JEE (Main) QUESTION PAPER 2025

2^{nd} April (Shift 1)

General Instructions: *Follow the same instructions as mentioned in 22^{nd} Jan Shift 1.*

Mathematics

Section A

Q. 1. The largest $n \in \mathbb{N}$ such that 3^n divides 50! is:

(1) 21 **(2)** 22 **(3)** 20 **(4)** 23

Q. 2. Let one focus of the hyperbola. H: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be at $(\sqrt{10}, 0)$ and the corresponding directrix be $x = \frac{9}{\sqrt{10}}$. If e and l respectively are the eccentricity and the length of the latus rectum of H, then $9(e^2 + l)$ is equal to:

(1) 14 **(2)** 15 **(3)** 16 **(4)** 12

Q. 3. The number of sequences of ten terms, whose terms are either 0 or 1 or 2, that contain exactly five 1*s* and exactly three 2*s*, is equal to:

(1) 360 **(2)** 45 **(3)** 2520 **(4)** 1820

Q. 4. Let $f: \mathbb{R} \to \mathbb{R}$ be a twice differentiable function such that $(\sin x \cos y)(f(2x + 2y) - f(2x - 2y)) = (\cos x \sin y)(f(2x + 2y) + f(2x - 2y))$, for all $x, y \in \mathbb{R}$ and $f'(0) = 1/2$. If $f(x) = \frac{1}{2}$, then the value of $24\, f''\left(\frac{5\pi}{3}\right)$ is:

(1) 2 **(2)** –3 **(3)** 3 **(4)** –2

Q. 5. Let A = $\begin{bmatrix} \alpha & -1 \\ 6 & \beta \end{bmatrix}$, $\alpha > 0$, such that det(A) = 0 and $\alpha + \beta = 1$. If I denotes 2×2 identity matrix, then the matrix $(1 + A)^8$ is:

(1) $\begin{bmatrix} 4 & -1 \\ 6 & -1 \end{bmatrix}$ **(2)** $\begin{bmatrix} 257 & -64 \\ 514 & -127 \end{bmatrix}$

(3) $\begin{bmatrix} 1025 & -511 \\ 2024 & -1024 \end{bmatrix}$ **(4)** $\begin{bmatrix} 766 & -255 \\ 1530 & -509 \end{bmatrix}$

Q. 6. The term independent of x in the expansion of $\left(\frac{(x+1)}{\left(x^{\frac{2}{3}}+1-x^{\frac{1}{3}}\right)} - \frac{(x+1)}{\left(x-x^{\frac{1}{2}}\right)}\right)^{10}$, $x > 1$ is:

(1) 210 **(2)** 150 **(3)** 240 **(4)** 120

Q. 7. If $\theta \in [-2\pi, 2\pi]$, then the number of solutions of $2\sqrt{2}\cos^2\theta + (2-\sqrt{6})\cos\theta - \sqrt{3} = 0$, is equal to:

(1) 12 **(2)** 6 **(3)** 8 **(4)** 10

Q. 8. Let a_1, a_2, a_3 .... be in an AP such that $\sum_{k=1}^{12} a_{2k-1} = -\frac{72}{5}a_1, a_1 \neq 0$. If $\sum_{k=1}^{n} a_k = 0$, then n is equal to:

(1) 11 **(2)** 10 **(3)** 18 **(4)** 17

Q. 9. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$, attains its local maximum and local minimum values at p and q, respectively, such that $p^2 = q$, then $f(3)$ is equal to:

(1) 55 **(2)** 10 **(3)** 23 **(4)** 37

Q. 10. Let z be a complex number such that $|z| = 1$. If $\frac{2+k^2z}{k+\bar{z}} = kz$, $k \in \mathbb{R}$, then the maximum distance of $k + ik^2$ from the circle $|z - (1 + 2i)| = 1$ is:

(1) $\sqrt{5}+1$ **(2)** 2

(3) 3 **(4)** $\sqrt{3}+1$

Q. 11. If a is non-zero vector such that its projections on the vectors $2\hat{i} - \hat{j} + 2\hat{k}$, $\hat{i} + 2\hat{j} - 2\hat{k}$ and $\hat{k}$ are equal, then a unit vector along a is:

(1) $\frac{1}{\sqrt{155}}\left(-7\hat{i}+9\hat{j}+5\hat{k}\right)$

(2) $\frac{1}{\sqrt{155}}\left(-7\hat{i}+9\hat{j}-5\hat{k}\right)$

(3) $\frac{1}{\sqrt{155}}\left(7\hat{i}+9\hat{j}+5\hat{k}\right)$

(4) $\frac{1}{\sqrt{155}}\left(7\hat{i}+9\hat{j}-5\hat{k}\right)$

Q. 12. Let A be the set of all functions $f: \mathbb{Z} \to \mathbb{Z}$ and R be a relation on A such that R = $\{(f, g) : f(0) = g(1)$ and $f(1) = g(0)\}$. Then R is:

(1) Symmetric and transitive but not reflective.

(2) Symmetric but neither reflective nor transitive.
(3) Reflexive but neither symmetric nor transitive.
(4) Transitive but neither reflexive nor symmetric.

Q. 13. For α, β, γ, ∈ R, if $\lim_{x\to 0}\frac{x^2\sin\alpha x+(\gamma-1)e^{x^2}}{\sin 2x-\beta x}=3$, then β + γ – α is equal to:

(1) 7 (2) 4 (3) 6 (4) –1

Q. 14. If the system of linear equations

$3x + y + \beta z = 3$

$2x + \alpha y - z = -3$

$x + 2y + z = 4$

has infinitely many solution, then the value of 22β – 9α is:

(1) 49 (2) 31 (3) 43 (4) 37

Q. 15. Let $P_n = \alpha^n + \beta^n$, $n \in N$, if $P_{10} = 123$, $P_9 = 76$, $P_8 = 47$ and $P_1 = 1$, then the quadratic equation having roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is:

(1) $x^2 - x + 1 = 0$ (2) $x^2 + x - 1 = 0$
(3) $x^2 - x - 1 = 0$ (4) $x^2 + x + 1 = 0$

Q. 16. If S and S′ are the foci of the ellipse $\frac{x^2}{18}+\frac{y^2}{9}=1$ and P be a point on the ellipse, then min(SP·S′P) + max(SP·S′P) is equal to:

(1) $3(1+\sqrt{2})$ (2) $3(6+\sqrt{2})$
(3) 9 (4) 27

Q. 17. Let the vertices Q and R of the triangle PQR lie on the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$, QR = 5 and the coordinates of the point P be (0, 2, 3). If the area of the triangle PQR is $\frac{m}{n}$ then:

(1) $m-5\sqrt{21}n=0$ (2) $2m-5\sqrt{21}n=0$
(3) $5m-2\sqrt{21}n=0$ (4) $5m-21\sqrt{2}n=0$

Q. 18. Let ABCD be a tetrahedron such that the edges AB, AC and AD are mutually perpendicular. Let the areas of the triangles ABC, ACD and ADB be 5, 6 and 7 square units, respectively. Then the area (in square units) of the ΔBCD is equal to:

(1) $\sqrt{340}$ (2) 12 (3) $\sqrt{110}$ (4) $7\sqrt{3}$

Q. 19. Let $a \in R$ and A be a matrix of order 3 × 3 such that det(A) = –4 and $A + I = \begin{bmatrix}1 & a & 1\\ 2 & 1 & 0\\ a & 1 & 2\end{bmatrix}$, where I is the identity matrix of order 3 × 3.

If $\det((a+1)\text{adj}((a-1)A))$ is 2^m3^n, $m, n \in \{0, 1, 2 .... 20\}$, then $m + n$ is equal to:

(1) 14 (2) 17 (3) 15 (4) 16

Q. 20. Let the focal chord PQ of the parabola $y^2 = 4x$ make an angle of 60° with the positive x-axis, where P lies in the first quadrant. If the circle, whose one diameter is PS, S being the focus of the parabola, touches the y-axis at the point (0, α), then $5\alpha^2$ is equal to:

(1) 15 (2) 25 (3) 30 (4) 20

Section B

Q. 21. Let [.] denote the greatest integer function. If $\int_0^{e^3}\left[\frac{1}{e^{x-1}}\right]dx = \alpha - \log_e 2$, then α^3 is equal to ______.

Q. 22. Let $f : R \to R$ be a thrice differentiable odd function satisfying $f'(x) \ge 0$, $f''(x) = f(x)$, $f(0) = 0$, $f'(0) = 3$. Then 9 $f(\log 3)$ is equal to _____.

Q. 23. If the area of the region $\{(x, y): |4 - x^2| \le y \le x^2, y \le 4, x \ge 0\}$ is $\left(\frac{80\sqrt{2}}{\alpha}-\beta\right)$, α, β ∈ N, then α + β is equal to ______.

Q. 24. Three distinct numbers are selected randomly from the set {1, 2, 3, ..., 40}. If the probability that the selected numbers are in an increasing GP. is $\frac{m}{n}$, gcd(m, n) = 1, then m + n is equal to ______.

Q. 25. The absolute difference between the squares of the radii of the two circles passing through the point (–9, 4) and touching the lines $x + y = 3$ and $x - y = 3$, is equal to ______.

■■■

Answer Key

Q. No.	Answer	Topic Name	Chapter Name
1	(2)	Power of exponent	Permutation and combination
2	(3)	Latus rectum and eccentricity	Hyperbola
3	(3)	Number of sequence	Permutation and combination
4	(2)	Higher-order derivative	Differentiation
5	(4)	Product of matrix	Matrix
6	(1)	Independent term	Binomial thorem
7	(3)	Solution of trigonometric equation	Trigonometric equation
8	(1)	Sum of AP	Sequence and series
9	(4)	Maxima minima	Application of derivative
10	(1)	Distance from circle	Complex number
11	(3)	Projection	Vector
12	(2)	Types of relation	Set and relation
13	(1)	Limit with series expansion	Limit
14	(2)	System of equation	Determinant
15	(2)	Framing	Quadratic equation
16	(4)	Focal distance	Ellipse
17	(2)	Area of triangle	3D
18	(3)	Area of triangle	3D
19	(4)	Properties of adjoint	Matrix
20	(1)	Focal chord	Parabola
21	[8]	Properties of definite integration	Definite integration
22	[36]	Functional equation	Function
23	[22]	Area between curves	Applications of integration
24	[2477]	Probability	Probability
25	[768]	Circle equation	Circle

JEE (Main) SOLUTIONS

2025
2^{nd} April (Shift 1)

Answers with Explanation

Section A

1. Option (2) is correct.

$$n = \left[\frac{50}{3}\right]+\left[\frac{50}{3^2}\right]+\left[\frac{50}{3^3}\right]+\left[\frac{50}{3^4}\right]........$$

{[ ] = Greatest integer function}

$$n = 16 + 5 + 1 + 0 ......$$
$$= 22$$

Maximum value of $n = 22$

2. Option (3) is correct.

$$\frac{x^2}{a^2}-\frac{y^2}{b^2} = 1$$

focus $= (\sqrt{10}, 0)$

Transverse axis $= 2a$

Minor axis $= 2b$

Eccentricity $= e$

Directrix $\Rightarrow x = \frac{a}{e}$

$$ae = \sqrt{10} \text{ and } \frac{a}{e} = \frac{9}{\sqrt{10}}$$

$$\Rightarrow \quad a^2 = 9 \text{ and } e = \frac{\sqrt{10}}{3}$$

Now, $(ae)^2 = a^2 + b^2$

$10 = 9 + b^2$

$\therefore \quad b = 1$

latus rectum $(l) = \frac{2b^2}{a} = \frac{2\times 1}{3}$

$\Rightarrow 9(e^2 + 1)$

$$= 9\left(\frac{10}{9}+\frac{2}{3}\right)$$
$$= 10 + 6$$
$$= 16$$

3. Option (3) is correct.

Sequence contains five 1*s*, three 2*s*, two 0*s*.

$\therefore$ Total number of sequences

$$= \frac{10!}{5!3!2!}$$
$$= 2,520$$

4. Option (2) is correct.

$(\sin x \cos y)(f(2x + 2y)) - f(2x - 2y) = (\cos x \sin y)$

$(f(2x + 2y) + f(2x - 2y))$

$f(2x + 2y)\cdot(\sin x\cos y - \cos x \sin y)$

$= f(2x - 2y)\cdot(\sin x \cos y + \cos x \sin y)$

$f(2x + 2y)\cdot\sin(x - y) = f(2x - 2y)\cdot\sin(x + y)$

$$\frac{f(2x+2y)}{\sin(x+y)} = \frac{f(2x-2y)}{\sin(x-y)}$$

Put $\quad 2x + 2y = m,\ 2x - 2y = n$

$$\frac{f(m)}{\sin\left(\frac{m}{2}\right)} = \frac{f(n)}{\sin\left(\frac{n}{2}\right)} = \text{K}$$

$$\therefore \quad \frac{f(m)}{\sin\left(\frac{m}{2}\right)} = \text{K}$$

$$f(m) = \text{K}\sin\left(\frac{m}{2}\right)$$

$$f(x) = \text{K}\sin\left(\frac{x}{2}\right)$$

$$f'(x) = \frac{\text{K}}{2}\cos\left(\frac{x}{2}\right)$$

Put $\quad x = 0;\ \frac{1}{2} = \frac{\text{K}}{2} \Rightarrow \text{K} = 1$

$$f'(x) = \frac{1}{2}\cos\frac{x}{2}$$

$$f''(x) = -\frac{1}{4}\sin\frac{x}{2}$$

$$24f''\left(\frac{5\pi}{3}\right) = \left(-\frac{1}{4}\sin\left(\frac{5\pi}{6}\right)\right)24$$

$$= -\frac{24}{4}\sin\left(\frac{5\pi}{6}\right)$$

$$= -6\sin\left(\pi-\frac{\pi}{6}\right)$$

$$= -6\sin\frac{\pi}{6} = -6\times\frac{1}{2} = -3$$

5. Option (4) is correct.

$$A = \begin{bmatrix} \alpha & -1 \\ 6 & \beta \end{bmatrix}, \alpha > 0,$$

$$\det(A) = 0 \Rightarrow |A| = 0$$

$$\begin{vmatrix} \alpha & -1 \\ 6 & \beta \end{vmatrix} = 0$$

$$\alpha\beta + 6 = 0$$

$$\alpha\beta = -6$$

Given, $\alpha + \beta = 1$

$$\therefore \quad \alpha + \left(\frac{-6}{\alpha}\right) = 1$$

$$\alpha^2 - 6 = \alpha$$

$$\alpha^2 - \alpha - 6 = 0$$

$$\alpha = 3, -2$$

But, $\alpha > 0 \Rightarrow \alpha = 3$

$$\beta = -2$$

$$A = \begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix}$$

$$\therefore \quad A^2 = A$$

$$A = A^2 = A^3 = A^4 = A^5$$

$$(I + A)^8 = I + {}^8C_1A^7 + {}^8C_2A^6 + \ldots + {}^8C_8A^8$$

$$= I + A({}^8C_1 + {}^8C_2 + \ldots + {}^8C_8)$$

$$= I + A(2^8 - 1)$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 765 & -255 \\ 1530 & -510 \end{bmatrix}$$

$$= \begin{bmatrix} 766 & -255 \\ 1530 & -509 \end{bmatrix}$$

6. Option (1) is correct.

$$\left(\frac{(x+1)}{(x^{2/3}+1-x^{1/3})} - \frac{(x+1)}{(x-x^{1/2})}\right)^{10}, x > 1$$

$$= \left[\frac{(x^{\frac{1}{3}})^3 + 1^3}{x^{\frac{2}{3}} + 1 - x^{\frac{1}{3}}} - \frac{(\sqrt{x})^2 - 1^2}{\sqrt{x}(\sqrt{x}-1)}\right]^{10}$$

We know that, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$$\therefore \quad (x^{\frac{1}{3}})^3 + 1^3 = (x^{\frac{1}{3}} + 1)(x^{\frac{2}{3}} + 1 - x^{\frac{1}{3}})$$

$$(\sqrt{x})^2 - 1 = (\sqrt{x} + 1)(\sqrt{x} - 1)$$

$$\Rightarrow \left(\frac{(x+1)}{x^{\frac{2}{3}} + 1 - x^{\frac{1}{3}}} - \frac{(x-1)}{\left(x - x^{\frac{1}{2}}\right)}\right)^{10}$$

$$= \left((x^{\frac{1}{3}} + 1) - \left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10}$$

$$= \left(x^{\frac{1}{3}} - \frac{1}{\sqrt{x}}\right)^{10}$$

$$T_{r+1} = {}^{10}C_r(x)^{\frac{10-r}{3}}(-1)^r(x)^{-\frac{r}{2}}$$

$$= {}^{10}C_r(-1)^r(x)^{\frac{10-r}{3} - \frac{r}{2}}$$

Term independent of $x = \dfrac{10-r}{3} - \dfrac{r}{2} = 0$

$$(20 - 2r) - 3r = 0$$

$$r = 4$$

$$\Rightarrow \quad {}^{10}C_4(-1)^4 = 210$$

7. Option (3) is correct.

$$2\sqrt{2}\cos^2\theta + 2\cos\theta - \sqrt{6}\cos\theta - \sqrt{3} = 0$$

$$(2\cos\theta - \sqrt{3})(\sqrt{2}\cos\theta + 1) = 0$$

$$\cos\theta = \frac{\sqrt{3}}{2}, \frac{-1}{\sqrt{2}}$$

In $[0, 2\pi]$, $\cos\theta = \dfrac{\sqrt{3}}{2}$, then $\theta = \pm\dfrac{\pi}{6}, \pm\dfrac{11\pi}{6}$

If $\cos\theta = \dfrac{-1}{\sqrt{2}}$, then $\theta = \pm\dfrac{3\pi}{4}, \pm\dfrac{5\pi}{4}$

Hence, there are a total of 8 solutions because $\cos x$ is an even function.

8. Option (1) is correct.

Let first term $a_1 = a$

Common difference $= d$

$$\sum_{k=1}^{12} a_{2k-1} = -\frac{72}{5}a_1, a_1 \neq 0$$

$$\frac{12}{2}[2a+11\times 2d] = -\frac{72}{5}a_1$$

$$12a + 132d = -\frac{72}{5}a_1$$

$$132a + 132 \times 5d = 0$$

$$a = -5d$$

If $$\sum_{k=1}^{n} a_k = 0,$$

$$\frac{n}{2}(2a+(n-1)d) = 0$$

$$\Rightarrow \quad -10d + nd - d = 0$$

$$n = 11$$

9. Option (4) is correct.

$$f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$$

$$f'(x) = 6x^2 - 18ax + 12a^2$$

$$f'(x) = 6(x^2 - 3ax + 2a^2)$$

$$f'(x) = 0$$

$$x^2 - 3ax + 2a^2 = 0$$

$$x = a, 2a$$

$$f''(x) = 6(2x - 3a)$$

$$f''(a) = 6(2a - 3a)$$

$$= -6a < 0$$

$\therefore$ Local maxima at $x = a$,

Local minima at $x = 2a$

Given: $$p^2 = q$$

$$\Rightarrow \quad a^2 = 2a$$

$$a(a - 2) = 0$$

$$\Rightarrow \quad a = 0 \text{ or } 2$$

Since $a > 0$, take $a = 2$

$$f(x) = 2x^3 - 18x^2 + 48x + 1$$

$$f(3) = 37$$

10. Option (1) is correct.

$$\frac{2+k^2z}{k+\bar{z}} = kz$$

$$2 + k^2z = k^2z + kz\bar{z}$$

$$2 = kz\bar{z}$$

$$\{|z|^2 = z\bar{z} \ (\because |z| = 1)\}$$

$$\Rightarrow \quad 2 = k \times 1$$

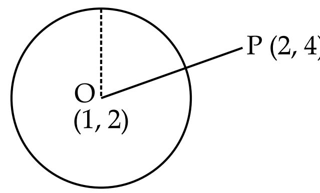
$$k + ik^2 = 2 + 4i$$

Let $$P = (2, 4)$$

Given, $|z - (1 + 2i)| = 1$

Centre of circle $= (1, 2)$

Radius $(r) = 1$



Maximum distance of P from circle = OP + r

$$(\text{OP} + r) = \sqrt{1+4}+1 = \sqrt{5}+1$$

11. Option (3) is correct.

Let $$\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$$

$$a_1^2 + a_2^2 + a_3^2 = 1$$

Let $$\vec{b} = 2\hat{i}-\hat{j}+2\hat{k}, \ \vec{c} = \hat{i}+2\hat{j}-2\hat{k}$$

$$\vec{d} = \hat{k}$$

Given projections are equal therefore,

$$\frac{\vec{a}\cdot\vec{b}}{|b|} = \frac{\vec{a}\cdot\vec{c}}{|c|} = \frac{\vec{a}\cdot\vec{d}}{|d|}$$

$$\frac{2a_1 - a_2 + 2a_3}{3} = \frac{a_1 + 2a_2 - 2a_3}{3} = a_3$$

$$\therefore \quad 2a_1 - a_2 + 2a_3 = a_1 + 2a_2 - 2a_3$$

$$a_1 - 3a_2 + 4a_3 = 0 \quad \text{...(i)}$$

$$2a_1 - a_2 + 2a_3 = 3a_3$$

$$2a_1 - a_2 - a_3 = 0 \quad \text{...(ii)}$$

$$a_1 + 2a_2 - 2a_3 = 3a_3$$

$$a_1 + 2a_2 - 5a_3 = 0 \quad \text{...(iii)}$$

From Eq. (ii), we get

$$a_2 + a_3 = 2t \quad \{\text{Let, } a_1 = t\}$$

From Eq. (i), we get

$$3a_2 - 4a_3 = t$$

$$\therefore \quad 3a_2 - 4(2t - a_2) = t$$

$$3a_2 + 4a_2 - 8t = t$$

$$7a_2 = 9t$$

$$a_2 = \frac{9t}{7}$$

Now, $$a_2 + a_3 = 2t$$

$$\frac{9t}{7}+a_3 = 2t$$

$$\therefore \quad a_3 = \frac{5t}{7}$$

Hence, $$a_1 : a_2 : a_3 = t : \frac{9t}{7} : \frac{5t}{7}$$

$$= 7 : 9 : 5$$

$\therefore$ Unit vector along $\vec{a}$ $$= \frac{1}{\sqrt{155}}(7\hat{i}+9\hat{j}+5\hat{k})$$

12. Option (2) is correct.

$R = \{(f, g) : f(0) = g(1) \text{ and } f(1) = g(0)\}$

Reflexive: $(f, f) \in R$

$= f(0) = f(1)$ and $f(1) = f(0) \rightarrow$ must hold

$\Rightarrow$ but this is not true for all function.

So, it is not reflexive.

Symmetric: If $(f, g) \in R \Rightarrow (g, f) \in R$

Now, $g(0) = f(1)$ and $g(1) = f(0) \rightarrow$ true

$\therefore$ symmetric

Transitive: If $(f, g) \in R$ and $(g, h) \in R$

$\Rightarrow (f, h) \in R$

Now, $(f, g) \in R \Rightarrow f(0) = g(1)$ and $f(1) = g(0)$

$(g, h) \in R \Rightarrow g(0) = h(1)$ and $g(1) = h(0)$

For $(f, h) \in R$, we need $f(0) = h(1)$ and $f(1) = h(0)$

Now, $f(0) = g(1) = h(0)$ and $f(1) = g(0) = h(1)$

Hence, it is not transitive.

It is symmetric but neither reflective nor transitive.

13. Option (1) is correct.

$$\lim_{x\to 0} \frac{x^2 \sin \alpha x + (\gamma - 1)e^{x^2}}{\sin 2x - \beta x} = 3$$

Applying following Frenies expansion

$$\sin t = t - \frac{t^3}{3!} + \ldots$$

$$e^{x^2} = 1 + \frac{x^2}{1} + \frac{x^4}{2!} \ldots.$$

Neglecting higher degree terms,

$$\lim_{x\to 10} \frac{x^2(\alpha x) + (\gamma - 1)\left(1 + \frac{x^2}{1}\right)}{2x - \frac{8x^3}{6} - \beta x} = 3$$

$$\lim_{x\to 0} \frac{(\gamma - 1) + (\gamma - 1)x^2 + \alpha x^3}{(2 - \beta)x - \frac{4}{3}x^3}$$

Limit will exist if coefficients of x^0, x^1, x^2 are zero.

$$\gamma - 1 = 0, 2 - \beta = 0$$

and
$$\frac{\alpha}{\left(\frac{-4}{3}\right)} = 3$$

$$\gamma = 1, \beta = 2, \frac{-3\alpha}{4} = 3$$

$$\Rightarrow \qquad \alpha = -4$$

$$\beta + \gamma - \alpha = 7$$

14. Option (2) is correct.

$$3x + y + \beta z = 3$$
$$2x + \alpha y - z = -3$$
$$x + 2y + z = 4$$

$$D = \begin{vmatrix} 3 & 1 & \beta \\ 2 & \alpha & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$D_x = \begin{vmatrix} 3 & 1 & \beta \\ -3 & \alpha & -1 \\ 4 & 2 & 1 \end{vmatrix}$$

$$D_y = \begin{vmatrix} 3 & 3 & \beta \\ 2 & -3 & -1 \\ 1 & 4 & 1 \end{vmatrix}$$

$$D_z = \begin{vmatrix} 3 & 1 & 3 \\ 2 & \alpha & -3 \\ 1 & 2 & 4 \end{vmatrix}$$

There are infinite many solution, therefore, $D = 0 = D_x = D_y = D_z$

$$D = 0 = \begin{vmatrix} 3 & 1 & \beta \\ 2 & \alpha & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$3\alpha + 4\beta - \alpha\beta + 3 = 0$$

$$D_z = 0 = \begin{vmatrix} 3 & 1 & 3 \\ 2 & \alpha & -3 \\ 1 & 2 & 4 \end{vmatrix}$$

$$0 = 9\alpha + 19$$

$$\alpha = \frac{-19}{9}, \beta = \frac{6}{11}$$

$$\Rightarrow \qquad 22\beta - 9\alpha = 31$$

15. Option (2) is correct.

$$P_n = \alpha^n + \beta^n$$
$$P_{10} = \alpha^{10} + \beta^{10} = 123$$
$$P_9 = \alpha^9 + \beta^9 = 76$$
$$P_8 = \alpha^8 + \beta^9 = 47$$
$$P_1 = \alpha + \beta = 1$$
$$P_9 + P_8 = \alpha^9 + \beta^9 + \alpha^8 + \beta^8$$
$$= 76 + 47$$
$$\alpha^8(\alpha + 1) + \beta^8(\beta + 1) = 123$$
$$\therefore \qquad \alpha^8(\alpha + 1) + \beta^8(\beta + 1) = \alpha^{10} + \beta^{10}$$
$$\Rightarrow \qquad \alpha^2 = \alpha + 1$$
and
$$\beta^2 = \beta + 1$$

Given quadratic equation is,

$$x^2 = x + 1$$
$$x^2 - x - 1 = 0$$
$$\alpha + \beta = 1, \alpha\beta = -1$$
$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{1}{-1}$$
$$\frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{\alpha\beta} = -1$$

$\therefore$ Quadratic equation is

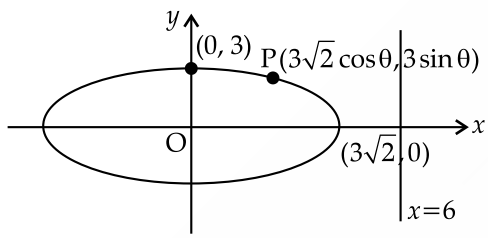
$$x^2 - x(-1) - 1 = 0$$
$$x^2 + x - 1 = 0$$

16. Option (4) is correct.

$$\frac{x^2}{18} + \frac{y^2}{9} = 1$$
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

On comparison, we get

$$a^2 = 18, b^2 = 9 \Rightarrow a > b$$



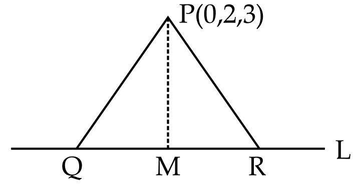
$$PS + PS' = 2 \times 3\sqrt{2}$$
$$b^2 = a^2(1 - e^2) = 18(1 - e^2)$$
$$\Rightarrow \quad e = \frac{1}{\sqrt{2}}$$

Directrix $$x = \frac{3\sqrt{2}}{\frac{1}{\sqrt{2}}} = 6$$

$$SP = a(1 - e\cos\theta)$$
$$S'P = a(1 + e\cos\theta)$$
$$SP \cdot S'P = a^2(1 - e\cos\theta)(1 + e\cos\theta)$$
$$= a^2(1 - e^2\cos^2\theta)$$
$$= 18\left(1 - \frac{\cos^2\theta}{2}\right)$$

$$(SP \cdot S'P)_{max} = 18 \text{ if } \cos\theta = 0$$
$$(SP \cdot S'P)_{min} = 9 \text{ if } \cos\theta = 1$$
$$\text{Sum} = 18 + 9 = 27$$

17. Option (2) is correct.



$$QR = 5$$
$$\therefore \quad \frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = \lambda$$

Any point on line L is: $5\lambda - 3, 2\lambda + 1, 3\lambda - 4$

Let, M = $5\lambda - 3, 2\lambda + 1, 3\lambda - 4$

Drs of PM $\Rightarrow 5\lambda - 3, 2\lambda - 1, 3\lambda - 7$

Drs of line L $\Rightarrow 5, 2, 3$

L and PM are perpendicular,

$$\therefore \quad 5(5\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 7) = 0$$
$$25\lambda - 15 + 4\lambda - 2 + 9\lambda - 21 = 0$$
$$38\lambda - 38 = 0$$
$$\lambda = 1$$
$$\therefore \quad M = (2, 3, -1)$$
$$PM = \sqrt{4+1+16} = \sqrt{21}$$
$$\text{Area} = \frac{1}{2} \times 5 \times \sqrt{21} = \frac{m}{n}$$
$$2m - 5\sqrt{21}n = 0$$

18. Option (3) is correct.

Edges AB, AC and AD are mutually perpendicular,

$\therefore$ Ar(ΔABCD)

$$= \sqrt{(\text{Ar}(\Delta ABC))^2 + (\text{Ar}(\Delta ACD))^2 + (\text{Ar}(\Delta ADB))^2}$$
$$= \sqrt{5^2 + 6^2 + 7^2}$$
$$= \sqrt{110}$$

19. Option (4) is correct.

$$A + I = \begin{bmatrix} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{bmatrix}$$
$$\therefore \quad A = \begin{bmatrix} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{bmatrix} - I = \begin{bmatrix} 0 & a & 1 \\ 2 & 0 & 0 \\ a & 1 & 1 \end{bmatrix}$$
$$|A| = \begin{vmatrix} 0 & a & 1 \\ 2 & 0 & 0 \\ a & 1 & 1 \end{vmatrix}$$
$$|A| = 2 - 2a$$

Given, $$|A| = -4$$
$$\therefore \quad 2 - 2a = -4$$
$$a = 3$$
$$|(a + 1)\text{adj}(a - 1)A| = |4 \text{ adj } 3A|$$

$= 4^3|\text{adj } 3A|$

$= 4^3 \times |3A|^{3-1} = 64|3A|^2$

$= 64 \times (3^3)^2\ |A|^2$

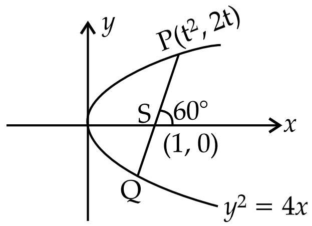
$= 2^6 \times 3^6 \times 16 \quad (\because |A| = -4)$

$2^m \times 3^n = 2^{10} \times 3^6$

$\therefore \quad m = 10, n = 6$

$\therefore \quad m + n = 16$

20. Option (1) is correct.



Parabola $\Rightarrow y^2 = 4x$

Let, parametric coordinate of P $= (t^2, 2t)$

Focus (S) $= (1, 0)$

$$\tan 60° = \frac{2t-0}{t^2-1} = \sqrt{3} \Rightarrow t = \sqrt{3}$$

$\therefore$ P$(3, 2\sqrt{3})$

Equation of circle in diameter for m is:

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

$(x - 1)(x - 3) + (y - 0)(y - 2\sqrt{3}) = 0$

at $x = 0$

$\Rightarrow \quad 3 + y^2 - 2\sqrt{3}y = 0$

$\Rightarrow \quad y = \sqrt{3} = \alpha$

$5\alpha^2 = 15$

Section B

21. Correct answer is [8].

Let $\quad f(x) = \dfrac{1}{e^{x-1}}$

$f(x) = e^{1-x}$

If, $\quad f(x) = 1 \Rightarrow 1 - x = 0$

$\therefore \quad x = 1$

$f(x) = 2$

$\dfrac{1}{e^{x-1}} = 2$

$x = 1 - \ln 2$

$f(0) = e^1 = 2.71$

$f(e^3) = e^{1-e^3} \in (0, 1)$

$$I = \int_0^{e^3} [e^{1-x}]dx = \int_0^{1-\ln 2} [e^{1-x}]dx + \int_{1-\ln 2}^{1} [e^{1-x}]dx + \int_1^{e^3} [e^{1-x}]dx$$

$$I = \int_0^{1-\ln 2} 2dx + \int_{1-\ln 2}^{1} 1dx + \int_1^{e^3} 0dx$$

$= 2(1 - \ln 2 - 0) + 1(1 - 1 + \ln 2) + 0$

$\alpha - \ln 2 = 2 - \ln 2$

$\Rightarrow \quad \alpha = 2$

$\Rightarrow \quad \alpha^3 = 8$

22. Correct answer is [36].

$f''(x) = f(x)$

Multiplying $f'(x)$ on both sides, we get

$f''(x)\,f'(x) = f(x)f'(x)$

On integration both sides, we get

$$\int f''(x)f'(x)dx = \int f(x)f'(x)dx$$

$$\Rightarrow \quad \frac{(f'(x))^2}{2} = \frac{(f(x))^2}{2} + C$$

C is the constant of integration,

Take $x = 0$

$$\frac{(f'(0))^2}{2} = \frac{(f(0))^2}{2} + C$$

$$\frac{9}{2} = C$$

$(f'(x))^2 = (f(x))^2 + 9$

$f(x) = \sqrt{(f(x))^2 + 9} \quad \because f(x) \geq 0$

$\int dx \Rightarrow \ln y + \sqrt{y^2 + 9} = x + C \qquad \{\text{Let } f(x) = y\}$

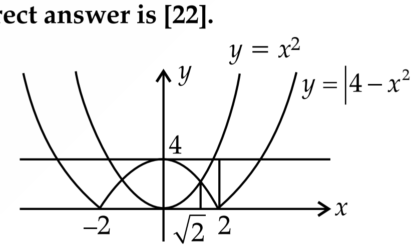
$\Rightarrow \quad f(0) = 0 \Rightarrow C = \ln 3$

$\Rightarrow \quad y + \sqrt{y^2 + 9} = 3e^x$

at $\quad x = \ln 3; y = 4$

$\therefore \quad 9f(\ln 3) = 36$

23. Correct answer is [22].



$$A = \int_0^4 \sqrt{4+y}\,dy - \int_0^2 \sqrt{4-y}\,dy - \int_0^4 \sqrt{y}\,dy$$

$$= \left(\frac{(4+y)^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^4 + \left(\frac{(4-y)^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^2 - \left(\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^4$$

$$= \frac{40\sqrt{2}}{3} - 16 = \frac{80\sqrt{2}}{6} - 16$$

$$A = \frac{80\sqrt{2}}{\alpha} - \beta$$

$\therefore \quad \alpha = 6, \beta = 16$

$\alpha + \beta = 22$

24. Correct answer is [2477].

$1 \le a < ar < ar^2 \le 40$

Let first term of GP = a

Common ratio = r

Case I: If r is natural number.

If $r = 2$

$1 \le a < 2a < 4a \le 40$

$a \in \{1, ..., 10\}$ (10 G.P.)

If $r = 3$

$1 \le a < 3a < 9a \le 40$

$a \in \{1, 2, 3, 4\}$ (4 G.P.)

If $r = 4$

$1 \le a < 4a < 16a \le 40$

$a \in \{1, 2\}$ (2 GP)

If $r = 5$

$1 \le a < 5a < 25a \le 40$

$a \in \{1\}$ (1 GP)

If $r = 6$

$1 \le a < 6a < 36a \le 40$

$a \in \{1\}$ (1 GP)

Case II: If r is fraction > 1

If $r = \frac{3}{2}$

then $a, \frac{3a}{2}, \frac{9a}{4}$

A must be multiple of 4

$a \in \{4, 8, 12, 16\}$

$$\left.\begin{matrix}(4,6,9)\\(8,12,18)\\(12,18,27)\\(16,24,36)\end{matrix}\right\} 4, \text{ GP}$$

If $r = \frac{5}{2}$, then $a, \frac{5a}{2}, \frac{25a}{4}$

$a = 4$ ...(1) GP

$r = \frac{4}{3}, \quad ar^2 = \frac{16a}{9} \rightarrow a = 9k$

(9, 12, 16), (18, 24, 32) ...(2) GP

$r = \frac{5}{3}, \quad ar^2 = \frac{25a}{9}; a = 9k$

(9, 15, 25) ...(1) GP

$r = \frac{5}{4} \quad ar^2 = \frac{25a}{16}; a = 16k$

(16, 20, 25) ...(1) GP

$r = \frac{6}{5} \quad ar^2 = \frac{36a}{25}; a = 25k$

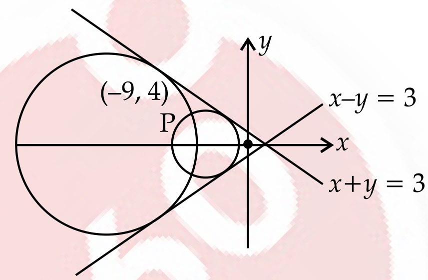
(25, 30, 36) ...(1) GP

Total = 18 + 10 = 28

$$P = \frac{28}{{}^{40}C_3} = \frac{28}{9880} = \frac{7}{2470}$$

$m + n = 2477$

25. Correct answer is [768].



Let centre of circle = $(a, 0)$

radius = r

Distance of centre from line

$x - y - 3 = 0$ is r

$$\therefore \quad r = \left|\frac{a-3}{\sqrt{2}}\right|$$

Equation of circle,

$$(x-a)^2 + y^2 = \left(\frac{a-3}{\sqrt{2}}\right)^2$$

$\because$ It passes through (–9, 4)

$2(a^2 + 18a + 81 + 16) = (a^2 - 6a + 9)$

$a^2 + 42a + 185 = 0$

$(a + 37)(a + 5) = 0$

$\Rightarrow \quad a = -37, -5$

As, $r = \left|\frac{a-3}{\sqrt{2}}\right|$

$$r_1 = \left|\frac{-37-3}{\sqrt{2}}\right| = 20\sqrt{2}$$

$$r_2 = \left|\frac{-5-3}{\sqrt{2}}\right| = 4\sqrt{2}$$

$$|r_1^2 - r_2^2| = |800 - 32| = 768$$

■■■