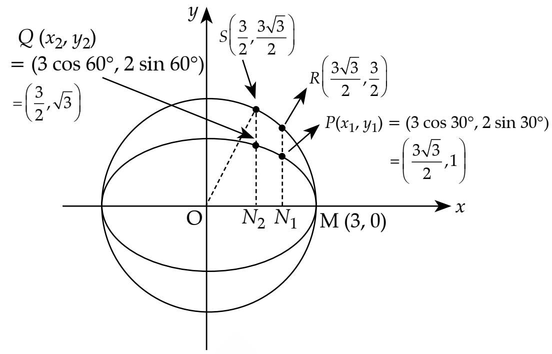
**7. Correct options are (A, C).**

$$\frac{x^2}{9}+\frac{y^2}{4}=1 \quad ...(1)$$

$$x^2 + y^2 = 9 \quad ...(2)$$

$$m_{PQ}=\frac{\sqrt{3}-1}{\frac{3}{2}(1-\sqrt{3})}$$

$$\Rightarrow\quad m_{PQ}=-\frac{2}{3}$$




Equation of line $PQ$,

$$y-\sqrt{3}=\frac{-2}{3}\left(x-\frac{3}{2}\right)$$

$\Rightarrow\quad 3y-3\sqrt{3} = -2x + 3$

$\Rightarrow\quad 2x + 3y = 3+3\sqrt{3}$

$\because\quad |N_2Q| = \sqrt{3}$

and $\quad |N_2S| = \frac{3\sqrt{3}}{2}$

$\therefore\quad 3|N_2Q| = 2|N_2S|$

$|N_1P| = 1$

and $\quad |N_1R| = \frac{3}{2}$

$\therefore\quad |3N_1P| = 2|N_1R|$

**8. Correct answer is [B, C, D].**

$$f(x)=\begin{cases}\dfrac{6x+\sin x}{2x+\sin x} & \text{if } x\neq 0,\\[2mm] \dfrac{7}{3} & \text{if } x=0.\end{cases}$$

$$\lim_{x\to 0}f(x)=\lim_{x\to 0}\left(\frac{6x+\sin x}{2x+\sin x}\right)$$

$$=\left(\lim_{x\to 0}\frac{6+\frac{\sin x}{x}}{2+\frac{\sin x}{x}}\right)=\frac{6+1}{2+1}=\frac{7}{3}=f(0)$$

$\therefore f(x)$ is continuous at $x = 0$.

$$f'(x)=\frac{(2x+\sin x)(6+\cos x)-(2+\cos x)(6x+\sin x)}{(2x+\sin x)^2}:$$

$$=\frac{4(\sin x-x\cos x)}{(2x+\sin x)^2}$$

$$f'(x)=\frac{4\cos x(\tan x-x)}{(2x+\sin x)^2}$$

$f'(x) = 0$ at $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$

$$\pi < \alpha_1 < \frac{3\pi}{2}$$

$$2\pi < \alpha_2 < \frac{5\pi}{2}$$

$$3\pi < \alpha_3 < \frac{7\pi}{2}$$

$$4\pi < \alpha_4 < \frac{9\pi}{2}$$

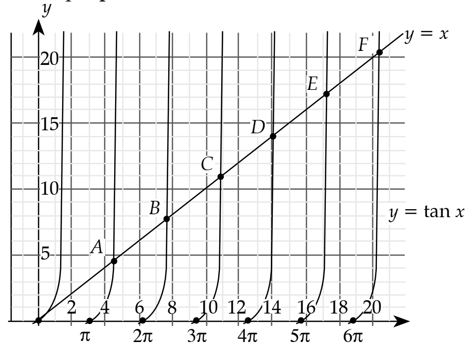
$$5\pi < \alpha_5 < \frac{11\pi}{2}$$

$$f'(x)=\frac{4\cos x(\tan x-x)}{(2x+\sin x)^2}$$

$$f'(\alpha_1^-)=\frac{4(-\text{ve})(-\text{ve})}{(+\text{ve})}>0$$

$$f'(\alpha_1^+) = \frac{4(-\text{ve})(+\text{ve})}{(+\text{ve})} < 0$$

$\therefore$ $x = \alpha_1$ is point of local maxima




Similarly $x = \alpha_2$ is point of minima & so on.

So $[\pi, 6\pi]$ points of local maxima are $A, C, E \rightarrow 3$ Points &

$[2\pi, 4\pi]$ points of local minima are $B$ only $\rightarrow 1$ Point.

**9. Correct answer is [0.75].**

$$x^2\frac{dy}{dx} + xy = x^2 + y^2$$

$\Rightarrow$ $$x^2\frac{dy}{dx} = x^2 + y^2 - xy$$

$\Rightarrow$ $$\frac{dy}{dx} = 1 + \frac{y^2 - xy}{x^2}$$

Let $y = vx.$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

then

$\Rightarrow$ $$v + x\frac{dv}{dx} = 1 + \frac{v^2x^2 - vx^2}{x^2}$$

$\Rightarrow$ $$v + x\frac{dv}{dx} = 1 + v^2 - v$$

$\Rightarrow$ $$x\frac{dv}{dx} = 1 + v^2 - 2v$$

$\Rightarrow$ $$\int\frac{dv}{(v-1)^2} = \int\frac{dx}{x}$$

$\Rightarrow$ $$\frac{-1}{v-1} = \ln x + \ln c \qquad (\because x > 0)$$

$\Rightarrow$ $$\frac{-1}{\frac{y}{x} - 1} = \ln cx$$

$\Rightarrow$ $$\frac{x}{x-y} = \ln cx$$

$\because$ $y(1) = 0$

$\therefore$ $$\frac{1}{1-0} = \ln c$$

$\Rightarrow$ $c = e$

$\therefore$ $$\frac{x}{x-y} = \ln xe$$

Let $x = e,$

$\therefore$ $$\frac{e}{e-y} = \ln e^2$$

$$e = 2e - 2y$$

$$y = \frac{e}{2}$$

Now, let $x = e^2$

$\Rightarrow$ $$\frac{e^2}{e^2 - y} = \ln e^3$$

$\Rightarrow$ $$e^2 = 3e^2 - 3y$$

$\Rightarrow$ $$y = \frac{2e^2}{3}$$

$\therefore$ $$\frac{2(y(e))^2}{y(e^2)} = \frac{2\times\left(\frac{e}{2}\right)^2}{\frac{2e^2}{3}}$$

$$= \frac{3}{4} = 0.75$$

**10. Correct answer is [6].**

$$\left(1 + \frac{2}{5}x\right)^{23} = \sum_{i=0}^{23} a_i x^i$$

Let $T_{r+1} = a_r x^r$

If $T_{r+1}$ is numerically the greatest term, then,

$$r \geq \frac{(n+1)|x|}{|x|+1} - 1$$

$\Rightarrow$ $$r \geq \frac{(23+1)\left|\frac{2x}{5}\right|}{1 + \left|\frac{2x}{5}\right|} - 1$$

We want to find the largest value of $a_r$,

$\therefore$ $|x| = 1$

$\Rightarrow$ $$r \geq \frac{24\times 2}{7} - 1$$

$\Rightarrow$ $r \geq 6.8 - 1$

$\Rightarrow$ $r \geq 5.8$

$\therefore$ $r_{\min} = 6$

$\Rightarrow T_7$ is the largest.

$\Rightarrow a_6$ is the largest

**11. Correct answer is [0.3].**

$$P(D) = P(M_1)\cdot P\left(\frac{D}{M_1}\right) + P(M_2)\cdot P\left(\frac{D}{M_2}\right) + P(M_3)\cdot P\left(\frac{D}{M_3}\right)$$

$\Rightarrow$ $$\frac{20}{100} = \frac{40}{100}\times\frac{15}{100} + \frac{40}{100}\times D_2 + \frac{20}{100}\times D_3$$

$\Rightarrow$ $$\frac{14}{100} = \frac{40}{100}D_2 + \frac{20}{100}D_3 \qquad \text{...(1)}$$

and $$P\left(\frac{M_2}{D}\right) = \frac{P(M_2)\cdot P\left(\frac{D}{M_2}\right)}{20\%}$$

$\Rightarrow$ $$\frac{2}{5}\times\frac{100}{20} = \frac{40}{100}\times D_2$$

$\Rightarrow$ $$\frac{8}{100} = \frac{40}{100}D_2$$

$\Rightarrow$ $$D_2 = \frac{1}{5}$$

From Eq. (1), we get,

$$\frac{14}{100} = \frac{40}{100}\times\frac{1}{5} + \frac{20}{100}D_3$$

$\Rightarrow$ $6 = 20\,D_3$

$\Rightarrow$ $$D_3 = \frac{6}{20} = \frac{3}{10}$$

**12. Correct answer is [–2].**

$\vec{X}, \vec{Y}, \vec{Z}$ lie in a plane.

$\therefore$ $[\vec{X}\ \vec{Y}\ \vec{Z}] = 0$

$$\begin{vmatrix} \alpha & \beta & -1 \\ -1 & \alpha & \beta \\ \beta & -1 & \alpha \end{vmatrix} \times \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix} = 0$$

$\alpha^3 + \beta^3 - 1 + 3\alpha\beta = 0$

$\alpha^3 + \beta^3 + (-1)^3 = 3\alpha\beta(-1)$

{if $A^3 + B^3 + C^3 = 3ABC \Rightarrow A = B = C$ Or, $A + B + C + 0$

$\because\ \alpha = \beta = -1$ (Rejected)}

$\therefore\ \alpha + \beta - 1 = 0 \Rightarrow \alpha + \beta - 3 = -2$

**13. Correct answer is (–2).**

$$\because\ S = \sum_{n=1}^{2025} (-\omega)^n = (-\omega) + (-\omega)^2 + (-\omega)^3 + \ldots\ldots + (-\omega)^{2025}$$

Applying sum of G.P.,

$$(-\omega)\left(\frac{(-\omega)^{2025} - 1}{-\omega - 1}\right) = \omega\left(\frac{-\omega^{2025} - 1}{\omega + 1}\right) \qquad \left\{\begin{array}{r}\because 1 + \omega + \omega^2 = 0 \\ \omega^3 = 1\end{array}\right\}$$

$$\therefore \quad S = \frac{-2\omega}{1+\omega} = \frac{-2\omega}{-\omega^2} = \frac{2}{\omega} = 2\omega^2$$

$\therefore\ \omega^{-1} = \omega^2$, because $\omega \cdot \omega^2 = 1$

$$S = -1 - \sqrt{3}i$$

$$\text{Arg}(S) = \arg\left(-1 - \sqrt{3}i\right)$$

$$\alpha = \frac{-2\pi}{3}$$

$$\therefore \quad \frac{3\alpha}{\pi} = -2$$

**14. Correct answer is (0.25).**

$$g(x) = \frac{4}{1 + e^{-2x}}$$

$$\therefore \quad g(0) = \frac{4}{1+1} = 2 = 2$$

$$\Rightarrow \quad g^{-1}(2) = 0$$

$$\frac{d}{dx}\left(f\left(g^{-1}(x)\right)\right) = f'(g^{-1}(x))(g^{-1}(x))'$$

Put $x = 2$

$f'(g^{-1}(2)) \times (g^{-1}(2))'$

$f'(0) \times (g^{-1}(2))'$ ... (1)

$f(x) = \log_e(x^2 + 2x + 4)$

Now
$$f'(x) = \frac{2x+2}{x^2 + 2x + 4}$$

$$f'(0) = \frac{1}{2} \quad \text{...(2)}$$

$$g^{-1}(g(x)) = x$$

$$(g^{-1}(g(x)))' = \frac{1}{g'(x)}$$

Put $x = 0$

$$(g^{-1}(g(0)))' = \frac{1}{g'(0)}$$

$$(g^{-1}(2))' = \frac{1}{g'(0)}$$

Now
$$g'(x) = \frac{8e^{-2x}}{\left(1 + e^{-2x}\right)^2}$$

$$\Rightarrow \quad g'(0) = \frac{8}{4} = 2 \quad \text{... (3)}$$

From Eqs (1), (2) and (3), we get,

$$f'(0) \times (g^{-1}(2))' = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

**15. Correct answer is [3].**

$$\frac{\sin(A-B)}{\sin A \sin B} = \frac{\sin A \cos B - \cos A \sin B}{\sin A \sin B} = \cot\beta - \cot\alpha$$

$$\alpha \times \sin 1^\circ = \frac{\sin(61^\circ - 60^\circ)}{\sin 60^\circ \cdot \sin 61^\circ} + \frac{\sin(63^\circ - 62^\circ)}{\sin 62^\circ - \sin 63^\circ} + \ldots + \frac{\sin(119^\circ - 118^\circ)}{\sin 118^\circ \cdot \sin 119^\circ}$$

$= (\cot 60^\circ - \cot 61^\circ) + (\cot 62^\circ - \cot 63^\circ) + \ldots + (\cot 89^\circ - \cot 90^\circ) + (\cot 91^\circ - \cot 92^\circ) + \ldots + (\cot 116^\circ - \cot 117^\circ) + (\cot 118^\circ - \cot 119^\circ)$

$$= (\cot 60^\circ - \cot 90^\circ) = \frac{1}{\sqrt{3}} = \left(\frac{\alpha}{\text{cosec}1^\circ}\right)^{-2} = \left(\frac{1}{3}\right)^{-1} = 3$$

**16. Correct answer is [21].**

$$\alpha = \int_{\frac{1}{2}}^{2} \frac{\tan^{-1}x}{2x^2 - 3x + 2}\,dx \quad \ldots (1)$$

Let $x = \frac{1}{t}$

$$\Rightarrow \quad dx = \frac{1}{t^2}dt$$

$$\therefore \quad \alpha = -\int_{2}^{\frac{1}{2}} \frac{\tan^{-1}\left(\frac{1}{t}\right)}{\frac{2}{t^2} - \frac{3}{t} + 2} \cdot \frac{dt}{t^2}$$

$$\Rightarrow \quad \alpha = \int_{\frac{1}{2}}^{2} \frac{\tan^{-1}\left(\frac{1}{t}\right)}{2 - 3t + 2t^2}\,dt \quad \ldots (2)$$

Adding Eqs (1) and (2), we get,

$$2\alpha = \int_{\frac{1}{2}}^{2} \frac{\tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right)}{2x^2 - 3x + 2}\,dx$$

$$\left(\because\ \tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2}\right)$$

$$2\alpha = \int_{\frac{1}{2}}^{2} \frac{\frac{\pi}{2}}{(2x^2 - 3x + 2)}\,dx$$

$$\alpha = \frac{\pi}{8}\int_{\frac{1}{2}}^{2} \frac{dx}{\left(x - \frac{3}{4}\right)^2 + \left(\frac{\sqrt{7}}{4}\right)^2}$$

$$= \frac{\pi}{8} \cdot \frac{1}{\frac{\sqrt{7}}{4}}\left(\tan^{-1}\left(\frac{4x-3}{\sqrt{7}}\right)\right)_{\frac{1}{2}}^{2}$$

$$= \frac{\pi}{2\sqrt{7}}\left[\tan^{-1}\left(\frac{5}{\sqrt{7}}\right) - \tan^{-1}\left(-\frac{1}{\sqrt{7}}\right)\right]$$

$$= \frac{\pi}{2\sqrt{7}}\left[\tan^{-1}\left(\frac{5}{\sqrt{7}}\right) + \tan^{-1}\left(-\frac{1}{\sqrt{7}}\right)\right]$$

$$\left\{\because\ \tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right) \text{ If } 1 - AB > 0\right\}$$

$$= \frac{\pi}{2\sqrt{7}}\tan^{-1}\left(\frac{\frac{5}{\sqrt{7}} + \frac{1}{\sqrt{7}}}{1 - \frac{5}{7}}\right) = \frac{\pi}{2\sqrt{7}}\tan^{-1}\left(\frac{6}{\sqrt{7}} \times \frac{7}{2}\right)$$

$$\alpha = \frac{\pi}{2\sqrt{7}}\tan^{-1}(3\sqrt{7}) \Rightarrow \sqrt{7}\tan\left(\frac{2\alpha\sqrt{7}}{\pi}\right)$$

$$= \sqrt{7}\tan(\tan^{-1}3\sqrt{7}) = 21$$