Solved Paper 2017
Mathematics
Class-XII

Time : 3 Hours **Max. Marks : 100**

General Instructions:

1. *All questions are compulsory.*
2. *The question paper consists of 29 questions divided into four sections A, B, C and D. Section A comprises of **4** questions of **one mark** each, Section B comprises of **8** questions of **two marks** each, Section C comprises of **11** questions of **four marks** each and Section D comprises of **6** questions of **six marks** each.*
3. *All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.*
4. *There is no overall choice. However, internal choice has been provided in 3 questions of four marks each and 3 questions of six marks each. You have to attempt only one of the alternatives in all such questions.*
5. *Use of calculators is not permitted. You may ask for logarithmic tables, if required.*

Delhi Set I — Code No. 65/1/1

SECTION A

1. If A is a 3×3 invertible matrix, then what will be the value of k if $\det(A^{-1}) = (\det A)^k$.

Ans. $|A^{-1}| = \dfrac{1}{|A|} \Rightarrow 1$

$k = -1$ **1 [CBSE Marking Scheme 2017]**

2. Determine the value of the constant 'k' so that the function $f(x) = \begin{cases} \dfrac{kx}{|x|} & \text{, if } x < 0 \\ 3 & \text{, if } x \geq 0 \end{cases}$ is continuous at $x = 0$.

Ans. $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \dfrac{kx}{|x|} = -k$ ½

$k = -3$ ½

[CBSE Marking Scheme 2017]

3. Evaluate: $\int_2^3 3^x \, dx$

Ans. $\int_2^3 3^x dx = \left[\dfrac{3^x}{\log 3}\right]_2^3 = \dfrac{3^3 - 3^2}{\log 3} = \dfrac{18}{\log 3}$ ½+½

[CBSE Marking Scheme 2017]

4. If a line makes angles 90° and 60° respectively with the positive direction of X and Y axes, find the angle which makes with the positive direction of Z-axis.

Ans. $\cos^2 90° + \cos^2 60° + \cos^2\gamma = 1$ ½

$\cos\gamma = \pm\dfrac{\sqrt{3}}{2}, \ \gamma = \dfrac{\pi}{6} \text{ or } \dfrac{5\pi}{6}$ ½

[CBSE Marking Scheme 2017]

SECTION B

5. Show that all the diagonal elements of a skew symmetric matrix are zero.

Ans. Let $A = [a_{ij}]_{n \times n}$ be skew symmetric matrix

A is skew symmetric

$\therefore \quad A = -A'$ 1

$\Rightarrow \quad a_{ij} = -a_{ji} \ \forall i, j$

For diagonal elements $i = j$,

$\Rightarrow \quad 2a_{ii} = 0$

$\Rightarrow a_{ii} = 0 \Rightarrow$ diagonal elements are zero. 1

[CBSE Marking Scheme 2017]

6. Find $\dfrac{dy}{dx}$ at $x = 1, y = \dfrac{\pi}{4}$ if

$\sin^2 y + \cos xy = K$

Ans. From the given equation

$2\sin y \cos y \dfrac{dy}{dx} - \sin xy.\left[x.\dfrac{dy}{dx} + y.1\right] = 0$ 1

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{y \sin xy}{\sin 2y - x\sin(xy)}$

$\therefore \quad \left.\dfrac{dy}{dx}\right|_{x=1, y=\frac{\pi}{4}} = \dfrac{\pi}{4(\sqrt{2}-1)} = \dfrac{\pi}{4}(\sqrt{2}+1)$ 1

[CBSE Marking Scheme 2017]

7. The volume of a sphere is increasing at the rate of 3 cubic centimeter per second. Find the rate of increase of its surface area, when the radius is 2 cm.

Ans. $V = \frac{4}{3}\pi r^3$

$\Rightarrow \frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{3}{4\pi r^2}$ **1**

$S = 4\pi r^2$

$\Rightarrow \frac{dS}{dt} = 8\pi r.\frac{dr}{dt} = 8\pi r.\frac{3}{4\pi r^2}$ ½

$\Rightarrow \left.\frac{dS}{dt}\right|_{r=2} = 3 \text{ cm}^2/\text{s}$ ½

[CBSE Marking Scheme 2017]

8. Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ is always increasing on R.

Ans. $f(x) = 4x^3 - 18x^2 + 27x - 7$

$f'(x) = 12x^2 - 36x + 27$ ½

$= 3(2x - 3)^2 \geq 0 \ \forall\ x \in R$ **1**

$\Rightarrow f(x)$ is increasing on R ½

[CBSE Marking Scheme 2017]

9. Find the vector equation of the line passing through the point $A(1, 2, -1)$ and parallel to the line

$5x - 25 = 14 - 7y = 35z$.

Ans. Equation of given line is

$\frac{x-5}{1/5} = \frac{y-2}{-1/7} = \frac{z}{1/35}$ ½

Its DR's $\left\langle \frac{1}{5}, -\frac{1}{7}, \frac{1}{35} \right\rangle$ or $\langle 7, -5, 1 \rangle$ ½

Equation of required line is

$\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(7\hat{i} - 5\hat{j} + \hat{k})$ **1**

[CBSE Marking Scheme 2017]

10. Prove that if E and F are independent events, then the events E' and F' are also independent.

Ans. $P(E' \cap F') = P(E) - P(E \cap F)$ **1**

$= P(E) - P(E) \,.\, P(F)$ ½

$= P(E)[1 - P(F)]$

$= P(E)P(F')$ ½

$\Rightarrow$ E' and F' are independent events

[CBSE Marking Scheme 2017]

11. A small firm manufactures necklaces and bracelets. The total number of necklaces and bracelets that it can handle per day is at most 24. It takes one hour to make a bracelet and half an hour to make a necklace. The maximum number of hours available per day is 16. If the profit on a necklace is ₹ 100 and that on a bracelet is ₹ 300. Formulate on L.P.P. for finding how many of each should be produced daily to maximize the profit ? It is being given that at least one of each must be produced.

Ans. Let x necklaces and y bracelets are manufactured

∴ L.P.P. is

Maximize profit, P = $100x + 300y$ ½

subject to constraints

$x + y \leq 24$

$\frac{1}{2}x + y \leq 16$ or $x + 2y \leq 32$ **½×3=1½**

$x, y, \geq 1$

[CBSE Marking Scheme 2017]

Detailed Answer:

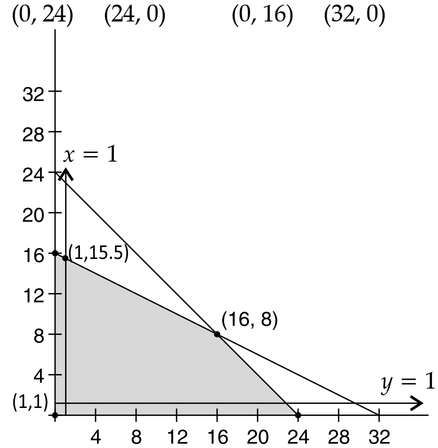
Let x necklace and y bracelets to be produced

$x + y \leq 24$

$\frac{1}{2}x + y \leq 16 \Rightarrow x + 2y \leq 32$

$z = 100x + 300y$

$x + y = 24$		$x + 2y = 32$	
(0, 24)	(24, 0)	(0, 16)	(32, 0)



Points	Z = $100x + 300y$	
(1, 1)	Z = 100 + 300	400
(24, 0)	Z = 2400 + 0	2400
(16, 8)	Z = 1600 + 2400	4000
(1, 15.5)	Z = 100 + 4650	4750

Maximum is 4750 when 16 bracelets and 0 necklaces but condition is given that at least one of each must be produced.

∴ 15 necklaces and 1 bracelets to be produced to make maximum profit of ₹ 4600

12. Find $\int \frac{dx}{x^2 + 4x + 8}$.

Ans. $\int \frac{dx}{x^2 + 4x + 8} = \int \frac{dx}{(x+2)^2 + (2)^2}$ **1**

$= \frac{1}{2}\tan^{-1}\frac{x+2}{2} + C$ **1**

[CBSE Marking Scheme 2017]

SECTION C

13. Prove that :

$$\tan\left\{\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}+\tan\left\{\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}=\frac{2b}{a}$$

Ans. Let $\frac{1}{2}\cos^{-1}\frac{a}{b}=x$ ½

$$\text{LHS}=\tan\left(\frac{\pi}{4}+x\right)+\tan\left(\frac{\pi}{4}-x\right)$$

$$=\frac{1+\tan x}{1-\tan x}+\frac{1-\tan x}{1+\tan x}\quad 1\tfrac{1}{2}$$

$$=\frac{2(1+\tan^2 x)}{1-\tan^2 x}=\frac{2}{\cos 2x}\quad 1$$

$$=\frac{2b}{a}=\text{RHS}\quad 1$$

[CBSE Marking Scheme 2017]

14. Using properties of determinants, prove that :

$$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}=9y^2(x+y).$$

OR

Let $A=\begin{pmatrix}2 & -1\\ 3 & 4\end{pmatrix}$, $B=\begin{pmatrix}5 & 2\\ 7 & 4\end{pmatrix}$, $C=\begin{pmatrix}2 & 5\\ 3 & 8\end{pmatrix}$, **find a matrix** D **such that** $CD-AB=0$.

Ans.

$$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3$

$$=3(x+y)\begin{vmatrix} 1 & x+y & x+2y \\ 1 & x & x+y \\ 1 & x+2y & x \end{vmatrix}\quad 1$$

$R_1 \to R_1 - R_2,\ R_3 \to R_3 - R_2$

$$=3(x+y)\begin{vmatrix} 0 & y & y \\ 1 & x & x+y \\ 0 & 2y & -y \end{vmatrix}\quad 1+1$$

$$=-3(x+y)(-y^2-2y^2)$$

$$=9y^2(x+y)\quad 1$$

[CBSE Marking Scheme 2017]

OR

Let $D=\begin{bmatrix} x & y \\ z & w \end{bmatrix}$ **1 + 1**

$CD = AB$

$$\Rightarrow \begin{bmatrix} 2x+5z & 2y+5w \\ 3x+8z & 3y+8w \end{bmatrix}=\begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix}$$

$2x + 5z = 3,\ 3x + 8z = 43;$

$2y + 5w = 0,\ 3y + 8w = 22.$

Solving, we get $x = -191$, $y = -110$, $z = 77$, $w = 44$ **1**

$$\therefore\quad D=\begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}\quad ½$$

[CBSE Marking Scheme 2017]

15. Differentiate the function $(\sin x)^x+\sin^{-1}\sqrt{x}$ **with respect to** x.

OR

If $x^m y^n = (x+y)^{m+n}$, **prove that** $\frac{d^2y}{dx^2}=0$.

Ans. $y=(\sin x)^x+\sin^{-1}\sqrt{x}$

$$y=u+v\Rightarrow\frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx}\quad 1$$

$u=(\sin x)^x$

$\Rightarrow \log u = x\log\sin x$ ½

$$\frac{du}{dx}=(\sin x)^x[x\cot x+\log\sin x]\quad 1$$

$v=\sin^{-1}\sqrt{x}$

$$\Rightarrow\frac{dv}{dx}=\frac{1}{2\sqrt{x-x^2}}\quad 1$$

$$\therefore\frac{dy}{dx}=(\sin x)^x[x\cot x+\log\sin x]+\frac{1}{2\sqrt{x-x^2}}\quad ½$$

[CBSE Marking Scheme 2017]

OR

$x^m \cdot y^n = (x+y)^{m+n}$

$\Rightarrow m\log x + n\log y = (m+n)\log(x+y)$ **1**

$$\Rightarrow\frac{m}{x}+\frac{n}{y}\cdot\frac{dy}{dx}=\frac{m+n}{x+y}\left(1+\frac{dy}{dx}\right)\quad 1$$

$$\frac{dy}{dx}=\frac{y}{x}\quad ...(i)\ 1$$

$$\frac{d^2y}{dx^2}=\frac{x\frac{dy}{dx}-y}{x^2}=0\quad ...(ii)\ (\text{using (i)})\ 1$$

[CBSE Marking Scheme 2017]

16. Find $\int\frac{2x}{(x^2+1)(x^2+2)^2}dx$

Ans. $$\int\frac{2x}{(x^2+1)(x^2+2)^2}=\int\frac{dy}{(y+1)(y+2)^2}$$

[by substituting $x^2 = y$] **1**

$$= \int \frac{dy}{y+1} - \int \frac{dy}{y+2} - \int \frac{dy}{(y+2)^2}$$ **1½**

(using partial fraction)

$$= \log(y+1) - \log(y+2) + \frac{1}{y+2} + C$$ **1**

$$= \log(x^2+1) - \log(x^2+2) + \frac{1}{x^2+2} + C$$ **½**

[CBSE Marking Scheme 2017]

17. Evaluate : $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

OR

Evaluate : $\int_0^{3/2} |x \sin \pi x| \, dx$

Ans.

$$I = \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$$

$$= \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x} dx$$ **1**

$$\Rightarrow \quad 2I = \pi \int_0^{\pi} \frac{\sin x \, dx}{1+\cos^2 x}$$

Put $\cos x = t$ and $-\sin x \, dx = dt$ **1**

$$2I = -\pi \int_1^{-1} \frac{dt}{1+t^2}$$

$$= \pi[\tan^{-1} t]_{-1}^{1} = \frac{\pi^2}{2}$$ **1½**

$$\Rightarrow \quad I = \frac{\pi^2}{4}$$ **½**

[CBSE Marking Scheme 2017]

OR

$$I = \int_0^{3/2} |x \sin \pi x| \, dx$$

$$= \int_0^{1} x \sin \pi x . dx - \int_1^{3/2} x \sin \pi x \, dx$$ **1½**

$$= \left[-x \frac{\cos \pi x}{\pi} + \frac{\sin \pi x}{\pi^2} \right]_0^1$$

$$- \left[-\frac{x \cos \pi x}{\pi} + \frac{\sin \pi x}{\pi^2} \right]_1^{3/2}$$ **1½**

$$= \frac{2}{\pi} + \frac{1}{\pi^2}$$ **1**

[CBSE Marking Scheme 2017]

18. Prove that $x^2 - y^2 = C(x^2 + y^2)^2$ is the general solution of the differential equation $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dy$, where C is a parameter.

Ans. $x^2 - y^2 = C(x^2 + y^2)^2 \Rightarrow 2x - 2yy'$

$$= 2C(x^2 + y^2)(2x + 2yy')$$ **1**

$$\Rightarrow \quad (x - yy') = \frac{x^2 - y^2}{y^2 + x^2}(2x + 2yy')$$

$$\Rightarrow (y^2 + x^2)(x - yy') = (x^2 - y^2)(2x + 2yy')$$ **1**

$$\Rightarrow [-2y(x^2 - y^2) - y(y^2 + x^2)] \frac{dy}{dx}$$

$$= 2x(x^2 - y^2) - x(y^2 + x^2)$$ **1**

$$\Rightarrow \quad (y^3 - 3x^2y) \frac{dy}{dx} = (x^3 - 3xy^2)$$

$$\Rightarrow \quad (y^3 - 3x^2y)dy = (x^3 - 3xy^2)dx$$ **1**

Hence $x^2 - y^2 = C(x^2 + y^2)^2$ is the solution of given differential equation.

[CBSE Marking Scheme 2017]

* **19. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$, then**

(a) Let $c_1 = 1$ and $c_2 = 2$, find c_3 which makes $\vec{a}$, $\vec{b}$ and $\vec{c}$ coplanar.

(b) If $c_2 = -1$ and $c_3 = 1$, show that no value of c_1 can make $\vec{a}$, $\vec{b}$ and $\vec{c}$ coplanar.

20. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$. Also, find the angle which $\vec{a} + \vec{b} + \vec{c}$ makes with $\vec{a}$ or $\vec{b}$ or $\vec{c}$.

Ans. $|\vec{a}| = |\vec{b}| = |\vec{c}|$ and $\vec{a}.\vec{b} = 0 = \vec{b}.\vec{c} = \vec{c}.\vec{a}$ ...(i)**1**

Let α, β and γ be the angles made by $(\vec{a} + \vec{b} + \vec{c})$ with $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively

$$(\vec{a} + \vec{b} + \vec{c})\vec{a} = |\vec{a} + \vec{b} + \vec{c}||\vec{a}| \cos\alpha$$

$$\Rightarrow \quad \alpha = \cos^{-1}\left(\frac{|\vec{a}|}{|\vec{a} + \vec{b} + \vec{c}|}\right)$$

Similarly, $\beta = \cos^{-1}\left(\frac{|\vec{b}|}{|\vec{a} + \vec{b} + \vec{c}|}\right)$

and $\gamma = \cos^{-1}\left(\frac{|\vec{c}|}{|\vec{a} + \vec{b} + \vec{c}|}\right)$ **1**

Using (i), we get $\alpha = \beta = \gamma$ **½**

* *Out of Syllabus*

Now $|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2 + 2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})$ **1**

$\Rightarrow \quad |\vec{a}+\vec{b}+\vec{c}|^2 = \sqrt{3}\,|\vec{a}|^2$ (using (i))

$|\vec{a}+\vec{b}+\vec{c}| = \sqrt{3}\,|\vec{a}|$

$\therefore \quad \alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = \beta = \gamma$ **½**

[CBSE Marking Scheme 2017]

21. The random variable X can take only the values 0, 1, 2, 3. Given that $P(X = 0) = P(X = 1) = p$ and $P(X = 2) = P(X = 3)$ such that $\Sigma p_1 x_2^2 = 2\Sigma p_1 x_1$, find the value of p.

Ans.

x	0	1	2	3
$P(x)$	p	p	k	k

$\Sigma p(x) = 1 \Rightarrow 2p + 2k = 1 \Rightarrow k = \frac{1}{2} - p$ **1**

x_i	p_i	$p_i x_i$	$p_i x_i^2$
0	p	0	0
1	p	p	p
2	$\frac{1}{2}-p$	$1-2p$	$2-4p$
3	$\frac{1}{2}-p$	$\frac{3}{2}-3p$	$\frac{9}{2}-9p$
		$\frac{5}{2}-4p$	$\frac{13}{2}-12p$

As per problem, $\Sigma p_i x_i^2 = 2\Sigma p_i x_i$ **2**

$\Rightarrow \quad p = \frac{3}{8}$ **1**

[CBSE Marking Scheme 2017]

22. Often it is taken that a truthful person commands, more respect in the society. A man is known to speak the truth 4 out of 5 times, He throws a die and reports that it is a six. Find the probability that it is actually a six.

Do you also agree that the value of truthfulness leads to more respect in the society ?

Ans. Let H_1 be the event that 6 appears on throwing a die

H_2 be the event that 6 does not appear on throwing a die

E be the event that he reports it is six **1**

$P(H_1) = \frac{1}{6}, P(H_2) = 1-\frac{1}{6} = \frac{5}{6}$

$P(E/H_1) = \frac{4}{5}, P(E/H_2) = \frac{1}{5}$ **1**

$P(H_1/E) = \frac{P(H_1).P(E/H_1)}{P(H_1).P(E/H_1)+P(H_2)P(E/H_2)}$ **½**

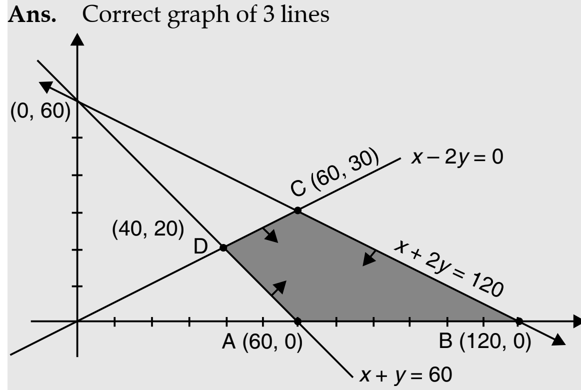
$= \frac{\frac{1}{6}\times\frac{4}{5}}{\frac{1}{6}\times\frac{4}{5}+\frac{5}{6}\times\frac{1}{5}} = \frac{4}{9}$ **½**

Relevant value : Yes, Truthness leads to more respect in society. **1**

[CBSE Marking Scheme 2017]

23. Solve the following L.P.P. graphically :
Minimize $Z = 5x + 10y$
Subject to $x + 2y \le 120$
Constraints $x + y \ge 60$
$x - 2y \ge 0$
and $x, y \ge 0$

Ans. Correct graph of 3 lines **1½**



Correct shade of 3 lines **1½**

$Z = 5x + 10y$

$Z|_{A\,(60,\,0)} = 300$

$Z|_{B\,(120,\,0)} = 600$

$Z|_{C\,(60,\,30)} = 600$

$Z|_{D\,(40,\,20)} = 400$

Minimum value of $Z = 300$ at $x = 60, y = 0$ **1**

[CBSE Marking Scheme 2017]

SECTION D

24. Use product $\begin{bmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\9 & 2 & -3\\6 & 1 & -2\end{bmatrix}$ to solve the system of equations $x + 3z = 9$, $-x + 2y - 2z = 4$, $2x - 3y + 4z = -3$

Ans. $A = \begin{bmatrix}1 & -1 & 2\\0 & 2 & -3\\3 & -2 & 4\end{bmatrix}.\begin{bmatrix}-2 & 0 & 1\\9 & 2 & -3\\6 & 1 & -2\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$ **1**

$AB = I \Rightarrow A^{-1} = B = \begin{bmatrix}-2 & 0 & 1\\9 & 2 & -3\\6 & 1 & -2\end{bmatrix}$ **1**

Given equations in matrix form are :

$$\begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & -2 \\ 2 & -3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix}$$ **1**

$$A'X = C$$ ½

$\Rightarrow X = (A')^{-1}, C = (A^{-1})' C$ **1**

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 9 & 6 \\ 0 & 2 & 1 \\ 1 & -3 & -2 \end{bmatrix}\begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$$ **1**

$\Rightarrow x = 0, y = 5, z = 3$ ½

[CBSE Marking Scheme 2017]

25. * **Consider $f : R_+ \rightarrow [-5, \infty]$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{y+6}-1}{3}\right)$.**

Hence Find

(i) $f^{-1}(10)$

(ii) y if $f^{-1}(y) = \frac{4}{3}$.

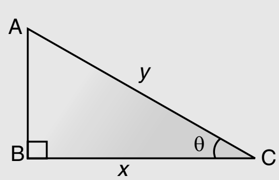
Where R_+ is the set of all non-negative real numbers.

OR

* **Discuss the commutativity and associativity of binary operation '*' defined on $A = Q - \{1\}$ by the rule $a * b = a - b + ab$ for all $a, b\ A$. Also find the identity element of * in A and hence find the invertible elements of A.**

26. **If the sum of lengths of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum, when the angle between them is $\frac{\pi}{3}$.**

Ans. **Given** $x + y = k$ **1**



Area of $\Delta = \frac{1}{2}x\sqrt{y^2 - x^2}$

Let $Z = \frac{1}{4}x^2(y^2 - x^2)$

$= \frac{1}{4}x^2[(k-x)^2 - x^2]$ **1**

$= \frac{1}{4}[k^2x^2 - 2kx^3]$

$\frac{dz}{dx} = \frac{1}{4}[2k^2x - 6kx^2] = 0$

$\Rightarrow k - 3x = 0 \Rightarrow x = \frac{k}{3}$ **1½**

$\Rightarrow x + y - 3x = 0$ or $y = 2x$

$\frac{d^2z}{dx^2} = \frac{1}{4}[2k^2 - 12kx]$ **1**

$\left.\frac{d^2z}{dx^2}\right|_{x=\frac{k}{3}} = \frac{1}{4}[2k^2 - 4k^2] = -\frac{k^2}{2} < 0$

$\therefore$ Area will be maximum for $2x = y$ **1**

but $\frac{x}{y} = \cos\theta \Rightarrow \cos\theta$

$= \frac{x}{2x} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$ ½

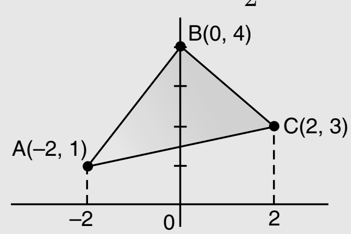
[CBSE Marking Scheme 2017]

27. **Using integration, find the area of region bounded by the triangle whose vertices are (– 2, 1), (0, 4) and (2, 3).**

OR

Find the area bounded by the circle $x^2 + y^2 = 16$ and the line $\sqrt{3}y = x$ in the first quadrant, using integration.

Ans. Equation of AB : $y = \frac{3}{2}x + 4$ **1**



Equation of BC ; $y = 4 - \frac{x}{2}$

Equation of AC ; $y = \frac{1}{2}x + 2$ **1½**

Required area $= \int_{-2}^{0}\left(\frac{3}{2}x + 4\right)dx + \int_{0}^{2}\left(4 - \frac{x}{2}\right)dx$

$-\int_{-2}^{2}\left(\frac{1}{2}x + 2\right)dx$ **1**

$= \left[\frac{3x^2}{4} + 4x\right]_{-2}^{0} + \left[4x - \frac{x^2}{4}\right]_{0}^{2}$

$-\left[\frac{x^2}{4} + 2x\right]_{-2}^{2}$ **1½**

$= 5 + 7 - 8$ **1**

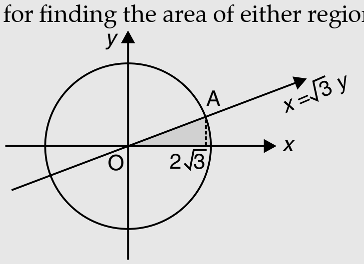
$= 4$ sq.unit

[CBSE Marking Scheme 2017]

* *Out of Syllabus*

OR

Note: In this problem, two regions are possible instead of a unique one, so full 6 marks may be given for finding the area of either region correctly. **1**



X-coordinate of points of intersection is $x = \pm 2\sqrt{3}$

Required area **1**

$$= \int_0^{2\sqrt{3}} \frac{x}{\sqrt{3}}.dx + \int_{2\sqrt{3}}^{4} \sqrt{4^2 - x^2}\,dx$$ **1½**

$$= \left[\frac{x^2}{2\sqrt{3}}\right]_0^{2\sqrt{3}} + \left[\frac{x\sqrt{16-x^2}}{2} + 8\sin^{-1}\frac{x}{4}\right]_{2\sqrt{3}}^{4}$$ **1½**

$$= 2\sqrt{3} + 8\left(\frac{\pi}{2} - \frac{\pi}{3}\right) - 2\sqrt{3}$$

$$= \frac{4\pi}{3} \text{ sq.units}$$ **1**

[CBSE Marking Scheme 2017]

28. Solve the differential equation $x\frac{dy}{dx} + y = x\cos x + \sin x$, given that $y = 1$ when $x = \frac{\pi}{2}$.

Ans. The given equation can be written as

$$\frac{dy}{dx} + \frac{y}{x} = \cos x + \frac{\sin x}{x}$$ **1**

$$\text{I.F.} = e^{\int \frac{1}{x} dx} = e^{\log x} = x$$

$\therefore$ Solution is $y \,.\, x = \int (x\cos x + \sin x)dx + c$ **1**

$\Rightarrow \quad y \,.\, x = x \sin x + c$ **1**

or $\quad y = \sin x + \frac{c}{x}$

when $\quad x = \frac{\pi}{2}, y = 1$ **1**

we get $\quad c = 0$ **1**

Required solution is $y = \sin x$ **1**

[CBSE Marking Scheme 2017]

* **29. Find the equation of the plane through the line of intersection of $\vec{r}\cdot(2\hat{i} - 3\hat{j} + 4\hat{k}) = 1$ and $\vec{r}\cdot(\hat{i} - \hat{j}) + 4 = 0$ and perpendicular to the plane $\vec{r}\cdot(2\hat{i} - \hat{j} + \hat{k}) + 8 = 0$. Hence find whether the plane thus obtained contains the line**

$$x - 1 = 2y - 4 = 3z - 12.$$

OR

Find the vector and Cartesian equations of a line passing through (1, 2, – 4) and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

OR

Equation of line L_1 passing through (1, 2, –4) is

$$\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+4}{c}$$ **1**

$$L_2: \quad \frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$$

$$L_3: \quad \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

$\because L_1 \perp L_2 \Rightarrow 3a - 16b + 7c = 0$ **1**

$L_1 \perp L_3 \Rightarrow 3a + 8b - 5c = 0$ **1**

Solving, we get

$$\frac{a}{24} = \frac{b}{36} = \frac{c}{72}$$

$$\Rightarrow \quad \frac{a}{2} = \frac{b}{3} = \frac{c}{6}$$ **1**

$\therefore$ Required cartesian equation of line

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$$ **1**

Vector equation

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$ **1**

[CBSE Marking Scheme 2017]

Delhi Set II — Code No. 63/1/2

Note: Expect these, all other questions are from Delhi Set-I

SECTION B

12. For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/s, then find the rate of change of the slope of the curve when $x = 3$.

Ans. Given curve is $y = 5x - 2x^3$

$$\Rightarrow \quad \frac{dy}{dx} = 5 - 6x^2$$

$$\Rightarrow \quad m = 5 - 6x^2$$ **1**

* *Out of Syllabus*

$$\frac{dm}{dt} = -12x\frac{dx}{dt} = -24x$$

$$\left.\frac{dm}{dt}\right|_{x=3} = -72 \qquad 1$$

[CBSE Marking Scheme 2017]

SECTION C

20. The random variable X can take only the values, 0, 1, 2, 3. Given that P(2) = P(3) = p and P(0) = 2P(1), If $\Sigma p_i x_i^2 = 2\Sigma p_i x_i$, find the value of p.

Ans.

x_i	p_i	$p_i x_i$	$p_i x_i^2$
0	$2q$	0	0
1	q	q	q
2	p	$2p$	$4p$
3	p	$3p$	$9p$

2

$\Sigma p_i = 1 \Rightarrow 3q + 2p$ ...(1) ½

$\Sigma p_i x_i^2 = 2\Sigma p_i x_i \Rightarrow q - 3p$ ...(2) ½

from (I) and (2), $p = \frac{1}{11}$ **1**

[CBSE Marking Scheme 2017]

21. Using vectors find the area of triangle ABC with vertices $A(1, 2, 3)$, $B(2, -1, 4)$ and $C(4, 5, -1)$.

Ans. $\overrightarrow{AB} = \hat{i} - 3\hat{j} + \hat{k}, \overrightarrow{AC} = 3\hat{i} + 3\hat{j} - 4\hat{k}$ **1**

Area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$ **1**

$$= \frac{1}{2} \text{ magnitude of } \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 3 & 3 & -4 \end{vmatrix}$$

$$= \frac{1}{2}\left|9\hat{i} + 7\hat{j} + 12\hat{k}\right|$$

$$= \frac{\sqrt{274}}{2} \text{ sq.units}$$ **1+1**

[CBSE Marking Scheme 2017]

22. Solve the following L.P.P. graphically :

Minimize $Z = 4x + y$

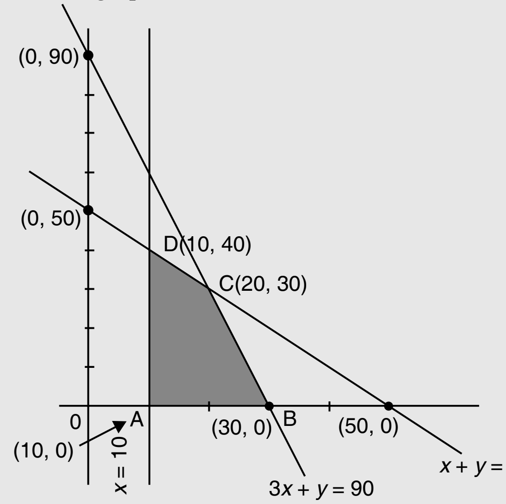
Subject to following constraints $x + y \le 50$,

$3x + y \le 90$,

$x \ge 10$

and $x, y \ge 0$

Ans. Correct graph of 3 lines **1½**



Correct shade of 3 lines **1½**

$Z|_{A(10,\,0)} = 40$

$Z|_{B(30,\,0)} = 120$

$Z|_{C(20,\,30)} = 110$

$Z|_{D(10,\,40)} = 80$

Maximum value of Z = 120 at (30, 0) **1**

[CBSE Marking Scheme 2017]

23. Find : $\int \frac{2x}{(x^2+1)(x^4+4)}dx$

Ans. $$\int \frac{2x\,dx}{(x^2+1)(x^4+4)} = \int \frac{dy}{(y+1)(y^2+4)}$$

[put $x^2 = y \Rightarrow 2x\,dx = dy$] ½

$$\frac{1}{(y+1)(y^2+4)} = \frac{1}{5(y+1)} + \frac{\frac{1}{5} - \frac{1}{5}y}{y^2+4}$$ 1½

$$\int \frac{dy}{(y+1)(y^2+4)} = \frac{1}{5}\log|y+1| + \frac{1}{10}\tan^{-1}\frac{y}{2} - \frac{1}{10}\log(y^2+4) + C$$ 1½

$$= \frac{1}{5}\log(x^2+1) + \frac{1}{10}\tan^{-1}\frac{x^2}{2} - \frac{1}{10}\log(x^4+4) + C$$ ½

[CBSE Marking Scheme 2017]

SECTION D

28. A metal box with a square base and vertical sides is to contain 1024 cm^3. The material for the top and bottom costs ₹ 5 per cm^2 and the material for the sides costs ₹ 2·50 per cm^2. Find the least cost of the box.

Ans. Let side of square base be x cm and height of the box be y cm.

$$x^2y = 1024 \Rightarrow y = \frac{1024}{x^2} \quad 1$$

cost of the box. $C = 5 \times 2x^2 + 2.5 \times 4xy$

$$= 10x^2 + \frac{10240}{x} \quad 1$$

$$\frac{dC}{dx} = 20x - \frac{10240}{x^2} \quad 1$$

$$\frac{dC}{dx} = 0 \Rightarrow x = 8 \quad 1$$

$$\frac{d^2C}{dx^2} = 20 + \frac{20480}{x^3}$$

$$\left.\frac{d^2C}{dx^2}\right|_{x=8} > 0 \quad 1$$

$\Rightarrow C$ is minimum at $x = 8$ cm

$\therefore$ Minimum cost C = ₹ 1920 **1**

[CBSE Marking Scheme 2017]

29. **If** $A = \begin{pmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{pmatrix}$**, find** A^{-1}**. Using** A^{-1} **solve the system of equations**

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2,\ \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5,\ \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = -4.$$

Ans. Here $|A| = 1200$ **1**

Co-factors are

$C_{11} = 75,\ C_{21} = 150,\ C_{31} = 75$
$C_{12} = 110,\ C_{22} = -100,\ C_{32} = 30$
$C_{13} = 72,\ C_{23} = 0,\ C_{33} = -24$ **2**

$$A^{-1} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \quad ½$$

Given equation in matrix form is :

$$\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}\begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$$

$\Rightarrow$ A X = B

$\Rightarrow$ X = A^{-1} B **1**

$$\Rightarrow \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{-1}{3} \\ \frac{1}{5} \end{bmatrix} \quad 1$$

$\Rightarrow x = 2, y = -3, z = 5$ **½**

[CBSE Marking Scheme 2017]

Delhi Set III — Code No. 63/1/3

Note: Expect these, all other questions are from Delhi Set-I & II

SECTION B

12. **If** $y = \sin^{-1}(6x\sqrt{1-9x^2}),\ -\frac{1}{3\sqrt{2}} < x < \frac{1}{3\sqrt{2}}$**, then find** $\frac{dy}{dx}$.

Ans. $y = \sin^{-1}(6x\sqrt{1-9x^2}), -\frac{1}{3\sqrt{2}} < x < \frac{1}{3\sqrt{2}}$

Put $3x = \sin\theta = \Rightarrow \theta = \sin^{-1} 3x$ **½**

$y = \sin^{-1}(\sin 2\theta)$

$= 2\theta = 2\sin^{-1} 3x$ **½**

$$\therefore \frac{dy}{dx} = \frac{6}{\sqrt{1-9x^2}} \quad 1$$

[CBSE Marking Scheme 2017]

SECTION C

20. **Solve the following L.P.P. graphically:**

Maximize $Z = 20x + 10y$

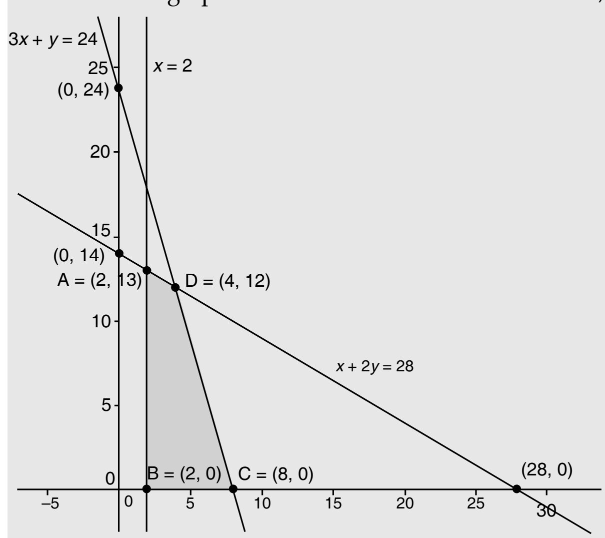
Subject to following constraints $x + 2y \le 28$,

$3x + y \le 24$,

$x \ge 2$,

and $x, y \ge 0$

Ans. Correct graph of 3 lines **1½**



Correct shade of 3 lines **1½**

$$Z = 20x + 10y$$

$$Z|_{A(2,\,13)} = 170$$

$Z|_{B(2,0)} = 40$
$Z|_{D(4,12)} = 200$
$Z|_{C(8,0)} = 160$

Maximum value of Z = 200 at $x = 4, y = 12$ **1**

[CBSE Marking Scheme 2017]

21. Show that the family of curves for which $\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$, is given by $x^2 - y^2 = cx$.

Ans. $x^2 - y^2 = cx \Rightarrow \frac{x^2-y^2}{x} = c$ **1**

$$\Rightarrow \frac{x\left(2x - 2y\frac{dy}{dx}\right) - (x^2-y^2)}{x^2} = 0$$ **1**

$$\Rightarrow 2x^2 - 2xy\frac{dy}{dx} - x^2 + y^2 = 0$$ **1**

$$\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$$ **1**

Hence proved.

[CBSE Marking Scheme 2017]

22. Find $\int \frac{(3\sin x - 2)\cos x}{13 - \cos^2 x - 7\sin x}\, dx$

Ans. $$\int \frac{(3\sin x - 2)\cos x}{13 - \cos^2 x - 7\sin x} dx = \int \frac{(3\sin x - 2)\cos x}{\sin^2 x - 7\sin x + 12}$$

put $\sin x = y$, $\cos x\, dx = dy$ **1**

$$= \int \frac{(3y-2)dy}{y^2 - 7y + 12}$$

$$= \int \frac{(3y-2)dy}{(y-4)(y-3)}$$ ½

$$= \int \left(\frac{10}{y-4} - \frac{7}{y-3}\right) dy$$ **1**

$= 10 \log | y - 4 | - 7 \log | y - 3 | + C$ **1**

$= 10 \log | \sin x - 4 | - 7 \log | \sin x - 3 | + C$ ½

[CBSE Marking Scheme 2017]

23. Solve the following equation for x :

$$\cos(\tan^{-1} x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$$

Ans. $$\cos(\tan^{-1} x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$$

$$\Rightarrow \cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right) = \sin\left(\sin^{-1}\frac{4}{5}\right)$$ **1 + 1**

$$\frac{1}{\sqrt{1+x^2}} = \frac{4}{5}$$ **1**

$$x = \pm\frac{3}{4}$$ **1**

[CBSE Marking Scheme 2017]

SECTION D

28. If $A = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ -3 & 1 & -1 \end{pmatrix}$, find A^{-1} and hence solve the system of equations $2x + y - 3z = 13$, $3x + 2y + z = 4$, $x + 2y - z = 8$.

Ans. $|A| = -16$ **1**

Co-factors are

$C_{11} = -4, C_{21} = 4, C_{31} = 4$
$C_{12} = -5, C_{22} = 1, C_{32} = -3$ **2**
$C_{13} = 7, C_{23} = -11, C_{33} = 1$

$$A^{-1} = \frac{-1}{16}\begin{bmatrix} -4 & 4 & 4 \\ -5 & 1 & -3 \\ 7 & -11 & 1 \end{bmatrix}$$ ½

given equations can be written as

$AX = C\ X = (A^{-1})' C$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{16}\begin{bmatrix} -4 & -5 & 7 \\ 4 & 1 & -11 \\ 4 & -3 & 1 \end{bmatrix}\begin{bmatrix} 13 \\ 4 \\ 8 \end{bmatrix}$$ 1½

$$= \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$$ ½

$\Rightarrow x = 1, y = 2, z = -3$ ½

[CBSE Marking Scheme 2017]

29. Find the particular solution of the differential equation $\tan x \cdot \frac{dy}{dx} = 2x\tan x + x^2 - y$; $(\tan x \neq 0)$ given that $y = 0$ when $x = \frac{\pi}{2}$.

Ans. Given equation can be written as

$$\Rightarrow \frac{dy}{dx} + (\cot x)y = 2x + x^2 \cot x$$ **1**

$$\text{I.F.} = e^{\int \cot x\, dx} = e^{\int \log \sin x} = \sin x$$ **1**

Solution is, $y \times \sin x = \int (2x\sin x + x^2\cos x)dx$ **1**

$\Rightarrow \quad y \sin x = x^2 \sin x + C$ 1½

when $x = \frac{\pi}{2}, y = 0$

we get $c = \frac{-\pi^2}{4}$ **1**

$\therefore$ Required solution is, $4y \sin x = 4x^2 \sin x - \pi^2$

or, $y = x^2 - \pi^2/4 \operatorname{cosec} x$ ½

[CBSE Marking Scheme 2017]

Outside Delhi Set I Code No. 65/1

SECTION - A

1. If for any 2 × 2 square matrix A, A(adj A) = $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of |A|.

Ans. $|A| = 8$. **[CBSE Marking Scheme 2017] 1**

2. Determine the value of 'k' for which the following function is continuous at $x = 3$:

$$f(x) = \begin{cases} \dfrac{(x+3)^2 - 36}{x-3} &, x \neq 3 \\ k &, x = 3 \end{cases}$$

Ans. $k = 12$. **[CBSE Marking Scheme 2017] 1**

3. Find: $\int \dfrac{\sin^2 x - \cos^2 x}{\sin x \cos x} dx$

Ans. $-\log|\sin 2x| + c$ **OR** $\log|\sec x| - \log|\sin x| + c$.
[CBSE Marking Scheme 2017] 1

* **4. Find the distance between the planes $2x - y + 2z = 5$ and $5x - 2\cdot5y + 5z = 20$.**

SECTION B

5. If A is a skew-symmetric matrix of order 3, then prove that det A = 0.

Ans. **Any skew symmetric matrix of order 3 is**

$$A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} \quad 1$$

$\Rightarrow |A| = 0 - a(bc) + b(ac) = 0$ 1

OR

Since A is a skew-symmetric matrix $\therefore A^T = -A$ ½

$\Rightarrow |A^T| = |-A| = (-1)^3.|A|$ ½

$\Rightarrow |A| = -|A|$ ½

$\Rightarrow 2|A| = 0$ or $|A| = 0$. ½

[CBSE Marking Scheme 2017]

* **6. Find the value of c in Rolle's theorem for the function $f(x) = x^3 - 3x$ in $[-\sqrt{3}, 0]$.**

7. The volume of a cube is increasing at the rate of 9 cm³/s. How fast is its surface area increasing when the length of an edge is 10 cm ?

Ans. Let V be the volume of cube,

then $\dfrac{dV}{dt} = 9$ cm³/s.

Surface area (S) of cube = $6x^2$, where x is the side.

then $V = x^3$

$$\Rightarrow \frac{dV}{dt} = 3x^2 \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{1}{3x^2}.\frac{dV}{dt} \quad 1$$

$$S = 6x^2 \Rightarrow \frac{dS}{dt} = 12x\frac{dx}{dt}$$

$$= 12x.\frac{1}{3x^2}\frac{dV}{dt} \quad ½$$

$$= 4.\frac{1}{10}.9 = 3.6 \text{ cm}^2/\text{s}. \quad ½$$

[CBSE Marking Scheme 2017]

8. Show that the function $f(x) = x^3 - 3x^2 + 6x - 100$ is increasing on R.

Ans. $f(x) = x^3 - 3x^2 + 6x - 100$

$f'(x) = 3x^2 - 6x + 6$ ½

$= 3[x^2 - 2x + 2]$

$= 3[(x-1)^2 + 1]$ 1

since $f'(x) > 0 \; \forall \; x \in R$

$\therefore f(x)$ is increasing on R ½

[CBSE Marking Scheme 2017]

9. The X-coordinate of a point on the line joining the points P(2, 2, 1) and Q(5, 1, – 2) is 4. Find its z-coordinate.

Ans. Equation of line PQ is

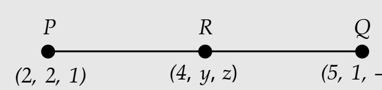
$$\frac{x-2}{3} = \frac{y-2}{-1} = \frac{z-1}{-3} \quad ½$$

Any point on the line is $(3\lambda + 2, -\lambda + 2, -3\lambda + 1)$ ½

$3\lambda + 2 = 4 \Rightarrow \lambda = \dfrac{2}{3}$ ½

$\therefore$ z coordinate $= -3\left(\dfrac{2}{3}\right) + 1 = -1$. ½

OR



Let R(4, y, z) lying on PQ divides PQ in the ratio k :1

$\Rightarrow \quad 4 = \dfrac{5k+2}{k+1} \Rightarrow k = 2.$ 1

$\therefore \quad z = \dfrac{2(-2)+1(1)}{2+1} = \dfrac{-3}{3} = -1.$ 1

[CBSE Marking Scheme 2017]

10. A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red". Find the A and B are independent events.

* *Out of Syllabus*

Ans. Event A: Number obtained is even

B: Number obtained is red.

$$P(A) = \frac{3}{6} = \frac{1}{2},\ P(B) = \frac{3}{6} = \frac{1}{2}$$ ½+½

$P(A \cap B)$ = P (getting an even red number)

$$= \frac{1}{6}$$ ½

Since $P(A).P(B) = \frac{1}{2}.\frac{1}{2} = \frac{1}{4} \neq P(P \cap B)$ which is $\frac{1}{6}$

$\therefore$ A and B are not independent events. ½

[CBSE Marking Scheme 2017]

11. Two tailors, A and B, earn ₹ 300 and ₹ 400 per day respectively. A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day, To find how many days should each of them work and if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost, formulate this as an LPP.

Ans. Let A works for x day and B for y days.

$\therefore$ L.P.P. is Minimize C = $300x + 400y$ ½

$$\text{Subject to : } \begin{cases} 6x + 10y \geq 60 \\ 4x + 4y \geq 32 \\ x \geq 0, y \geq 0 \end{cases}$$ 1½

[CBSE Marking Scheme 2017]

12. Find $\int \frac{dx}{5 - 8x - x^2}$

Ans.
$$\int \frac{dx}{5 - 8x - x^2} = \int \frac{dx}{(\sqrt{21})^2 - (x+4)^2}$$
$$= \frac{1}{2\sqrt{21}} \log \left| \frac{\sqrt{21} + (x+4)}{\sqrt{21} - (x+4)} \right| + c$$

[CBSE Marking Scheme 2017]

SECTION C

13. If $\tan^{-1}\frac{x-3}{x-4} + \tan^{-1}\frac{x+3}{x+4} = \frac{\pi}{4}$, **then find the value of** x.

Ans.
$$\tan^{-1}\frac{x-3}{x-4} + \tan^{-1}\frac{x+3}{x+4} = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{x-3}{x-4} + \frac{x+3}{x+4}}{1 - \frac{x-3}{x-4}.\frac{x+3}{x+4}} \right) = \frac{\pi}{4}$$ 1½

$$\Rightarrow \frac{2x^2 - 24}{-7} = 1 \Rightarrow x^2 = \frac{17}{2}$$ 1½

$$\Rightarrow x = \pm\sqrt{\frac{17}{2}}$$ 1

[CBSE Marking Scheme 2017]

14. Using properties of determinants, prove that

$$\begin{vmatrix} a^2 + 2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$$

OR

Find matrix A such that

$$\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} A = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}$$

Ans.
$$\Delta = \begin{vmatrix} a^2 + 2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

$R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$\Delta = \begin{vmatrix} a^2 - 1 & a-1 & 0 \\ 2(a-1) & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$ 1 + 1

$$= (a-1)^2 \begin{vmatrix} a+1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$ 1

Expanding along C_3

$(a-1)^2.(a-1) = (a-1)^3$. 1

[CBSE Marking Scheme 2017]

OR

Let
$$\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}$$ 1

$$\Rightarrow \begin{pmatrix} 2a - c & 2b - d \\ a & b \\ -3a + 4c & -3b + 4d \end{pmatrix} = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}$$ 1

$\Rightarrow$ $2a - c = -1,\ 2b - d = -8$

$a = 1,\ b = -2$ 1

$-3a + 4c = 9,\ -3b + 4d = 22$

Solving to get $a = 1,\ b = -2,\ c = 3,\ d = 4$

$$\therefore \quad A = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}$$ 1

[CBSE Marking Scheme 2017]

15. If $x^y + y^x = a^b$, **then find** $\frac{dy}{dx}$.

OR

If $e^y(x+1) = 1$, **then show that** $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$.

Ans.
$$x^y + y^x = a^b$$

Let $u + v = a^b$, where $x^y = u$ and $y^x = v$.

$$\therefore \quad \frac{du}{dx} + \frac{dv}{dx} = 0$$ ...(i) ½

$$y \log x = \log u$$

$$\Rightarrow \quad \frac{du}{dx} = x^y\left[\frac{y}{x}+\log x.\frac{dy}{dx}\right] \qquad 1$$

$$x \log y = \log v \Rightarrow \frac{dv}{dx}$$

$$\Rightarrow \quad \frac{dv}{dx} = y^x\left[\frac{x}{y}\frac{dy}{dx}+\log y\right] \qquad 1$$

Putting in (i)

$$x^y\left[\frac{y}{x}+\log x\frac{dy}{dx}\right]+y^x\left[\frac{x}{y}\frac{dy}{dx}+\log y\right]=0 \qquad ½$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{y^x\log y+y.x^{y-1}}{x^y.\log x+x.y^{x-1}} \qquad 1$$

[CBSE Marking Scheme 2017]

OR

$e^y.(x + 1) = 1$

$$\Rightarrow e^y\,.\,1 + (x+1).\,e^y\,\frac{dy}{dx} = 0 \qquad 1½$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{1}{(x+1)} \qquad 1$$

$$\frac{d^2y}{dx^2} = +\frac{1}{(x+1)^2}=\left(\frac{dy}{dx}\right)^2 \qquad 1½$$

[CBSE Marking Scheme 2017]

16. Find : $\displaystyle\int\frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)}d\theta$

Ans. $$I = \int\frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)}\,d\theta$$

$$= \int\frac{\cos\theta}{(4+\sin^2\theta)(1+4\sin^2\theta)}\,d\theta \qquad ½$$

$$= \int\frac{dt}{(4+t^2)(1+4t^2)}$$

Where $\sin\theta = t$ 1

$$= \int\frac{-\frac{1}{15}}{4+t^2}dt+\int\frac{\frac{4}{15}}{1+4t^2}dt \qquad 1$$

$$= -\frac{1}{30}\tan^{-1}\left(\frac{t}{2}\right)+\frac{4}{30}\tan^{-1}(2t)+c \qquad 1$$

$$= -\frac{1}{30}\tan^{-1}\left(\frac{\sin\theta}{2}\right)+\frac{2}{15}\tan^{-1}(2\sin\theta)+c \qquad ½$$

[CBSE Marking Scheme 2017]

17. Evaluate: $\displaystyle\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}dx$

OR

Evaluate: $\displaystyle\int_1^4\{|x-1|+|x-2|+|x-4|\}dx$

Ans. $$I = \int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}\,dx$$

$$= \int_0^{\pi}\frac{(\pi-x)\tan x}{\sec x+\tan x}\,dx \qquad 1$$

$$\Rightarrow 2I = \pi\int_0^{\pi}\frac{\tan x}{\sec x+\tan x}\,dx$$

$$= \pi\int_0^{\pi}\tan x(\sec x-\tan x)dx \qquad 1$$

$$I = \frac{\pi}{2}\int_0^{\pi}(\sec x\tan x-\sec^2 x+1)\,dx$$

$$= \frac{\pi}{2}[\sec x-\tan x+x]_0^{\pi} \qquad 1$$

$$I = \frac{\pi(\pi-2)}{2} \qquad 1$$

[CBSE Marking Scheme 2017]

OR

$$I = \int_1^4\{|x-1|+|x-2|+|x-4|\}\,dx$$

$$= \int_1^4(x-1)dx-\int_1^2(x-2)dx$$

$$+\int_2^4(x-2)dx-\int_1^4(x-4)dx \qquad 2$$

$$= \left[\frac{(x-1)^2}{2}\right]_1^4-\left[\frac{(x-2)^2}{2}\right]_1^2+\left[\frac{(x-2)^2}{2}\right]_2^4-\left[\frac{(x-4)^2}{2}\right]_1^4 \qquad 1$$

$$= \frac{9}{2}+\frac{1}{2}+2+\frac{9}{2} = 11\frac{1}{2}\ or\ \frac{23}{2} \qquad 1$$

[CBSE Marking Scheme 2017]

18. Solve the differential equation
$(\tan^{-1}x - y)dx = (1 + x^2)dy$.

Ans. Given differential equation can be written as

$$(1+x^2)\frac{dy}{dx}+y = \tan^{-1}x \Rightarrow \frac{dy}{dx}+\frac{1}{1+x^2}y$$

$$= \frac{\tan^{-1}x}{1+x^2} \qquad 1$$

Integrating factor $= e^{\tan^{-1}x}$. 1

$\therefore$ Solution is

$$y\,.e^{\tan^{-1}x} = \int\tan^{-1}x.e^{\tan^{-1}x}\frac{1}{1+x^2}dx \qquad 1$$

$$\Rightarrow y.e^{\tan^{-1}x} = e^{\tan^{-1}x}.(\tan^{-1}x-1)+c \qquad 1$$

or $\quad y = (\tan^{-1}x-1)+c.e^{-\tan^{-1}x}$

[CBSE Marking Scheme 2017]

19. Show that the point A, B, C with position vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$ respectively, are the vertices of a right-angled triangle, Hence find the area of the triangle.

Ans. $\overrightarrow{AB} = -\hat{i} - 2\hat{j} - 6\hat{k}$,

$\overrightarrow{BC} = 2\hat{i} - \hat{j} + \hat{k}$,

$\overrightarrow{CA} = -\hat{i} + 3\hat{j} + 5\hat{k}$ **1**

Since $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CA}$, are not parallel vectors, and

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$

$\therefore$ A, B, C form a triangle **1**

Also $\overrightarrow{BC}.\overrightarrow{CA} = 0$ **1**

$\therefore$ A, B, C form a right triangle

Area of $\Delta = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{BC}| = \frac{1}{2}\sqrt{210}$ **1**

[CBSE Marking Scheme 2017]

* **20. Find the value of λ, if four points with position vectors $3\hat{i}+6\hat{j}+9\hat{k}$, $\hat{i}+2\hat{j}+3\hat{k}$, $2\hat{i}+3\hat{j}+\hat{k}$ and $4\hat{i}+6\hat{j}+\lambda\hat{k}$ are coplanar.**

* **21. There are 4 cards numbered 1, 3, 5 and 7, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two drawn cards. Find the mean and variance of X.**

22. Of the students in a school, it is known that 30% have 100% attendance and 70% students are irregular. Previous year results report that 70% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one students is chosen at random from the school and he was found to have an A grade. What is the probability that the student has 100% attendance? Is regularity required only in school ? justify your answer.

Ans. Let E_1 : Selecting a student with 100% attendance

E_2 : Selecting a student who is not regular **1**

A : selected student attains A grade.

$P(E_1) = \frac{30}{100}$ and $P(E_2) = \frac{70}{100}$ **½**

$P(A/E_1) = \frac{70}{100}$ and $P(A/E_2) = \frac{10}{100}$ **½**

$$P(E_1/A) = \frac{P(E_1).P(A/E_1)}{P(E_1).P(A/E_1) + P(E_2)P(A/E_2)}$$

$$= \frac{\frac{30}{100} \times \frac{70}{100}}{\frac{30}{100} \times \frac{70}{100} + \frac{70}{100} \times \frac{10}{100}}$$

$$= \frac{3}{4}$$ **1**

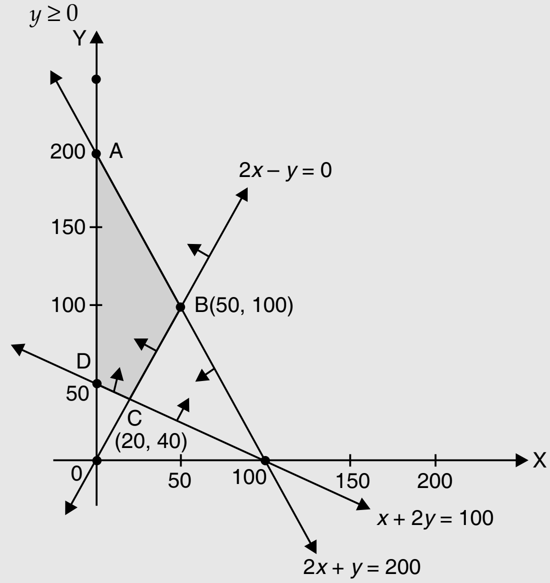
Regularity is required everywhere or any relevant value **1 [CBSE Marking Scheme 2017]**

23. Maximise $Z = x + 2y$
Subject to the constraints

$$x + 2y \geq 100$$
$$2x - y \leq 0$$
$$2x + y \leq 200$$
$$x, y \geq 0$$

Solve the above LPP graphically.

Ans. $Z = x + 2y$ $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x, y \geq 0$



For correct graph of three lines **1½**

For correct shading **1½**

$Z(A) = 0 + 400 = 400$

$Z(B) = 50 + 200 = 250$

$Z(C) = 20 + 80 = 100$

$Z(D) = 0 + 100 = 100$

$\therefore$ Max (= 400) at $x = 0, y = 200$ **1**

[CBSE Marking Scheme 2017]

SECTION - D

24. Determine the product

$$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$ **and use it to solve the system of equations $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$.**

Ans. Getting

$$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$ ...(i) **1½**

Given equations can be written as

$$\begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \\ 1 \end{pmatrix}$$ **1**

* *Out of Syllabus*

$\therefore \qquad AX = B$

From (i) $A^{-1} = \dfrac{1}{8}\begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix}$ 1½

$\Rightarrow \qquad X = A^{-1}B$

$= \dfrac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$

$= \dfrac{1}{8}\begin{pmatrix} 24 \\ -16 \\ -8 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$ 1

$\Rightarrow x = 3, y = -2, z = -1$ 1

[CBSE Marking Scheme 2017]

* **25. Consider $f : R - \left\{-\dfrac{4}{3}\right\} \to R - \left\{\dfrac{4}{3}\right\}$ given by $f(x) = \dfrac{4x+3}{3x+4}$. Show that f is bijective. Find the inverse of f and hence find $f^{-1}(0)$ and x such that $f^{-1}(x) = 2$.**

OR

* **Let A = Q × Q and let * be a binary operation on A defined by $(a, b) * (c, d) = (ac, b + ad)$ for (a, b), $(c, d) \in$ A. Determine, whether * is commutative and associative. Then, with respect to * on A**

(i) Find the identity element is A.

(ii) Find the invertible elements of A.

26. Show that the surface area of a closed cuboid with square base and given volume is minimum, when it is a cube.

Ans. Let the sides of cuboid be x, x, y

$\Rightarrow x^2y = k$ and $S = 2(x^2 + xy + xy)$

$= 2(x^2 + 2xy)$ 1½

$\therefore \quad S = 2\left[x^2 + 2x\dfrac{k}{x^2}\right] = 2\left[x^2 + \dfrac{2k}{x}\right]$ 1

$\dfrac{ds}{dx} = 2\left[2x - \dfrac{2k}{x^2}\right]$ 1

$\dfrac{ds}{dx} = 0 \Rightarrow x^3 = k = x^2y \Rightarrow x = y$ 1

$\therefore \quad \dfrac{ds}{dx} = 2\left[2 + \dfrac{4k}{x^3}\right] > 0$

$\therefore \quad x = y$ will given minimum surface area 1

and $x = y$, means sides are equal

$\therefore$ Cube will have minimum surface area ½

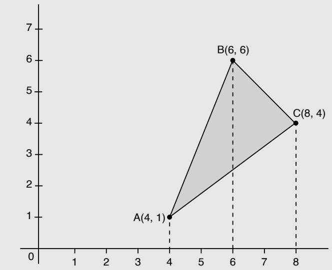
[CBSE Marking Scheme 2017]

27. Using the method of integration, find the area of the triangle ABC, coordinates of whose vertices are A(4, 1), B(6, 6) and C(8, 4).

OR

Find the area enclosed between the parabola $4y = 3x^2$ and the straight line $3x - 2y + 12 = 0$.

Ans.



1

Equation of AB : $y = \dfrac{5}{2}x - 9$

Equation of BC : $y = 12 - x$

Equation of AC : $y = \dfrac{3}{4}x - 2$ 1½

$\therefore \quad$ Area $(A) = \int_4^6 \left(\dfrac{5}{2}x - 9\right)dx + \int_6^8 (12 - x)dx$

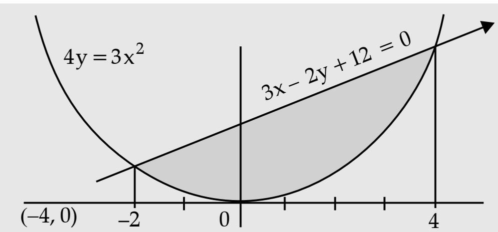
$- \int_4^8 \left(\dfrac{3}{4}x - 2\right)dx$ 1

$= \left[\dfrac{5}{4}x^2 - 9x\right]_4^6 + \left[12x - \dfrac{x^2}{2}\right]_6^8$

$- \left[\dfrac{3}{8}x^2 - 2x\right]_4^8$ 1½

$= 7 + 10 - 10 = 7$ sq.units 1

OR



1

$4y = 3x^2$ and $3x - 2y + 12 = 0$

$\Rightarrow \quad 4\left(\dfrac{3x+12}{2}\right) = 3x^2$

$\Rightarrow \quad 3x^2 - 6x - 24 = 0$ or $x^2 - 2x - 8 = 0$

$\Rightarrow \quad (x - 4)(x + 2) = 0$

$\Rightarrow$ X-coordinates of points of intersection are

$x = -2, x = 4$ 1

$\therefore$ Area $(A) = \int_{-2}^{4}\left[\dfrac{1}{2}(3x+12) - \dfrac{3}{4}x^2\right]dx$ 1½

$= \left[\dfrac{1}{2}\dfrac{(3x+12)^2}{6} - \dfrac{3}{4}\dfrac{x^3}{3}\right]_{-2}^{4}$ 1½

$= 32 - 5 = 27$ sq.units 1

[CBSE Marking Scheme 2017]

* *Out of Syllabus*

28. Find the particular solution of the differential equation $(x-y)\dfrac{dy}{dx}=(x+2y)$, given that $y = 0$ when $x = 1$.

Ans. $$\frac{dy}{dx} = \frac{x+2y}{x-y} = \frac{1+\frac{2y}{x}}{1-\frac{y}{x}} \qquad \tfrac{1}{2}$$

$$\frac{y}{x} = v \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx} \qquad \tfrac{1}{2}$$

$$\therefore \quad v + x\frac{dv}{dx} = \frac{1+2v}{1-v} \qquad 1$$

$$x\frac{dv}{dx} = -\frac{1+2v-v+v^2}{v-1}$$

$$\Rightarrow \int \frac{v-1}{v^2+v+1}dv = \frac{-dx}{x}$$

$$\Rightarrow \int \frac{2v+1-3}{v^2+v+1}dv = \int -\frac{2}{x}dx \Rightarrow \int \frac{2v+1}{v^2+v+1}dv$$

$$-3\int \frac{1}{\left(v+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dv = -\int \frac{2}{x}dx \qquad 1 + 1$$

$$\Rightarrow \log|v^2+v+1| - 3.\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2v+1}{\sqrt{3}}\right)$$

$$= -\log|x|^2 + C \qquad 1$$

$$\Rightarrow \log|y^2+xy+x^2| - 2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) \qquad \tfrac{1}{2}$$

$$x = 1, y = 0 \Rightarrow c = -2\sqrt{3}.\frac{\pi}{6} = -\frac{\sqrt{3}}{3}\pi \qquad \tfrac{1}{2}$$

$$\Rightarrow \log|y^2+xy+x^2| - 2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) + \frac{\sqrt{3}}{3}\pi = 0$$

[CBSE Marking Scheme 2017]

* **29. Find the coordinates of the point where the line through the points (3, – 4, – 5) and (2, – 3, 1), crosses the plane determined by the points (1, 2, 3), (4, 2, – 3) and (0, 4, 3).**

OR

* **A variable plane which remains at a constant distance $3p$ from the origin cuts the coordinate axes at A, B, C. Show that the locus of the centroid of triangle ABC is $\dfrac{1}{x^2}+\dfrac{1}{y^2}+\dfrac{1}{z^2}=\dfrac{1}{p^2}$.**

Outside Delhi Set II — Code No. 65/2

Note: Expect these, all other questions are from Outside Delhi Set-I

SECTION - B

12. The length x, of a rectangle is decreasing at the rate of 5 cm/minute and the width y, is increasing at the rate of 4 cm/minute. When x = 8 cm and y = 6 cm, find the rate of change of the area of the rectangle.

Ans. Given $$\frac{dx}{dt} = -5 \text{ cm/m.,}$$

$$\frac{dy}{dt} = 4 \text{ cm/m.}$$

$$A = xy \Rightarrow \frac{dA}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt} \qquad 1$$

$$= 8(4) + 6(-5) = 2$$

$\therefore$ Area is increasing at the rate of 2 cm^2/minute. **1**

[CBSE Marking Scheme 2017]

SECTION - C

20. Find : $\displaystyle\int \frac{\sin\theta d\theta}{(4+\cos^2\theta)(2-\sin^2\theta)}$

Ans. $$I = \int \frac{\sin\theta d\theta}{(4+\cos^2\theta)(2-\sin^2\theta)}$$

$$= \int \frac{\sin\theta d\theta}{(4+\cos^2\theta)(1+\cos^2\theta)} \qquad \tfrac{1}{2}$$

$$= -\int \frac{dt}{(4+t^2)(1+t^2)},$$

Where $\cos\theta = t$ **1**

$$= \int \frac{1/3}{4+t^2}dt - \int \frac{1/3}{1+t^2}dt \qquad 1$$

$$= \frac{1}{6}\tan^{-1}\frac{t}{2} - \frac{1}{3}\tan^{-1}t + c \qquad 1$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{\cos\theta}{2}\right)$$

$$-\frac{1}{3}\tan^{-1}(\cos\theta) + c \qquad \tfrac{1}{2}$$

[CBSE Marking Scheme 2017]

21. Solve the following linear programming problem graphically:

Maximise Z = 34x + 45y

under the following constraints

$x + y \le 300, 2x + 3y \le 70, x \ge 0, y \ge 0$

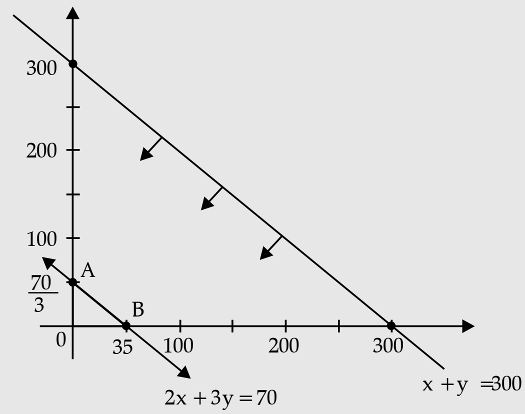
Ans. Maximise : $z = 34x + 45y$ subject to

$x + y \le 300, 2x + 3y \le 70, x \ge 0, y \ge 0$

Plotting the two lines. **2**

Correct shading **1**

* *Out of Syllabus*



$z(A) = z\left(0, \frac{70}{3}\right) = 1050$

$z(B) = z(35, 0) = 1190$

$\Rightarrow$ max (1190) at $x = 35, y = 0$. **1**

[CBSE Marking Scheme 2017]

* **22. Find the value of x such that the points A(3, 2, 1), B(4, x, 5), C(4, 2, – 2) and D(6, 5, – 1) are coplanar.**

23. Find the general solution of the differential equation $y\,dx - (x + 2y^2)\,dy = 0$

Ans. Given differential equation can be written as

$y\frac{dx}{dy} - x = 2y^2$ or $\frac{dx}{dy} - \frac{1}{y}.x = 2y$ **1**

Integrating factor is $e^{-\log y} = \frac{1}{y}$ **1**

$\therefore$ Solution is $x.\frac{1}{y} = \int 2dy = 2y + c$ **2**

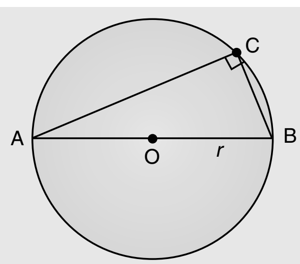
or $x = 2y^2 + cy$.

[CBSE Marking Scheme 2017]

SECTION - D

28. AB is the diameter of a circle and C is any point on the circle. Show that the area of triangle ABC is maximum, when it is an isosceles triangle.

Ans.



1

Let the length of sides of ΔABC are, $AC = x$ and $BC = y$

$\Rightarrow x^2 + y^2 = 4r^2$ and Area $A = \frac{1}{2}xy$ **1**

$A = \frac{1}{2}x\sqrt{4r^2 - x^2}$

or $S = \frac{x^2}{4}(4r^2 - x^2)$ **1**

$\therefore S = \frac{1}{4}[4r^2x^2 - x^4]$

$\frac{dS}{dx} = \frac{1}{4}[8r^2x - 4x^3]$

$\therefore \frac{dS}{dx} = 0 \Rightarrow 2r^2 = x^2 \Rightarrow x = \sqrt{2}r$ **1**

and $y = \sqrt{4r^2 - 2r^2} = \sqrt{2}r$ **½**

and $\frac{d^2S}{dx^2} = \frac{1}{4}[8r^2 - 12x^2]$

$= \frac{1}{4}[8r^2 - 24r^2] < 0$ **1**

$\therefore$ For maximum area, $x = y$ *i.e.*, Δ is isosceles. **½**

[CBSE Marking Scheme 2017]

29. If A = $\begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}$, find A^{-1}. Hence using A^{-1} solve the system of equations $2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$, $x + y - 2z = -3$

Ans. $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

$\Rightarrow |A| = 2(0) + 3(-2) + 5(1)$

$= -1 \neq 0$ **1**

$A_{11} = 0, A_{12} = 2, A_{13} = 1$

$A_{21} = -1, A_{22} = -9, A_{23} = -5$ **2**

$A_{31} = 2, A_{32} = 23, A_{33} = 13$

$A^{-1} = -1\begin{pmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{pmatrix}$

$= -1\begin{pmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{pmatrix}$

$= \begin{pmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{pmatrix}$ **½**

Given equations can be written as

$\begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11 \\ -5 \\ -3 \end{pmatrix}$ or $AX = B$

$\Rightarrow X = A^{-1}B$ **1**

* *Out of Syllabus*

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{pmatrix}\begin{pmatrix} 11 \\ -5 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$\Rightarrow \quad x = 1, y = 2, z = 3.$ 1½

[CBSE Marking Scheme 2017]

Outside Delhi Set III — Code No. 65/3

Note: Expect these, all other questions are from Outside Delhi Set-I & II

SECTION B

12. The volume of a sphere is increasing at the rate of 8 cm³/s. Find the rate at which its surface area is increasing when the radius of the sphere is 12 cm.

Ans. $\frac{dV}{dt} = 8 \text{ cm}^3/\text{s}$, where V is the volume of sphere

i.e.,

$$V = \frac{4}{3}\pi r^3$$

$$\Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{4\pi r^2}.\frac{dV}{dt}$$ 1

$$S = 4\pi r^2$$

$$\Rightarrow \frac{dS}{dt} = 8\pi r\frac{dr}{dt} = 8\pi r.\frac{1}{4\pi r^2}.8$$ ½

$$= \frac{2\times 8}{12} = \frac{4}{3}\text{cm}^2/\text{s}$$ ½

[CBSE Marking Scheme 2017]

SECTION C

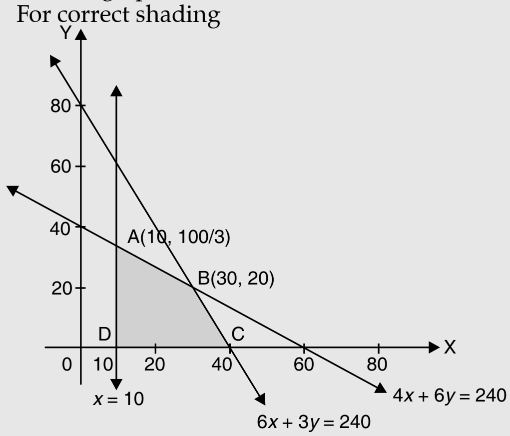
20. Solve the following linear programming problem graphically :
Maximise Z = 7x + 10y
subject to the constraints

$$4x + 6y \le 240$$
$$6x + 3y \le 240$$
$$x \ge 10$$
$$x \ge 0, y \ge 0$$

Ans. Maximise $z = 7x + 10y$, subject to $4x + 6y \le 240$;
$6x + 3y \le 240$; $x \ge 10$, $x \ge 0$, $y \ge 0$

Correct graph of three lines 1½

For correct shading 1½



$$Z(A) = Z\left(10, \frac{100}{3}\right) = 70 + 10 \times \frac{100}{3}$$

$$= 403\frac{1}{3}$$

$Z(B) = Z(30, 20) = 210 + 200 = 410$

$Z(C) = Z(40, 0) = 280 + 0 = 280$

$Z(D) = Z(10, 0) = 70 + 0 = 70$

$\Rightarrow$ Max (= 410) at $x = 30$, $y = 20$ 1

[CBSE Marking Scheme 2017]

21. Find : $\displaystyle\int \frac{e^x dx}{(e^x - 1)^2(e^x + 2)}$

Ans.

$$I = \int \frac{e^x dx}{(e^x - 1)^2(e^x + 2)}$$

$$= \int \frac{dt}{(t+2)(t-1)^2}$$

Where $e^x = t$ ½

$$= \int \frac{1/9}{(t+2)}dt - \int \frac{1/9}{(t-1)}dt + \int \frac{1/3}{(t-1)^2}dt$$ 1½

$$= \frac{1}{9}[\log|t+2| - \log|t-1|] - \frac{1}{3(t-1)} + c$$ 1½

$$= \frac{1}{9}\log\left|\frac{e^x+2}{e^x-1}\right| - \frac{1}{3(e^x-1)} + c$$ ½

[CBSE Marking Scheme 2017]

22. If $\vec{a} = 2\hat{i} - \hat{j} - 2\hat{k}$ and $\vec{b} = 7\hat{i} + 2\hat{j} - 3\hat{k}$, then express $\vec{b}$ in the form of $\vec{b} = \vec{b_1} + \vec{b_2}$, where $\vec{b_1}$ is parallel to $\vec{a}$ and $\vec{b_2}$ is perpendicular to $\vec{a}$.

Ans. $\vec{b_1} \,||\, \vec{a} \Rightarrow$ let $\vec{b_1} = \lambda(2\hat{i} - \hat{j} - 2\hat{k})$

$$\vec{b_2} = \vec{b} - \vec{b_1}$$ ½

$$= (7\hat{i} + 2\hat{j} - 3\hat{k}) - (2\lambda\hat{i} - \lambda\hat{j} - 2\lambda\hat{k})$$ ½

$$= (7 - 2\lambda)\hat{i} + (2 + \lambda)\hat{j} - (3 - 2\lambda)\hat{k})$$ 1

$\vec{b_2} \perp \vec{a} \Rightarrow 2(7 - 2\lambda) - 1(2 + \lambda) + 2(3 - 2\lambda) = 0$

$\Rightarrow \quad \lambda = 2$ 1

$\therefore\ \vec{b}_1 = 4\hat{i} - 2\hat{j} - 4\hat{k}$ and $\vec{b}_2 = 3\hat{i} + 4\hat{j} + \hat{k}$ **1**

$\Rightarrow (7\hat{i} + 2\hat{j} - 3\hat{k}) = (4\hat{i} - 2\hat{j} - 4\hat{k}) + (3\hat{i} + 4\hat{j} + \hat{k})$

[CBSE Marking Scheme 2017]

23. Find the general solution of the differential equation

$$\frac{dy}{dx} - y = \sin x.$$

Ans. Given differential equation is $\frac{dy}{dx} - y = \sin x$ ½

$\Rightarrow$ Integrating factor $= e^{-x}$

$\therefore$ Solution is : $ye^{-x} = \int \sin x e^{-x} dx = I_1$ **1**

$$I_1 = -\sin x e^{-x} + \int \cos x e^{-x} dx$$

$$= -\sin x e^{-x} + [-\cos x e^{-x} - \int + \sin x e^{-x} dx]$$

$$I_1 = \frac{1}{2}[-\sin x - \cos x]e^{-x}$$ **1½**

$\therefore$ Solution is $ye^{-x} = \frac{1}{2}(-\sin x - \cos x)e^{-x} + c$ **1**

or $y = -\frac{1}{2}(\sin x + \cos x) + ce^{x}$

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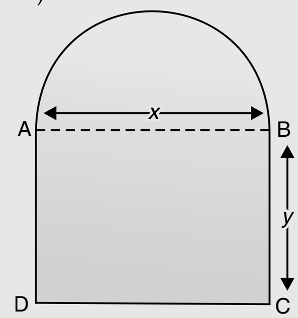
SECTION - D

29. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

Ans. Let dimensions of the rectangle be x and y (as shown)



1

$\therefore$ Perimeter of window

$p = 2y + x + \pi\frac{x}{2} = 10$ m ...(i) ½

Area of window $A = xy + \frac{1}{2}\pi\frac{x^2}{4}$ ½

$$A = x\left[5 - \frac{x}{2} - \pi\frac{x}{4}\right] + \frac{1}{2}\pi\frac{x^2}{4}$$

$$= 5x - \frac{x^2}{2} - \pi\frac{x^2}{8}$$ **1**

$\frac{dA}{dx} = 5 - x - \pi\frac{x}{4} = 0 \Rightarrow x = \frac{20}{4+\pi}$ **1**

$\frac{d^2A}{dx^2} = \left(-1 - \frac{\pi}{4}\right) < 0$ **1**

$\Rightarrow x = \frac{20}{4+\pi}$,

$y = \frac{10}{4+\pi}$ will give maximum light. **1**

[CBSE Marking Scheme 2017]

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