Solved Paper 2017

Mathematics (Standard)

CLASS-X

Time : 3 Hours **Max. Marks : 90**

General Instructions :

*(i) All questions are **compulsory**.*

*(ii) This question paper consists of **31** questions divided into four sections – **A, B, C** and **D**.*

*(iii) Section **A** contains **4** questions of **1** mark each. Section **B** contains **6** questions of **2** marks each. Section **C** contains **10** questions of **3** marks each. Section **D** contains **11** questions of **4** marks each.*

*(iv) Use of calculator is **not** permitted.*

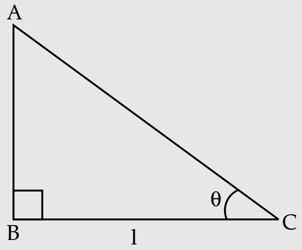
Delhi Set-I **Code No. 30/1/1**

SECTION - A

Question numbers 1 to 4 carry 1 mark each.

1. The ratio of the height of a tower and the length of its shadow on the ground is $\sqrt{3}$: 1. What is the angle of elevation of the sun ?

Sol.



$\tan\theta = \frac{AB}{BC} = \frac{\sqrt{3}}{1}$ **1**

$\Rightarrow \theta = 60°$ **(CBSE Marking Scheme, 2017)**

2. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere ?

Sol. $\frac{2}{3}\pi r^3 = 3\pi r^2 \Rightarrow r = \frac{9}{2}$ units

$\therefore d = 9$ units **1**

(CBSE Marking Scheme, 2017)

3. A number is chosen at random from the numbers –3, –2, –1, 0, 1, 2, 3. What will be the probability that square of this number is less than or equal to 1 ?

Sol. Favourable outcomes are –1, 0, 1

$\therefore$ Required Probability $= \frac{3}{7}$ **1**

(CBSE Marking Scheme, 2017)

4. If the distance between the points (4, k) and (1, 0) is 5, then what can be the possible values of k ?

Sol. $\left|\sqrt{(4-1)^2+(k-0)^2}\right| = 5$

$\Rightarrow k = \pm 4$ **1**

(CBSE Marking Scheme, 2017)

SECTION - B

Question numbers 5 to 10 carry 2 marks each.

5. Find the roots of the quadratic equation $\sqrt{2}\,x^2 + 7x + 5\sqrt{2} = 0$.

Sol. $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$ **1**

$\Rightarrow (\sqrt{2}x + 5) + (x + \sqrt{2})$

$\Rightarrow x = \frac{-5}{\sqrt{2}}, -\sqrt{2}$

or $\frac{-5\sqrt{2}}{2}, -\sqrt{2}$ **1**

(CBSE Marking Scheme, 2017)

6. Find how many integers between 200 and 500 are divisible by 8.

Sol. A.P. formed is 208, 216, 224, ..., 496 **1**

$a_n = 496$ ½

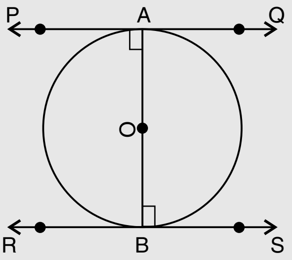
$\Rightarrow$ $208 + (n - 1) \times 8 = 496$

$\Rightarrow$ $n = 37$ ½

(CBSE Marking Scheme, 2017)

7. Prove that tangents drawn at the ends of a diameter of a circle are parallel to each other.

Sol.



1

$\angle PAO = \angle OBS = 90°$

As, these are alternate interior angles

$\therefore$ PQ || RS 1

(CBSE Marking Scheme, 2017)

8. Find the value of k for which the equation $x^2 + k(2x + k - 1) + 2 = 0$ has real and equal roots.

Sol. $x^2 + k(2x + k - 1) + 2 = 0$

$\Rightarrow$ $x^2 + 2kx + (k^2 - k + 2) = 0$ ½

For equal roots, $b^2 - 4ac = 0$ ½

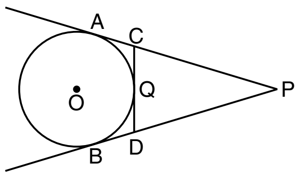
$\Rightarrow$ $4k^2 - 4k^2 + 4k - 8 = 0$ **1**

$\Rightarrow$ $k = 2$ **1**

(CBSE Marking Scheme, 2017)

*** 9. Draw a line segment of length 8 cm and divide it internally in the ratio 4 : 5.**

10. In the given figure, *PA* and *PB* are tangents to the circle from an external point *P*. *CD* is another tangent touching the circle at *Q*. If *PA* = 12 cm, *QC* = *QD* = 3 cm, then find *PC* + *PD*.



Sol. $PA = PC + CA = PC + CQ$ **1**

$\Rightarrow 12 = PC + 3 \Rightarrow PC = 9$ cm

$PD = 9$ cm

$\therefore PC + PD = 18$ cm **1**

(CBSE Marking Scheme, 2017)

SECTION - C

Question numbers 11 to 20 carry 3 marks each.

11. If m^{th} term of an A.P. is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$, then find the sum of its first mn terms.

Sol. $a_m = \frac{1}{n}$.

$\Rightarrow$ $a + (m-1)d = \frac{1}{n}$ ...(1) ½

$a_n = \frac{1}{m}$

$\Rightarrow$ $a + (n-1)d = \frac{1}{m}$ ...(2) ½

1

Solving (1) and (2), $a = \frac{1}{mn}$ and $d = \frac{1}{mn}$

$$S_{mn} = \frac{mn}{2}\left[2 \times \frac{1}{mn} + (mn-1) \times \frac{1}{mn}\right]$$

$$= \frac{1}{2}(mn+1)$$ **1**

(CBSE Marking Scheme, 2017)

12. Find the sum of n terms of the series $\left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + \ldots\ldots\ldots\ldots$

Sol. $S_n = \left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right)$ + ... upto n terms

$$= \left(\underbrace{4 + 4 + \ldots + 4}_{n\text{times}}\right) - \frac{1}{n}(1 + 2 + 3 + \ldots + n)$$ **1**

$$= 4n - \frac{1}{n} \times \frac{n(n+1)}{2}$$ ½ + 1

$$= \frac{7n-1}{2}.$$ ½

(CBSE Marking Scheme, 2017)

13. If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots then show that $c^2 = a^2(1 + m^2)$.

Sol. $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$

For equal roots, $b^2 - 4ac = 0$ ½

$\Rightarrow$ $4m^2c^2 - 4(1 + m^2)(c^2 - a^2) = 0$ **1**

$\Rightarrow$ $m^2c^2 - c^2 - m^2c^2 + a^2 + m^2a^2 = 0$ **1**

$\Rightarrow$ $c^2 = a^2(1 + m^2)$ ½

(CBSE Marking Scheme, 2017)

14. The $\frac{3}{4}^{th}$ part of a conical vessel of internal radius 5 cm and height 24 cm is full of water. The water is emptied into a cylindrical vessel with internal radius 10 cm. Find the height of water in cylindrical vessel.

Sol. Radius of conical vessel $r = 5$ cm

Height of conical vessel $h = 24$ cm ½

The volume of water

$= \frac{3}{4} \times$ volume of conical vessel. **1**

$= \frac{3}{4} \times \frac{1}{3}\pi r^2 h$

$= \frac{3}{4} \times \frac{1}{3}\pi. \times 25 \times 24$ ½

$= 150\,\pi$

Let h' be the height of cylindrical vessel, which filled by the water of conical vessel,

Radius of cylindrical vessel = 10 cm

Clearly,

* *Out of Syllabus*

Volume of cylindrical vessel = volume of water

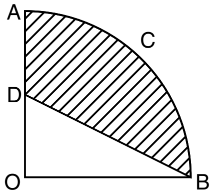
$$\pi(10)^2h' = 150\pi \qquad \mathbf{1}$$

$$\Rightarrow \quad h' = \frac{150\pi}{100\pi}$$

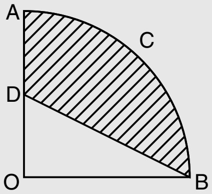
$$\Rightarrow \quad h' = 1.5 \text{ cm}$$

Thus, the height of water level in cylindrical vessel is 1.5 cm.

15. In the given figure, *OACB* is a quadrant of a circle with centre *O* and radius 3.5 cm. If *OD* = 2 cm, find the area of the shaded region.



Sol.



Area of shaded region = Area of quadrant *OACB* – Area of Δ*ODB* **1**

$$= \left(\frac{22}{7} \times \frac{3.5 \times 3.5}{4} - \frac{1}{2} \times 3.5 \times 2\right) \text{cm}^2 \qquad \mathbf{1}$$

$$= \frac{49}{8} \text{ or } 6.125 \text{ cm}^2 \qquad \mathbf{1}$$

(CBSE Marking Scheme, 2017)

16. Two tangents *TP* and *TQ* are drawn to a circle with centre *O* from an external point *T*. Prove that ∠*PTQ* = 2 ∠*OPQ*.

Sol. Refer **to** 2020 year Delhi Set-I Q.2. OR part

17. Show that Δ *ABC*, where *A* (–2, 0), *B* (2, 0), *C* (0, 2) and Δ *PQR* where *P* (–4, 0), *Q* (4, 0), *R* (0, 4) are similar triangles.

Sol. $A(-2, 0), B(2, 0), C(0, 2)$

$AB = 4$ units, $BC = 2\sqrt{2}$ units, $AC = 2\sqrt{2}$ units **1**

$P(-4, 0), Q(4, 0), R(0, 4)$

$PQ = 8$ units, $QR = 4\sqrt{2}$ units, $PR = 4\sqrt{2}$ units **1**

$$\therefore \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{1}{2}$$

As all the corresponding sides are in same ratio

∴ Hence, $\Delta ABC \sim \Delta PQR$ **1**

(CBSE Marking Scheme, 2017)

*** 18. The area of a triangle is 5 sq unit. Two of its vertices are (2, 1) and (3, – 2). If the third vertex is $\left(\frac{7}{2}, y\right)$, find the value of *y*.**

19. Two different dice are thrown together. Find the probability that the numbers obtained

(i) have a sum less than 7.

(ii) have a product less than 16.

(iii) is a doublet of odd numbers.

Sol. Total number of outcomes = 36

(i) Favourable outcomes are

(1, 1,) (1, 2) (1, 3) (1, 4) (1, 5) (2, 1) (2, 2) (2, 3) (2, 4) (3, 1) (3, 2) (3, 3) (4, 1) (4, 2) (5, 1)

i.e., 15

$$\therefore \text{P(sum less than 7)} = \frac{15}{36} \text{ or } \frac{5}{12} \qquad \mathbf{1}$$

(ii) Favourable outcomes are

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6) (2, 1) (2, 2) (2, 3)
(2, 4) (2, 5) (2, 6) (3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (4, 1)
(4, 2) (4, 3) (5, 1) (5, 2) (5, 3) (6, 1) (6, 2)

i.e., 25 **1**

$$\therefore P(\text{product less than 16}) = \frac{25}{36}.$$

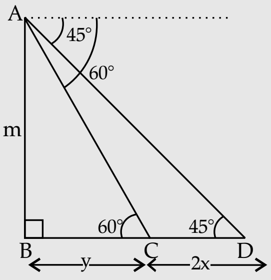
(iii) Favourable outcomes are (1, 1), (3, 3), (5, 5)

$$\therefore \quad P(\text{doublet of odd number}) = \frac{3}{36} \text{ or } \frac{1}{12}$$

(CBSE Marking Scheme, 2017)

20. A moving boat is observed from the top of a 150 m high cliff moving away from the cliff. The angle of depression of the boat changes from 60° to 45° in 2 minutes. Find the speed of the boat in m/h.

Sol.



½

Let the speed of boat be x m/min

$\therefore \quad CD = 2x$

$$\frac{150}{y} = \tan 60° \Rightarrow y = \frac{150}{\sqrt{3}} = 50\sqrt{3} \qquad \mathbf{1}$$

$$\frac{150}{y+2x} = \tan 45° \Rightarrow 150 = 50\sqrt{3} + 2x$$

$$\Rightarrow \quad x = 25\left(3-\sqrt{3}\right) \qquad \mathbf{1}$$

* *Out of Syllabus*

$\therefore$ Speed $= 25(3-\sqrt{3})$ m / min

$= 1500(3-\sqrt{3})$ m / h. ½

(CBSE Marking Scheme, 2017) 3

SECTION - D

Question numbers 21 to 31 carry 4 marks each.

*** 21. Construct an isosceles triangle with base 8 cm and altitude 4 cm. Construct another triangle whose sides are $\frac{2}{3}$ times the corresponding sides of the isosceles triangle.**

22. Prove that the lengths of tangents drawn from an external point to a circle are equal.

Sol. Refer to 2018 year Delhi Set-I Q.18.

23. The ratio of the sums of first m and first n terms of an A.P. is $m^2 : n^2$. Show that the ratio of its m^{th} and n^{th} terms is $(2m - 1) : (2n - 1)$.

Sol. $\frac{S_m}{S_n} = \frac{m^2}{n^2}$

$$\Rightarrow \frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \frac{m^2}{n^2} \quad \textbf{1}$$

$$\Rightarrow \frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n} \quad \textbf{1}$$

Solving we get $d = 2a$ **1**

$$\frac{a_m}{a_n} = \frac{a+(m-1)d}{a+(n-1)d} = \frac{a+(m-1)\times 2a}{a+(n-1)\times 2a}$$

$$= \frac{2m-1}{2n-1} \quad \textbf{1}$$

(CBSE Marking Scheme, 2017)

24. Speed of a boat in still water is 15 km/h. It goes 30 km upstream and returns back at the same point in 4 hours 30 minutes. Find the speed of the stream.

Sol. Let the speed of stream be x km/h

$\therefore$ Speed of boat upstream = $(15 - x)$ km/h ½

Speed of boat downstream = $(15 + x)$ km/h ½

$$\frac{30}{15-x} + \frac{30}{15+x} = 4\frac{1}{2} = \frac{9}{2}$$

$$\Rightarrow \frac{30(15+x+15-x)}{(15-x)(15+x)} = \frac{9}{2} \quad \textbf{1}$$

$$\Rightarrow 200 = 225 - x^2 \quad \textbf{1}$$

$$x = 5 \text{ (Rejecting } -5)$$

$\therefore$ Speed of stream = 5 km/h **1**

(CBSE Marking Scheme, 2017)

* *Out of Syllabus*

*** 25. If $a \neq b \neq 0$, prove that the points (a, a^2), (b, b^2) (0, 0) will not be collinear.**

*** 26. The height of a cone is 10 cm. The cone is divided into two parts using a plane parallel to its base at the middle of its height. Find the ratio of the volumes of the two parts.**

27. Peter throws two different dice together and finds the product of the two numbers obtained. Rina throws a die and squares the number obtained. Who has the better chance to get the number 25.

Sol. For Peter,

Total number of outcomes = 36

Favourable outcome is (5, 5)

$\therefore$ P(Peter getting the number 25) $= \frac{1}{36}$ **1½**

For Rina, Total number of outcomes = 6

Favourable outcome is 5.

$\therefore$ P (Rina getting the number 25) $= \frac{1}{6}$ **1½**

$\therefore$ Rina has the better chance **1**

(CBSE Marking Scheme, 2017)

28. A chord PQ of a circle of radius 10 cm subtends an angle of 60° at the centre of circle. Find the area of major and minor segments of the circle.

Sol. Area of minor segment

$$= \frac{22}{7}\times 10\times 10\times \frac{\cancel{60}^{1}}{\cancel{360}_{6}} - \frac{\sqrt{3}}{4}\times 10\times 10$$

$$= 10\times 10\left[\frac{22}{7}\times\frac{1}{6} - \frac{\sqrt{3}}{4}\right]$$

$$= \frac{100}{84}(44-21\sqrt{3})\text{ cm}^2 \text{ or } \frac{25}{21}(44-21\sqrt{3})\text{ cm}^2$$

2½

Area of major segment

$$= \left[\frac{22}{7}\times 10\times 10 - \frac{100}{84}(44-21\sqrt{3})\right]\text{ cm}^2$$

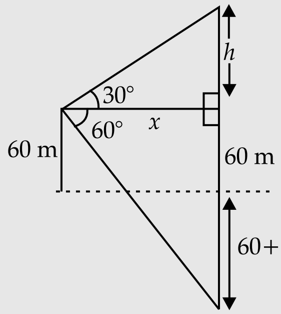
$$= \frac{100}{84}(220+21\sqrt{3})\text{ cm}^2$$

$$= \frac{25}{21}(220+21\sqrt{3})\text{ cm}^2 \quad \textbf{1½}$$

(CBSE Marking Scheme, 2017) 4

29. The angle of elevation of a cloud from a point 60 m above the surface of the water of a lake is 30° and the angle of depression of its shadow in water of lake is 60°. Find the height of the cloud from the surface of water.

Sol.



1

$\frac{h}{x} = \tan 30° \Rightarrow x = h\sqrt{3}$ 1

$\frac{60+60+h}{x} = \tan 60°$ 1

$\Rightarrow \frac{120+h}{x} = \sqrt{3}$ ½

$\Rightarrow 120 + h = h\sqrt{3}\times\sqrt{3}$ ½

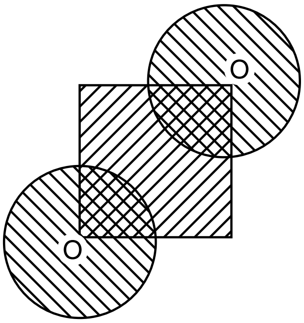
$\Rightarrow h = 60$

$\therefore$ height of cloud from surface of water

$= (60 + 60)$ m $= 120$ m

(CBSE Marking Scheme, 2017)

30. In the given figure, the side of square is 28 cm and radius of each circle is half of the length of the side of the square where *O* and *O′* are centres of the circles. Find the area of shaded region.



Sol. Area of shaded region 1½

Area of square + Area of 2 major sectors.

$= \left[28\times28+2\times\frac{22}{7}\times14\times14\times\frac{270°}{360°}\right]$ cm^2 1½

$= 28\times28\left(1+\frac{33}{28}\right) = 1708$ cm^2 1

(CBSE Marking Scheme, 2017)

31. In a hospital used water is collected in a cylindrical tank of diameter 2 m and height 5 m. After recycling this water is used to irrigate a park of hospital whose length is 25 m and breadth is 20 m. If tank is filled completely then what will be the height of standing water used for irrigating the park. Write your views on recycling of water.

Sol. Volume of water in cylindrical tank.

= Volume of water in park.

$\Rightarrow \frac{22}{7}\times1\times1\times5 = 25 \times 20 \times h$, where h is the height of standing water. 1½

$\Rightarrow h = \frac{11}{350}$ m or $\frac{22}{7}$ cm 1½

Conservation of water or any other relevant value. 1

(CBSE Marking Scheme, 2017)

Delhi Set-II Code No. 30/1/2

SECTION - B

Question numbers 5 to 10 carry 2 marks each.

*** 10. Draw a line segment of length 7 cm and divide it internally in the ratio 2 : 3.**

SECTION - C

Question numbers 11 to 20 carry 3 marks each.

18. If the m^{th} term of an A.P is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$ then show that its $(mn)^{\text{th}}$ term is 1.

Sol. $a_m = \frac{1}{n} \Rightarrow a + (m-1)\,d = \frac{1}{n}$ ...(1) ½

$a_n = \frac{1}{n} \Rightarrow a + (n-1)\,d = \frac{1}{n}$ ...(2) ½

Solving (1) and (2) we get, $a = \frac{1}{mn}$, $d = \frac{1}{mn}$ 1

$a_{mn} = a + (mn-1)\,d$

$= \frac{1}{mn}+(mn-1)\times\frac{1}{mn} = 1$ 1

(CBSE Marking Scheme, 2017)

19. A metallic solid sphere of radius 10.5 cm is melted and recast into smaller solid cones, each of radius 3.5 cm and height 3 cm. How many cones will be made ?

Sol. Let the number of cones be n

Volume of solid sphere = Volume of n solid cones

1

* *Out of Syllabus*

$$\Rightarrow \frac{4}{3}\pi \times 10.5 \times 10.5 \times 10.5 \qquad \textbf{1}$$

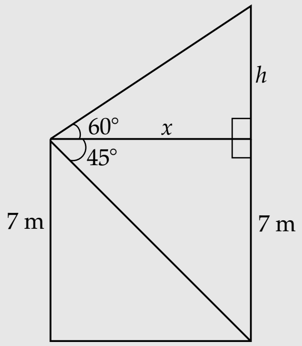
$$= n \times \frac{1}{3} \times \pi \times 3.5 \times 3.5 \times 3$$

$$\Rightarrow n = \frac{4 \times 10.5 \times 10.5 \times 10.5}{3.5 \times 3.5 \times 3.5} \qquad \textbf{1}$$

$$= 108$$ **(CBSE Marking Scheme, 2017)**

20. From the top of a 7 m high building the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Find the height of the tower.

Sol.



½

$$\frac{7}{x} = \tan 45^\circ$$

$$\Rightarrow \quad x = 7 \text{ m} \qquad \textbf{1}$$

$$\frac{h}{x} = \tan 60^\circ$$

$$h = x\sqrt{3}$$

$$= 7\sqrt{3} \qquad \textbf{1}$$

$\therefore$ Height of tower $= (7\sqrt{3}+7)$ m

$$= 7(\sqrt{3}+1) \text{ m} \qquad ½$$

(CBSE Marking Scheme, 2017)

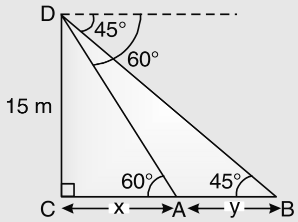
SECTION - D

Question numbers 21 to 31 carry 4 marks each.

*** 28. Draw a right triangle in which the sides (other than the hypotenuse) are of lengths 4 cm and 3 cm. Now construct another triangle whose sides are $\frac{3}{5}$ times the corresponding sides of the given triangle.**

30. Two points A and B are on the same side of a tower and in the same straight line with its base. The angles of depression of these points from the top of the tower are 60° and 45° respectively. If the height of the tower is 15 m, then find the distance between these points.

Sol.



1

$$\frac{15}{x} = \tan 60^\circ$$

$$\Rightarrow \quad x = \frac{15}{\sqrt{3}} = 5\sqrt{3} \text{ m} \qquad \textbf{1}$$

$$\frac{15}{x+y} = \tan 45^\circ \qquad \textbf{1}$$

$$\Rightarrow \quad 15 = 5\sqrt{3}+y$$

$$\Rightarrow \quad y = 15-5\sqrt{3}$$

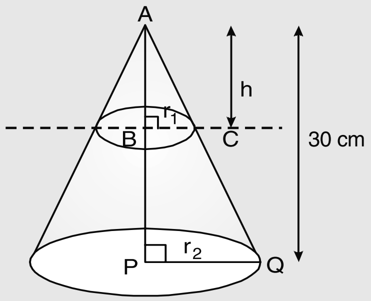
$$= 5(3-\sqrt{3})$$

$\therefore$ Distance between two points $= 5(3-\sqrt{3})$ m.

(CBSE Marking Scheme, 2017) 1

31. The height of a cone is 30 cm. From its topside a small cone is cut by a plane parallel to its base. If volume of smaller cone is $\frac{1}{27}$ of the given cone then at what height it is cut from its base ?

Sol.



1

$$\Delta ABC \sim \Delta APQ \qquad \textbf{1}$$

$$\Rightarrow \quad \frac{h}{30} = \frac{r_1}{r_2} \qquad ...(i)$$

Volume of smaller cone

$$= \frac{1}{27} \times \text{Volume of larger cone}$$

$$\Rightarrow \quad \frac{1}{3}\pi r_1^2 \times h = \frac{1}{27} \times \frac{1}{3}\pi r_2^2 \times 30 \qquad ½$$

$$\Rightarrow \quad \left(\frac{r_1}{r_2}\right)^2 \times \frac{h}{30} = \frac{1}{27}$$

$$\Rightarrow \quad \left(\frac{h}{30}\right)^3 = \frac{1}{27} \qquad \text{[using (i)] } ½$$

$$\Rightarrow \quad h = 10 \text{ cm}$$

$\therefore$ Required height = (30 – 10) cm = 20 cm **1**

(CBSE Marking Scheme, 2017)

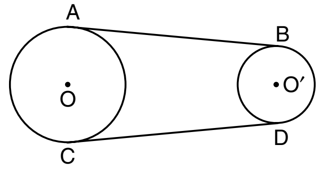
* *Out of Syllabus*

Delhi Set-III — Code No. 30/1/3

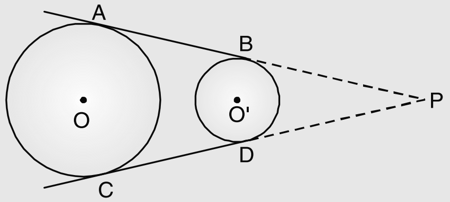
SECTION - B

Question numbers 5 to 10 carry 2 marks each.

10. In the figure, AB and CD are common tangents to two circles of unequal radii. Prove that $AB = CD$.



Sol.



½

Construction : Extend AB and CD to meet at P

$$PA = PC \quad ½$$
$$PB = PD \quad ½$$
$$\Rightarrow \quad PA - PB = PC - PD \quad ½$$
$$\Rightarrow \quad AB = CD$$

(CBSE Marking Scheme, 2017)

SECTION - C

Question numbers 11 to 20 carry 3 marks each.

18. If the p^{th} term of an A.P is q and q^{th} term is p. Prove that its n^{th} term is $(p + q - n)$.

Sol. $a_p = q \Rightarrow a + (p-1)d = q$ ...(1) ½

$a_q = p \Rightarrow a + (q-1)d = p$ ...(2) ½

Solving (1) and (2) we get, $a = p + q - 1$, $d = -1$ **1**

$a_n = a + (n-1)d$

$= (p + q - 1) + (n - 1)(-1)$ **1**

$= p + q - n$

(CBSE Marking Scheme, 2017)

19. A solid metallic sphere of diameter 16 cm is melted and recast into smaller solid cones, each of radius 4 cm and height 8 cm. Find the number of cones so formed.

Sol. Let the number of cones be n **1**

Volume of sphere = Volume of n cones

$$\Rightarrow \frac{4}{3}\pi\times8\times8\times8 = n\times\frac{1}{3}\times\pi\times4\times4\times8 \quad \textbf{1}$$

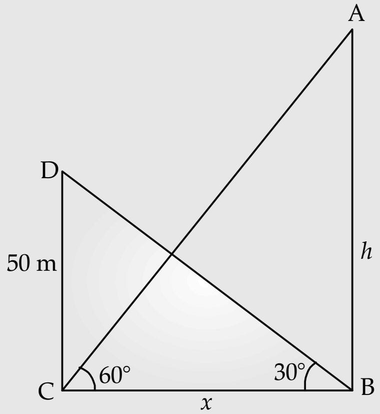
$$\Rightarrow \quad n = \frac{4\times8\times8\times8}{4\times4\times8}$$

$$= 16 \quad \textbf{1}$$

(CBSE Marking Scheme, 2017)

20. The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30°. If height of the tower is 50 m, find the height of the hill.

Sol.



½

$$\frac{50}{x} = \tan 30° \quad \textbf{1}$$

$$\Rightarrow \quad x = 50\sqrt{3}$$

$$\frac{h}{x} = \tan 60°$$

$$\Rightarrow \quad h = 50\sqrt{3}\times\sqrt{3} = 150 \quad \textbf{1}$$

$\therefore$ Height of hill = 150 m ½

(CBSE Marking Scheme, 2017)

SECTION - D

Question numbers 21 to 31 carry 4 marks each.

*** 28. Construct a triangle ABC with sides BC = 7 cm, $\angle B$ = 45° and $\angle A$ = 105°. Then construct a triangle whose sides are $\frac{3}{4}$ times the corresponding sides of ΔABC.**

29. If the p^{th} term of an A.P is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$, prove that the sum of first pq terms of the A.P is $\left(\frac{pq+1}{2}\right)$.

Sol. $a_p = \frac{1}{q} \Rightarrow a + (p-1)d = \frac{1}{q}$ ...(1) **1**

$a_q = \frac{1}{p} \Rightarrow a + (q-1)d = \frac{1}{p}$ ...(2) **1**

Solving (1) and (2) we get, $a = \frac{1}{pq}$, $d = \frac{1}{pq}$ **1**

* *Out of Syllabus*

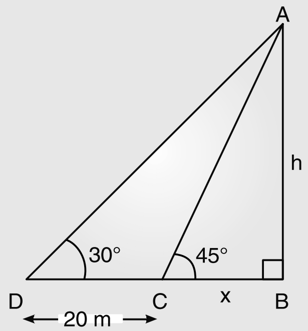
$$S_{pq} = \frac{pq}{2}\left[2\times\frac{1}{pq} + (pq-1)\times\frac{1}{pq}\right]$$

$$= \frac{(pq+1)}{2}$$ **1**

(CBSE Marking Scheme, 2017)

30. An observer finds the angle of elevation of the top of the tower from a certain point on the ground as 30°, If the observer moves 20m towards the base of the tower the angle of elevation of the top increases by 15°, find the height of the tower.

Sol.



1

$$\frac{h}{x} = \tan 45°$$

$\Rightarrow$ $h = x$

$$\frac{h}{x+20} = \tan 30°$$

$\Rightarrow$ $h\sqrt{3} = x + 20$ **1**

$\Rightarrow$ $h\sqrt{3} = h + 20$

$\Rightarrow$ $h = \dfrac{20}{\sqrt{3}-1}$

or $10(\sqrt{3}+1)$ **1**

$\therefore$ Height of tower $= 10(\sqrt{3}+1)$ m

(CBSE Marking Scheme, 2017)

Outside Delhi Set-I — Code No. 30/1

SECTION - A

Question numbers 1 to 4 carry 1 mark each.

1. What is the common difference of an A.P. in which $a_{21} - a_7 = 84$? **1**

Sol. $a_{21} - a_7 = 84$

$\Rightarrow (a + 20d) - (a + 6d) = 84$ ½

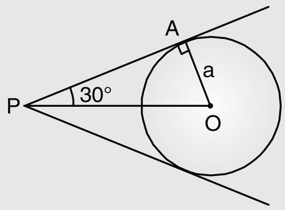
$\Rightarrow 14d = 84$

$d = 6$ ½

(CBSE Marking Scheme, 2017)

2. If the angle between two tangents drawn from an external point *P* to a circle of radius a and centre O, is 60°, then find the length of OP. **1**

Sol.



$\angle OPA = 30°$ ½

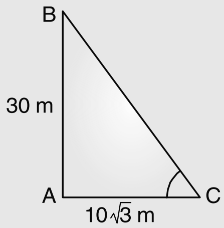
$\sin 30° = \dfrac{a}{OP}$

$\Rightarrow$ $OP = 2a$ ½

(CBSE Marking Scheme, 2017)

3. If a tower 30 m high, casts a shadow $10\sqrt{3}$ m long on the ground, then what is the angle of elevation of the sun ? **1**

Sol.



$\tan\theta = \dfrac{30}{10\sqrt{3}} = \sqrt{3}$ ½

$\Rightarrow$ $\theta = 60°$ ½

(CBSE Marking Scheme, 2017)

4. The probability of selecting a rotten apple randomly from a heap of 900 apples is 0.18 What is the number of rotten apples in the heap ?

Sol. Let the number of rotten apples in the heap be n.

$\therefore$ $\dfrac{n}{900} = 0.18$ ½

$\Rightarrow$ $n = 162$ ½

(CBSE Marking Scheme, 2017)

SECTION - B

Question numbers 5 to 10 carry 2 marks each.

5. Find the value of p, for which one root of the quadratic equation $px^2 - 14x + 8 = 0$ is 6 times the other.

Sol. Let the roots of the given equation be α and 6α. ½

Thus the quadratic equation is $(x - \alpha)(x - 6\alpha) = 0$

$\Rightarrow \quad x^2 - 7\alpha x + 6\alpha^2 = 0$ ...(i) ½

Given equation can be written as

$$x^2 - \frac{14}{p}x + \frac{8}{p} = 0 \quad \text{...(ii) ½}$$

Comparing the co-efficients in (i) & (ii)

$$7\alpha = \frac{14}{p} \text{ and } 6\alpha^2 = \frac{8}{p} = 6 \times \frac{4}{p^2} = \frac{8}{p}$$

Solving to get $p = 3$ ½

(CBSE Marking Scheme, 2017)

6. Which term of the progression 20, $19\frac{1}{4}$, $18\frac{1}{2}$, $17\frac{3}{4}$, ... is the first negative term ?

Sol. Here $d = \frac{-3}{4}$ ½

Let the n^{th} term be first negative term

$\therefore \quad 20 + (n-1)\left(\frac{-3}{4}\right) < 0$ ½

$\Rightarrow \quad 3n > 83$

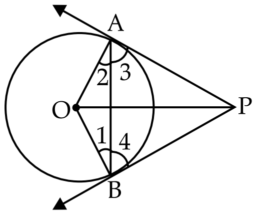
$\Rightarrow \quad n > 27\frac{2}{3}$ ½

Hence 28th term is first negative term. ½

(CBSE Marking Scheme, 2017)

7. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord.

Sol.



½

Given: A circle with center O,PA and PB are tangents drawn at ends A and B on chord AB.

To prove: $\angle PAB = \angle PBA$

Construction: Join OA and OB ½

Proof: In ΔAOB, we have

$OA = OB$ (Radii of the same circle)

$\angle OAB = \angle OBA$ ...(1) (Angles opposite to equal sides)

$\angle OAP = \angle OBP = 90°$ ($\because$ Radius $\perp$ Tangent)

$\Rightarrow \quad \angle OAB + \angle PAB = \angle OBA + \angle PBA$ ½

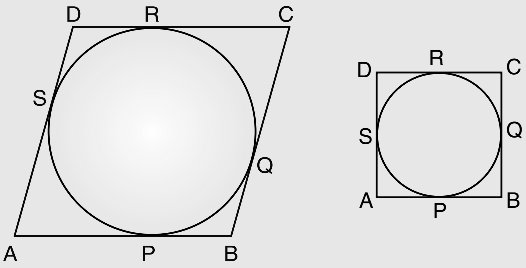
$\Rightarrow \quad \angle OAB + \angle PAB = \angle OAB + \angle PBA$ (From (1))

$\Rightarrow \quad \angle PAB = \angle PBA$ ½

Hence proved.

8. A circle touches all the four sides of a quadrilateral *ABCD*. Prove that $AB + CD = BC + DA$.

Sol.



Here $AP = AS$

$BP = BQ$ 1

$CR = CQ$

$DR = DS$ ½

Adding $(AP + PB) + (CR + RD)$

$= (AS + SD) + (BQ + QC)$

$\Rightarrow \quad AB + CD = AD + BC$ ½

(CBSE Marking Scheme, 2017)

9. A line intersects the *y*-axis and *x*-axis at the points *P* and *Q* respectively. If (2, – 5) is the mid-point of *PQ*, then find the coordinates of *P* and *Q*. **2**

Sol. Let the coordinates of points P and Q be $(0, b)$ and $(a, 0)$ resp. ½

$\therefore \quad \frac{a}{2} = 2 \Rightarrow a = 4$ ½

$\frac{b}{2} = -5 \Rightarrow b = -10$ ½

$\therefore p(0, -10)$ and $Q(4, 0)$ ½

(CBSE Marking Scheme, 2017)

10. If the distances of *P* (*x*, *y*) from *A* (5, 1) and *B* (–1, 5) are equal, then prove that $3x = 2y$.

Sol. $PA^2 = PB^2$

$\Rightarrow (x-5)^2 + (y-1)^2 = (x+1)^2 + (y-5)^2$ 1

$\Rightarrow \quad 12x = 8y$

$\Rightarrow \quad 3x = 2y$ 1

(CBSE Marking Scheme, 2017)

SECTION - C

Question numbers 11 to 20 carry 3 marks each.

11. If $ad \neq bc$, then prove that the equation $(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$ has no real roots.

Sol. $D = 4(ac + bd)^2 - 4(a^2 + b^2)(c^2 + d^2)$ 1

$= -4(a^2d^2 + b^2c^2 - 2abcd)$ 1

$= -4(ad - bc)^2$

Since $ad \neq bc$ ½

Therefore $D < 0$

The equation has no real roots ½

(CBSE Marking Scheme, 2017)

12. The first term of an A.P. is 5, the last term is 45 and the sum of all its terms is 400. Find the number of terms and the common difference of the A.P.

Sol. Here $a = 5$, $l = 45$ and $S_n = 400$ **1**

$\therefore \frac{n}{2}(a+l) = 400$ or $\frac{n}{2}(5+45) = 400$ ½

$\Rightarrow n = 16$ **1**

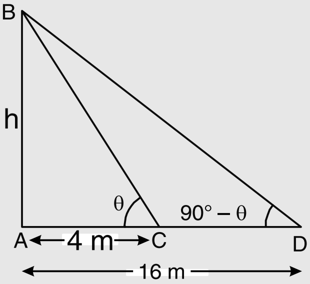
Also $5 + 15d = 45$

$\Rightarrow d = \frac{8}{3}$ ½

(CBSE Marking Scheme, 2017)

13. On a straight line passing through the foot of a tower, two points *C* and *D* are at distances of 4 m and 16 m from the foot respectively. If the angles of elevation from *C* and *D* of the top of the tower are complementary, then find the height of the tower.

Sol.



½

$$\tan\theta = \frac{h}{4} \quad \text{...(i)} \ ½$$

$$\tan(90-\theta) = \frac{h}{16}$$

$$\Rightarrow \cot\theta = \frac{h}{16} \quad \text{...(ii)} \ \mathbf{1}$$

Multiplying (i) and (ii) to get

$$h^2 = 64$$

$$\Rightarrow h = 8\text{m} \quad \mathbf{1}$$

(CBSE Marking Scheme, 2017)

14. A bag contains 15 white and some black balls. If the probability of drawing a black ball from the bag is thrice that of drawing a white ball, find the number of black balls in the bag.

Sol. Let the number of black balls in the bag be n.

$\therefore$ Total number of balls are $15 + n$ **1**

Prob(Black ball) = 3 × Prob(White ball)

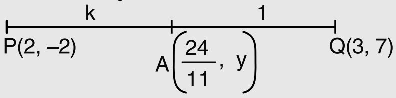
$\Rightarrow \frac{n}{15+n} = 3\times\frac{15}{15+n}$ **1**

$\Rightarrow n = 45$ **1**

(CBSE Marking Scheme, 2017)

15. In what ratio does the point $\left(\frac{24}{11}, y\right)$ divide the line segment joining the points *P* (2, – 2) and *Q* (3, 7) ? Also find the value of *y*.

Sol. Let $PA : AQ = k : 1$



1

$\therefore \frac{2+3k}{k+1} = \frac{24}{11}$ ½

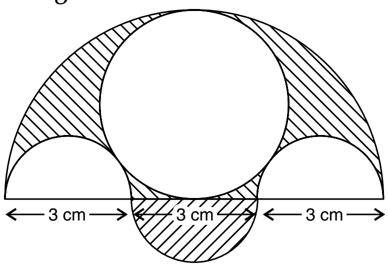
$\Rightarrow k = \frac{2}{9}$ ½

Hence the ratio is 2 : 9.

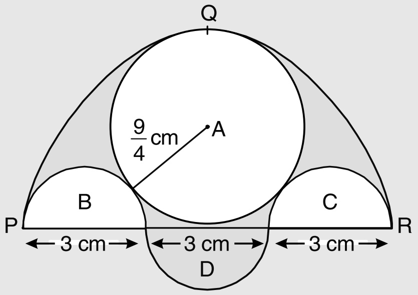
Therefore $y = \frac{-18+14}{11} = \frac{-4}{11}$ **1**

(CBSE Marking Scheme, 2017)

16. Three semicircles each of diameter 3 cm, a circle of diameter 4.5 cm and a semicircle of radius 4.5 cm are drawn in the given figure. Find the area of the shaded region.



Sol.



Area of semi-circle *PQR*

$$= \frac{\pi}{2}\left(\frac{9}{2}\right)^2 = \frac{81}{8}\pi \text{ cm}^2 \quad ½$$

Area of region $A = \pi\left(\frac{9}{4}\right)^2 = \frac{81}{16}\pi \text{ cm}^2$ ½

Area of region $(B + C) = \pi\left(\frac{3}{2}\right)^2 = \frac{9}{4}\pi \text{ cm}^2$ ½

Area of region $D = \frac{\pi}{2}\left(\frac{3}{2}\right)^2 = \frac{9}{8}\pi \text{ cm}^2$ ½

Area of shaded region =

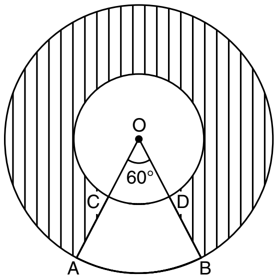
$$\left(\frac{81}{8}\pi - \frac{81}{16}\pi - \frac{9}{4}\pi + \frac{9}{8}\pi\right) \text{cm}^2$$

$= \frac{63}{16}\pi \text{ cm}^2$ or $\frac{99}{8}$ cm^2 = 12.375 cm^2 **1**

(CBSE Marking Scheme, 2017)

17. In the given figure, two concentric circles with centre O have radii 21 cm and 42 cm. If $\angle AOB$ = 60°, find the area of the shaded region.

[Use $\pi = \frac{22}{7}$].



Sol. Area of region $ABDC = \pi \times \frac{60}{360} \times (42^2 - 21^2)$

$= \frac{22}{7} \times \frac{1}{6} \times 63 \times 21$ **1**

$= 693 \text{ cm}^2$

Area of shaded region $= \pi(42^2 - 21^2)$ – region $ABCD$ **1**

$= \frac{22}{7} \times 63 \times 21 - 693$

$= 4158 - 693$

$= 3465 \text{ cm}^2$ **1**

(CBSE Marking Scheme, 2017)

18. Water in a canal, 5.4 m wide and 1.8 m deep, is flowing with a speed of 25 km/hour. How much area can it irrigate in 40 minutes, if 10 cm of standing water is required for irrigation ?

Sol. Volume of water flowing in 40 min

$= 5.4 \times 1.8 \times 25000 \times \frac{40}{60} \text{ m}^3$ **1**

$= 162000 \text{ m}^3$ ½

Height of standing water = 10 cm = 0.10 m **1**

$\therefore$ Area to be irrigated $= \frac{162000}{0.10}$

$= 1620000 \text{ m}^2$ ½

(CBSE Marking Scheme, 2017)

*** 19. The slant height of a frustum of a cone is 4 cm and the perimeters of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.**

20. The dimensions of a solid iron cuboid are 4.4 m × 2.6 m × 1.0 m. It is melted and recast into a hollow cylindrical pipe of 30 cm inner radius and thickness 5 cm. Find the length of the pipe.

Sol. Volume of cuboid $= 4.4 \times 2.6 \times 1 \text{ m}^3$ ½

Inner and outer radii of cylindrical pipe = 30 cm, 35 cm ½

$\therefore$ Volume of material used

$= \frac{\pi}{100^2}(35^2 - 30^2) \times h\text{m}^3$

$= \frac{\pi}{100^2} \times 65 \times 5h$

Volume of hollow pipe = Volume of cuboid ½

Now $\frac{\pi}{100^2} \times 65 \times 5h = 4.4 \times 2.6 \times 1$

$\Rightarrow \quad h = \frac{7 \times 4.4 \times 2.6 \times 100 \times 100}{22 \times 65 \times 5}$ ½ + ½

$\Rightarrow \quad h = 112 \text{ m}$ ½

(CBSE Marking Scheme, 2017)

SECTION - D

Question numbers 21 to 31 carry 4 marks each.

21. Solve for x :

$$\frac{1}{x+1} + \frac{3}{5x+1} = \frac{5}{x+4}, \; x \neq -1, -\frac{1}{5}, -4$$

Sol. Here $[(5x + 1) + (x + 1)3]\,(x + 4)$

$= 5(x + 1)(5x + 1)$ **1**

$\Rightarrow (8x + 4)(x + 4) = 5(5x^2 + 6x + 1)$ **1**

$\Rightarrow 17x^2 - 6x - 11 = 0$

$\Rightarrow 17x^2 - 17x + 11x - 11 = 0$ **1**

$\Rightarrow (17x + 11)(x - 1) = 0$

$\Rightarrow x = \frac{-11}{17}, x = 1$ **1**

(CBSE Marking Scheme, 2017)

22. Two taps running together can fill a tank in $3\frac{1}{13}$ hours. If one tap takes 3 hours more than the other to fill the tank, then how much time will each tap take to fill the tank ?

Sol. Let one tap fill the tank in x hrs.

Therefore, other tap fills the tank in $(x + 3)$ hrs. ½

Work done by both the taps in one hour is

$\frac{1}{x} + \frac{1}{x+3} = \frac{13}{40}$ **1**

$\Rightarrow \quad (2x + 3)\,40 = 13\,(x^2 + 3x)$

$\Rightarrow \quad 13x^2 - 41x - 120 = 0$ ½

$13x^2 - 65x + 24x - 120 = 0$

$\Rightarrow 13x\,(x - 5) + 24\,(x - 5) = 6$ **1**

$\Rightarrow \quad (13x + 24)(x - 5) = 0$

$\Rightarrow \quad x = 5$ ½

(rejecting the negative value)

Hence one tap takes 5 hrs and another 8 hrs separately to fill the tank. ½

(CBSE Marking Scheme, 2017)

* *Out of Syllabus*

23. If the ratio of the sum of the first n terms of two A.P. s is $(7n + 1) : (4n + 27)$, then find the ratio of their 9th terms.

Sol. Let the first terms be a and a' and d and d' be their respective common differences.

$$\frac{S_n}{S'_n} = \frac{\frac{n}{2}(2a+(n-1)d)}{\frac{n}{2}(2a'+(n-1)d')} = \frac{7n+1}{4n+27} \qquad \textbf{1}$$

$$\Rightarrow \frac{a+\left(\frac{n-1}{2}\right)d}{a'+\left(\frac{n-1}{2}\right)d'} = \frac{7n+1}{4n+27} \qquad \textbf{1}$$

To get ratio of 9th terms, replacing $\frac{n-1}{2} = 8$

$\Rightarrow n = 17$ **1**

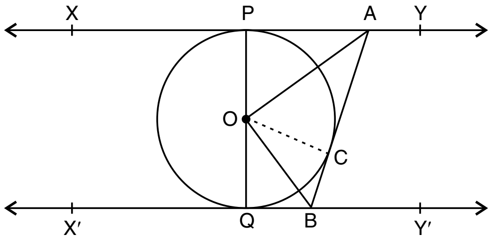
Hence $\frac{t_9}{t'_9} = \frac{a+8d}{a'+8d'} = \frac{120}{95}$ or $\frac{24}{19}$ **1**

(CBSE Marking Scheme, 2017)

24. Prove that the lengths of two tangents drawn from an external point to a circle are equal.

Sol. Refer to 2018 year Delhi Set-I Q. 18.

25. In the given figure, XY and $X'Y'$ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C, is intersecting XY at A and $X'Y'$ at B. Prove that $\angle AOB = 90°$.



Sol. In right angled ΔPOA and ΔOCA

$$\Delta OPA \cong \Delta OCA$$

$\therefore \quad \angle POA = \angle AOC$ ...(i) **1**

Also $\quad \Delta OQB \cong \Delta OCB$

$\therefore \quad \angle QOB = \angle BOC$ ...(ii) **1**

And, $\quad \angle AOB = \angle AOC + \angle COB$

$$= \frac{1}{2}\angle POC + \frac{1}{2}\angle COQ \qquad \textbf{1}$$

$$= \frac{1}{2}(\angle POC + \angle COQ)$$

$$= \frac{1}{2}\times 180°$$

$$= 90° \qquad \textbf{1}$$

(CBSE Marking Scheme, 2017)

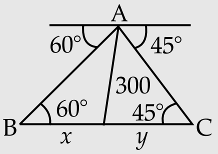
*** 26. Construct a triangle ABC with side $BC = 7$ cm, $\angle B = 45°$, $\angle A = 105°$. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the ΔABC.**

27. An aeroplane is flying at a height of 300 m above the ground. Flying at this height, the angles of depression from the aeroplane of two points on both banks of a river in opposite directions are 45° and 60° respectively. Find the width of the river. [Use $\sqrt{3} = 1.732$]

Sol. $\tan 45° = \frac{300}{y}$ **1**

$\Rightarrow \quad 1 = \frac{300}{y}$ or $y = 300$ **1**

$\tan 60° = \frac{300}{x}$



$\Rightarrow \sqrt{3} = \frac{300}{x}$ or $x = \frac{300}{\sqrt{3}} = 100\sqrt{3}$ **1**

Width of river $= 300 + 100\sqrt{3} = 300 + 173.2$

$= 473.2$ m **1**

(CBSE Marking Scheme, 2017)

*** 28. If the points $A (k + 1, 2k)$, $B (3k, 2k + 3)$ and $C (5k – 1, 5k)$ are collinear, then find the value of k.**

29. Two different dice are thrown together. Find the probability that the numbers obtained have
(i) even sum, and
(ii) even product.

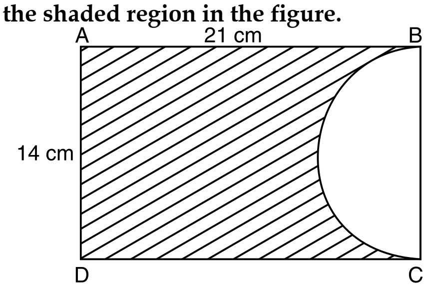
Sol. Total number of outcomes = 36 **1**

(i) P(even sum) $= \frac{18}{36} = \frac{1}{2}$ **1½**

(ii) P(even product) $= \frac{27}{36} = \frac{3}{4}$ **1½**

(CBSE Marking Scheme, 2017)

30. In the given figure, $ABCD$ is a rectangle of dimensions 21 cm × 14 cm. A semicircle is drawn with BC as diameter. Find the area and the perimeter of the shaded region in the figure.



* *Out of Syllabus*

Sol. Area of shaded region

$$= (21 \times 14) - \frac{1}{2} \times \pi \times 7 \times 7 \qquad 1$$

$$= 294 - \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$$

$$= 294 - 77 = 217 \text{ cm}^2 \qquad 1$$

Perimeter of shaded region

$$= 21 + 14 + 21 + \frac{22}{7} \times 7 \qquad 1$$

$$= 56 + 22 = 78 \text{ cm} \qquad 1$$

(CBSE Marking Scheme, 2017)

31. In a rain-water harvesting system, the rain-water from a roof of 22 m × 20 m drains into a cylindrical tank having diameter of base 2 m and height 3.5 m. If the tank is full, find the rainfall in cm. Write your views on water conservation. 4

Sol. Rectangular roof: l = 22 m, b = 20 m

Cylindrical tank: r = 1 m, h = 3.5 m

The volume of tank $= \pi r^2 h$

$$= \frac{22}{7} \times 1 \times 1 \times 3.5$$

$$= \frac{22}{7} \times \frac{3.5}{10}$$

$$= 11 \text{ m}^3$$

Area of roof $= l \times b$

$$= 22 \times 20 = 440 \text{ m}^2$$

$$\text{Rainfall} = \frac{\text{Volume of tank}}{\text{Area of roof}}$$

$$\therefore \quad \text{Rainfall} = \frac{11}{440} = 0.025 \text{ m}$$

$\therefore$ Rainfall is 2.5 cm

Outside Delhi Set-II — Code No. 30/2

SECTION - B

Question numbers 5 to 10 carry 2 marks each.

10. Which term of the A.P. 8, 14, 20, 26, ... will be 72 more than its 41st term ?

Sol. Here $a = 8, d = 6$

Let $a_n = 72 + a_{41}$

$\Rightarrow \quad 8 + (n-1)6 = 72 + 8 + 40 \times 6 \qquad 1$

$\Rightarrow \quad 6n = 318$

$\Rightarrow \quad n = 53. \qquad 1$

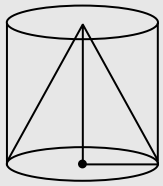
(CBSE Marking Scheme, 2017)

SECTION - C

Question numbers 11 to 20 carry 3 marks each.

18. From a solid right circular cylinder of height 2.4 cm and radius 0.7 cm, a right circular cone of same height and same radius is cut out. Find the total surface area of the remaining solid.

Sol.



Total surface area of remaining solid 1

$= \pi rl + \pi r^2 + 2\pi rh$

$l = \sqrt{(2.4)^2 + (0.7)^2} = 2.5$ cm ½

$\therefore$ TSA $= \pi r(l + r + 2h)$

$$= \frac{22}{7} \times 0.7(2.5 + 0.7 + 4.8)$$

$= 17.6 \text{ cm}^2$ 1½

(CBSE Marking Scheme, 2017)

19. If the 10th term of an A.P. is 52 and the 17th term is 20 more than the 13th term, find the A.P.

Sol. Here $a_{10} = 52$

$\Rightarrow \quad a + 9d = 52$ ...(i) 1

Also $\quad a_{17} = 20 + a_{13}$

$\Rightarrow \quad a + 16d = 20 + a + 12d$

$\Rightarrow \quad 4d = 20$ ...(ii) 1

Solving to get $d = 5$ and $a = 7$

or A.P. is 7, 12, 17, 22, ... ½ + ½

(CBSE Marking Scheme, 2017)

20. If the roots of the equation $(c^2 - ab)\,x^2 - 2\,(a^2 - bc)\,x + b^2 - ac = 0$ in x are equal, then show that either $a = 0$ or $a^3 + b^3 + c^3 = 3\,abc$.

Sol. For equal roots D = 0 1

Therefore $4(a^2 - bc)^2 - 4(c^2 - ab)\,(b^2 - ac) = 0$ 1

$4[a^4 + b^2c^2 - 2a^2bc - b^2c^2 + ac^3 + ab^3 - a^2bc] = 0$

$\Rightarrow a(a^3 + b^3 + c^3 - 3abc) = 0$ ½

$\Rightarrow a = 0$ or $a^3 + b^3 + c^3 = 3abc$ ½

(CBSE Marking Scheme, 2017)

SECTION - D

Question numbers 21 to 31 carry 4 marks each.

28. Solve for x :

$$\frac{1}{2x-3} + \frac{1}{x-5} = 1\frac{1}{9},\ x \neq \frac{3}{2},\ 5.$$

Sol. $[(x-5) + (2x-3)]9 = 10(2x-3)(x-5)$ 1

$\Rightarrow 20x^2 - 157x + 222 = 0$ 1

$\Rightarrow (x-6)\,(20x-37) = 0$ 1

$\Rightarrow x = 6, \dfrac{37}{20}$ 1

(CBSE Marking Scheme, 2017)

29. A train covers a distance of 300 km at a uniform speed. If the speed of the train is increased by 5 km/hour, it takes 2 hours less in the journey. Find the original speed of the train.

Sol. Let original speed of train be x km/h

Therefore $\dfrac{300}{x} - \dfrac{300}{x+5} = 2$ **1½**

$\Rightarrow \quad x^2 + 5x - 750 = 0$ **1**

$\Rightarrow \quad (x + 30)(x - 25) = 0$

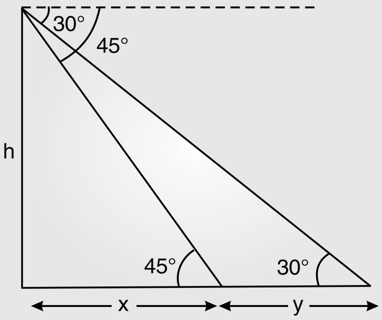
$\Rightarrow \quad x = 25 \text{ or } x = -30$ **1**

$\therefore \quad \text{Speed} = 25 \text{ km/h}$ **½**

(CBSE Marking Scheme, 2017)

30. A man observes a car from the top of a tower, which is moving towards the tower with a uniform speed. If the angle of depression of the car changes from 30° to 45° in 12 minutes, find the time taken by the car now to reach the tower.

Sol.



1

$$\frac{h}{x} = \tan 45° = 1$$

$\Rightarrow \quad h = x$ ...(i) **½**

$$\frac{h}{x+y} = \tan 30° = \frac{1}{\sqrt{3}}$$

$\Rightarrow \quad \sqrt{3}h = x + y$ ...(ii) **½**

Therefore from (i) & (ii) $\sqrt{3}x = x + y$

$\Rightarrow \quad y = x(\sqrt{3} - 1)$ **1**

To cover a distance of $x(\sqrt{3}-1)$, car takes 12 min.

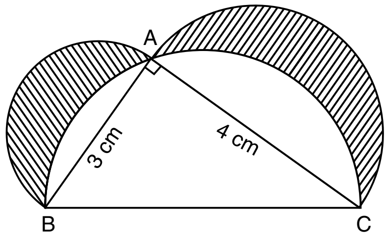
$\therefore$ Time taken by car to cover a distance of x units

$= \dfrac{12}{\sqrt{3}-1}$ minutes

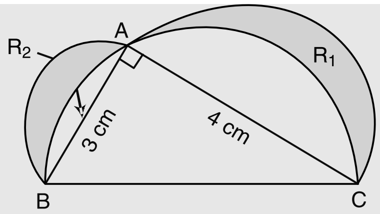
$= 6(\sqrt{3}+1)$ min or 16.4 min (approx). **1**

(CBSE Marking Scheme, 2017)

31. In the given figure, ΔABC is a right-angled triangle in which $\angle A$ is 90°. Semicircles are drawn on AB, AC and BC as diameters. Find the area of the shaded region.



Sol.



$$BC = \sqrt{3^2 + 4^2} = 5 \text{ cm}$$ **½**

$$\text{Area } (R_1 + R_2) = \frac{\pi}{2}\left(\frac{5}{2}\right)^2 - \frac{1}{2} \times 3 \times 4 \text{ cm}^2$$

$$= \left(\frac{25}{8}\pi - 6\right) \text{cm}^2$$ ...(i) **1**

Area of shaded region

$$= \frac{\pi}{2}\left(\frac{3}{2}\right)^2 + \frac{\pi}{2}(2)^2 - \left[\frac{25}{8}\pi - 6\right] \text{cm}^2$$ **1**

$$= \frac{\pi}{2}\left(\frac{9}{4} + 4 - \frac{25}{4}\right) + 6$$ **1**

$$= 6 \text{ cm}^2$$ **½**

(CBSE Marking Scheme, 2017)

Outside Delhi Set-III — Code No. 30/3

SECTION - B

Question numbers 5 to 10 carry 2 marks each.

10. For what value of n, are the n^{th} terms of two A.Ps 63, 65, 67,.... and 3, 10, 17,.... equal ?

Sol. Here $a_n = a'_n$

$\Rightarrow \quad 63 + (n-1)2 = 3 + (n-1)7$ **1**

$\Rightarrow \quad 5n = 65$

$\Rightarrow \quad n = 13.$ **1**

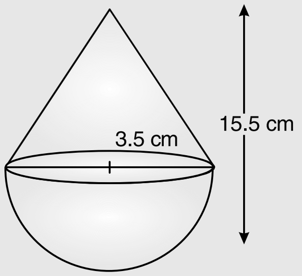
(CBSE Marking Scheme, 2017)

SECTION - C

Question numbers 11 to 20 carry 3 marks each.

18. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius on its circular face. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Sol.



Height of cone = 15.5 – 3.5 = 12 cm ½

$\therefore \quad l = \sqrt{(3.5)^2 + 12^2} = 12.5\text{cm}$ **1**

Total surface area = $\pi rl + 2\pi r^2$

$= \frac{22}{7} \times 3.5(12.5 + 3.5 \times 2)$ **1**

$= 214.5 \text{ cm}^2$ ½

(CBSE Marking Scheme, 2017)

19. How many terms of an A.P. 9, 17, 25, ... must be taken to give a sum of 636 ?

Sol. Here $a = 9$, $d = 8$, $S_n = 636$ **1**

Therefore $636 = \frac{n}{2}[18 + (n-1)8]$

$\Rightarrow \quad 4n^2 + 5n - 636 = 0$ **1**

$\Rightarrow \quad (4n + 53)(n - 12) = 0$

$n = 12$ **1**

(CBSE Marking Scheme, 2017)

20. If the roots of the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal, prove that $\frac{a}{b} = \frac{c}{d}$.

Sol. For equal roots $D = 0$

$\Rightarrow 4(ac + bd)^2 - 4(a^2 + b^2)(c^2 + d^2) = 0$ **1**

$\Rightarrow 4(a^2c^2 + b^2d^2 + 2abcd - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2) = 0$ **1**

$\Rightarrow -4(a^2d^2 + b^2c^2 - 2abcd) = 0$

$\Rightarrow (ad - bc)^2 = 0$ ½

$\Rightarrow ad = bc$

$\Rightarrow \frac{a}{b} = \frac{c}{d}$ ½

(CBSE Marking Scheme, 2017)

SECTION - D

Question numbers 28 to 31 carry 4 marks each.

28. Solve for x :

$$\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = 2, \text{ where } x \neq -\frac{1}{2}, 1.$$

Sol. $(x-1)^2 + (2x+1)^2 = 2(2x+1)(x-1)$ **1**

$x^2 + 1 - 2x + 4x^2 + 1 + 4x = 4x^2 - 4x + 2x - 2$ **1**

$\Rightarrow x^2 + 4x + 4 = 0$ ½

$\Rightarrow (x+2)^2 = 0$ **1**

$\Rightarrow x = -2$ ½

(CBSE Marking Scheme, 2017)

29. A takes 6 days less than B to do a work. If both A and B working together can do it in 4 days, how many days will B take to finish it ?

Sol. Let B take x days to finish the work.

Therefore number of days taken by A = $x - 6$ ½

Work done by both in one day is **1**

$$\frac{1}{x} + \frac{1}{x-6} = \frac{1}{4}$$ **1**

$\Rightarrow \quad x^2 - 14x + 24 = 0$ **1**

$\Rightarrow \quad (x-12)(x-2) = 0$

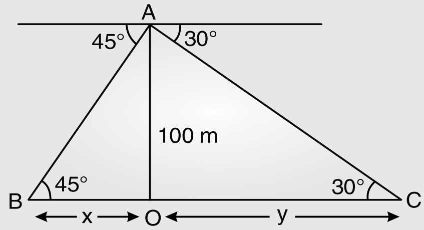
$\Rightarrow \quad x = 12$ or $x = 2$ ½

$x \neq 2 \therefore$ B takes 12 days to complete the work

(CBSE Marking Scheme, 2017)

30. From the top of a tower, 100 m high, a man observes two cars on the opposite sides of the tower and in same straight line with its base, with angles of depression 30° and 45°. Find the distance between the cars. [Take $\sqrt{3}$ = 1.732]

Sol.



1

$$\frac{100}{x} = \tan 45° = 1$$

$\Rightarrow \quad x = 100$ ...(i) ½

$\frac{100}{y} = \tan 30° = \frac{1}{\sqrt{3}}$ ½

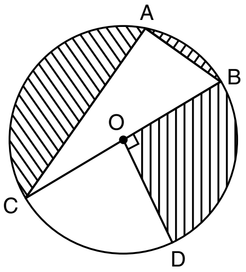
$\Rightarrow \quad y = 100\sqrt{3}$ ...(ii) **1**

Distance between the cars = $x + y = 100(\sqrt{3} + 1)$ ½

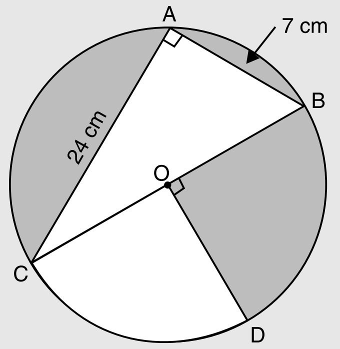
$= 273.2$ m ½

(CBSE Marking Scheme, 2017)

31. In the given figure, O is the centre of the circle with $AC = 24$ cm, $AB = 7$ cm and $\angle BOD = 90°$. Find the area of the shaded region.



Sol.



Diameter $BC = \sqrt{24^2 + 7^2} = 25$ cm ½

Area $\Delta CAB = \frac{1}{2} \times 24 \times 7 = 84 \text{ cm}^2$ **1**

Area of shaded region

$$= \frac{\pi}{2}\left(\frac{25}{2}\right)^2 - 84 + \frac{\pi}{4}\left(\frac{25}{2}\right)^2$$ **1**

$$= \left(\frac{1875\pi}{16} - 84\right) \text{cm}^2$$ **1**

$$= (117.18\,\pi - 84) \text{ cm}^2$$

or $= 283.94 \text{ cm}^2$ ½

(CBSE Marking Scheme, 2017)

❑❑