$\therefore \quad I = \int \frac{3t^2 - 2t}{t^2 + 2t - 8} \frac{dt}{t}$

$= \int \frac{3t - 2}{t^2 + 2t - 8} dt$

Now, $\frac{3t-2}{t^2+2t-8} = \frac{3t-2}{(t+4)(t-2)}$

So, $\frac{3t-2}{(t+4)(t-2)} = \frac{A}{(t+4)} + \frac{B}{(t-2)}$

$3t - 2 = A(t - 2) + B(t + 4)$

$3t - 2 = (A + B)t + (4B - 2A)$

On comparing, we get

$A + B = 3$ ...(i)

and $4B - 2A = -2$

or $2B - A = -1$ ...(ii)

On solving eqs. (i) & (ii), we get

$A = \frac{7}{3}$ and $B = \frac{2}{3}$

$\therefore \quad \frac{3t-2}{t^2+2t-8} = \frac{7}{3}\frac{1}{(t+4)} + \frac{2}{3}\frac{1}{t-2}$

$\therefore \quad I = \frac{7}{3}\int \frac{1}{t+4} dt + \frac{2}{3}\int \frac{1}{t-2} dt$

$= \frac{7}{3}\log|t+4| + \frac{2}{3}\log|t-2| + C$

$= \frac{7}{3}\log|e^x+4| + \frac{2}{3}\log|e^x-2| + C$

[substituting $t = e^x$]

OR

(ii) Let $I = \int \frac{2}{(1-x)(1+x^2)}$

We can write integrand as

$\frac{2}{(1-x)(1+x^2)} = \frac{2}{-(x-1)(1+x^2)}$

$= \frac{-2}{(x-1)(1+x^2)}$

Applying partial fraction,

$\frac{-2}{(x-1)(1+x^2)} = \frac{A}{(x-1)} + \frac{Bx+C}{(1+x^2)}$

$\frac{-2}{(x-1)(1+x^2)} = \frac{A(1+x^2)+(Bx+C)(x-1)}{(x-1)(1+x^2)}$

$-2 = A(1 + x^2) + (Bx + C)(x - 1)$ ...(i)

Putting $x = 1$, we get

$-2 = 2A + 0$

$\Rightarrow \quad A = -1$

Putting $x = 0$, we get

$-2 = A + (-C)$

$\Rightarrow \quad -2 = -1 - C$ $[\because A = -1]$

$\Rightarrow \quad C = 1$

Putting $A = -1$, $C = 1$ in eq (i), we get

$B = 1$

So, $\frac{-2}{(x-1)(1+x^2)} = \frac{-1}{(x-1)} + \frac{x+1}{(x^2+1)}$

$\therefore \quad I = \int -\frac{1}{(x-1)} dx + \int \frac{x+1}{x^2+1} dx$

$= -\int \frac{1}{(x-1)} dx + \int \frac{x}{x^2+1} dx + \int \frac{1}{x^2+1} dx$

$= -\int \frac{1}{(x-1)} dx + I_1 + \int \frac{1}{x^2+1} dx$

where $I_1 = \int \frac{x}{x^2+1} dx$

let $x^2 + 1 = t$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad x dx = \frac{dt}{2}$

$\therefore \quad I_1 = \frac{1}{2}\int \frac{dt}{t}$

$= \frac{1}{2}\log|t| + C$

$= \frac{1}{2}\log|x^2 + 1| + C_1$ ...(ii)

Thus, $I = -\int \frac{1}{(x-1)} dx + \int \frac{x}{x^2+1} dx + \int \frac{1}{x^2+1} dx$

$= -\log|x-1| + \frac{1}{2}\log|x^2+1| + \tan^{-1} x + C_1 + C_2$

[from eq. (ii)]

$= -\log|x-1| + \frac{1}{2}\log|x^2+1| + \tan^{-1} x + C$

$[C = C_1 + C_2]$

14. Total number of fruits = 30

Number of rotten (spoiled) fruits = 10

Number of unspoiled fruits = 20

Probability of rotten fruits $P(\text{rotten}) = \frac{10}{30}$

Probability of unspoiled fruits $P(\text{unspoiled}) = \frac{20}{30}$

Let X be the random variable of number of unspoiled fruit.

So, $X = 0, 1, 2$

Two fruit can be drawn in 30 fruits

$P(X = 0)$ = Two fruits are spoiled fruits

$= \frac{^{10}C_2}{^{30}C_2} = \frac{10 \times 9 \times 2}{2 \times 30 \times 29}$

$= \frac{9}{87}$

$P(X = 1)$ = 1 fruit is unspoiled and 1 fruit is spoiled

$= \frac{^{20}C_1 . ^{10}C_1}{^{30}C_2} = \frac{20 \times 10 \times 2}{30 \times 29}$

$= \frac{40}{87}$

$P(X = 2)$ = Two fruits are unspoiled

$= \frac{^{20}C_2}{^{30}C_2} = \frac{20 \times 19 \times 2}{30 \times 29 \times 2}$

$= \frac{38}{87}$

Mean of probability distribution

$$= \sum X_i P(X_i)$$

$$= X_0P(X_0) + X_1P(X_1) + X_2P(X_2)$$

$$= 0\times\frac{9}{87}+1\times\frac{40}{87}+2\times\frac{38}{87}$$

$$= \frac{116}{87} = 1.33$$

SECTION - B

15. (i) Option (b) is correct

Explanation: Given, $|\vec{a}|=3,\ |\vec{b}|=\frac{\sqrt{2}}{3}$

and $|\vec{a}\times\vec{b}| = 1$

We know that,

$$|\vec{a}\times\vec{b}| = \left||\vec{a}||\vec{b}|\sin\theta\,\hat{n}\right|$$

where θ is the angle between $\vec{a}$ and $\vec{b}$

$$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta.1$$

$$\Rightarrow \quad 1 = 3\times\frac{\sqrt{2}}{3}\sin\theta$$

$$\Rightarrow \quad \sin\theta = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$$

$$\Rightarrow \quad \theta = \frac{\pi}{4}$$

(ii) Option (b) is correct

Explanation: We know that

$$\text{distance} = \frac{|\vec{a}.\vec{n}-d|}{|\vec{n}|}$$

Given, $\vec{a} = 2\hat{i}+\hat{j}-\hat{k},\ \vec{n} = \hat{i}-2\hat{j}+4\hat{k},\ d = 9$

$$\therefore \quad \text{distance} = \frac{|(2\hat{i}+\hat{j}-\hat{k}).(\hat{i}-2\hat{j}+4\hat{k})-9|}{|\hat{i}-2\hat{j}+4\hat{k}|}$$

$$= \frac{|2-2-4-9|}{\sqrt{1+4+16}}$$

$$= \frac{13}{\sqrt{21}}$$

(iii) Let $\vec{d_1} = \hat{i}-3\hat{j}+\hat{k}$ and $\vec{d_2} = \hat{i}+\hat{j}+\hat{k}$

$$\text{Area of parallelogram} = \frac{1}{2}|\vec{d_1}\times\vec{d_2}|$$

$$\vec{d_1}\times\vec{d_2} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -3 & 1\\ 1 & 1 & 1\end{vmatrix}$$

$$= \hat{i}(-3-1)-\hat{j}(1-1)+\hat{k}(1+3)$$

$$= -4\hat{i}+4\hat{k}$$

$$\therefore \quad |\vec{d_1}\times\vec{d_2}| = \sqrt{(-4)^2+(4)^2}$$

$$= \sqrt{16+16} = \sqrt{32}$$

Thus, area of parallelogram

$$= \frac{1}{2}\sqrt{32}\text{ unit}^2$$

$$= \sqrt{8}\text{ unit}^2$$

(iv) The equation of the plane intercepts on the coordinate axes area a, b and c is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1$$

Given, $\quad a = b = c$

$$\therefore \quad \frac{x}{a}+\frac{y}{a}+\frac{z}{a} = 1$$

$$\Rightarrow \quad x+y+z = a$$

This plane passes through point (2, 4, 6).

$$\therefore \quad 2+4+6 = a$$

$$\Rightarrow \quad a = 12$$

Thus, required equation of plane is $x + y + z = 12$.

(v) For perpendicular vectors $\vec{a}.\vec{b} = 0$

$$\therefore (3\hat{i}+\alpha\hat{j}+\hat{k}).(2\hat{i}-\hat{j}+8\hat{k}) = 0$$

$$\Rightarrow \quad 6-\alpha+8 = 0$$

$$\Rightarrow \quad \alpha = 14$$

16. (i) We have, $A(1, 2, -3)$ and $B(-1, -2, 1)$

$$\vec{AB} = (-1-1)\hat{i}+(-2-2)\hat{j}+(1+3)\hat{k}$$

$$\therefore \quad \vec{AB} = -2\hat{i}-4\hat{j}+4\hat{k}$$

Unit vector in direction of $\vec{AB}$

$$= \frac{-2\hat{i}-4\hat{j}+4\hat{k}}{\sqrt{(-2)^2+(-4)^2+(4)^2}}$$

$$= \frac{-2(\hat{i}+2\hat{j}-2\hat{k})}{\sqrt{4+16+16}}$$

$$= \frac{-2(\hat{i}+2\hat{j}-2\hat{k})}{6}$$

$$= -\frac{1}{3}(\hat{i}+2\hat{j}-2\hat{k})$$

OR

(ii) Given, $\quad (2\vec{x}-3\hat{a}).(2\vec{x}+3\hat{a}) = 91$

$$\Rightarrow \quad 4|\vec{x}|^2+6\vec{x}.\hat{a}-6\hat{a}.\vec{x}-9|\hat{a}|^2 = 91$$

$$\Rightarrow \quad 4|\vec{x}|^2+6.\vec{x}.1-6.1.\vec{x}-9.1 = 91 \quad [\because \hat{a}=1]$$

$$\Rightarrow \quad 4|\vec{x}|^2 = 100$$

$\Rightarrow \qquad |\vec{x}|^2 = 25$

$\Rightarrow \qquad |\vec{x}| = 5$

17. Let the equation of plane passing through point (1, 1, – 1) be

$a(x-1) + b(x-1) + c(x+1) = 0 \qquad \ldots(i)$

Eq (i) is perpendicular to the plane $x + 2y + 3z - 7 = 0$

$\therefore \qquad 1.a + 2.b + 3.c = 0$

$\Rightarrow \qquad a + 2b + 3c = 0 \qquad \ldots(ii)$

Again eq (i) is perpendicular to plane $2x - 3y + 4z = 0$

$\therefore \qquad 2.a - 3.b + 4.c = 0$

$\Rightarrow \qquad 2a - 3b + 4c = 0 \qquad \ldots(iii)$

On solving eqs (ii) and (iii), we get

$$\frac{a}{8+9} = \frac{b}{6-4} = \frac{c}{-3-4}$$

$$\Rightarrow \qquad \frac{a}{17} = \frac{b}{2} = \frac{c}{-7} = k$$

$\Rightarrow a = 17k, b = 2k$ and $c = -7k$

Putting the values of a, b and c in eq (i), we get

$17k(x-1) + 2k(y-1) - 7k(z+1) = 0$

$\Rightarrow \quad 17(x-1) + 2(y-1) - 7(z+1) = 0$

$\Rightarrow \quad 17x + 2y - 7z - 17 - 2 - 7 = 0$

$\Rightarrow \quad 17x + 2y - 7z - 26 = 0$

OR

Let eq. of required line passing Through (2, –1, 3) is

$$\vec{r} = (2\vec{i} - \vec{j} - 3\vec{k}) + \mu(\vec{i} + 2\vec{j} + 2\vec{k})$$

eq. (1) is perpendicular to

$$\vec{r} = (\vec{i} - \vec{j} - \vec{k}) + \lambda(2\vec{i} + \vec{j} + \vec{k})$$

$\therefore \qquad 2x - 2y + 2 = 0$

Again eq (i) is perpendicular to

$$\vec{r} = (2\vec{i} - \vec{j} - 3\vec{k}) + \pi(\vec{i} + 2\vec{j} + 2\vec{k})$$

$\therefore \qquad k + 2y + 22 = 0$

On solving eq. (2) and (3) we get

$$\frac{x}{-4-2} = \frac{y}{1-4} = \frac{z}{4-2}$$

$$\frac{x}{-6} = \frac{y}{-3} = \frac{z}{2} = k$$

$\Rightarrow x = -6k, y = 3k$ and $z = 2k$

Putting the values of x, y and z k eq

we get $\vec{r} = (2\vec{i} - 3\vec{j} + 3\vec{k}) + \lambda_1(-6\vec{i} - 3\vec{j} + 3\vec{k})$

18. Given curve $x^2 = 4y$ ... (i) represents an upward parabola with vertex (0, 0) and axis along y-axis

Given equation of line is $x = 4y - 2 \qquad \ldots(ii)$

On solving eqs (i) & (ii), we get

$x^2 = x + 2$

$\Rightarrow \qquad x^2 - x - 2 = 0$

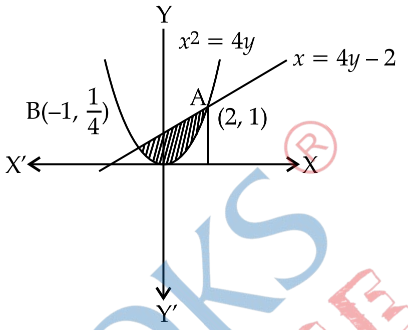
$\Rightarrow \qquad (x-2)(x+1) = 0$

$\Rightarrow \qquad x = 2, -1$

when $\qquad x = 2, y = 1$

and $\qquad x = -1, y = \frac{1}{4}$

Thus, line meets the parabola at the points A(2, 1) and B$\left(-1, \frac{1}{4}\right)$.



Required area = (Area under the line $x = 4y - 2$) – (Area under the parabola $x^2 = 4y$)

$$= \int_{-1}^{2}\left(\frac{x+2}{4}\right)dx - \int_{-1}^{2}\frac{x^2}{4}dx$$

[From eq. (ii), $y = \frac{x+2}{4}$ and from eq. (i), $y = \frac{x^2}{4}$]

$$= \frac{1}{4}\left[\frac{x^2}{2} + 2x\right]_{-1}^{2} - \frac{1}{4}\left[\frac{x^3}{3}\right]_{-1}^{2}$$

$$= \frac{1}{4}\left[\left\{\frac{2^2}{2} + 2(2)\right\} - \left\{\frac{(-1)^2}{2} + 2(-1)\right\}\right] - \frac{1}{12}(2^3 - (-1)^3)$$

$$= \frac{1}{4}\left(6 + \frac{3}{2}\right) - \frac{1}{12} \times 9$$

$$= \frac{15}{8} - \frac{9}{12}$$

$$= \frac{9}{8} \text{ sq. unit}$$

SECTION - C

19. (i) Option (a) is correct.

Explanation: Given, $\quad p = 1500 - 2x - x^2$

Revenue function, $\quad R = px$

$R = 1500x - 2x^2 - x^3$

Marginal revenue $= \frac{dR}{dx}$

$= \frac{d}{dx}(1500x - 2x^2 - x^3)$

$= 1500 - 4x - 3x^2$

Marginal revenue at $x = 10$ is

$= 1500 - 4(10) - 3(10)^2$

$= 1500 - 40 - 300$

$= 1160$

(ii) Option (c) is correct.

Explanation: $\quad r = \sqrt{0.8 \times 0.2}$

$= \sqrt{0.16}$

$$= \sqrt{(0.4)^2}$$
$$= 0.4$$

Here, correlation coefficient will be positive because both the coefficients are positive.

(iii) $x + 2y - 5 = 0$ ...(i)

$2x + 3y = 8$ ...(ii)

Let eq (i), be on x and eq. (ii) be x on

$$\text{Slope of eq (i)} = -\frac{1}{2}$$
$$\text{Slope of eq (ii)} = -\frac{2}{3}$$
$$\Rightarrow \quad b_{yx} = -\frac{1}{2}, \& + \frac{1}{b_{xy}} = \frac{-1}{3}$$
$$\Rightarrow \quad b_{yx} = -\frac{1}{2}, \& \ b_{xy} = \frac{-3}{2}$$

Since both b_{yx} and b_{xy} are of since sign and

$$b_{yx} \times b_{xy} = -\frac{1}{2} \times \frac{-3}{2} = \frac{3}{4} < 1$$

∴ Our assumption is true

Hence eq. (i) *i.e.*, $x + 2y - 5 = 0$ is a line of regression of y on x.

From eq (ii), the regression line of y on x is

(iv) Given, $C(x) = 3x^2 - 6x + 5$

$$\text{Average cost} = \frac{C(x)}{x}$$
$$= 3x - 6 + \frac{5}{x}$$
$$(\text{Average cost})_{\text{at } x = 2} = 3(2) - 6 + \frac{5}{2}$$
$$= \frac{5}{2} = 2.5$$

(v) Total cost = fixed cost + variable cost

$C(x) = ₹\ 30{,}000 + ₹\ 800x$

where x = total unit

Also, revenue function, $R(x) = p.x$

$= (4500 - 100x)x$

[∵ Given $p = 4500 - 100x$]

$= 4500x - 100x^2$

Profit function $P(x) = R(x) - C(x)$

$= 4500x - 100x^2 - 30000 - 800x$

$= -100\,x^2 + 3700x - 30000$

At break even point, $P(x) = 0$

∴ $-100x^2 + 3700x - 30000 = 0$

or, $x^2 - 37x + 300 = 0$

$$x = \frac{+37 \pm \sqrt{(-37)^2 - 4(1)(300)}}{2 \times 1}$$
$$= \frac{37 \pm \sqrt{1369 - 1200}}{2}$$
$$= \frac{37 \pm 13}{2}$$
$$= \frac{37 + 13}{2} \text{ and } \frac{37 - 13}{2}$$
$$= \frac{50}{2} \text{ and } \frac{24}{2}$$
$$= 25 \text{ and } 12$$

So, break even values are 25 and 12.

20. (i) Given, $C(x) = \sqrt{6x + 5} + 2500$

$$MC = \frac{dC}{dx}$$
$$= \frac{d}{dx}[\sqrt{6x + 5} + 2500]$$

or, $$MC = \frac{1}{2}(6x + 5)^{-1/2}(6)$$

or, $$MC = \frac{3}{\sqrt{6x + 5}}$$

Now, put $x = 2$,

$$MC = \frac{3}{\sqrt{12 + 5}} = \frac{3}{\sqrt{17}}$$
$$= \frac{3}{4.12} = 0.72$$

Put $x = 3$,

$$MC = \frac{3}{\sqrt{18 + 5}} = \frac{3}{\sqrt{23}}$$
$$= \frac{3}{4.79} = 0.62$$

So, it is clear, as we increase output x, MC decreases.

OR

(ii) Given, average revenue $= AR = 25 - \frac{x}{4}$

Since, $$AR = \frac{R}{x}$$
$$= \frac{p.x}{x} = p$$

∴ $$p = 25 - \frac{x}{4}$$

$$\text{Total revenue } R(x) = p.x = 25x - \frac{x^2}{4}$$

$$\text{Marginal revenue, MR} = \frac{d}{dx}R(x)$$
$$= \frac{d}{dx}\left(25x - \frac{x^2}{4}\right)$$
$$= 25 - \frac{x}{2}$$

21. Given LPP is

Max $z = 5x + 2y$

Subject to: $x - 2y \le 2$

$3x + 2y \le 12$

$-3x + 2y \le 3$

$x \ge 0, y \ge 0$

Converting the inequations into equations, we get

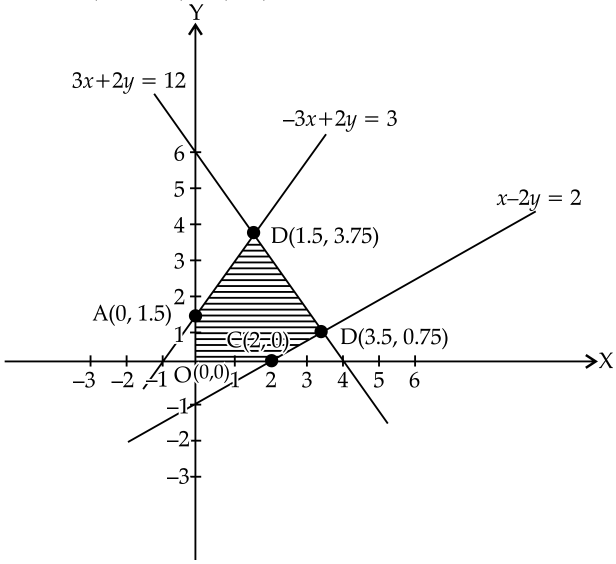
$x - 2y = 2$ ...(i)

$3x + 2y = 12$ ...(ii)

$-3x + 2y = 3$ ...(iii)

$x = 0, y = 0$ ...(iv)

On plotting the above set of equation, we get the corner points as A(0, 1.5), B(3.5, 0.75), C(2, 0), D(1.5, 3.75), O(0, 0).



The value of the objective function are:

Point (x, y)	$z = 5x + 2y$
A(0, 1.5)	$5 \times 0 + 2 \times 1.5 = 3$
B(3.5, 0.75)	$5 \times 3.5 + 2 \times 0.75 = 19$ (max)
C(2, 0)	$5 \times 2 + 2 \times 0 = 10$
D(1.5, 3.75)	$5 \times 1.5 + 2 \times 3.75 = 15$
O(0, 0)	$5 \times 0 + 2 \times 0 = 0$

So, maximum value of z is 19.

22. (i) Given, $r = 0.6$

Mean of $x = \bar{x} = 8$

Mean of $y = \bar{y} = 6$

S.D. of $x = \sigma_x = 12$

S.D. of $y = \sigma_y = 4$

(i) $b_{xy} = \dfrac{r\sigma_x}{\sigma_y} = \dfrac{0.6 \times 12}{(4)}$

$$= \frac{0.6 \times 12}{4} = 1.8$$

$$b_{yx} = \frac{r\sigma_y}{\sigma_x} = \frac{0.6 \times 4}{(12)}$$

$$= \frac{0.6 \times 4}{12} = 0.2$$

(ii) Regression line y on x is given by

$$y - \bar{y} = b_{yx}(x - \bar{x})$$
$$y - 6 = 0.2\,(x - 8)$$
$$y = 0.2x - 1.6 + 6$$
$$= 0.2x + 4.4$$

at $x = 20$

$$= 0.2 \times 20 + 4.4$$
$$= 4 + 4.4$$
$$y = 8.4$$

(ii) Given, $n = 102$, $\Sigma x = 510$, $\Sigma y = 7140$, $\Sigma x^2 = 4150$, $\Sigma y^2 = 740200$, $\Sigma xy = 54900$

We know that, regression equation of y on x is

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

So, $\bar{x} = \dfrac{\Sigma x}{n} = \dfrac{510}{102} = 5$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{7140}{102} = 70$$

$$b_{yx} = \frac{\Sigma xy - n\bar{x}\,\bar{y}}{\Sigma x^2 - n(\bar{x})^2}$$

$$= \frac{54900 - (102)(5)(70)}{4150 - 102(5)^2}$$

$$= \frac{54900 - 35700}{4150 - 2550}$$

$$= \frac{19200}{1600} = 12$$

Regression line y on x is

$$y - 70 = 12(x - 5)$$

$\Rightarrow$ $y = 12x - 60 + 70$

$\Rightarrow$ $y = 12x + 10$

$\Rightarrow$ $y = 2(6x + 5)$

■■