# JEE (Main) MATHEMATICS SOLVED PAPER

## 2023
## $24^{th}$ Jan. Shift 2

**General Instructions :**

1. *In mathematics Section, there are 30 Questions (Q. no. 1 to 30).*
2. *In mathematics, Section A consists of 20 single choice questions & Section B consists of 10 numerical value type questions. In Section B, candidates have to attempt any five questions out of 10.*
3. *There will be only one correct choice in the given four choices in Section A. For each question for Section A, 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice question and zero mark will be awarded for unattempted question.*
4. *For Section B questions, 4 marks will be awarded for correct answer and zero for unattempted and incorrect answer.*
5. *Any textual, printed or written material, mobile phones, calculator etc. are not allowed for the students appearing for the test.*
6. *All calculations / written work should be done in the rough sheet provided with Question Paper.*

## Section A

**Q. 1.** If, $f(x) = x^3 - x^2 f'(1) + xf''(2) - f'''(3), x \in \mathbb{R}$ then

**(A)** $f(1) + f(2) + f(3) = f(0)$
**(B)** $2f(0) - f(1) + f(3) = f(2)$
**(C)** $3f(1) + f(2) = f(3)$
**(D)** $f(3) - f(2) = f(1)$

**Q. 2.** If the system of equations

$$x + 2y + 3z = 3$$
$$4x + 3y - 4z = 4$$
$$8x + 4y - \lambda z = 9 + \mu$$

has infinitely many solutions, then the ordered pair $(\lambda, \mu)$ is equal to:

**(A)** $\left(-\frac{72}{5}, \frac{21}{5}\right)$ **(B)** $\left(-\frac{72}{5}, -\frac{21}{5}\right)$
**(C)** $\left(\frac{72}{5}, -\frac{21}{5}\right)$ **(D)** $\left(\frac{72}{5}, \frac{21}{5}\right)$

**Q. 3.** If, $f(x) = \frac{2^{2x}}{2^{2x}+2}, x \in R,$ then

$f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + \ldots + f\left(\frac{2022}{2023}\right)$ is equal to

**(A)** 1011 **(B)** 2010
**(C)** 1010 **(D)** 2011

**Q. 4.** Let $\vec{\alpha} = 4\hat{i} + 3\hat{j} + 5\hat{k}$ and $\vec{\beta} = \hat{i} + 2\hat{j} - 4\hat{k}$. Let $\vec{\beta}_1$ be parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ be perpendicular to $\vec{\alpha}$. If $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, then the value of $5\vec{\beta}_2 . (\hat{i} + \hat{j} + \hat{k})$ is

**(A)** 7 **(B)** 9
**(C)** 6 **(D)** 11

**Q. 5.** Let $y = y(x)$ be the solution of the differential equation $(x^2 - 3y^2)\, dx + 3xydy = 0$, $y(1) = 1$. Then $6y^2(e)$ is equal to

**(A)** $2e^2$ **(B)** $3e^2$
**(C)** $e^2$ **(D)** $\frac{3}{2}e^2$

**Q. 6.** The locus of the mid points of the chords of the circle $C_1$: $(x - 4)^2 + (y - 5)^2 = 4$ which subtend an angle $\theta_1$ at the centre of the circle $C_1$, is a circle of radius $r_i$. If $\theta_1 = \frac{\pi}{3}, \theta_3 = \frac{2\pi}{3}$ and $r_1^2 = r_2^2 + r_3^2$, then $\theta_2$ is equal to

**(A)** $\frac{\pi}{4}$ **(B)** $\frac{\pi}{2}$
**(C)** $\frac{\pi}{6}$ **(D)** $\frac{3\pi}{4}$

**Q. 7.** The number of real solutions of the equation $3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$, is

**(A)** 0 **(B)** 3
**(C)** 4 **(D)** 2

**Q. 8.** Let A be a $3 \times 3$ matrix such that $|\text{adj}(\text{adj}(\text{adj}A))| = 12^4$ Then $|A^{-1}\text{adj}A|$ is equal to

**(A)** $\sqrt{6}$ **(B)** $2\sqrt{3}$
**(C)** 12 **(D)** 1

**Q. 9.** $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}} dx$

**(A)** $2\pi$ **(B)** $\frac{\pi}{6}$
**(C)** $\frac{\pi}{3}$ **(D)** $\frac{\pi}{2}$

**Q. 10.** The number of square matrices of order 5 with entries form the set {0, 1}, such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1, is

**(A)** 125 **(B)** 225
**(C)** 150 **(D)** 120

**Q. 11.** If $({}^{30}C_1)^2 + 2\,({}^{30}C_2)^2 + 3\,({}^{30}C_3)^2 + \ldots. + 30\,({}^{30}C_{30})^2 = \frac{\alpha 60!}{(30!)^2}$ then $\alpha$ is equal to:

**(A)** 30 **(B)** 10
**(C)** 60 **(D)** 15

**Q. 12.** Let the plane containing the line of intersection of the planes $P_1 : x + (\lambda + 4)y + z = 1$ and $P_2: 2x + y + z = 2$ pass through the points (0, 1, 0) and (1, 0, 1). Then the distance of the point $(2\lambda, \lambda, -\lambda)$ from the plane $P_2$ is

**(A)** $4\sqrt{6}$ **(B)** $3\sqrt{6}$
**(C)** $5\sqrt{6}$ **(D)** $2\sqrt{6}$

**Q. 13.** Let $f(x)$ be a function such that $f(x + y) = f(x).f(y)$ for all, $x, y \in N$. If $f(1) = 3$ and $\sum_{k=1}^{n} f(k) = 3279$, then the value of $n$ is

**(A)** 9 **(B)** 6
**(C)** 8 **(D)** 7

**Q. 14.** Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6$, be in A.P. and $a_1 + a_3 = 10$. If the mean of these six numbers is $\frac{19}{2}$ and their variance is $\sigma^2$, then $8\sigma^2$ is equal to:

**(A)** 210 **(B)** 220
**(C)** 200 **(D)** 105

**Q. 15.** The equations of the sides AB and AC of a triangle ABC are $(\lambda + 1)x + \lambda y = 4$ and $\lambda x + (1 - \lambda)y + \lambda = 0$ respectively. Its vertex A is on the y-axis and its orthocentre is (1, 2). The length of the tangent from the point C to the part of the parabola $y^2 = 6x$ in the first quadrant is :

**(A)** 4 **(B)** 2
**(C)** $\sqrt{6}$ **(D)** $2\sqrt{2}$

**Q. 16.** Let $p$ and $q$ be two statements. Then $\sim (p \wedge (p \Rightarrow \sim q))$ is equivalent to

**(A)** $p \vee (p \wedge q)$ **(B)** $p \vee (p \wedge (\sim q))$
**(C)** $(\sim p) \vee q$ **(D)** $p \vee ((\sim p) \wedge q)$

**Q. 17.** The set of all values of a for which $\lim_{x \to a} ([x - 5] - [2x + 2]) = 0$, where $[\alpha]$ denotes the greatest integer less than or equal to $\alpha$ is equal to

**(A)** [–7.5, –6.5) **(B)** [–7.5, –6.5]
**(C)** (–7.5, – 6.5] **(D)** (–7.5, –6.5)

**Q. 18.** If the foot of the perpendicular drawn from (1, 9, 7) to the line passing through the point (3, 2, 1) and parallel to the planes $x + 2y + z = 0$ and $3y - z = 3$ is $(\alpha, \beta, \gamma)$, then $\alpha + \beta + \gamma$ is equal to

**(A)** 3 **(B)** 1
**(C)** –1 **(D)** 5

**Q. 19.** The number of integers, greater than 7000 that can be formed, using the digits 3, 5, 6, 7, 8 without repetition, is

**(A)** 168 **(B)** 220
**(C)** 120 **(D)** 48

**Q. 20.** The value of $\left(\dfrac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$

**(A)** $-\frac{1}{2}(\sqrt{3}-i)$ **(B)** $-\frac{1}{2}(1-i\sqrt{3})$
**(C)** $\frac{1}{2}(1-i\sqrt{3})$ **(D)** $\frac{1}{2}(\sqrt{3}+i)$

## Section B

**Q. 21.** If the shortest distance between the lines $\frac{x+\sqrt{6}}{2} = \frac{y-\sqrt{6}}{3} = \frac{z-\sqrt{6}}{4}$ and $\frac{x-\lambda}{3} = \frac{y-2\sqrt{6}}{4} = \frac{z+2\sqrt{6}}{5}$ is 6, then the square of sum of all possible values of $\lambda$ is

**Q. 22.** Three urns A, B and C contain 4 red, 6 black; 5 red, 5 black; and $\lambda$ red, 4 black balls respectively. One of the urns is selected at random and a ball is drawn. If the ball drawn is red and the probability that it is drawn from urn C is 0.4 then the square of the length of the side of the largest equilateral triangle, inscribed in the parabola $y^2 = \lambda x$ with one vertex at the vertex of the parabola, is

**Q. 23.** Let $S = \{\theta \in [0, 2\pi) : \tan(\pi\cos\theta) + \tan(\pi\sin\theta) = 0\}$

Then $\sum_{\theta \in s} \sin^2\left(\theta + \frac{\pi}{4}\right)$ is equal to

**Q. 24.** If $\frac{1^3 + 2^3 + 3^3 + \ldots \text{ up to } n \text{ terms}}{1.3+2.5+3.7+\ldots\text{up to } n \text{ terms}} = \frac{9}{5}$, then the value of $n$ is

**Q. 25.** Let the sum of the coefficients of the first three terms in the expansion of $\left(x - \frac{3}{x^2}\right)^n, x \neq 0, n \in N$, be 376. Then the coefficient of $x^4$ is

**Q. 26.** The equations of the sides AB, BC and CA of a triangle ABC are : $2x + y = 0$, $x + py = 21a$, $(a \neq 0)$ and $x - y = 3$ respectively. Let $P(2, a)$ be the centroid of $\Delta$ ABC. Then $(BC)^2$ is equal to

**Q. 27.** Let $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}, \vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}$,

$\vec{a}.\vec{c} = 7, 2\vec{b}.\vec{c} + 43 = 0, \ \vec{a} \times \vec{c} = \vec{b} \times \vec{c}$.

Then $|\vec{a}.\vec{b}|$ is equal to

**Q. 28.** The minimum number of elements that must be added to the relation $R = \{(a, b), (b, c), (b, d)\}$ on the set $\{a, b, c, d\}$ so that it is an equivalence relation, is

**Q. 29.** If the area of the region bounded by the curves $y^2 - 2y = -x$, $x + y = 0$ is A, then 8 A is equal to

**Q. 30.** Lef $f$ be a differentiable function defined on $\left[0, \frac{\pi}{2}\right]$ such that $f(x) > 0$ and

$f(x) + \int_0^x f(t)\sqrt{1-(\log_e f(t))^2}\,dt = e, \forall x \in \left[0, \frac{\pi}{2}\right]$.

Then $\left(6\log_e f\left(\frac{\pi}{6}\right)\right)^2$ is equal to

# Answer Key

| Q. No. | Answer | Topic Name | Chapter Name |
|---|---|---|---|
| 1 | B | Higher Order Derivatives | Differential Calculus |
| 2 | C | System of linear equations | Matrices and Determinants |
| 3 | A | Algebra of Functions | Function |
| 4 | A | Scalar and Vector Products | Vector Algebra |
| 5 | A | Linear Differential Equations | Differential Equations |
| 6 | B | Interaction between Circle and a Line | Circle |
| 7 | A | Quadratic Equation and its Solution | Quadratic Equations |
| 8 | B | Adjoint of a Matrix | Matrices and Determinants |
| 9 | A | Basics of Definite Integration | Definite Integration |
| 10 | D | Permutations | Permutation and Combination |
| 11 | D | Properties of Binomial Coefficients | Binomial Theorem |
| 12 | B | Plane and a Point | Three Dimensional Geometry |
| 13 | D | Geometric Progressions | Sequences and Series |
| 14 | A | Measures of Dispersion | Statistics |
| 15 | D | Tangent to a Parabola | Parabola |
| 16 | C | Logical Operations | Mathematical Reasoning |
| 17 | D | Algebra of Limits | Limits |
| 18 | D | Lines in 3D | Three Dimensional Geometry |
| 19 | A | Permutations | Permutations and Combinations |
| 20 | A | Representation of Complex Numbers | Complex Numbers |
| 21 | [384] | Skew Lines | Three Dimensional Geometry |
| 22 | [432] | Bayes' Theorem | Probability |
| 23 | [2] | Trigonometric Equations | Trigonometric Equations and Inequalities |
| 24 | [5] | Series of Natural Numbers and other Miscellaneous Series | Sequences and Series |
| 25 | [405] | Binomial Theorem for Positive Integral Index | Binomial Theorem |
| 26 | [122] | Interaction between Two Lines | Point and Straight Line |
| 27 | [8] | Scalar and Vector Products | Vector Algebra |
| 28 | [13] | Algebra of Relations | Set Theory and Relations |
| 29 | [36] | Area Bounded by Curves | Area under Curves |
| 30 | [27] | Variable Separable Form | Differential Equations |

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# Solutions

## Section A

**1. Option (B) is correct.**

Given, $f(x) = x^3 - x^2 f'(1) + xf''(2) - f'''(3), x \in \mathbb{R}$

Let $f'(1) = p, f''(2) = q$ and $f'''(3) = r$

$\Rightarrow f(x) = x^3 - px^2 + qx - r$

$\Rightarrow f'(x) = 3x^2 - 2px + q$

$\Rightarrow f''(x) = 6x - 2p$

$\Rightarrow f'''(x) = 6$

So, $f'''(3) = 6 = r$

Now, $f'(1) = 3(1)^2 - 2p(1) + q$

$\Rightarrow p = 3 - 2p + q$

$\Rightarrow 3p = 3 + q$ ...(i)

And $f''(2) = 6(2) - 2p$

$\Rightarrow q = 12 - 2p$

$\Rightarrow 2p + q = 12$ ...(ii)

On solving equation (i) and equation (ii), we get

$p = 3, q = 6$

$\therefore f(x) = x^3 - 3x^2 + 6x - 6$

So, $f(0) = -6, f(1) = -2, f(2) = 2, f(3) = 12$

Now, $2f(0) - f(1) + f(3) = 2(-6) - (-2) + 12$

$= 2 = f(2)$

**HINT:**

Find $f'(x), f''(x)$ and $f'''(x)$ using $\frac{d}{dx}(x^n) = nx^{n-1}$ and solve further.

**2. Option (C) is correct.**

**Given:** System of equations

$x + 2y + 3z = 3$

$4x + 3y - 4z = 4$

$8x + 4y - \lambda z = 9 + \mu$

As we know for infinite many solutions, $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

Now, $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 3 & -4 \\ 8 & 4 & -\lambda \end{vmatrix} = 0$

$\Rightarrow 1(-3\lambda + 16) - 2(-4\lambda + 32) + 3(16 - 24) = 0$

$\Rightarrow -3\lambda + 16 + 8\lambda - 64 - 24 = 0$

$\Rightarrow 5\lambda - 72 = 0$

$\Rightarrow \lambda = \frac{72}{5}$

Now, $\Delta_3 = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 3 & 4 \\ 8 & 4 & 9+\mu \end{vmatrix} = 0$

$\Rightarrow 1(27 + 3\mu - 16) - 2(36 + 4\mu - 32) + 3(16 - 24) = 0$

$\Rightarrow 11 + 3\mu - 8\mu - 8 - 24 = 0$

$\Rightarrow -5\mu - 21 = 0$

$\Rightarrow \mu = -\frac{21}{5}$

**HINT:**

Consider, $a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

To solve this system we first define the following determinants

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}, \Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$$

$$\Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

System of linear equations have infinite solutions if $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

**3. Option (A) is correct.**

Given: $f(x) = \frac{2^{2x}}{2^{2x} + 2}, x \in R$

$\Rightarrow f(x) = \frac{4^x}{4^x + 2}$

Now, $f(1-x) = \frac{4^{(1-x)}}{4^{(1-x)} + 2}$

$\Rightarrow f(1-x) = \frac{4}{4 + 2.4^x}$

$\Rightarrow f(1-x) = \frac{2}{2 + 4^x}$

So, $f(x) + f(1-x) = \frac{4^x}{4^x + 2} + \frac{2}{4^x + 2} = 1$

Let $A = f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + .... + f\left(\frac{2022}{2023}\right)$

$$\Rightarrow A = f\left(\frac{1}{2023}\right) + f\left(\frac{2022}{2023}\right) + f\left(\frac{2}{2023}\right) + f\left(\frac{2021}{2023}\right) + .... + f\left(\frac{1011}{2023}\right) + f\left(\frac{1012}{2023}\right)$$

$\Rightarrow A = 1 + 1 + 1 + ....$ up to 1011 terms $\{\because f(x) + f(1-x) = 1\}$

$\Rightarrow A = 1011$

**HINT:**

Use $f(x) + f(1-x) = 1$ and solve further.

4. **Option (A) is correct.**

Given: $\vec{\alpha} = 4\hat{i} + 3\hat{j} + 5\hat{k}$

$\vec{\beta} = \hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$

$\because\ \vec{\beta}_1$ is parallel to $\vec{\alpha}$

$\Rightarrow \beta_1 = \mu(4\hat{i} + 3\hat{j} + 5\hat{k}); \mu \in R$

Also given that $\vec{\beta}_2$ is perpendicular to $\alpha$

$\Rightarrow \vec{\beta}_2\ \alpha = 0$

Since, $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$

$\Rightarrow \vec{\beta} = \mu\vec{\alpha} + \vec{\beta}_2$

$\Rightarrow \vec{\beta}.\vec{\alpha} = \mu\,|\vec{\alpha}^2| + \vec{\beta}_2.\vec{\alpha}$

$\Rightarrow (4\hat{i} + 3\hat{j} + 5\hat{k}).(\hat{i} + 2\hat{j} - 4\hat{k}) = \mu(\sqrt{16+9+25})^2 + 0$

$\Rightarrow 4 + 6 - 20 = \mu(50)$

$\Rightarrow \mu = -\frac{1}{5}$

Now, $\vec{\beta} = -\frac{1}{5}\vec{\alpha} + \vec{\beta}_2$

$\Rightarrow 5\vec{\beta}_2 = 5\vec{\beta} + \vec{\alpha}$

$\Rightarrow 5\vec{\beta}_2 = 5(\hat{i} + 2\hat{j} - 4\hat{k}) + (4\hat{i} + 3\hat{j} + 5\hat{k})$

$\Rightarrow 5\vec{\beta}_2 = 9\hat{i} + 13\hat{j} - 15\hat{k}$

Now, $5\vec{\beta}_2.(\hat{i} + \hat{j} + \hat{k}) = 9 + 13 - 15 = 7$

**HINT:**

**(1)** Use if $\vec{a}$ is parallel to $\vec{b}$, then $\vec{b} = k\vec{a}; k \in R$

**(2)** Use if $\vec{u}$ is perpendicular to $\vec{v}$, then $\vec{u}.\vec{v} = 0$

**(3)** If $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$ and $\vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$, then $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

5. **Option (A) is correct.**

Given, differential equation $(x^2 - 3y^2)\,dx + 3xy\,dy = 0$, $y(1) = 1$

$\Rightarrow 3xy\frac{dy}{dx} - 3y^2 = -x^2$

$\Rightarrow y\frac{dy}{dx} - \frac{y^2}{x} = -\frac{x}{3}$

$\Rightarrow 2y\frac{dy}{dx} - \frac{2y^2}{x} = -\frac{2x}{3}$

Let $y^2 = v$

$\Rightarrow 2y\frac{dy}{dx} = \frac{dv}{dx}$

So, $\frac{dv}{dx} - \frac{2v}{x} = \frac{-2x}{3}$ which is linear differential equation

Now, I.F. $= e^{\int -\frac{2}{x}dx}$

$\Rightarrow$ I.F. $= e^{-2\ln x}$

$\Rightarrow$ I.F. $= e^{\ln x^{-2}} \Rightarrow$ I.F. $= \frac{1}{x^2}$

Now, solution of linear differential equation is

$v\,(\text{I.F.}) = \int \frac{-2x}{3}(\text{I.F.})\,dx + c$

$\Rightarrow v\frac{(1)}{(x^2)} = \int \frac{-2x}{3} \times \frac{1}{x^2}dx + c$

$\Rightarrow \frac{v}{x^2} = -\frac{2}{3}\int \frac{1}{x}dx + c$

$\Rightarrow \frac{v}{x^2} = -\frac{2}{3}\ln x + c$

$\Rightarrow \frac{y^2}{x^2} = -\frac{2}{3}\ln x + c \qquad \because y(1) = 1$

$\Rightarrow c = 1$

So, $\frac{y^2}{x^2} = -\frac{2}{3}\ln x + 1$

$\Rightarrow y^2 = -\frac{2}{3}x^2\ln x + x^2$

$\Rightarrow y^2(e) = -\frac{2}{3}e^2\ln e + e^2$

$\Rightarrow y^2(e) = \frac{e^2}{3}$

$\Rightarrow 6y^2(e) = 2e^2$

**HINT:**

**(1)** Convert given differential equation into linear differential equation by substituting $y^2 = v$.

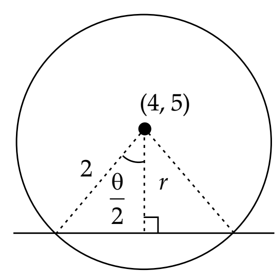
**(2)** Solution of linear differential equation $\frac{dy}{dx} + Px = Q$ is given by $y(\text{I.F.}) = \int Q(\text{I.F.})dx + c$, where I.F. $= e^{\int P dx}$

6. **Option (B) is correct.**

**Given:** Circle $c_1$: $(x-4)^2 + (y-5)^2 = 4$

$\Rightarrow$ Centre = (4, 5) and radius = 2

Also given that $\theta_1 = \frac{\pi}{3}, \theta_3 = \frac{2\pi}{3}$ and $r_1^2 = r_2^2 + r_3^2$




So, $\cos\left(\frac{\theta_i}{2}\right) = \frac{r_i}{2}$

$\Rightarrow r_i = 2\cos\left(\frac{\theta_i}{2}\right) \qquad [\because r_1^2 = r_2^2 + r_3^2]$

$\Rightarrow \cos^2\frac{\theta_1}{2} = \cos^2\frac{\theta_2}{2} + \cos^2\frac{\theta_3}{2}$

$$\Rightarrow \cos^2\left(\frac{\pi}{6}\right) = \cos^2\left(\frac{\theta_2}{3}\right) + \cos^2\left(\frac{\pi}{3}\right)$$

$$\Rightarrow \frac{3}{4} = \frac{1}{4} + \cos^2\frac{\theta_2}{2}$$

$$\Rightarrow \cos^2\frac{\theta_2}{2} = \frac{1}{2}$$

$$\Rightarrow \frac{\theta_2}{2} = \frac{\pi}{4}$$

$$\Rightarrow \theta_2 = \frac{\pi}{2}$$

**HINT:**

**(1)** Use radius of locus will be perpendicular from centre of the circle to chord.
**(2)** Use perpendicular drawn from centre to the chord bisects the chord.

**7. Option (A) is correct.**

Given: $3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$

$$\Rightarrow 3\left[\left(x + \frac{1}{x}\right)^2 - 2\right] - 2\left(x + \frac{1}{x}\right) + 5 = 0$$

Let $x + \frac{1}{x} = v$
$\Rightarrow 3\,[v^2 - 2] - 2v + 5 = 0$
$\Rightarrow 3v^2 - 2v - 1 = 0$
$\Rightarrow (3v + 1)(v - 1) = 0$

$\Rightarrow v = 1, -\frac{1}{3}$

As we know $x + \frac{1}{x} \geq 2$ or $x + \frac{1}{x} \leq -2$

But $x + \frac{1}{x} = v = 1, -\frac{1}{3}$

So, no real solution of the given equation is possible.

**HINT:**

**(1)** Convert given equation into quadratic equation by substituting $x + \frac{1}{x} = v$ and solve further.
**(2)** Use $x + \frac{1}{x} \in (-\infty, -2] \cup [2, \infty)$

**8. Option (B) is correct.**

Given $|\text{adj (adj (adj A))}| = 12^4$

$\Rightarrow |A|^{(n-1)^3} = 12^4$

$\Rightarrow |A|^{(2)^3} = 12^4$

$\Rightarrow |A|^8 = 12^4$

$\Rightarrow |A| = \sqrt{12}$

Now, $|A^{-1}\,\text{adj}A| = |A^{-1}|\ |\text{adj A}|$

$= \frac{1}{|A|}|A|^2$ $\qquad \{\because |\text{adj A}| = |A|^{n-1};$ where $n$ = order of matrix A$\}$

$= |A| = \sqrt{12} = 2\sqrt{3}$

**HINT:**

**(1)** Use $|\text{adj (adj (adj A))}| = |A|^{(n-1)^3}$; where $n$ = order of square matrix.
**(2)** Use $|\text{adj A}| = |A|^{n-1}$; where $n$ = order of square matrix.
**(3)** Use $|AB| = |A|\ |B|$

**9. Option (A) is correct.**

Let I = $\displaystyle\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9 - 4x^2}}dx$

As we know $\displaystyle\int \frac{1}{\sqrt{a^2 - (bx)^2}}dx = \frac{1}{b}\sin^{-1}\left(\frac{bx}{a}\right)$

$$\Rightarrow I = \frac{48}{2}\left[\sin^{-1}\left(\frac{2x}{3}\right)\right]_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}}$$

$$\Rightarrow I = 24\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]$$

$$\Rightarrow I = 24\left[\frac{\pi}{3} - \frac{\pi}{4}\right] \Rightarrow I = 24\left[\frac{\pi}{12}\right] \Rightarrow I = 2\pi$$

**HINT:**

**(1)** Use $\int_a^b f(x)dx = F(b) - F(a)$,
where $\int f(x)dx = F(x) + c$

**(2)** Use $\displaystyle\int \frac{1}{\sqrt{a^2 - (bx)^2}}dx = \frac{1}{b}\sin^{-1}\left(\frac{bx}{a}\right)$

**10. Option (D) is correct.**

$$\begin{bmatrix} - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \end{bmatrix}$$

$\because$ Sum of all the elemetns in each row and in each column is 1
$\therefore$ In every row and every column there would be exactly one 1 and four zeroes.
So, number of required matrices
$= {}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times {}^1C_1$
$= 5 \times 4 \times 3 \times 2 \times 1 = 120$

**HINT:**

**(1)** In every row and every column there would be exactly one 1 and four zeroes.
**(2)** Recall multiplication principle of counting.

**11. Option (D) is correct.**

Given, $({}^{30}C_1)^2 + 2({}^{30}C_2)^2 + 3({}^{30}C_3)^2 + .... + 30({}^{30}C_{30})^2$

$= \dfrac{\alpha 60!}{(30!)^2}$

Let $P = 0({}^{30}C_0)^2 + 1({}^{30}C_1)^2 + 2({}^{30}C_2)^2 + .... + 30({}^{30}C_{30})^2$ ...(i)

$P = 30({}^{30}C_{30})^2 + 29({}^{30}C_{29})^2 + 28({}^{30}C_{28})^2 + .... + 0({}^{30}C_0)^2$ ...(ii)

Adding equation (i) and equation (ii), we get

$2P = 30\left[\left({}^{30}C_0^2\right)+\left({}^{30}C_1^2\right)+\left({}^{30}C_2^2\right)+....+\left({}^{30}C_{30}^2\right)\right]$

As we know $\sum_{r=0}^{n}\left({}^{n}C_r\right)^2 = {}^{2n}C_n$

So, $P = 15\ {}^{60}C_{30}$

$\Rightarrow P = 15\ \dfrac{60!}{(30!)^2}$

$\Rightarrow \alpha = 15$

**HINT:**

**(1)** Let $P = 0({}^{30}C_0)^2 + 1({}^{30}C_1)^2 + ... + 30({}^{30}C_{30})^2$ and make another equation by reversing the term and add both the equations.

**(2)** Use $\sum_{r=0}^{n}\left({}^{n}C_r\right)^2 = {}^{2n}C_n$

**12. Option (B) is correct.**

Given planes $P_1: x + (\lambda + 4)y + z = 1$

$P_2: 2x + y + z = 2$

Equation of plane containing the line of intersection of the plane $P_1$ and $P_2$ is given by

$P: [x + (\lambda + 4)y + z - 1] + k[2x + y + z - 2] = 0$

$\because$ Plane P passes through (0, 1, 0)

$\Rightarrow \lambda + 4 - 1 + k(1 - 2) = 0$

$\Rightarrow \lambda - k + 3 = 0$ ...(i)

Plane P also passes through (1, 0, 1)

$\Rightarrow 1 + k(2 + 1 - 2) = 0$

$\Rightarrow k = -1$

Put the value of $k = -1$ in equation (i), we get

$\lambda = -4$

So, point $(2\lambda, \lambda, -\lambda) = (-8, -4, 4)$

Now, distance of $(-8, -4, 4)$ from plane $P_2$ is

$d = \left|\dfrac{2(-8)-4+4-2}{\sqrt{2^2+1^2+1^2}}\right|$

$\Rightarrow d = \left|\dfrac{-18}{\sqrt{6}}\right|$

$\Rightarrow d = 3\sqrt{6}$

**HINT:**

**(1)** Equation of plane containing the line of intersection of the plane $P_1$ and $P_2$ is given by $P_1 + \lambda P_2 = 0$.

**(2)** Perpendicular distance of point $(x_1, y_1, z_1)$ from plane $ax + by + cz + d = 0$ is given by

$D = \left|\dfrac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$

**13. Option (D) is correct.**

Given: $f(x + y) = f(x).f(y)$

$\Rightarrow f(x) = p^x$ $[\because f(1) = 3]$

$\Rightarrow p = 3$

So, $f(x) = 3^x$

Also given that $\sum_{k=1}^{n} f(k) = 3279$

$\Rightarrow f(1) + f(2) + ..... + f(n) = 3279$

$\Rightarrow 3 + 3^2 + .... + 3^n = 3279$

$\Rightarrow \dfrac{3(3^n-1)}{3-1} = 3279$

$\Rightarrow 3^n = 2187$

$\Rightarrow 3^n = 3^7$

$\Rightarrow n = 7$

**HINT:**

**(1)** Consider $f(x) = P^x$ and find the value of P by given condition.

**(2)** Sum of GP whose first term is $a$, common ratio $= r$ is given by $S = \dfrac{a(r^n-1)}{r-1}; r > 1$ where $n$ = number of terms.

**14. Option (A) is correct.**

Given $a_1, a_2, a_3, a_4, a_5, a_6$ are in A.P. and $a_1 + a_3 = 10$

And mean of $a_1, a_2, a_3, a_4, a_5, a_6 = \dfrac{19}{2}$

So, $\dfrac{a_1+a_2+a_3+a_4+a_5+a_6}{6} = \dfrac{19}{2}$

$\Rightarrow a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 57$

Let common difference of AP be $d$.

So, $\dfrac{6}{2}(2a_1 + 5d) = 57$

$\{\because$ Sum of $n$ terms of AP is given by $\dfrac{n}{2}[2a+(n-1)d]$

where $a$ = first term and $d$ = common difference$\}$

$\Rightarrow 2a_1 + 5d = 19$ ...(i)

$[\because a_1 + a_3 = 10]$

$\Rightarrow a_1 + a_1 + 2d = 10$

$\Rightarrow 2a_1 + 2d = 10$

$\Rightarrow a_1 + d = 5$ ...(ii)

On solving equation (i) and equation (ii), we get

$a_1 = 2$ and $d = 3$

Now, variance $= \sigma^2 = \dfrac{\Sigma x_i^2}{n} - (\bar{x})^2$

$\Rightarrow \sigma^2 = \dfrac{2^2+5^2+8^2+11^2+14^2+17^2}{6} - \left(\dfrac{19}{2}\right)^2$

$\Rightarrow \sigma^2 = \dfrac{699}{6} - \dfrac{361}{4}$

$\Rightarrow \sigma^2 = \dfrac{105}{4} \Rightarrow 8\sigma^2 = 210$

**HINT:**

**(1)** mean of $a_1, a_2, a_3, ...., a_n$ is given by

$x = \dfrac{a_1+a_2+a_3+....+a_n}{n}$

**(2)** Sum of $n$ terms of AP is given by

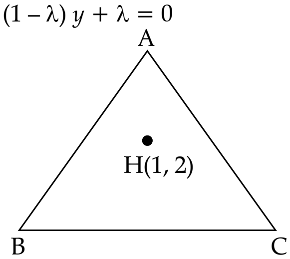
$S_n = \frac{n}{2}[2a+(n-1)d]$; where $a$ = first term and $d$ = common difference

**(3)** Variance $= \frac{\sum x_i^2}{n} - (\bar{x})^2$

**15. Option (D) is correct.**

**Given:** Equation of AB : $(\lambda + 1)\, x + \lambda y = 4$

AC : $\lambda x + (1 - \lambda)\, y + \lambda = 0$




$\because$ Vertex A lies on $y$-axis

$\therefore$ $x$-coordinate of point A = 0

So, $x = 0$ will satisfy the equation of AB and AC

So, from equation of AB, $y = \frac{4}{\lambda}$

And from equation of AC, $y = \frac{\lambda}{\lambda - 1}$

So, $\frac{4}{\lambda} = \frac{\lambda}{\lambda-1}$

$\Rightarrow 4\lambda - 4 = \lambda^2$

$\Rightarrow (\lambda - 2)^2 = 0$

$\Rightarrow \lambda = 2$

So, A = (0, 2)

Now, AB : $3x + 2y = 4$ and AC : $2x - y = -2$

Slope of AB, $m_{AB} = -\frac{3}{2}$

$\because$ $\mu(1, 2)$ is orthocentre of $\Delta$ABC

$\therefore m_{CH} \,.\, m_{AB} = -1$

$\Rightarrow m_{CH} = \frac{2}{3}$

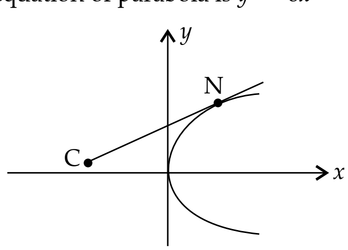
Let the coordinates of point C be (P, 2P + 2)

$\Rightarrow \frac{2P+2-2}{P-1} = \frac{2}{3}$

$\Rightarrow P = -\frac{1}{2}$

$\therefore C = \left(-\frac{1}{2}, 1\right)$

Given equation of parabola is $y^2 = 6x$




Now, equation of tangent to the parabola $y^2 = 6x$ in parametric form is given by $ty = x + \frac{3}{2}t^2$.

$\because$ Tangent is passing through $C\left(-\frac{1}{2}, 1\right)$

$\therefore t = -\frac{1}{2} + \frac{3}{2}t^2$

$\Rightarrow 3t^2 - 2t - 1 = 0$

$\Rightarrow (3t + 1)(t - 1) = 0 \Rightarrow t = 1$

So, coordinates of point of contact N = $(at^2, 2at)$

$= \left(\frac{3}{2}, 3\right)$

Now, NC $= \sqrt{\left(\frac{3}{2}+\frac{1}{2}\right)^2 + (3-1)^2}$

$\Rightarrow$ NC $= \sqrt{4+4} = 2\sqrt{2}$

**HINT:**

**(1)** Find the coordinates of point A by using the condition of vertex A lies on $y$-axis.

**(2)** Find the coordinates of point B and C by using the definition of orthocentre.

**(3)** Equation of tangent to parabola $y^2 = 4ax$ in parametric form is given by $ty = x + at^2$

**16. Option (C) is correct.**

As we know $A \Rightarrow B = \sim A \vee B$

So, $p \Rightarrow \sim q = \sim p \vee \sim q$

Now, $p \wedge (p \Rightarrow \sim q) = p \wedge (\sim p \vee \sim q)$

$= (p \wedge \sim p) \vee (p \wedge \sim q) = F \vee (p \wedge \sim q)$

Now, $\sim [p \wedge (p \Rightarrow \sim q)] = \sim [F \vee (p \wedge \sim q)]$

$= \sim F \wedge \sim (p \wedge \sim q) = T \wedge (\sim p \vee q) = \sim p \vee q$

**HINT:**

**(1)** Use $A \Rightarrow B = \sim A \vee B$

**(2)** Use $A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C)$

**17. Option (D) is correct.**

Given, $\lim_{x \to a}([x-5]-[2x+2]) = 0$

$\Rightarrow [a - 5] - [2a + 2] = 0$

$\Rightarrow [a] - 5 - [2a] - 2 = 0$

$\Rightarrow [a] - [2a] = 7$ ...(i)

If $a \in z$, we have $a = -7$

For $a \in (-7.5, -7)$, $[a] - [2a] = -8 + 15 = 7$

So, $a \in (-7.5, -7)$ satisfy the given equation.

For $a \in (-7, -6.5)$, $[a] - [2a] = -7 + 14 = 7$

So, $a \in (-7, -6.5)$ satisfy the given equation

At $a = -7.5$

$[a] - [2a] = -8 + 15 = 7$

So, $a = -7.5$ satisfy the equation (i)

Now, at $a = -6.5$

$[a] - [2a] = -7 + 13 = 6$

So, $a = -6.5$ doesn't satisfy the equation (i)

$\because x \to a$

$\therefore a \neq -6.5$ or $-7.5$

So, $a \in (-7.5, -6.5)$

**HINT:**

Solve given limit using the definition of greatest integer function.

**18. Option (D) is correct.**

Let the normals of the plane $x + 2y + z = 0$ and $3y - z = 3$ be $\vec{n}_1$ & $\vec{n}_2$

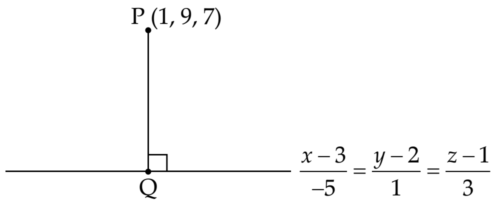
$\Rightarrow \vec{n}_1 = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{n}_2 = 3\hat{j} - \hat{k}$

And the direction ratio of the line = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{vmatrix}$

$= \hat{i}(-2-3) - \hat{j}(-1+0) + \hat{k}(3-0) = -5\hat{i} + \hat{j} + 3\hat{k}$

So the equation of the line passing through (3, 2, 1) is

$\frac{x-3}{-5} = \frac{y-2}{1} = \frac{z-1}{3} = k$




Let the coordinates of point Q be $(-5k + 3, k + 2, 3k + 1)$

Now, direction ratios of PQ = $-5k + 3 - 1, k + 2 - 9, 3k + 1 - 7$

$= -5k + 2, k - 7, 3k - 6$

$\because$ PQ $\perp$ Line

So, $(-5k + 2)(-5) + (k - 7)(1) + (3k - 6)3 = 0$

$\Rightarrow 35k = 35$

$\Rightarrow k = 1$

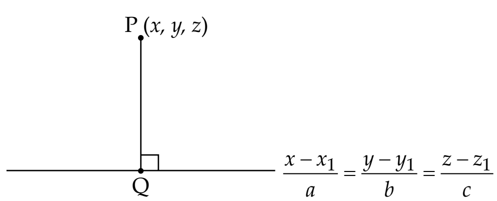
$\therefore$ Foot of perpendicular Q = $(-5 + 3, 1 + 2, 3 + 1)$ = $(-2, 3, 4)$

So, $\alpha + \beta + \gamma = -2 + 3 + 4 = 5$

**HINT:**

**(1)** Direction ratio of the line will be $\vec{n}_1 \times \vec{n}_2$; where $\vec{n}_1$ and $\vec{n}_2$ are normal vectors of given planes.

**(2)**




Assume coordinates of point Q on the line in parametric form and find the value of unknown using PQ $\perp$ line

**19. Option (A) is correct.**

Given digits : 3, 5, 6, 7, 8

All five digits number is greater than 7000

So, number of five digits number = 5! = 120

For 4 digits number greater than 7000

For 1000$^{th}$ place we can take only 7 or 8 from given digits and for remaining places we can take any digit from given digits.

So, number of 4 digits number greater than 7000

$= 2 \times 4 \times 3 \times 2 = 48$

$\therefore$ Number of integer, greater than 7000

$= 120 + 48 = 168$

**HINT:**

First find number of 5 digits numbers and then find 4 digit numbers of taking 7 or 8 on 1000$^{th}$ place using the fundamental principle of counting.

**20. Option (A) is correct.**

Let A = $\left(\frac{1 + \sin\left(\frac{2\pi}{9}\right) + i\cos\left(\frac{2\pi}{9}\right)}{1 + \sin\left(\frac{2\pi}{9}\right) - i\cos\left(\frac{2\pi}{9}\right)}\right)^3$

$\Rightarrow A = \left(\frac{1 + \cos\left(\frac{\pi}{2} - \frac{2\pi}{9}\right) + i\sin\left(\frac{\pi}{2} - \frac{2\pi}{9}\right)}{1 + \cos\left(\frac{\pi}{2} - \frac{2\pi}{9}\right) - i\sin\left(\frac{\pi}{2} - \frac{2\pi}{9}\right)}\right)^3$

$\Rightarrow A = \left(\frac{1 + \cos\left(\frac{5\pi}{18}\right) + i\sin\left(\frac{5\pi}{18}\right)}{1 + \cos\left(\frac{5\pi}{18}\right) - i\sin\left(\frac{5\pi}{18}\right)}\right)^3$

$\Rightarrow A = \left(\frac{2\cos^2\frac{5\pi}{36} + 2i\sin\left(\frac{5\pi}{36}\right)\cos\left(\frac{5\pi}{36}\right)}{2\cos^2\frac{5\pi}{36} - 2i\sin\left(\frac{5\pi}{36}\right)\cos\left(\frac{5\pi}{36}\right)}\right)^3$

$\Rightarrow A = \left(\frac{\cos\frac{5\pi}{36} + i\sin\frac{5\pi}{36}}{\cos\frac{5\pi}{36} - i\sin\frac{5\pi}{36}}\right)^3$

$\Rightarrow A = \left(\frac{e^{i\frac{5\pi}{36}}}{e^{-i\frac{5\pi}{36}}}\right)^3$

$\Rightarrow A = \left(e^{i\frac{5\pi}{18}}\right)^3 \Rightarrow A = e^{i\frac{5\pi}{6}}$

$\Rightarrow A = \cos\frac{5\pi}{6} + i\sin\left(\frac{5\pi}{6}\right)$

$\Rightarrow A = -\frac{\sqrt{3}}{2} + i\frac{1}{2}$

**HINT:**

**(1)** Simplify given expression using trigonometric identities and try to convert given expression as $\cos\theta + i\sin\theta$ in numerator and denominator and then solve further using Euler form.

**(2)** Use $e^{i\theta} = \cos\theta + i\sin\theta$

**21. The correct answer is (384).**

Given lines $L_1$: $\frac{x+\sqrt{6}}{2} = \frac{y-\sqrt{6}}{3} = \frac{z-\sqrt{6}}{4}$

$\Rightarrow L_1$: $\vec{r} = (-\sqrt{6}\hat{i} + \sqrt{6}\hat{j} + \sqrt{6}\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$

And $L_2$: $\frac{x-\lambda}{3} = \frac{y-2\sqrt{6}}{4} = \frac{z+2\sqrt{6}}{5}$

$\Rightarrow L_2$: $\vec{r} = (\lambda\hat{i} + 2\sqrt{6}\hat{j} - 2\sqrt{6}\hat{k}) + \mu(3\hat{i} + 4\hat{j} + 5\hat{k})$

As we know shortest distance between two lines

$\vec{r} = \vec{a} + \lambda\vec{p}$ and $\vec{r} = \vec{b} + \mu\vec{q}$ is given by

$d = \left|\frac{(\vec{b} - \vec{a}).(\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|}\right|$

So, $\vec{a} = -\sqrt{6}\hat{i} + \sqrt{6}\hat{j} + \sqrt{6}\hat{k}$

$\vec{b} = \lambda\hat{i} + 2\sqrt{6}\hat{j} - 2\sqrt{6}\hat{k}$

$\vec{p} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{q} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

Now, $\vec{b} - \vec{a} = (\lambda + \sqrt{6})\hat{i} + \sqrt{6}\hat{j} - 3\sqrt{6}\hat{k}$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$\Rightarrow \vec{p} \times \vec{q} = -\hat{i} + 2\hat{j} - \hat{k}$

$\Rightarrow |\vec{p} \times \vec{q}| = \sqrt{1+4+1} = \sqrt{6}$

Now, $(\vec{b} - \vec{a}).(\vec{p} \times \vec{q}) = -\lambda - \sqrt{6} + 2\sqrt{6} + 3\sqrt{6} = -\lambda + 4\sqrt{6}$

So, shortest distance = $\left|\dfrac{-\lambda + 4\sqrt{6}}{\sqrt{6}}\right| = 6$

$\Rightarrow |-\lambda + 4\sqrt{6}| = 6\sqrt{6}$

$\Rightarrow -\lambda + 4\sqrt{6} = \pm 6\sqrt{6}$

$\Rightarrow \lambda = 4\sqrt{6} \mp 6\sqrt{6}$

$\Rightarrow \lambda = -2\sqrt{6}, 10\sqrt{6}$

Sum of all possible values of $\lambda = -2\sqrt{6} + 10\sqrt{6} = 8\sqrt{6}$

$\therefore (8\sqrt{6})^2 = 384$

**HINT:**

**(1)** Write the given equation of line vector form and use distance between two lines $\vec{r} = \vec{a} + \lambda\vec{p}$ and $\vec{r} = \vec{b} + \mu\vec{q}$ is given by

$$d = \left|\frac{(\vec{b} - \vec{a}).(\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|}\right|$$

**(2)** If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**22. The correct answer is (432).**

Given, Urn A contains 4 Red, 6 Black

Urn B contains 5 Red, 5 Black

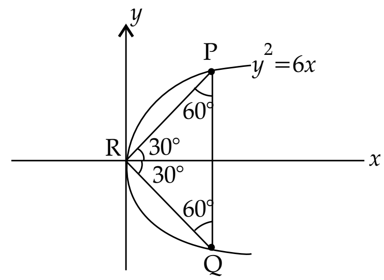
Urn C contains $\lambda$ Red, 4 Black

Also P(Red ball from urn C) = 0.4

$$\Rightarrow \frac{\frac{1}{3} \times \frac{\lambda}{\lambda+4}}{\frac{1}{3} \times \frac{4}{10} + \frac{1}{3} \times \frac{5}{10} + \frac{1}{3} \times \frac{\lambda}{\lambda+4}} = \frac{4}{10}$$

$$\Rightarrow \frac{\frac{\lambda}{\lambda+4}}{\frac{2}{10} + \frac{\lambda}{\lambda+4}} = \frac{4}{10} \Rightarrow 24\lambda = 144 \Rightarrow \lambda = 6$$

So, equation of parabola is $y^2 = 6x$




Let parametric coordinates of point P be $\left(\frac{3}{2}t^2, 3t\right)$

Now, slope of PR = tan30°

$$\Rightarrow \frac{3t}{\frac{3}{2}t^2} = \frac{1}{\sqrt{3}} \Rightarrow t = 2\sqrt{3}$$

$\therefore$ Coordinates of P = $(18, 6\sqrt{3})$

Now, PR = $\sqrt{(18)^2 + (6\sqrt{3})^2}$

$\Rightarrow$ PR = $\sqrt{432} \Rightarrow (\text{PR})^2 = 432$

**HINT:**

**(1)** Find the value of $\lambda$ using Bayes theorem.

**(2)** **Bayes Theorem:** Let $\varepsilon_1$, $\varepsilon_2$, $\varepsilon_n$ be a set of events associated with a sample space S, where all the events $\varepsilon_1$, $\varepsilon_2$,.. $\varepsilon_n$ have non zero probability of occurence and they form a partition of S. Let B be any event associated with S, then according to Bayes theorem.

$$P\left(\frac{\varepsilon_i}{B}\right) = \frac{P(\varepsilon_i).P(B \mid \varepsilon_i)}{\sum_{k=1}^{n} P(\varepsilon_k)P(A \mid \varepsilon_k)}; k = 1, 2, \ldots, n$$

**(3)** Parametric coordinates of any point on parabola $y^2 = 4ax$ is $(at^2, 2at)$

**23. The correct answer is (2).**

Given: S = {$\theta \in [0, 2\lambda)$; tan ($\pi$ cos $\theta$) + tan ($\pi$ sin $\theta$) = 0}

So, tan ($\pi$ cos $\theta$) = – tan ($\pi$ sin $\theta$)

$\Rightarrow$ tan ($\pi$ cos $\theta$) = tan ($-\pi$ sin $\theta$)

As we know if tan $\theta$ = tan $\alpha$, then $\theta = n\pi + \alpha$; $n \in$ I

$\therefore \pi \cos\theta = n\pi - \pi\sin\theta$; $n \in$ I

$\Rightarrow \pi\cos\theta + \pi\sin\theta = n\pi$

$\Rightarrow \cos\theta + \sin\theta = n$

Since, $-\sqrt{2} \le \cos\theta + \sin\theta \le \sqrt{2}$

$\therefore n = -1, 0, 1$

**Case 1:** If $n = -1$

$\cos\theta + \sin\theta = -1$

$$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = \cos\left(\frac{3\pi}{4}\right)$$

$$\Rightarrow \theta - \frac{\pi}{4} = 2k\pi \pm \frac{3\pi}{4}$$

$$\Rightarrow \theta = 2k\pi + \pi \text{ or } \theta = 2k\pi - \frac{\pi}{2}$$

$\Rightarrow \theta = \pi, \dfrac{3\pi}{2}$

**Case-2:** If $n = 0$

$\cos\theta + \sin\theta = 0$

$\Rightarrow \cos\left(\theta - \dfrac{\pi}{4}\right) = 0$

$\Rightarrow \theta - \dfrac{\pi}{4} = 2k\pi \pm \dfrac{\pi}{2}$

$\Rightarrow \theta = 2k\pi + \dfrac{3\pi}{4}$ or $\theta = 2k\pi - \dfrac{\pi}{4}$

$\Rightarrow \theta = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}$

**Case-3:** If $n = 1$

$\cos\theta + \sin\theta = \dfrac{1}{\sqrt{2}}$

$\Rightarrow \cos\left(\theta - \dfrac{\pi}{4}\right) = \cos\left(\dfrac{\pi}{4}\right)$

$\Rightarrow \theta - \dfrac{\pi}{4} = 2k\pi \pm \dfrac{\pi}{4}$

$\Rightarrow \theta = 2k\pi + \dfrac{\pi}{2}$ or $\theta = 2k\pi$

$\Rightarrow \theta = \dfrac{\pi}{2}, 0$

$\therefore \theta = \left\{0, \dfrac{\pi}{2}, \pi, \dfrac{3\pi}{2}, \dfrac{3\pi}{4}, \dfrac{7\pi}{4}\right\}$

So, $\sin\left(\theta + \dfrac{\pi}{4}\right) = \dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}, -\dfrac{1}{\sqrt{2}}, -\dfrac{1}{\sqrt{2}}$

Now, $\sum_{\theta \in S} \sin^2\left(\theta + \dfrac{\pi}{4}\right) = \dfrac{1}{2} + \dfrac{1}{2} + \dfrac{1}{2} + \dfrac{1}{2} = 2$

**HINT:**

**(1)** General solution of $\tan\theta = \tan\alpha$ is $\theta = n\pi + \alpha$; $\alpha \in \left[\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right]$ $n \in I$

**(2)** Use $-\sqrt{a^2 + b^2} \le a\sin x + b\cos x \le \sqrt{a^2 + b^2}$

**24. The correct answer is (5).**

Given: $\dfrac{1^3 + 2^3 + 3^3 + .... \text{ upto } n \text{ terms}}{1.3 + 2.5 + 3.7 + .... \text{upto } n \text{ terms}} = \dfrac{9}{5}$

As we know, sum of cubes of $n$ natural numbers

$= \left\{\dfrac{n(n+1)}{2}\right\}^2$

$$\Rightarrow \frac{\left\{\frac{n(n+1)}{2}\right\}^2}{\sum_{x=1}^{n} x(2x+1)} = \frac{9}{5}$$

$$\Rightarrow \frac{\frac{n^2(n+1)^2}{4}}{\sum_{x=1}^{n} (2x^2 + x)} = \frac{9}{5}$$

$$\Rightarrow \frac{\frac{n^2(n+1)^2}{4}}{2\sum_{x=1}^{n} x^2 + \sum_{x=1}^{n} x} = \frac{9}{5}$$

$$\Rightarrow \frac{\frac{n^2(n+1)^2}{4}}{2\left\{\frac{n(n+1)(2n+1)}{6}\right\} + \left\{\frac{n(n+1)}{2}\right\}} = \frac{9}{5}$$

{$\because$ Sum of squares of $n$ natural numbers $= \dfrac{n(n+1)(2n+1)}{6}$ and sum of $n$ natural numbers $= \dfrac{n(n+1)}{2}$}

$$\Rightarrow \frac{\frac{n^2(n+1)^2}{4}}{\frac{n(n+1)(2n+1)}{3} + \frac{n(n+1)}{2}} = \frac{9}{5}$$

$$\Rightarrow \frac{\frac{n^2(n+1)^2}{4}}{n(n+1)\left\{\frac{2n+1}{3} + \frac{1}{2}\right\}} = \frac{9}{5}$$

$$\Rightarrow \frac{\frac{n(n+1)}{4}}{\frac{(4n+2+3)}{6}} = \frac{9}{5} \Rightarrow \frac{3n(n+1)}{2(4n+5)} = \left(\frac{9}{5}\right)$$

$\Rightarrow 5n^2 + 5n = 24n + 30$

$\Rightarrow 5n^2 - 19n - 30 = 0$

$\Rightarrow 5n^2 + 6n - 25n - 30 = 0$

$\Rightarrow n(5n + 6) - 5(5n + 6) = 0$

$\Rightarrow (n - 5)(5n + 6) = 0$

$\Rightarrow n = 5$

**HINT:**

**(1)** Use $1 + 2 + 3 + .... + n = \dfrac{n(n+1)}{2}$

**(2)** Use $1^2 + 2^2 + 3^2 + .... + n^2 = \dfrac{n(n+1)(2n+1)}{6}$

**(3)** Use $1^3 + 2^3 + 3^3 + .... + n^3 = \left[\dfrac{n(n+1)}{2}\right]^2$

**25. The correct answer is (405).**

**Given:** Sum of coefficients of first 3 terms of $\left(x - \dfrac{3}{x^2}\right)^n = 376$

General term of given expansion is

$T_{r+1} = {}^nC_r x^{n-r} \left(\dfrac{-3}{x^2}\right)^r$.

So, coefficients of first three terms are ${}^nC_0$, $-3{}^nC_1$, $9{}^nC_2$

$\therefore {}^nC_0 - 3{}^nC_1 + 9{}^nC_2 = 376$

$\Rightarrow 1 - 3n + \dfrac{9n(n-1)}{2} = 376$

$\Rightarrow 3n^2 - 5n - 250 = 0$

$\Rightarrow (3n + 25)(n - 10) = 0$

$\Rightarrow n = 10, \dfrac{-25}{3}$ (not possible)

For coefficient of $x^4$, $n - 3r = 4$

$\Rightarrow 10 - 3r = 4$

$\Rightarrow r = 2$

$\therefore$ Coefficient of $x^4 = {}^{10}C_2(-3)^2 = \dfrac{10\times 9}{2\times 1}\times 9 = 405$

**HINT:**

General term of binomial expansion $(a + b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$

**26. The correct answer is (122).**

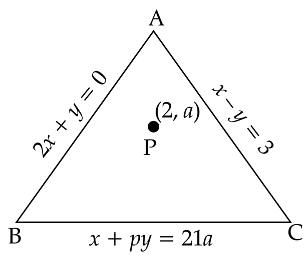
Given equation of sides are

AB : $2x + y = 0$ ...(i)

BC : $x + py = 21a$ ...(ii)

CA : $x - y = 3$ ...(iii)

Centroid of $\Delta$ABC = P $(2, a)$




On solving equation (i) and equation (iii), we get

A = (1, –2)

Let the coordinates of point B be $(m, -2m)$ and coordinates of point C be $(n + 3, n)$

$\because$ Centroid of $\Delta$ ABC = $(2, a)$

$\Rightarrow \dfrac{m+n+3+1}{3} = 2$ and $\dfrac{n-2m-2}{3} = a$

$\Rightarrow m + n = 2$ ...(iv)

& $n - 2m = 3a + 2$ ...(v)

Put $n = 2 - m$ from equation (iv) to (v), we get

$m = -a$

Point B satisfy the equation of BC

So, $m - 2mp = 21a$

$\Rightarrow m(1 - 2p) = 21a$

$\Rightarrow 2p - 1 = 21$

$\Rightarrow p = 11$

Point C also satisfy the equation of BC

So, $n + 3 + p(n) = 21a$

$\Rightarrow 12n + 3 = -21m$

$\Rightarrow 12n + 21m + 3 = 0$ ...(vi)

On solving equation (iv) and equation (vi), we get

$m = -3, n = 5$

$\therefore$ B = (–3, 6) and C = (8, 5)

Now, BC = $\sqrt{(11)^2 + 1^2}$

$\Rightarrow$ BC = $\sqrt{122}$

$\Rightarrow BC^2 = 122$

**HINT:**

**(1)** Find the coordinates of A by solving equation of side AB and AC.

**(2)** Find the coordinates of B and C using given condition and solve further.

**27. The correct answer is (8).**

Given: $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}$

$\vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}$

$\vec{a}.\vec{c} = 7$

$2\vec{b}.\vec{c} + 43 = 0$ and $\vec{a}\times\vec{c} = \vec{b}\times\vec{c}$

$\because\ \vec{a}\times\vec{c} = \vec{b}\times\vec{c}$

$\Rightarrow (\vec{a} - \vec{b})\times\vec{c} = 0$

$\Rightarrow (\vec{a} - \vec{b}) \,||\, \vec{c}$

$\Rightarrow \vec{c} = k(\vec{a} - \vec{b}) \Rightarrow \vec{c} = k(-2\hat{i} + 7\hat{j} + 2\lambda\hat{k})$

$\because \vec{a}.\vec{c} = 7$

$\Rightarrow k(-2 + 14 + 2\lambda^2) = 7$

$\Rightarrow k(2\lambda^2 + 12) = 7$ ...(i)

Also, $2\vec{b}.\vec{c} = -43$

$\Rightarrow 2k(-6 - 35 - 2\lambda^2) = -43$

$\Rightarrow 2k(-41 - 2\lambda^2) = -43$ ...(ii)

From equation (i) and equation (ii), we get

$\dfrac{2\lambda^2 + 12}{2(41 + 2\lambda^2)} = \dfrac{7}{43}$

$\Rightarrow 43(\lambda^2 + 6) = 7(2\lambda^2 + 41)$

$\Rightarrow 29\lambda^2 = 29$

$\Rightarrow \lambda^2 = 1$

Now, $\vec{a}.\vec{b} = 3 - 10 - \lambda^2 = -8$

$\Rightarrow |\vec{a}.\vec{b}| = 8$

**HINT:**

**(1)** Use if $\vec{a}$ is parallel to $\vec{b}$, then $\vec{a} = k\vec{b}; k \in R$

**(2)** If $\vec{a} = \vec{a}_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = \vec{b}_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

**28. The correct answer is (13).**

Given: Relation R = $\{(a, b), (b, c), (b, d)\}$ on set $\{a, b, c, d\}$ for a relation to be equivalence relation, it must be reflexive, symmetric and transitive.

For reflexive relation, $(a, a), (b, b), (c, c), (d, d)$ must be added in relation R.

So, R = $\{(a, a), (b, b), (c, c), (d, d), (a, b), (b, c), (b, d)\}$

For symmetric relation, if $(x, y) \in R \Rightarrow (y, x) \in R$

Now, as $(a, b) \in R \Rightarrow (b, a) \in R$

and $(b, c) \in R \Rightarrow (c, b) \in R$

and $(b, d) \in R \Rightarrow (d, b) \in R$

So, R = $\{(a, a), (b, b), (c, c), (d, d), (a, b), (b, c), (b, d), (b, a), (c, b), (d, b)\}$

For transitive relation, if $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$

So, R = $\{(a, a), (b, b), (c, c), (d, d), (a, b), (b, c), (b, d), (b, a), (c, b), (d, b), (a, c), (a, d), (c, a), (c, d), (d, c), (d, a)\}$

So, total number of elements added = 13

**HINT:**

**(1)** For a relation to be equivalence relation, it must be reflexive, symmetric and transitive.

**(2)** For reflexive relation, $(x, x) \in R$.

| | |
|---|---|
| **(3)** | For symmetric relation, if $(x, y) \in R \Rightarrow (y, x) \in R$ |
| **(4)** | For transitive relation, if $(x, y) \in R$ & $(y, z) R \Rightarrow (x, z) \in R$ |

**29. The correct answer is (36).**

Given curves $y^2 - 2y = -x$ and $x + y = 0$

Now, $y^2 - 2y + 1 = -x + 1$

$\Rightarrow (y - 1)^2 = -(x - 1)$ ...(i)

Let's find intersecting points of both curves

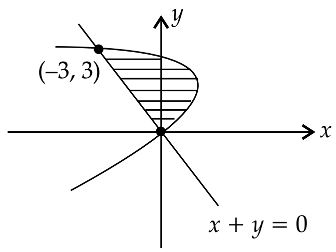
$y^2 - 2y - y = 0$

$\Rightarrow y^2 - 3y = 0$

$\Rightarrow y = 0, 3$

$\Rightarrow x = 0, -3$

$\therefore$ Intersecting points are (0, 0) and (–3, 3)




So, required area = $\int_0^3 \{(2y - y^2) - (-y)\}dy$

$\Rightarrow A = \int_0^3 (3y - y^2)dy$

$\Rightarrow A = \left[\frac{3y^2}{2} - \frac{y^3}{3}\right]_0^3$

$\Rightarrow A = \frac{27}{2} - 9$

$\Rightarrow A = \frac{9}{2}$

$\Rightarrow 8A = 36$

**HINT:**

Draw the figure of both curves and identify the bounded region and use the concept of vertical strip and solve further.

**30. The correct answer is (27).**

**Given:** $f(x) + \int_0^x f(t)\sqrt{1 - (\log_e f(t))^2}\,dt = e, \forall x \in \left[0, \frac{\pi}{2}\right]$ ...(i)

Differentiate the above equation, we get

$f'(x) + f(x)\sqrt{1 - [\log_e f(x)]^2} = 0$

$\Rightarrow \frac{dy}{dx} + y\sqrt{1 - (\log_e y)^2} = 0$

$\Rightarrow \frac{dy}{dx} = -y\sqrt{1 - \log_e^2 y}$

$\Rightarrow \int \frac{dy}{y\sqrt{1 - \log_e^2 y}} = \int -1\,dx$

Let $\log_e y = u$

$\Rightarrow \frac{1}{y}dy = du$

$\Rightarrow \int \frac{du}{\sqrt{1 - u^2}} = -x + c$

$\Rightarrow \sin^{-1} u = -x + c$

$\Rightarrow \sin^{-1} \log_e y = -x + c$

Put $x = 0$ in equation (i), we get

$f(0) = e$ i.e., $y(0) = e$

So, at $x = 0$, $\sin^{-1}(1) = c$

$\Rightarrow c = \frac{\pi}{2}$

$\therefore \sin^{-1} \log_e y = -x + \frac{\pi}{2}$

$\Rightarrow \log_e y = \sin\left(\frac{\pi}{2} - x\right)$

$\Rightarrow \log_e y = \cos x$

At $x = \frac{\pi}{6}, \log_e f\left(\frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

So, $\left[6\log_e f\left(\frac{\pi}{6}\right)\right]^2 = \left[6 \times \frac{\sqrt{3}}{2}\right]^2 = 27$

**HINT:**

**(1)** Differentiate given equation using newton leibnitz rule and solve further differential equation using variable separable form.

**(2)** If $I(x) = \int_{g(x)}^{h(x)} \phi(x)dx$, then

$$I'(x) = \phi(h(x))h'(x) - \phi(g(x))g'(x)$$