Solved Paper 2014
Mathematics
Class-XII

Time : 3 Hours **Max. Marks : 100**

General Instructions:

(i) All questions are compulsory.

(ii) The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.

(iii) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted.

Delhi Set I **Code No. 2/1/1**

SECTION - A

*** 1. Let * be binary operation, on the set of all non-zero real numbers, given by $a * b = \frac{ab}{5}$ for all $a, b * R - \{0\}$. Find the value of x, given that $2 * (x * 5) = 10$.** **1**

2. If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right) = 1$, then find the value of x. **1**

Sol. $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right) = 1$

$$\sin^{-1}\frac{1}{5}+\cos^{-1}x = \sin^{-1}1$$

$$\sin^{-1}\frac{1}{5}+\cos^{-1}x = \frac{\pi}{2}$$

$$\sin^{-1}\frac{1}{5} = \frac{\pi}{2}-\cos^{-1}x$$

$$\sin^{-1}\frac{1}{5} = \sin^{-1}x$$

$$\therefore \quad x = \frac{1}{5}$$

3. If $2\begin{bmatrix}3 & 4\\5 & x\end{bmatrix}+\begin{bmatrix}1 & y\\0 & 1\end{bmatrix}=\begin{bmatrix}7 & 0\\10 & 5\end{bmatrix}$, find $(x - y)$. **1**

Sol. $2\begin{bmatrix}3 & 4\\5 & x\end{bmatrix}+\begin{bmatrix}1 & y\\0 & 1\end{bmatrix} = \begin{bmatrix}7 & 0\\10 & 5\end{bmatrix}$

$$\begin{bmatrix}6 & 8\\10 & 2x\end{bmatrix}+\begin{bmatrix}1 & y\\0 & 1\end{bmatrix} = \begin{bmatrix}7 & 0\\10 & 5\end{bmatrix}$$

$$\begin{bmatrix}7 & 8+y\\10 & 2x+1\end{bmatrix} = \begin{bmatrix}7 & 0\\10 & 5\end{bmatrix}$$

$$8 + y = 0$$

$$\Rightarrow \quad y = -8$$

$$2x + 1 = 5$$

$$\Rightarrow \quad x = 2$$

$$x - y = 2 - (-8) = 10$$

4. Solve the following matrix equation for $x : [x \quad 1]\begin{bmatrix}1 & 0\\-2 & 0\end{bmatrix} = 0$. **1**

Sol. $[x \quad 1]\begin{bmatrix}1 & 0\\-2 & 0\end{bmatrix} = 0$

$$[x-2 \quad 0] = 0$$

$$\therefore \quad x - 2 = 0$$

$$\Rightarrow \quad x = 2$$

5. If $\begin{vmatrix}2x & 5\\8 & x\end{vmatrix} = \begin{vmatrix}6 & -2\\7 & 3\end{vmatrix}$, write the value of x. **1**

Sol. $\begin{vmatrix}2x & 5\\8 & x\end{vmatrix} = \begin{vmatrix}6 & -2\\7 & 3\end{vmatrix}$

$$2x^2 - 40 = 18 + 14$$

$$2x^2 = 72$$

$$x = \sqrt{36} = \pm 6$$

6. Write the antiderivative of $\left(3\sqrt{x}+\frac{1}{\sqrt{x}}\right)$. **1**

Sol. $= \int\left(3\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx$

$$= 3\int\sqrt{x}dx + \int\frac{1}{\sqrt{x}}dx$$

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* *Out of Syllabus*

Height of the cylinder = $h = 2R\sin\theta = \frac{2}{\sqrt{3}}R$

Hence Proved

$$\frac{d^2r}{d\theta^2} = 2\pi R^3[-\sin\theta(1-3\sin^2\theta) + \cos\theta(-6\sin\theta\cos\theta)]$$

$$\left(\frac{d^2r}{d\theta^2}\right)_{\theta=\sin^{-1}\frac{1}{3}} < 0$$

Hence volume is maximum

$$V = 2\pi R^3\cos^2\theta\sin\theta$$
$$= 2\pi R^3(1-\sin^2\theta)\sin\theta$$
$$= 2\pi R^3\left(1-\frac{1}{3}\right)\left(\frac{1}{\sqrt{3}}\right)$$
$$= \frac{4\pi R^3}{3\sqrt{3}} \text{ unit}^3$$

Outside Delhi Set III — Code No. 2/1/3

Note : Except for the following questions, all the remaining questions have been asked in previous set.

SECTION - A

9. If $\int_0^a \frac{1}{4+x^2} = \frac{\pi}{8}$ find the value of a. **1**

Sol.
$$\int_0^a \frac{1}{4+x^2}dx = \frac{\pi}{8}$$
$$\left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_0^a = \frac{\pi}{8}$$
$$\frac{1}{2}\tan^{-1}\frac{a}{2} - 0 = \frac{\pi}{8}$$
$$\tan^{-1}\frac{a}{2} = \frac{\pi}{4}$$
$$\frac{a}{2} = \tan\frac{\pi}{4}$$
$$\therefore \quad a = 2$$

10. If $\vec{a}$ and $\vec{b}$ are perpendicular vectors, $|\vec{a}+\vec{b}| = 13$ and $|\vec{a}| = 5$, find the value of $|\vec{b}|$.

Sol. Given $\vec{a} \perp \vec{b}$

$$\therefore \quad \vec{a}.\vec{b} = 0$$
$$(\vec{a}+\vec{b}).(\vec{a}+\vec{b}) = 13^2$$
$$\vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{b}.\vec{a} + \vec{b}.\vec{b} = 13^2$$
$$|a|^2 + 0 + 0 + |b|^2 = 169$$
$$25 + |b|^2 = 169$$
$$|b| = \sqrt{169-25} = 12 \text{ unit}$$

SECTION - B

19. Using properties of determinants, prove that:

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc + bc + ca + ab$$ **4**

Sol.
$$\Delta = \begin{vmatrix} 17a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$$

$R_1 \to \frac{R_1}{a}, R_2 \to \frac{R_2}{b}$ and $R_3 \to \frac{R_3}{c}$

$$\Delta = abc\begin{vmatrix} 1+\frac{1}{a} & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & 1+\frac{1}{b} & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & 1+\frac{1}{c} \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$\Delta = abc\begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & \frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1 \\ \frac{1}{b} & 1+\frac{1}{b} & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & 1+\frac{1}{c} \end{vmatrix}$$

$$\Delta = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{b} & 1+\frac{1}{b} & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & 1+\frac{1}{c} \end{vmatrix}$$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$\Delta = (abc + bc + ca + ab)\begin{vmatrix} 1 & 0 & 0 \\ \frac{1}{b} & 1 & 0 \\ \frac{1}{c} & 0 & 1 \end{vmatrix}$$

$$\Delta = (abc + bc + ca)(1-0)$$
$$\Delta = abc + bc + ca = \text{R.H.S.}$$ **Hence Proved**

20. If $x = \cos t(3 - 2\cos^2 t)$ and $y = \sin t(3 - 2\sin^2 t,)$ find the value of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$. **4**

Sol.
$$x = \cos t(3-2\cos^2 t)$$
and $$y = \sin t(3-2\sin^2 t)$$
$$\therefore \quad x = (3\cos t - 2\cos^3 t)$$
and $$y = (3\sin t - 2\sin^3 t)$$

$$y = 3\sin t - 2\sin^3 t$$

$$\frac{dy}{dt} = 3\cos t - 6\sin^2 t\cos t$$

$$= 3\cos t(1 - 2\sin^2 t)$$

$$= 3\cos t\cos 2t$$

$$x = 3\cos t - 2\cos^3 t$$

$$\frac{dx}{dt} = -3\sin t + 6\cos^2 t\sin t$$

$$= -3\sin t + (1 - 2\cos^2 t)$$

$$= 3\sin t\cos 2t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{3\cos t\cos 2t}{3\sin t\cos 2t}$$

$$\left(\frac{dy}{dx}\right)_{\frac{\pi}{4}} = \cot\frac{\pi}{4} = 1$$

21. Find the particular solution of the differential equation $\left(\frac{dy}{dx}\right) = 3x + 4y$, given that $y = 0$ when $x = 0$. **4**

Sol.

$$\frac{dy}{dx} = 3x + 4y$$

$$\frac{dy}{dx} = 3x$$

Equation in the form of $\frac{dy}{dx} + Py = Q(x)$

∴ Integrating factor

$$I.F. = e^{\int P dx} = e^{\int -4 dx}$$

$$I.F. = e^{-4x}$$

Solution of the differential equation

$$y + I.F. = \int I.F. \times Q(x)dx$$

$$e^{-4x} y = \int e^{-4x} \times 3x\, dx$$

$$ye^{-4x} = 3\left[\frac{xe^{-4x}}{-4} - \int\frac{e^{-4x}}{-4}dx\right]$$

$$ye^{-4x} = 3\left[-\frac{xe^{-4x}}{4} - \frac{e^{-4x}}{16}\right] + C$$

when $y = 0$, $x = 0$

$$0 = 3\left[-0 - \frac{1}{16}\right] + C$$

$$C = \frac{3}{16}$$

∴ Solution of differential equation is

$$ye^{-4x} = -\frac{3xe^{-4x}}{4} - \frac{e^{-4x}}{16} + \frac{3}{16}$$

$$16y = -12x - 1 + 3e^{4x}$$

$$12x + 16y + 1 = 3e^{4x}$$

22. Find the value of p, so that the line $l_1 : \frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2}$ and $l_2 : \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other. Also find the equations of a line passing through a point (3, 2, – 4) and parallel to line l_1. **4**

Sol.

$$l_1 = \frac{7y-14}{p} = \frac{z-3}{2} = \frac{1-x}{3}$$

$$= \frac{y-2}{\frac{p}{7}} = \frac{z-3}{2} = \frac{x-1}{-3}$$

$$l_2 = \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$$

$$= \frac{x-1}{\frac{-3p}{7}} = \frac{y-5}{1} = \frac{z-6}{-5}$$

lines are perpendicular to each other

$$\therefore \left(\frac{-3p}{7}\right)(-3) + \frac{p}{7}(1) + 2(-5) = 0$$

$$\frac{9p}{7} + \frac{p}{7} - 10 = 0$$

$$\frac{10p}{7} = 10$$

$$\Rightarrow \quad p = 7$$

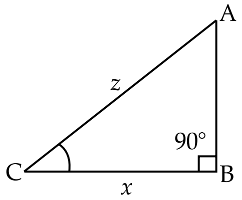
Equation of the line passing through a point (3, 2, –4) and parallel to line l_1 is

$$\frac{x-3}{-3} = \frac{y-2}{1} = \frac{z+4}{2}$$

SECTION - C

28. If the sum of the lengths of the hypotenuse and a side of a right triangle is given, show that the area of the triangle is maximum, when the angle of between them is 60°. **6**

Sol.



Let in ΔABC be right angled at B

and $x + z = \delta$ (Given)

$$\text{Area of } \Delta ABC = \frac{1}{2} BC \times AB$$

$$= \frac{1}{2}xy$$

$$A = \frac{1}{2}x\sqrt{z^2 - x^2}$$

$$A^2 = \frac{1}{4}x^2(z^2 - x^2)$$

$$\frac{dP}{dx} = \frac{1}{4}\frac{d}{dx}(z^2x^2 - x^4)$$

$$(\because A^2 = P)$$

$$\frac{dP}{dx} = \frac{1}{4}(2xz^2 - 4x^3)$$

$$\frac{dP}{dx} = \frac{1}{2}x(z^2 - 4x^2)$$

for maxima/minima

$$\frac{dP}{dx} = 0$$

$$\therefore \quad \frac{1}{2}x(z^2 - 4x^2) = 0$$

$$z - 4x^2 = 0$$

$$z^2 = 4x^2$$

$$z = 2x$$

$$\cos\theta = \frac{BC}{AC} = \frac{x}{2x} = \frac{1}{2}$$

$$\therefore \quad \theta = 60°$$

$$\frac{d^2P}{dx^2} = \frac{1}{4}(2z^2 - 12x^2)$$

$$= \frac{1}{4}(8x^2 - 12x^2)$$

$$= x^2 < 0$$

$\therefore$ Area is maximum when $\angle C = 60°$ **Hence Proved**

29. Evaluate: $\int \frac{1}{\sin^4 x + \sin^2 x\cos^2 x + \cos^4 x}dx$. **6**

Sol. $\int \frac{1}{\sin^4 x + \sin^2 x\cos^2 x + \cos^4 x}dx$

Divide by $\cos^4 x$

$$= \int \frac{\frac{1}{\cos^4 x dx}}{\frac{\sin 4x}{\cos 4x} + \frac{\sin^2 x\cos^2 x}{\cos^4 x} + \frac{\cos^4 x}{\cos^4 x}}$$

$$= \int \frac{\sec^4 x dx}{\tan^4 x + \tan^2 x + 1}$$

$$= \int \frac{(1+\tan^2 x)\sec^2 x}{\tan^4 x + \tan^2 x + 1}dx$$

let $\tan x = t$

$\sec^2 x\, dx = dt$

$$= \int \frac{1+t^2}{1^4 + t^2 + 1}dt$$

Divide by t^2

$$= \int \frac{1+\frac{1}{t^2}}{t^2 + \frac{1}{t^2} + 1}dt$$

$$= \int \frac{1+\frac{1}{t^2}}{\left(t - \frac{1}{t}\right)^2 + 3}dt$$

let $t - \frac{1}{t} = u \Rightarrow \left(1 + \frac{1}{t^2}\right)dt = du$

$$= \int \frac{du}{u^2 + (\sqrt{3})^2}$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\frac{4}{\sqrt{3}} + C$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\left|\frac{(\tan x - \cot x)}{\sqrt{3}}\right| + C$$

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