C.D.S.
Combined Defence Services

EXAM II 2025

Elementary Mathematics

Time Allowed: 2 Hours **Maximum Marks: 100**

Instructions

1. *This Test Booklet contains **100** items (questions). Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose **ONLY ONE** response for each item.*
2. *You have to mark all your responses **ONLY** on the separate Answer Sheet provided. See directions in the Answer Sheet.*
3. ***All** items carry equal marks.*
4. ***Penalty for wrong answers:***
   THERE WILL BE A PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
   (i). *There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as a penalty.*
   (ii). *If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.*
   (iii). *If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.*

1. In a triangle ABC, $\angle A = 30°$, $AB = 7$ cm and $AC = 12$ cm. What is the area of the triangle ABC?

(a) 21 cm^2 **(b)** $21\sqrt{3}$ cm^2

(c) 42 cm^2 **(d)** $42\sqrt{3}$ cm^2

2. ABC is a triangle right angled at B. D is a point on AC such that BD is perpendicular to AC. If $AB = p$ and $BC = \sqrt{3}p$, then what is BD equal to?

(a) $\frac{p}{3}$ **(b)** $\frac{p}{2}$

(c) $\frac{\sqrt{3}p}{2}$ **(d)** $\frac{\sqrt{3}p}{4}$

3. The difference between an interior angle and an exterior angle of a regular polygon is 120°. What is the number of sides of the polygon?

(a) 9 **(b)** 10

(c) 11 **(d)** 12

4. An angle θ is exactly one-fourth of its complementary angle. What is the value of angle θ?

(a) 12° **(b)** 15°

(c) 18° **(d)** 20°

5. The sides of a triangle are 11 cm, 60 cm and 61 cm. What is the area of the triangle formed by joining the mid- points of the sides of the triangle?

(a) 165 cm^2 **(b)** 110 cm^2

(c) 82.5 cm^2 **(d)** 72.5 cm^2

6. A pendulum swings through an angle of 30° and its end describes an arc of length 55 cm. What is the length of the pendulum?

(Take $\pi = 22/7$)

(a) 90 cm **(b)** 100 cm

(c) 105 cm **(d)** 110 cm

7. A conical tent has an angle of 60° at the vertex. If the curved surface area is 100 m^2, then what is the volume of the tent?

(a) $\frac{250\sqrt{2}}{\sqrt{3\pi}}$ m^3 **(b)** $\frac{500\sqrt{3}}{\sqrt{\pi}}$ m^3

(c) $\frac{1000\sqrt{3}}{\sqrt{2\pi}}$ m^3 **(d)** $\frac{1000\sqrt{3}}{\sqrt{\pi}}$ m^3

8. A right circular cone and a hemisphere have equal base and equal volume. What is the ratio of the height of the cone to the radius of the hemisphere?

(a) 1:1 **(b)** 1:2

(c) 2:1 **(d)** 3:2

9. A wire is in the form of an equilateral triangle with an area of $36\sqrt{3}$ cm^2. If it is changed into a semicircle, then what is its radius?

(a) $\frac{9}{\pi}$ cm **(b)** $\frac{18}{\pi+2}$ cm

(c) $\frac{18}{\pi}$ cm **(d)** None of the above

10. Let the area of the largest possible square inscribed in a circle of unit radius be x. Let the area of the largest possible circle inscribed in a square of unit side length be y. What is the relation between x and y?

(a) $\pi x = 2y$ **(b)** $2\pi x = y$

(c) $\pi x = 4y$ **(d)** $\pi x = 8y$

11. Let AD be the altitude of a triangle ABC. If $(AB + AC) = p$, $(AB - AC) = q$ and $(BD - CD) = r$, then what is BC equal to?

(a) $\frac{qr}{p}$ **(b)** $\frac{pr}{q}$

(c) $\frac{pq}{r}$ **(d)** $p + q - r$

12. The sum of the height and the radius of a right circular cylinder is 21 cm, and the radius is less than the height. If the curved surface area of the cylinder is 616 cm^2, then what is the volume of the cylinder? (Take $\pi = 22/7$)

(a) 1078 cm^3 **(b)** 1617 cm^3

(c) 1927 cm^3 **(d)** 2156 cm^3

13. ABC is a triangle right angled at B. P is the midpoint of AB and Q is the mid-point of BC. Consider the following:

I. $AQ = \sqrt{73}$ units

II. $CP = \sqrt{52}$ units

Which of the above is/are required to determine the area of the triangle?

(a) I only

(b) II only

(c) Both I and II

(d) More information is needed

14. The length, breadth and height of a cuboid are consecutive integers. If the volume of the cuboid is 336 cubic units, then what is the total surface area of the cuboid?

(a) 288 square units

(b) 292 square units

(c) 296 square units

(d) Cannot be determined due to insufficient data

15. In a circle of radius 14 cm, APB is a shorter arc and P is the midpoint of the arc. Let C be the midpoint of the chord AB and $PC = 7$ cm. What is the length of the chord AP?

(a) 3.5 cm **(b)** 7 cm

(c) 10.5 cm **(d)** 14 cm

16. Two poles of heights 10 m and 15 m are 25 m apart. What is the height of the point of intersection of the lines joining the tip of each pole to the foot of the other pole?

(a) 4.8 m **(b)** 5 m

(c) 6 m **(d)** 6.4 m

17. ABC is a triangle right angled at B. Further, $(AB + BC)$ exceeds AC by 10 units. If the perimeter of the triangle is 60 units, then what is the area of the triangle?

(a) 75 square units **(b)** 100 square units

(c) 125 square units **(d)** 150 square units

18. Two poles are situated 24 m apart and their heights differ by 10 m. What is the distance between their tips?

(a) 25 m

(b) 26 m

(c) 30 m

(d) Cannot be determined due insufficient data

19. Let X, Y and Z be the midpoints of the sides BC, CA and AB of a triangle ABC respectively. Consider the following statements:

I. The quadrilateral $AZXY$ is a parallelogram.

II. The area of the quadrilateral $AZXY$ is half of the area of the triangle ABC.

Which of the statements given above is/are correct?

(a) I only **(b)** II only

(c) Both I and II **(d)** Neither I nor II

20. Consider the following angles :

I. 4° **II.** 5°

III. 6° **IV.** 8°

How many of the above can be the exterior angle of a regular polygon?

(a) One **(b)** Two

(c) Three **(d)** All four

21. A Question is given followed by two Statements I and II. Consider the Question and the Statements.

Question: What is the remainder when $x^{2n} - y^{2n} + 1$ is divided by $x^n + y^n$, where n is a natural number?

Statement I: n is odd.

Statement II : n is even.

Which one of the following is correct in respect of the above Question and the Statements?

(a) The Question can be answered by using one of the Statements alone, but cannot be answered using the other Statement alone

(b) The Question can be answered by using either Statement alone

(c) The Question can be answered by using both the Statements together, but cannot be answered using either Statement alone

(d) The Question can be answered even without using any of the Statements

22. A Question is given followed by two Statements I and II. Consider the Question and the Statements.

Question: The product of a natural number N and the number M written by the same digits of N in the reverse order is 252. What is the number N?

Statement I : $N + M = 33$

Statement II: $N > M$

Which one of the following is correct in respect of the above Question and the Statements?

(a) The Question can be answered by using one of the Statements alone, but cannot be answered using the other Statement alone

(b) The Question can be answered by using either Statement alone

(c) The Question can be answered by using both the Statements together, but cannot be answered using either Statement alone

(d) The Question can be answered even without using any of the Statements

23. A Question is given followed by two Statements I and II. Consider the Question and the Statements.

Question: The last digit in the expansion of the number $(54D)^{100}$ is 1. What is the value of the digit D?

Statement I : $D > 5$

Statement II: D is a multiple of 3.

Which one of the following is correct in respect of the above Question and the Statements?

(a) The Question can be answered by using one of the Statements alone, but cannot be answered using the other Statement alone

(b) The Question can be answered by using either Statement alone

(c) The Question can be answered by using both the Statements together, but cannot be answered using either Statement alone

(d) The Question can be answered even without using any of the Statements

24. A Question is given followed by two Statements I and II. Consider the Question and the Statements.

Question: In a triangle ABC, $\angle A = \angle B - \angle C$. Is angle A acute?

Statement I: ΔABC is not an obtuse-angled triangle.

Statement II : Angle C is acute.

Which one of the following is correct in respect of the above Question and the Statements?

(a) The Question can be answered by using one of the Statements alone, but cannot be answered using the other Statement alone

(b) The Question can be answered by using either Statement alone

(c) The Question can be answered by using both the Statements together, but cannot be answered using either Statement alone

(d) The Question can be answered even without using any of the Statements

25. A Question is given followed by two Statements I and II. Consider the Question and the Statements.

Question: In a triangle ABC right angled at B, $AC = 20$ cm. What is the circum- radius of the triangle?

Statement I: $AB = 12$ cm

Statement II: $BC = 16$ cm

Which one of the following is correct in respect of the above Question and the Statements?

(a) The Question can be answered by using one of the Statements alone, but cannot be answered using the other Statement alone

(b) The Question can be answered by using either Statement alone

(c) The Question can be answered by using both the Statements together, but cannot be answered using either Statement alone

(d) The Question can be answered even without using any of the Statements

26. A Question is given followed by two Statements I and II. Consider the Question and the Statements.

Question: $ABCD$ is a parallelogram with $\angle ABC = 60°$. If the area of the parallelogram is $7\sqrt{3}$ square units, then what is the perimeter of the parallelogram?

Statement I: The lengths of the sides AB and DA are prime numbers.

Statement II: The lengths of the sides are natural numbers each greater than 1 unit.

Which one of the following is correct in respect of the above Question and the Statements?

(a) The Question can be answered by using one of the Statements alone, but cannot be answered using the other Statement alone

(b) The Question can be answered by using either Statement alone

(c) The Question can be answered by using both the Statements together, but cannot be answered using either Statement alone

(d) The Question can be answered even without using any of the Statements

27. A Question is given followed by two Statements I and II. Consider the Question and the Statements.

Question: AB and CD are chords of a circle intersecting at P. If $AP \times PB = 48$ square units, then what is $CP \times PD$ equal to?

Statement I: $AP = 8$ units

Statement II: $CP = 10$ units

Which one of the following is correct in respect of the above Question and the Statements?

(a) The Question can be answered by using one of the Statements alone, but cannot be answered using the other Statement alone

(b) The Question can be answered by using either Statement alone

(c) The Question can be answered by using both the Statements together, but cannot be answered using either Statement alone

(d) The Question can be answered even without using any of the Statements

28. A Question is given followed by two Statements I and II. Consider the Question and the Statements.

Question: In a quadrilateral $ABCD$, $AB = 6$ units, $BC = 18$ units, $CD = 6$ units, $DA = 9$ units. What is the length of diagonal BD?

Statement I: The length of BD is an integer greater than 13.

Statement II: The length of BD is an even integer.

Which one of the following is correct in respect of the above Question and the Statements?

(a) The Question can be answered by using one of the Statements alone, but cannot be answered using the other Statement alone

(b) The Question can be answered by using either Statement alone

(c) The Question can be answered by using both the Statements together, but cannot be answered using either Statement alone

(d) The Question can be answered even without using any of the Statements

29. A Question is given followed by two Consider the Statements I and II. Question and the Statements.

Question: ABC is an isosceles triangle with $AB = AC = 10$ units. If the area of the triangle is 48 square units, then what is the length of the base BC?

Statement I: The length of BC is an even integer.

Statement II: The height of the triangle is greater than the length of half of the base.

Which one of the following is correct in respect of the above Question and the Statements?

(a) The Question can be answered by using one of the Statements alone, but cannot be answered using the other Statement alone

(b) The Question can be answered by using either Statement alone

(c) The Question can be answered by using both the Statements together, but cannot be answered using either Statement alone

(d) The Question can be answered even without using any of the Statements

30. A Question is given followed by two Statements I and II. Consider the Question and the Statements.

Question: The diagonals of a rhombus $ABCD$ are in the ratio 5:12. Is one of the diagonals equal to side of the rhombus?

Statement I: The sum of the diagonals = 34 cm.

Statement II: The length of a side = 13 cm.

Which one of the following is correct in respect of the above Question and the Statements?

(a) The Question can be answered by using one of the Statements alone, but cannot be answered using the other Statement alone

(b) The Question can be answered by using either Statement alone

(c) The Question can be answered by using both the Statements together, but cannot be answered using either Statement alone,

(d) The Question can be answered even without using any of the Statements

31. Two trains X and Y are travelling in the same direction at 100 km/h and 60 km/h respectively. Train X crosses a man in train Y in 9 seconds. What is the length of train X?

(a) 80 m **(b)** 100 m

(c) 120 m **(d)** 150 m

32. Two persons X and Y leave place P for place Q at 7:00 a.m. and 7:10 a.m. same path. respectively along the X walks at a speed of 4.8 km/h and Y walks at a speed of 6 km/h. How many kilometres from place P will X meet Y?

(a) 3 km **(b)** 3.5 km

(c) 4 km **(d)** 4.5 km

33. There are two employees X and Y. X's salary is first increased by 12% and then decreased by 10%, and Y's salary is first increased by 10% and then decreased by 12%. If their salaries at present are equal, then what was the ratio of initial salary of X to initial salary of Y?

(a) 50: 53 **(b)** 51: 53

(c) 121: 126 **(d)** 121: 125

34. In a village consisting of p persons, x% can read and write. Of the males, only y% can read and write. Of the females, only z% can read and write. If $x, y > z$, then what is the number of males in the village?

(a) $p(x - z)/(y - z)$ **(b)** $p(y - z)/(x - z)$

(c) px/y **(d)** py/x

35. X and Y are two alloys of copper (Cu) and zinc (Zn). Alloy X is prepared by mixing Cu and Zn in the ratio 5:4, and alloy Y is prepared by mixing Cu and Zn in the ratio 5:13 respectively. If equal quantities of alloys X and Y are melted to form a third alloy Z, then what is the ratio of Cu to Zn in Z?

(a) 5:8 **(b)** 5:7

(c) 6:7 **(d)** 7:8

36. Two students X and Y appeared in a test. The score of X is 20 more than that of Y. If the score of X is 75% of the sum of the scores of X and Y, then what is the ratio of score of X to score of Y?

(a) 5:1 **(b)** 4:1

(c) 3:1 **(d)** 3:2

37. If one root of the equation

$$2x^2 - 5px + 2p^2 = 0$$

exceeds the other by 4, then what is the value of p?

(a) 8/3 **(b)** 4/3

(c) 2/3 **(d)** 1/3

38. An amount of ₹10,000 is borrowed at 10% per annum on compound interest for 3 years, compounded annually, and paid back in 3 equal annual installments during these years. What is the amount of each installment (approximately)?

(a) ₹ 4,437 **(b)** ₹ 4,237

(c) ₹ 4,021 **(d)** ₹ 3,811

39. What is the solution of the inequalities $5x+3< 8x - 9$ and $2x + 20 > 5x + 2$?

(a) $4 < x < 6$ **(b)** $3 < x < 5$

(c) $x < 3$ or $x > 5$ **(d)** $x < 4$ or $x > 6$

40. A shopkeeper gives three consecutive discounts 10%, 20% and 25% after which he sells the article at a profit of 8% on the cost price. Had he sold the article after the first discount, how much profit would he have got?

(a) 20% **(b)** 40%

(c) 50% **(d)** None of the above

41. Which measure of central tendency is least affected by the presence of extreme observations in the data?

(a) Arithmetic mean **(b)** Harmonic mean

(c) Geometric mean **(d)** Median

42. To find the average ratio like price/unit, work done/hour, kilometre/hour under certain conditions, the suitable measure of central tendency applicable is

(a) arithmetic mean **(b)** geometric mean

(c) harmonic mean **(d)** mode

43. The frequency distribution of marks of 100 candidates in a particular examination is as follows:

Marks	Number of Candidates
More than 10	100
More than 20	75
More than 30	60
More than 40	40

What are the (average marks of the candidates?

(a) 20.5 **(b)** 22.5

(c) 30.5 **(d)** 32.5

44. The arithmetic mean of 200 observations is 60. If 5 is multiplied to each observation, then what will be the new arithmetic mean?

(a) 500 **(b)** 300

(c) 60 **(d)** 40

45. A distribution consists of 3 components with frequencies 45, 40 and 55 having their means 2, 2.5 and 2 respectively. What is the mean of the combined distribution?

(a) 2.14 **(b)** 2.25

(c) 2.37 **(d)** 2.50

46. Which one of the following is a positional average?

(a) Arithmetic mean **(b)** Median

(c) Mode **(d)** Geometric mean

For the following two (02) items:

The following data represent the distance covered (in metres) by two groups of athletic children. It is known that the median distance in the first group is 20.8 metres while the mean distance in the second group is 17.3 metres. Some frequencies in both the groups are missing:

Distance Class	First Group	Second Group
0 – 5	u	$3u$
5 – 10	v	$2v$
10 – 15	11	40
15 – 20	52	50
20 – 25	75	30
25 – 30	22	28

47. What is the value of u?

(a) 11 **(b)** 2

(c) 3 **(d)** 4

48. What is the value of v?

(a) 5 **(b)** 6

(c) 7 **(d)** 8

For the following two (02) items:

Consider the following distribution having median value 24 :

Marks	Number of Students
Less than 10	5
Less than 20	30
Less than 30	$30 + k$
Less than 40	$48 + k$
Less than 50	$55 + k$

49. What is the value of k?

(a) 20 **(b)** 22

(c) 25 **(a)** 30

50. What is the mean of the distribution?

(a) 21.625 **(b)** 22.225

(c) 23.225 **(d)** 24.625

51. Let $p(x)$ be a polynomial. When $p(x)$ is divided by $(x-1)$, it leaves 2 as the remainder. When $p(x)$ is divided by $(x-2)$, it leaves 1 as the remainder. What is the remainder when $p(x)$ is divided by $(x-1)(x-2)$?

(a) 3 **(b)** -3

(c) $3-x$ **(d)** $3-2x$

52. Consider the following in respect of a positive real number x :

I. $x+\frac{1}{x}>1$ **II.** $\left(x+\frac{1}{x}\right)^2>2$

III. $\left(x+\frac{1}{x}\right)^4>9$

Which of the above are correct?

(a) I and II only **(b)** II and III only

(c) I and III only **(d)** I, II and III

53. Let p and q be natural numbers such that $q > p$. What is the largest value of p such that q^2-5p-4 is negative?

(a) 3 **(b)** 4

(c) 5 **(d)** 6

54. Let x and y be natural numbers, each less than 20, such that x, y, $x+y$ and $x-y$ are prime numbers. How many such combinations of (x, y, $x+y$, $x-y$) are possible?

(a) One **(b)** Two

(c) Three **(d)** None

55. If $(x+1)(x+p)(x^2+p^2)=x^4-1$, then what is the value of p?

(a) -1 **(b)** 0

(c) 1

(d) Cannot be determined

56. If $(2+\sqrt{3})^x+(2-\sqrt{3})^x=2$, then what is $(2+\sqrt{3})^x-(2-\sqrt{3})^x$ equal to?

(a) 0 **(b)** 0.5

(c) 1 **(d)** 1.5

57. If $\frac{1}{a}+\frac{1}{b}=\frac{5}{6}$ and $\frac{1}{a^2}+\frac{1}{b^2}=\frac{13}{36}$, then what is $\frac{1}{a^3}+\frac{1}{b^3}$ equal to?

(a) $\frac{31}{216}$ **(b)** $\frac{35}{216}$

(c) $\frac{37}{216}$ **(d)** $\frac{41}{216}$

58. What is the remainder when x^6 is divided by x^2+1?

(a) -1 **(b)** 0

(c) 1 **(d)** $x+1$

59. $(x+2)$ is a factor of which one of the following?

(a) $x^5-4x^4-3x^3+8x^2-14x+12$

(b) $x^5+4x^4-3x^3+8x^2-14x+12$

(c) $x^5-4x^4+3x^3+8x^2-14x+12$

(d) $x^5-4x-3x^3+8x^2+14x+12$

60. If $\log_{10}2=0.301$ and $\log_{10}3=0.477$, then what is the number of digits in the expansion of 60^{60}?

(a) 105 **(b)** 106

(c) 107 **(d)** 108

61. What is

$$\frac{(a+b)^2}{(c-a)(c+a+b)}+\frac{(a+b)c}{c^2+bc-a^2-ab}-\frac{(a+2b+c)}{2(c-a)}, a\neq b, b\neq c, c\neq a$$

equal to?

(a) $\frac{-1}{2}$ **(b)** 0

(c) $\frac{1}{2}$ **(d)** 1

62. If $a^b = b^a$, then what is

$$\frac{a\times\left(\frac{a}{b}\right)^{\frac{a}{b}}}{a^{\left(\frac{a}{b}\right)}}$$

equal to?

(a) 1 **(b)** ab

(c) b **(d)** a^b

63. If $x = 2 + 2^{1/2} + 2^{3/2}$, then what is $x^2 - 4x - 10$ equal to?

(a) 0 **(b)** 1

(c) 4 **(d)** 6

64. If $\frac{\sqrt{p+x}+\sqrt{p-x}}{\sqrt{p+x}-\sqrt{p-x}}=p$, then what is x equal to?

(a) $\frac{p}{p^2+1}$ **(b)** $\frac{2p}{p^2+1}$

(c) $\frac{p^2}{p^2+1}$ **(d)** $\frac{2p^2}{p^2+1}$

65. If $\left(\frac{a-b}{2}\right)x^2-\left(\frac{a+b}{2}\right)x+b=0$, then what are the roots of this equation?

(a) $1, \frac{b}{a-b}$ **(b)** $1, \frac{2b}{a-b}$

(c) $\frac{1}{2}, \frac{b}{a+b}$ **(d)** $\frac{1}{2}, \frac{2b}{a+b}$

66. If $x-\frac{1}{x}=2, x>0$; then what is $x^2-\frac{1}{x^2}$ equal to?

(a) 6 **(b)** $4\sqrt{2}$

(c) 4 **(d)** $2\sqrt{2}$

67. If $(a-b)^2+(b-c)^2+(c-a)^2=6$ and $a^2+b^2+c^2=29$, then what is $(a+b+c)$ equal to?

(a) ± 9 **(b)** ± 8

(c) ± 6 **(d)** ± 3

68. If $p=\frac{\sqrt{5}-2}{\sqrt{5}+2}$ and $q=\frac{\sqrt{5}+2}{\sqrt{5}-2}$, then what $\left(\frac{p}{q}+\frac{q}{p}\right)$ is equal to?

(a) 18 **(b)** $8\sqrt{5}$

(c) 322 **(d)** $72\sqrt{5}$

69. What is the digit at hundreds place of the number $(25)^{10}$?

(a) 1 **(b)** 2

(c) 5 **(d)** 6

70. A number N is such that when divided by 4, 6, 7 or 9, it leaves 3 as remainder. What is the smallest 4-digit number that satisfies this property?

(a) 1003 **(b)** 1005

(c) 1007 **(d)** 1011

71. What is the remainder when

$$(17^{25}+19^{25})$$

is divided by 18?

(a) 0 **(b)** 1

(c) 3 **(d)** 9

72. The HCF of x and y is H. Consider the following statements in respect of the HCF of $p=\frac{x^3+y^3}{x^2-xy+y^2}$ and $q=\frac{x^3-y^3}{x^2+xy+y^2}$:

I. The HCF of p and q can be H.

II. The HCF of p and q can be $2H$.

Which of the statements given above is/are correct?

(a) I only **(b)** II only

(c) Both I and II **(d)** Neither I nor II

73. If $x^4 = x^2 + 1$, where $x > 0$, then what is $2x^4$ equal to?

(a) $2+\sqrt{3}$ **(b)** $3+\sqrt{5}$

(c) $5-2\sqrt{3}$ **(d)** $3-\sqrt{5}$

74. If $\frac{p+q}{q+r}=\frac{r+s}{s+p}; (q+r)\neq 0, (s+p)\neq 0$, then which one of the following is correct?

(a) $p+q+r+s=0$

(b) $p=r$

(c) Either $p+q+r+s=0$ or $p=r$

(d) None of the above

75. If n is natural number less than 7, then what is the number of values of n for which $(12n+2)$ and $(8n+1)$ are relatively prime?

(a) 6 **(b)** 5

(c) 4 **(d)** 3

76. What is the HCF of $x^3 + y^2 + 3xy - 1$ and $(x + y)^4 - 1$?

(a) $x + y$ **(b)** $x + y + 1$

(c) $x + y - 1$ **(d)** 1

77. Let $x = n(n + 1)(n+2)$, where n is an even natural number. Which of the following statements is/ are correct?

I. x is always divisible by 48.

II. x^2 is always divisible by 144.

Select the answer using the code given below.

(a) I only **(b)** II only

(c) Both I and II **(d)** Neither I nor II

78. What is the LCM of $x^4 + x^2y^2 + y^4$, $x^3y + y^4$ and $x^4y^2 - x^3y^3$?

(a) $x^3y^3(x^6 - y^6)$ **(b)** $x^3y^2(x^6 - y^6)$

(c) $x^3y(x^6 - y^6)$ **(d)** $xy(x^6 - y^6)$

79. Let XYZ be a 3-digit number. Let D be the difference between XYZ and ZYX. What is the remainder when D is divided by 99?

(a) 0 **(b)** 1

(c) 7 **(d)** 9

80. Let p and q be two natural numbers such that $(p + q)^{p+q}$ is divisible by 512. What is the least value of $(p+q)$?

(a) 4 **(b)** 6

(c) 8 **(d)** 12

For the following two (02) items :

Let $p\sin^2\alpha + q\cos^2\alpha = m$, $q\sin^2\beta + p\cos^2\beta = n$; $p \neq m$, n and $q \neq m$, n.

81. What is $\left(\frac{\tan\alpha}{\tan\beta}\right)^2$ equal to?

(a) $-\frac{(m-q)(n-q)}{(m-p)(n-p)}$ **(b)** $-\frac{(m-q)(n-p)}{(m-p)(n-q)}$

(c) $\frac{(m-q)(n-q)}{(m-p)(n-p)}$ **(d)** $\frac{(m-q)(n-p)}{(m-p)(n-q)}$

82. If α and β are complementary angles, then which one of the following is correct?

(a) $mn - 1 = 0$ **(b)** $mn + 1 = 0$

(c) $m + n = 0$ **(d)** $m - n = 0$

For the following two (02) items :

Let $\text{cosec}\theta - \sin\theta = p$ and $\sec\theta - \cos\theta = q$.

83. What is $(p\sin\theta + q\cos\theta)$ equal to?

(a) –1 **(b)** 0

(c) 1 **(d)** 2

84. What is $p^2q^2(p^2 + q^2 + 3)$ equal to?

(a) 0 **(b)** 1

(c) 2 **(d)** 4

For the following two (02) items :

$\frac{\sin\alpha}{\sin\beta} = \frac{4\sqrt{2}}{3}$ and $\frac{\cos\alpha}{\cos\beta} = \frac{2\sqrt{3}}{9}$.

85. What is $\tan^2\alpha$ equal to?

(a) 8 **(b)** 6

(c) 4 **(d)** 3

86. What is $\tan^2\beta$ equal to?

(a) $\frac{1}{\sqrt{2}}$ **(b)** $\frac{3}{\sqrt{2}}$

(c) $\frac{1}{3}$ **(d)** $\frac{2}{3}$

For the following two (02) items :

Let $\frac{1+\sin\theta}{\cos\theta} = p + \sqrt{p^2+1}$.

87. What is $\sec\theta$ equal to?

(a) p **(b)** $\sqrt{p^2+1}$

(c) $\frac{1}{\sqrt{p^2+1}}$ **(d)** $\frac{p}{\sqrt{p^2+1}}$

88. What is $\tan\theta$ equal to?

(a) p **(b)** $\sqrt{p^2+1}$

(c) $\frac{1}{\sqrt{p^2+1}}$ **(d)** $\frac{p}{\sqrt{p^2+1}}$

For the following two (02) items:

Let $\sin\theta + \cos\theta = p$ and $\sec\theta + \text{cosec}\,\theta = q$, where $p \neq 1$.

89. What is the relation between p and q?

(a) $p = q(p^2 - 1)$ **(b)** $2p = q(p^2 - 1)$

(c) $q = p^2 - 1$ **(d)** $2q = p(p^2 - 1)$

90. What is $\tan\theta + \cot\theta$ equal to?

(a) $\frac{p}{q}$ **(b)** $\frac{q}{p}$

(c) $\frac{2p}{q}$ **(d)** $\frac{2q}{p}$

91. If $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\ldots}}}} = \text{cosec}\,\theta$, then what is $\sin\theta$ equal to?

(a) 1 **(b)** $\frac{\sqrt{3}}{2}$

(c) $\frac{1}{\sqrt{2}}$ **(d)** $\frac{1}{2}$

92. If $8\sin\theta - \cos\theta = 4$, where $0 < \theta < \pi/2$, then what is $\operatorname{cosec}\theta$ equal to?

(a) 1 **(b)** $\frac{3}{2}$

(c) $\frac{5}{3}$ **(d)** 2

93. If $2\tan\theta = \sec^2\theta - 2$, where $0 < \theta < \pi/2$, then what is $\cot$ equal to?

(a) $\sqrt{2}-1$ **(b)** $\sqrt{2}+1$

(c) $\sqrt{3}-1$ **(d)** $\sqrt{3}+2$

94. What is $(\sec\theta - \tan\theta) - \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$ equal to?

(a) 0 **(b)** $2\tan\theta$

(c) $2\sec\theta$ **(d)** $\sin\theta + \cos\theta$

95. If $\cot\theta = \sqrt{7}$, then what is

$$\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$$

equal to?

(a) $\frac{1}{2}$ **(b)** $\frac{1}{3}$

(c) $\frac{2}{3}$ **(d)** $\frac{3}{4}$

96. The difference between the two acute angles in a right-angled triangle is π radian. One of the acute angles of $\frac{\pi}{12}$ the triangle is

(a) 60° **(b)** 57.5°

(c) 52.5° **(d)** 47.5°

97. If α and β are the roots of the equation

$$\log_{10}\left[998 + \sqrt{x^2 - 18x + 76}\right] = 3$$

then what is $(\alpha - \beta)^2$ equal to?

(a) 16 **(b)** 25

(c) 36 **(d)** 49

98. If $x^4 + y^4 = 14x^2y^2$, then consider the following:

I. $\log_{10}(x^2 + y^2) = \log_{10} x + \log_{10} y + 2\log_{10} 2$

II. $\log_{10}(x^2 - y^2) = \log_{10} x + \log_{10} y + \log_{10} 2 + 0.5\log_{10} 3$

Which of the above is/are correct?

(a) I only **(b)** II only

(c) Both I and II **(d)** Neither I nor II

99. Which of the following is/are the factor(s) of $(3x + y)^2 + (3x + y)(x + 5y) - 20(x + 5y)^2$?

I. $(4x + 13y)$

II. $(x + 19y)$

Select the correct answer using the code given below.

(a) I only **(b)** II only

(c) Both I and II **(d)** Neither I nor II

100. What is

$$\frac{\frac{x}{x-y} + \frac{y}{y-z} + \frac{z}{z-x}}{\frac{x+y}{x-y} + \frac{y+z}{y-z} + \frac{z+x}{z-x} + 3}$$

equal to?

(a) 1 **(b)** $\frac{1}{2}$

(c) $\frac{1}{3}$ **(d)** $\frac{1}{4}$

Answer Key

Q. No	Answer Key	Topic's Name	Chapter's Name
1	(a)	Triangle	Geometry
2	(c)	Triangle	Geometry
3	(d)	Polygons	Geometry
4	(c)	Angle	Geometry
5	(c)	Triangle	Geometry
6	(c)	Circle	Mensuration
7	(a)	Cone	Mensuration
8	(c)	Cone	Mensuration
9	(d)	Circle	Geometry
10	(d)	Circle	Geometry

Q. No	Answer Key	Topic's Name	Chapter's Name
11	(c)	Triangle	Geometry
12	(d)	Cylinder	Mensuration
13	(c)	Triangle	Geometry
14	(b)	Cuboid	Mensuration
15	(d)	Circle	Geometry
16	(c)	Triangle	Geometry
17	(d)	Triangle	Geometry
18	(b)	Triangle	Geometry
19	(c)	Triangle	Geometry
20	(d)	Polygons	Geometry
21	(d)	Polynomials	Algebra
22	(c)	Number system	Arithmetic
23	(c)	Exponents	Algebra
24	(d)	Triangle	Geometry
25	(d)	Triangle	Geometry
26	(b)	Parallelograms	Geometry
27	(d)	Circle	Geometry
28	(b)	Quadrilaterals	Geometry
29	(a)	Triangle	Geometry
30	(d)	Rhombus	Geometry
31	(b)	Speed, Distance and Time	Arithmetic
32	(c)	Speed, Distance and Time	Arithmetic
33	(c)	Ratios	Arithmetic
34	(a)	Ratios	Arithmetic
35	(b)	Ratios	Arithmetic
36	(c)	Ratios	Arithmetic
37	(a)	Quadratic Equations	Algebra
38	(a)	Compound Interest	Arithmetic
39	(a)	Inequalities	Algebra
40	(d)	Profit and Loss	Arithmetic
41	(d)	Median	Statistics
42	(a)	Harmonic Mean	Statistics
43	(d)	Mean	Statistics
44	(b)	Mean	Statistics
45	(a)	Mean	Statistics
46	(b)	Median	Statistics
47	(c)	Median	Statistics
48	(c)	Median	Statistics
49	(c)	Median	Statistics
50	(d)	Mean	Statistics
51	(c)	Polynomials	Algebra
52	(d)	Exponents	Algebra
53	(a)	Inequalities	Algebra
54	(b)	Prime Nos.	Algebra
55	(a)	Polynomials	Algebra
56	(a)	Surds and Indices	Algebra

Q. No	Answer Key	Topic's Name	Chapter's Name
57	(b)	Polynomials	Algebra
58	(a)	Polynomials	Algebra
59	(a)	Polynomials	Algebra
60	(c)	Logarithms	Algebra
61	(a)	Algebraic Identities	Algebra
62	(a)	Exponents	Algebra
63	(c)	Surds and Indices	Algebra
64	(d)	Surds and Indices	Algebra
65	(b)	Quadratic Equations	Algebra
66	(b)	Surds and Indices	Algebra
67	(a)	Algebraic Identities	Algebra
68	(c)	Surds and Indices	Algebra
69	(d)	Exponents	Arithmetic
70	(d)	Number system	Arithmetic
71	(a)	Modular Arithmetic	Algebra
72	(c)	Number system	Arithmetic
73	(b)	Quadratic Equations	Algebra
74	(c)	Algebraic Identities	Algebra
75	(a)	Number system	Arithmetic
76	(c)	Algebraic Identities	Algebra
77	(b)	Number system	Arithmetic
78	(b)	Modular Arithmetic	Arithmetic
79	(a)	Factorial	Arithmetic
80	(c)	Permutations and Combinations	Arithmetic
81	(c)	Permutations and Combinations	Arithmetic
82	(d)	Permutations and Combinations	Arithmetic
83	(c)	Permutations and Combinations	Arithmetic
84	(b)	Probability	Arithmetic
85	(a)	Probability	Arithmetic
86	(c)	Probability	Arithmetic
87	(b)	Mean Deviation	Statistics
88	(a)	Variance	Statistics
89	(b)	Standard Deviation	Statistics
90	(b)	Variance	Statistics
91	(d)	Heights and Distances	Trigonometry
92	(c)	Heights and Distances	Trigonometry
93	(a)	Heights and Distances	Trigonometry
94	(a)	Trigonometric Identities	Trigonometry
95	(d)	Trigonometric Ratios	Trigonometry
96	(c)	Trigonometric Ratios	Trigonometry
97	(c)	Trigonometric Identities	Trigonometry
98	(c)	Trigonometric Identities	Trigonometry
99	(c)	Trigonometric Ratios	Trigonometry
100	(b)	Trigonometric Ratios	Trigonometry

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CDS
Combined Defence Services

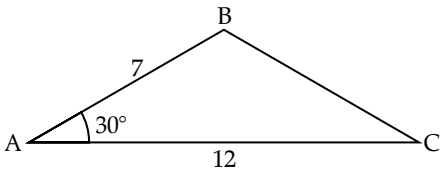
EXAM II 2025

Elementary Mathematics

ANSWERS WITH EXPLANATIONS

1. Option (a) is correct.

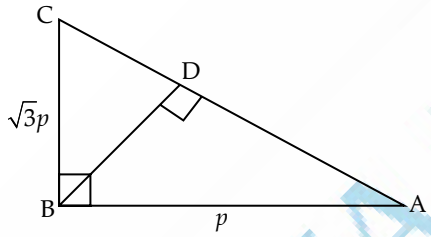
Explanation:



$$\text{Area of triangle} = \frac{1}{2} \times AB \times AC \times \sin\theta$$
$$= \frac{1}{2} \times 7 \times 12 \times \sin 30^\circ$$
$$= 7 \times 6 \times \frac{1}{2}$$
$$= 21\,\text{cm}^2$$

2. Option (c) is correct.

Explanation:



$$AC^2 = AB^2 + BC^2$$
$$= p^2 + 3p^2$$
$$AC^2 = 4p^2$$
$$AC = 2p$$
$$\text{Area of triangle } ABC = \frac{1}{2} \times AB \times BC$$
$$= \frac{1}{2} \times AC \times BD$$
$$\frac{1}{2} \times p \times \sqrt{3}p = \frac{1}{2} \times 2p \times BD$$
$$BD = \frac{\sqrt{3}p}{2}$$

3. Option (d) is correct.

Explanation:

Let interior angle be α

Exterior angle $= \beta$

$$\alpha - \beta = 120^\circ$$
$$\alpha + \beta = 180^\circ$$
$$2\alpha = 120^\circ + 180^\circ$$
$$\alpha = 150^\circ$$
$$\therefore \quad \beta = 30^\circ$$
$$\text{Number of sides} = \frac{360^\circ}{30^\circ}$$
$$= 12$$

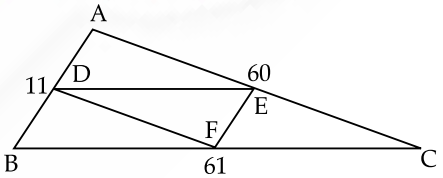
4. Option (c) is correct.

Explanation: Let complementary angle be α

$$\therefore \quad \alpha = 4\theta$$
$$4\theta + \theta = 90^\circ$$
$$5\theta = 90^\circ$$
$$\theta = 18^\circ$$

5. Option (c) is correct.

Explanation:



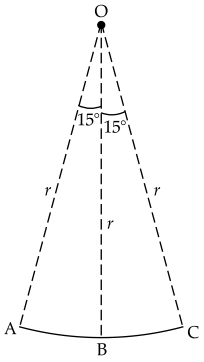
$$AB^2 + AC^2 = 11^2 + 60^2$$
$$= 121 + 3600$$
$$= 3721$$
$$= 61^2$$
$$AB^2 + AC^2 = BC^2$$

$\therefore \Delta ABC$ is a right angle triangle

$$\text{Area of } \Delta ABC = \frac{1}{2} \times 60 \times 11$$
$$= 330\ \text{cm}^2$$
$$\text{Area of } \Delta DEF = \frac{1}{4} \text{ of area of } \Delta ABC$$
$$= \frac{330}{4}$$
$$= 82.5\ \text{cm}^2$$

6. Option (c) is correct.

Explanation:



Arc length ABC = 55 cm

Arc length = $r\theta$

$$\therefore \quad 55 = r \times \frac{30^\circ}{180^\circ} \times \pi$$

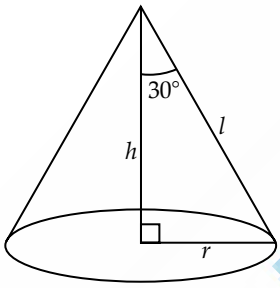
$$r = \frac{55 \times 180^\circ}{30^\circ \times \pi}$$

$$= \frac{55 \times 6}{22} \times 7$$

$$= 105 \text{ cm}$$

7. Option (a) is correct.

Explanation:



Curved surface area = πrl

$$100 = \pi rl \quad \ldots(i)$$

$$\frac{h}{r} = \cot 30°$$

$$h = \sqrt{3}r$$

$$\therefore \quad l^2 = h^2 + r^2$$

$$l^2 = 3r^2 + r^2$$

$$l^2 = 4r^2$$

$$\therefore \quad l = 2r \quad \ldots(ii)$$

From equation (i) and (ii) we get,

$$100 = \pi r \times 2r$$

$$r^2 = \frac{50}{\pi}$$

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi \times \frac{50}{\pi} \times \sqrt{3}r$$

$$= \frac{50}{\sqrt{3}} \times \sqrt{\frac{50}{\pi}}$$

$$= \frac{50 \times 5 \times \sqrt{2}}{\sqrt{3\pi}}$$

$$= \frac{250\sqrt{2}}{\sqrt{3\pi}} \text{ m}^3$$

8. Option (c) is correct.

Explanation:

Let height of cone = h

Radius of cone = r

Radius of sphere = R

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of hemisphere} = \frac{2}{3}\pi R^3$$

Base of cone = Base of hemisphere

$$\therefore \quad r = R$$

$$\therefore \quad \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$$

$$h = 2r$$

$$\frac{h}{r} = 2$$

9. Option (d) is correct.

Explanation:

Let side of an equilateral triangle = a

$$\text{Area of triangle} = \frac{\sqrt{3}}{4}a^2$$

$$\therefore \quad \frac{\sqrt{3}}{4}a^2 = 36\sqrt{3}$$

$$a^2 = 36 \times 4$$

$$a = 6 \times 2$$

$$a = 12$$

Perimeter of triangle = $3a$

$$= 3 \times 12$$

$$= 36 \text{ cm}$$

Let radius of circle = r

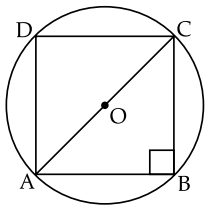
Perimeter of semicircle = $\pi r + 2r$

$$\pi r + 2r = 36$$

$$r = \frac{36}{\pi + 2} \text{ cm}$$

10. Option (d) is correct.

Explanation:



Let side length of square be a

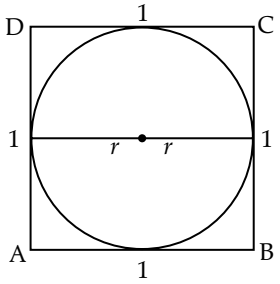
$\therefore$ Diagonal $AC = a\sqrt{2}$

$a\sqrt{2} = 1 \times 2$

$\therefore$ $a = \sqrt{2}$

Area of square $= a^2$

$\therefore$ $x = 2$ ...(i)



Let radius of largest circle $= r$

$\therefore$ $2r = 1$

$r = \frac{1}{2}$

Area of circle $= \pi r^2$

$y = \pi \times \frac{1}{4}$

$y = \frac{\pi}{4}$

$\therefore$ $\pi = 4y$ ...(ii)

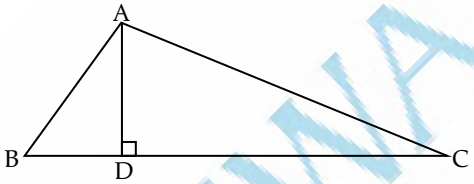
Now, from equation (i) and (ii),

$\pi \times x = 2 \times 4y$

$\pi x = 8y$

11. Option (c) is correct.

Explanation:



$AB + AC = p$

$AB - AC = q$

$\therefore$ $2AB = p + q$

$AB = \frac{p+q}{2}$

$AC = p - \left(\frac{p+q}{2}\right)$

$AC = \frac{p-q}{2}$

$BD - CD = r$

In ΔABD,

$AD^2 = AB^2 - BD^2$ ...(i)

In ΔACD,

$AD^2 = AC^2 - CD^2$. ..(ii)

From equation (i) and (ii) we get,

$AB^2 - BD^2 = AC^2 - CD^2$

$AB^2 - AC^2 = BD^2 - CD^2$

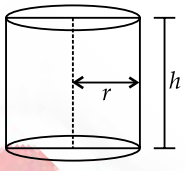
$(AB + AC)(AB - AC) = (BD + CD)(BD - CD)$

$pq = BC \times r$

$\therefore$ $BC = \frac{pq}{r}$

12. Option (d) is correct.

Explanation:



Let height $= h$

radius $= r$

$h + r = 21$ ...(i)

Curved surface area $= 2\pi rh$

$616 = 2 \times \frac{22}{7} \times rh$

$rh = \frac{616 \times 7}{22 \times 2}$

$rh = 98$ ...(ii)

Now from equation (i) and (ii),

$r(21 - r) = 98$

$21r - r^2 = 98$

$r^2 - 21r + 98 = 0$

$(r - 14)(r - 7) = 0$

$r = 7, 14$

If $r = 7 \Rightarrow h = 14$

If $r = 14 \Rightarrow h = 7$

$\because r < h$

$\therefore r = 7$ and $h = 14$

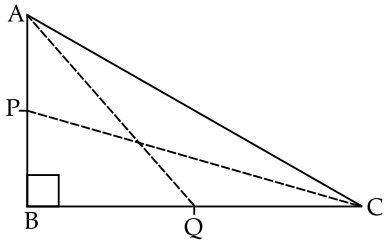
Volume $= \pi r^2 h$

$= \frac{22}{7} \times 7^2 \times 14$

$= 2156 \text{ cm}^3$

13. Option (c) is correct.

Explanation:



$AQ = \sqrt{73}$

$CP = \sqrt{52}$

Let $AB = c$,

$BC = a$

In ΔABQ,

$$AQ^2 = AB^2 + BQ^2$$

$$73 = c^2 + \frac{a^2}{4} \quad ...(i)$$

In ΔBCP,

$$CP^2 = BP^2 + BC^2$$

$$52 = \frac{c^2}{4} + a^2 \quad ...(ii)$$

From equation (i) and (ii),

$$52 = \frac{1}{4}\left(73 - \frac{a^2}{4}\right) + a^2$$

$$208 = 73 - \frac{a^2}{4} + 4a^2$$

$$208 - 73 = \frac{15a^2}{4}$$

$$135 = \frac{15a^2}{4}$$

$$a^2 = 9 \times 4$$

$$a = 6$$

Now from equation (i),

$$73 = c^2 + \frac{36}{4}$$

$$c^2 = 73 - 9$$

$$c^2 = 64$$

$$c = 8$$

$$\text{Area of } \Delta ABC = \frac{6 \times 8}{2}$$

$$= 24 \text{ unit}^2$$

Hence, Both I and II required

14. Option (b) is correct.

Explanation:

Let length = x

breadth = $x + 1$

height = $x + 2$

Volume = $x(x + 1)(x + 2)$

$336 = x(x + 1)(x + 2)$

$3 \times 112 = x(x + 1)(x + 2)$

$3 \times 4 \times 28 = x(x + 1)(x + 2)$

$3 \times 4 \times 4 \times 7 = x(x + 1)(x + 2)$

$6 \times 7 \times 8 = x(x + 1)(x + 2)$

$\therefore \quad x = 6$

Total surface area = $2(lb + bh + lh)$

$= 2\{x(x + 1) + x(x + 2) + (x + 1)(x + 2)\}$

$= 2\{6 \times 7 + 6 \times 8 + 7 \times 8\}$

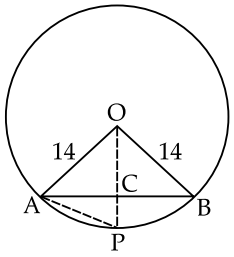
$= 2\{42 + 48 + 56\}$

$= 2\{146\}$

$= 292$ square units

15. Option (d) is correct.

Explanation:



$OA = OB = 14$

$CP = 7$

$OP = 14$

$OC = 14 - 7$

$OC = 7$

In ΔOAC,

$$14^2 = 7^2 + AC^2$$

$$AC^2 = 14^2 - 7^2 \quad ...(i)$$

In ΔACP,

$$AP^2 = AC^2 + CP^2$$

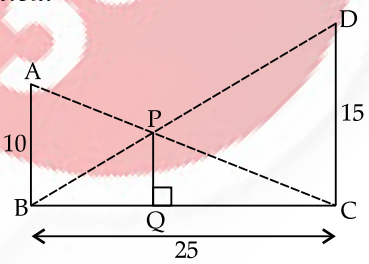
$$= 14^2 - 7^2 + 7^2$$

$$AP^2 = 14^2$$

$$AP = 14 \text{ cm}$$

16. Option (c) is correct.

Explanation:



Let $BQ = x$

$CQ = 25 - x$

$PQ = y$

$\Delta ABC \sim \Delta PQC$

$$\therefore \quad \frac{AB}{PQ} = \frac{BC}{QC}$$

$$\frac{10}{y} = \frac{25}{25 - x}$$

$$10(25 - x) = 25y$$

$$2(25 - x) = 5y$$

$$50 - 2x = 5y \quad ...(i)$$

$\Delta BPC \sim \Delta BPQ$

$$\therefore \quad \frac{DC}{PQ} = \frac{BC}{BQ}$$

$$\frac{15}{y} = \frac{25}{x}$$

$$15x = 25y$$

$$3x = 5y \quad \text{...(ii)}$$

From equation (i) and (ii)

$$50 - 2\left(\frac{5y}{3}\right) = 5y$$

$$50 = 5y + \frac{10y}{3}$$

$$10 = y + \frac{2y}{3}$$

$$30 = 3y + 2y$$

$$y = m$$

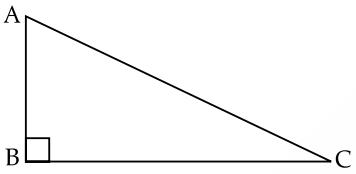
SHORTCUT: If the two poles have heights a and b, then the height of the intersection point is always

$$h = \frac{ab}{a+b}$$

$$h = \frac{10\times15}{10+15} = \frac{150}{25} = 6 \text{ m}$$

17. Option (d) is correct.

Explanation:



$$AB + BC = AC + 10$$

$$AB + BC + AC = 60$$

$$\therefore \quad AC + 10 + AC = 60$$

$$2AC = 60 - 10$$

$$2AC = 50$$

$$AC = 25$$

$$\therefore \quad AB + BC = 10 + 25$$

$$= 35$$

In ΔABC,

$$AB^2 + BC^2 = AC^2$$

$$AB^2 + (35 - AB)^2 = 25^2$$

$$AB^2 + AB^2 + 1225 - 70AB = 625$$

$$2AB^2 - 70AB + 600 = 0$$

$$AB^2 - 35AB + 300 = 0$$

$$(AB - 20)(AB - 15) = 0$$

$$\therefore \quad AB = 20 \text{ or } 15$$

$$\text{Area of } \Delta ABC = \frac{20\times15}{2}$$

$$= 150 \text{ square units}$$

SHORTCUT: Perimeter = 60, right-angled. The condition suggests a Pythagorean triple.

Spot 15-20-25 (scaled 3-4-5).

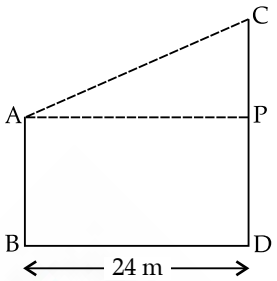
15 + 20 = 35, exceeds 25 by 10

Perimeter =60

$$\text{Area} = \frac{1}{2}\times15\times20 = 150$$

18. Option (b) is correct.

Explanation:



Let $AB = h$

$\therefore$ $CD = h + 10$

$CP = h + 10 - h$

$= 10\,\text{m}$

$AP = 24\,\text{m}$

In ΔACP,

$$AC^2 = 24^2 + 10^2$$

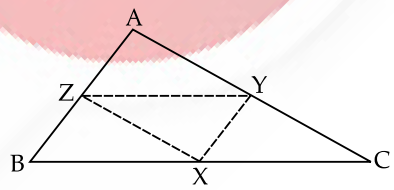
$$= 576 + 100$$

$$AC^2 = 676$$

$$AC = 26\,\text{m}$$

19. Option (c) is correct.

Explanation:



Z is mid-point of AB

X is mid-point of BC

$\therefore$ According to mid-point theorem

$ZX \parallel AC$

Similarly, $XY \parallel AB$

$\therefore$ $AZXY$ is a parallelogram

Area of ΔZYX = Area of ΔAZY

= Area of ΔBZX =Area of ΔZXC

Hence, area of quadrilateral $AZXY$ is half of area of ΔABC

20. Option (d) is correct.

Explanation:

Sum of all exterior angle of a regular polygon = 360°

Let number of sides = n

$\therefore\ n\theta = 360^\circ$

If $\theta = 4^\circ \Rightarrow n = 90$

If $\theta = 5^\circ \Rightarrow n = 72$

If $\theta = 6^\circ \Rightarrow n = 60$

If $\theta = 8^\circ \Rightarrow n = 45$

In all cases n is natural number hence, all of them are possible.

21. Option (d) is correct.

Explanation: Let

$$\begin{aligned} p &= x^{2n} - y^{2n} + 1 \\ &= (x^n)^2 - (y^n)^2 + 1 \\ &= (x^n + y^n)(x^n - y^n) + 1 \\ p - 1 &= (x^n + y^n)(x^n - y^n) \end{aligned}$$

$\therefore\ p - 1$ is divisible by $x^n + y^n \quad \forall\ n \in N$

Hence, if p is divided by $x^n + y^n$ then remainder will always be 1.

22. Option (c) is correct.

Explanation:

$$MN = 252$$

M and N both must be two digit numbers

Let $N = 10a + b$

$M = 10b + a$

$\therefore\ (10a + b)(10b + a) = 252 \quad ...(i)$

Statement - I : $M + N = 33$

$10a + b + 10b + a = 33$

$11(a + b) = 33$

$a + b = 3 \quad ...(ii)$

Now from equation (i) and (ii),

$$\begin{aligned} (10a + 3 - a)(10(3 - a) + a) &= 252 \\ (9a + 3)(30 - 10a + a) &= 252 \\ 3(3a + 1)(30 - 9a) &= 252 \\ 9(3a + 1)(10 - 3a) &= 252 \\ (3a + 1)(10 - 3a) &= 28 \\ 30a + 10 - 9a^2 - 3a &= 28 \\ 9a^2 - 27a + 18 &= 0 \\ a^2 - 3a + 2 &= 0 \\ (a - 2)(a - 1) &= 0 \end{aligned}$$

If $a = 1$, then $b = 2$

If $a = 2$, then $b = 1$

If $a = 1, b = 2$, then $N = 12$

If $a = 2, b = 1$, then $N = 21$

Statement-2:

$N > M$

$\therefore\ N = 21$

Hence, both statements required

23. Option (c) is correct.

Explanation:

Let $p = (54D)^{100}$

Unit place of p will only depend on D

$1^n = 1\ \forall n \in N$

Unit place of $3^{4n} = 1\ \forall n \in N$

Unit place of $7^{4n} = 1\ \forall n \in N$

Unit place of $9^{2n} = 1\ \forall n \in N$

D can be 1, 3, 7, 9

To define D uniquely we required both statements

24. Option (d) is correct.

Explanation:

$$\begin{aligned} \angle A &= \angle B - \angle C \\ \angle B &= \angle A + \angle C \\ \angle A + \angle B + \angle C &= 180 \\ (\angle B - \angle C) + \angle B + \angle C &= 180 \\ \angle B &= 90 \end{aligned}$$

So the triangle is right-angled at B.

Statement I: triangle ABC is not an obtuse-angled triangle.

So this is not sufficient to answer.

Statement II: Angle C is acute.

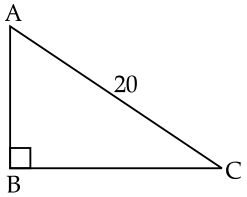
If C is acute (<90°), then A = 90° – ∠C is also acute.

So this is sufficient to answer.

Note: Without using both the statements (I and II), we can access the nature of angles.

25. Option (d) is correct.

Explanation:



$AC = 20\,\text{cm}$

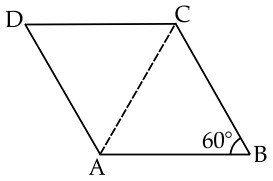
In right angle triangle hypotenuse is diameter of circumcircle

$\therefore$ Circumradius $= \dfrac{20}{2}$

$= 10$ cm

26. Option (b) is correct.

Explanation:



$$\text{Area of } \Delta ABC = \frac{AB \times BC \times \sin 60^\circ}{2}$$

$$= \frac{AB \times BC \times \sqrt{3}}{2 \times 2}$$

$$\text{Area of parallelogram} = 2 \times \text{Area of } \Delta ABC$$

$$= \frac{AB \times BC \times \sqrt{3}}{2}$$

$$\therefore \quad \frac{AB \times BC \times \sqrt{3}}{2} = 7\sqrt{3}$$

$$AB \times BC = 2 \times 7$$

If $AB = 2$ then, $BC = 7$

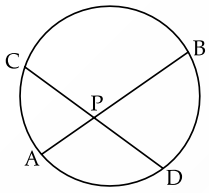
If $AB = 7$ then, $BC = 2$

$$\text{perimeter} = 2(7+2)$$

$$= 28 \text{ units}$$

27. Option (d) is correct.

Explanation:



$$AP \times PB = 48$$

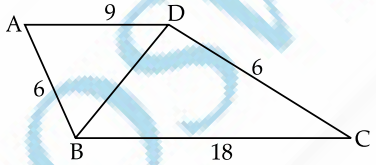
We know that,

$$AP \times PB = CP \times PD$$

$$\therefore \quad CP \times PD = 48 \text{ square units}$$

28. Option (b) is correct.

Explanation:



In ΔBCD,

$$BD > BC - DC$$

$$\therefore \quad BD > 18 - 6$$

$$BD > 12 \qquad \ldots(i)$$

In ΔABD,

$$BD < AB + AD$$

$$BD < 9 + 6$$

$$\therefore \quad BD < 15 \qquad \ldots(ii)$$

From equation (i) and (ii) we get,

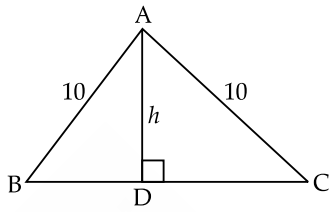
$$12 < BD < 15$$

Using statement – I, we get $BD = 14$

Using statement – *II*, we get $BD = 14$

29. Option (a) is correct.

Explanation:



$$\text{Area of } \Delta ABC = 48 \text{ square units}$$

Let $BC = 2x$

$$\text{(Semi perimeter)} \quad s = \frac{2x + 10 + 10}{2}$$

$$s = x + 10$$

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

Let, $a = 2x, b = 10, c = 10$

$$\therefore \quad 48 = \sqrt{(x+10) \cdot x \cdot x(10-x)}$$

$$48 \times 48 = x^2(100 - x^2)$$

$$x^4 - 100x^2 + 48 \times 48 = 0$$

$$(x^2 - 64)(x^2 - 36) = 0$$

$$x^2 = 64 \text{ or } 36$$

$$\therefore \quad x = 8 \text{ or } 6$$

Hence, $BC = 16$ or 12

If $BC = 16$

$$h^2 + 8^2 = 10^2$$

$$h^2 = 100 - 64$$

$$h^2 = 36$$

$$h = 6$$

$$h < \frac{BC}{2}$$

If $BC = 12$

$$h^2 + 6^2 = 10^2$$

$$h^2 = 100 - 36$$

$$h^2 = 64$$

$$h = 8$$

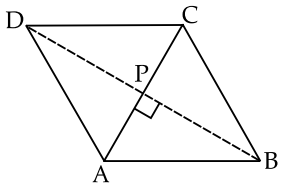
$$h > \frac{BC}{2}$$

According to Statement-I, BC have two values 12 & 16.

According to statement - II, $BC = 12$.

30. Option (d) is correct.

Explanation:



$$AC : BD = 5 : 12$$

Let $AC = 5x$

$BD = 12x$

In ΔPAB,

$$PA = \frac{5x}{2}$$

$$PB = \frac{12x}{2}$$

$$AB^2 = PA^2 + PB^2$$

$$= \frac{25x^2}{4} + \frac{144x^2}{4}$$

$$AB^2 = \frac{169x^2}{4}$$

$\therefore$ $AB = \frac{13x}{2}$

$AB \neq AC$

or $AB \neq BD$

Side of rhombus is not equal to any of the diagonals.

31. Option (b) is correct.

Explanation:

Let length of train $X = a$ m

Length of train $Y = b$ m

Speed of train $X = u = 100$ km/h

$$u = 100 \times \frac{1000}{3600} \text{ m/s}$$

$$= \frac{250}{9} \text{ m/s}$$

Speed of train $Y = v = 60$km/h

$$v = 60 \times \frac{1000}{3600} \text{ m/s}$$

$$= \frac{50}{3} \text{ m/s}$$

Relative speed of trains

$$= u - v$$

$$= \frac{250}{9} - \frac{50}{3}$$

$$= \frac{250 - 150}{9}$$

$$= \frac{100}{9} \text{ m/s}$$

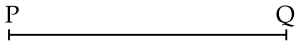
Time taken = 9 s

Length of train $X = \frac{100}{9} \times 9$

$= 100$ m

32. Option (c) is correct.

Explanation:



(7:00am) $\rightarrow$ 4.8 km/h

(7:10am) $\rightarrow$ 6 km/h

Let speed of person $X = u$

$u = 4.8$ km/h

$u = \frac{4.8}{60}$ km/min

$u = 0.08$ km/min

Speed of person $Y = v$

$v = 6$ km/hr

$v = \frac{6}{60}$ km/min

$v = 0.1$ km/min

Let both meet after t minute from 7:00 am

$\therefore$ $t \times 0.08 = (t - 10) \times 0.1$

$0.08t = 0.1t - 1$

$1 = 0.1t - 0.08t$

$1 = 0.02t$

$t = \frac{1}{0.02}$

$t = 50$ minutes

Distance travelled by $X = 50 \times 0.08$

$= 4$ km

33. Option (c) is correct.

Explanation:

Let initial salary of $X = a$

initial salary of $Y = b$

For employee X,

$$12\% \text{ increment} = a\left(1 + \frac{12}{100}\right)$$

$$= 1.12a$$

$$10\% \text{ decrease} = 1.12a\left(1 - \frac{10}{100}\right)$$

$$= 1.12a \times 0.9$$

$$= 1.008a$$

For Employee Y,

$$\text{Increased by } 10\% = b\left(1 + \frac{10}{100}\right)$$

$$= 1.1b$$

$$\text{Decreased by } 12\% = 1.1b\left(1-\frac{12}{100}\right)$$
$$= 1.1b \times 0.88$$
$$= 0.968b$$

According to problem,

$$1.008a = 0.968b$$
$$\frac{a}{b} = \frac{0.968}{1.008}$$
$$\frac{a}{b} = \frac{121}{126}$$

34. Option (a) is correct.

Explanation:

Let number of males $= m$

number of females $= p - m$

Total persons who can read and write $= \frac{px}{100}$

Number of males who can read and write $= \frac{my}{100}$

Number of females who can read and write $= (p-m)\frac{z}{100}$

$$\therefore \quad \frac{px}{100} = \frac{my}{100} + (p-m)\frac{z}{100}$$
$$px = my + pz - mz$$
$$px - pz = m(y-z)$$
$$m = p\frac{(x-z)}{(y-z)}$$

35. Option (b) is correct.

Explanation: In alloy X,

$$Cu : Zn = 5:4$$

Let Copper (Cu) $= 5a$

Zinc$(Zn) = 4a$

In alloy Y,

$$Cu : Zn = 5:13$$

Let Copper (Cu) $= 5b$

Zinc$(Zn) = 13b$

Alloy Z,

Let c gm of both alloy mixed

$$\text{Copper in alloy } z = \frac{5c}{9} + \frac{5c}{18}$$
$$\text{Zinc in alloy } z = \frac{4c}{9} + \frac{13c}{18}$$
$$\therefore \quad Cu : Zn = \left(\frac{5c}{9} + \frac{5c}{18}\right) : \left(\frac{4c}{9} + \frac{13c}{18}\right)$$
$$= \frac{15c}{18} : \frac{21c}{18}$$
$$= 5:7$$

36. Option (c) is correct.

Explanation:

$$\text{Let score of } Y = a$$
$$\text{Score of } X = a + 20$$
$$a + 20 = \frac{75}{100}(a + a + 20)$$
$$a + 20 = \frac{3}{4}(2a + 20)$$
$$4a + 80 = 6a + 60$$
$$6a - 4a = 80 - 60$$
$$2a = 20$$
$$a = 10$$
$$\text{Score of } X : \text{Score of } Y = 10 + 20 : 10$$
$$= 30:10$$
$$= 3:1$$

37. Option (a) is correct.

Explanation: Let the roots be α and β.

Given:

$$\alpha - \beta = 4$$

So, $\alpha + \beta = \frac{5p}{2}$

$$\alpha\beta = \frac{2p^2}{2} = p^2$$

Now, $(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

$$16 = \frac{25p^2}{4} - 4p^2$$
$$16 = \frac{25p^2}{4} - \frac{16p^2}{4} = \frac{9p^2}{4}$$
$$64 = 9p^2$$
$$p^2 = \frac{64}{9}$$
$$p = \pm\frac{8}{3}$$
$$p = \frac{8}{3}$$

38. Option (a) is correct.

$$p = \text{amount} = 10000$$
$$r = 10\%$$
$$n = 3 \text{ years}$$

$$\text{Total amount} = p\left(1+\frac{r}{100}\right)^n$$
$$= 10000\left(1+\frac{10}{100}\right)^3$$
$$= 10000(1\cdot1)^3$$
$$= 13310$$
$$\text{Each installment} = \frac{13310}{3}$$
$$\approx 4437$$

39. Option (a) is correct.

Explanation:

$$\begin{aligned} 5x+3 &< 8x-9 \\ 3+9 &< 8x-5x \\ 3x &> 12 \\ x &> 4 \quad \text{(i)} \\ 2x+20 &> 5x+2 \\ 20-2 &> 5x-2x \\ 3x &< 18 \\ x &< 6 \quad \text{(ii)} \end{aligned}$$

from equation (i) and (ii) we get,

$$4 < x < 6$$

40. Option (d) is correct.

Explanation:

$$\begin{aligned} \text{Let selling price} &= a \\ 10\%\text{ discounted price} &= a\left(1-\frac{10}{100}\right) \\ &= 0.9a \\ 20\%\text{ discounted price} &= 0.9a\left(1-\frac{20}{100}\right) \\ &= 0.9a\times 0.8 \\ &= 0.72a \\ 25\%\text{ discounted price} &= 0.72a\left(1-\frac{25}{100}\right) \\ &= 0.72a\times 0.75 \\ &= 0.54a \\ \text{Let Cost} &= c \\ 8\%\text{profit} &= c\times\frac{8}{100} \\ &= 0.08\text{c} \\ \text{Sold price} &= c+0.08c \\ \therefore \quad c(1.08) &= 0.54a \\ c &= \frac{0.54a}{1.08} \\ c &= \frac{a}{2} \\ \text{Profit after first discount} &= 0.9a-\frac{a}{2} \\ &= 0.9a-0.5a \\ &= 0.4a \\ \text{profit}(\%) &= \frac{0.4a}{0.5a}\times 100 \\ &= 80\% \end{aligned}$$

41. Option (d) is correct.

Explanation: Median is least affected by the presence of extreme observations in data.

42. Option (a) is correct.

Explanation: Arithmetic mean is suitable measure for given condition.

43. Option (d) is correct.

Explanation:

Marks	Number of Candidates
10 – 20	25
20 – 30	15
30 – 40	20
40 – 50	40

$$\begin{aligned} \text{Average marks} &= \frac{15\times 25+25\times 15+35\times 20+45\times 40}{100} \\ &= \frac{375+375+700+1800}{100} \\ &= \frac{3250}{100} \\ &= 32.5 \end{aligned}$$

44. Option (d) is correct.

Explanation:

Number of observations = 200

Arithmetic mean = 60

If 5 is multiplied to each observation, then mean also get multiplied by 5.

New Arithmetic mean = 60 × 5

= 300

45. Option (a) is correct.

Explanation: Let frequencies are f_1, f_2, f_3.

Means are m_1, m_2, m_3.

Mean of the combined distribution

$$\begin{aligned} &= \frac{m_1f_1+m_2f_2+m_3f_3}{f_1+f_2+f_3} \\ &= \frac{45\times 2+40\times 2.5+55\times 2}{45+40+55} \\ &= \frac{90+100+110}{140} \\ &= \frac{300}{140} \\ &= 2.14 \end{aligned}$$

46. Option (b) is correct.

Explanation: Median is positional average.

47. Option (c) is correct.

Explanation:

Distance Class	First Group	Second Group
0-5	u	$3u$
5-10	v	$2v$
10-15	11	40
15-20	52	50
20-25	75	30
25-30	22	28

48. Option (c) is correct.

Explanation:

Distance class	x	first group	$C.f.$	Second group
0 – 5	2.5	u	u	$3u$
0 – 10	7.5	v	$u + v$	$2v$
10 – 15	12.5	11	$u+v+11$	40
15 – 20	17.5	52	$u+v+63$	50
20 – 25	22.5	75	$u+v+138$	30
25 – 30	27.5	22	$u+v+160$	28

$$N = 160+u+v$$

$$m = l+\left(\frac{\frac{N}{2}-C\cdot f\cdot}{f}\right)h$$

$$N = 160+u+v$$

$$\frac{N}{2} = 80+\frac{u+v}{2}$$

$$c.f. = u+v+63$$

$$f = 75$$

$$l = 20$$

$$h = 5$$

$$\therefore \quad 20.8 = 20+\left(\frac{80+\frac{u+v}{2}-63-u-v}{75}\right)\times 5$$

$$0.8 = \frac{17-\left(\frac{u+v}{2}\right)}{15}$$

$$12 = 17-\left(\frac{u+v}{2}\right)$$

$$\frac{u+v}{2} = 17-12$$

$$\frac{u+v}{2} = 5$$

$$u+v = 10 \qquad \text{...(i)}$$

$$\text{Mean} = \frac{\Sigma(x_i f_i)}{\Sigma(f_i)}$$

$$17.3 = \frac{3u\times 2.5+2v\times 7.5+40\times 12.5+50\times 17.5+30\times 22.5+28\times 27.5}{3u+2v+40+50+30+28}$$

$$17.3 = \frac{7.5u+15v+2820}{3u+2v+148}$$

$$\because \quad u+v = 10$$

$$17.3 = \frac{15\times 10-7.5u+2820}{u+20+148}$$

$$17.3u+2906.4 = 2970-7.5u$$

$$17.3u+7.5u = 2970-2906.4$$

$$24.8u = 63.6$$

$$u = 2.56$$

$$\therefore \quad u = 3$$

$$v = 7$$

49. Option (c) is correct.

50. Option (d) is correct.

Explanation:

Marks	Number of tudents	c. f.
0 – 10	5	5
10 – 20	25	30
20 – 30	k	$30+k$
30 – 40	18	$48+k$
40 – 50	7	$55+k$

$$N = 55+k$$

$$\text{Median} = l+\left(\frac{\frac{N}{2}-c\cdot f}{f}\right)h$$

$$\frac{N}{2} = \frac{55+k}{2}$$

$$= 27.5+\frac{k}{2}$$

$$c\cdot f. = 30$$

$$f = k$$

$$h = 10$$

$$l = 20$$

$$24 = 20+\left(\frac{\frac{55+k}{2}-30}{k}\right)\times 10$$

$$24-20 = \frac{k-5}{2k}\times 10$$

$$4\times 2k = 10k-50$$

$$8k = 10k-50$$

$$50 = 10k-8k$$

$$2k = 50$$

$$\therefore \quad k = 25$$

$$\text{Mean} = \frac{5\times 5+25\times 15+25k+18\times 35+7\times 45}{55+25}$$

$$= \frac{25+375+625+630+315}{80}$$

$$= \frac{1970}{80}$$

$$= 24.625$$

51. Option (c) is correct.

Explanation:

Let $P(x) = Q(x)(x-1)(x-2)+ax+b$ ...(i)

$\therefore \quad P(1) = 2$

$P(2) = 1$

take $x = 1$ in equation (i)

$2 = a+b$ ...(ii)

take $x = 2$ in equation (i)

$1 = 2a+b$ ...(iii)

From equation (ii) and (iii);

$1 = a+a+b$

$1 = a+2$

$a = -1$

$b = 3$

$\therefore \quad P(x) = Q(x)(x-1)(x-2)-x+3$

If $P(x)$ divided by $(x-1)(x-2)$, then remainder is $3-x$.

52. Option (d) is correct.

Explanation:

$x > 0$

$\text{A.M} \geqslant G \cdot M$

$$\frac{x+\frac{1}{x}}{2} \geqslant \sqrt{x \times \frac{1}{x}}$$

$$x+\frac{1}{x} \geqslant 2$$

I. $x+\frac{1}{x} > 1$ true

II. $\left(x+\frac{1}{x}\right)^2 > 2$ true

III. $\left(x+\frac{1}{x}\right)^4 > 9$ true

53. Option (a) is correct.

Explanation:

$q > p$

$q^2-5p-4 < 0$

for largest value of $p, q \quad = p+1$

$\therefore \quad (p+1)^2-5p-4 < 0$

$p^2+2p+1-5p-4 < 0$

$p^2-3p-3 < 0$

If $p=4 \Rightarrow p^2-3p-3 > 0$

$\therefore$ largest value of $p = 3$.

54. Option (b) is correct.

Explanation:

$x < 20$

$y < 20$

$x+y = \text{prime}$

$x-y = \text{prime}$

one of x, y must be even to get $x + y$ and $x - y$ prime,

$\therefore \quad y = 2$

If $x = 5 \Rightarrow \begin{cases} x+y=7 \\ x-y=3 \end{cases}$

If $x = 9 \Rightarrow \begin{cases} x+y=11 \\ x-y=7 \end{cases}$

Therefore only two combinations possible

55. Option (a) is correct.

Explanation:

$(x+1)(x+p)(x^2+p^2) = (x^4-1)$

$(x+1)(x+p)(x^2+p^2) = (x^2+1)(x^2-1)$

$= (x^2+1)(x+1)(x-1)$

$\therefore \quad p = -1$

56. Option (a) is correct.

Explanation:

$(2+\sqrt{3})^x+(2-\sqrt{3})^x = 2$

$\text{Let}(2+\sqrt{3})^x = t$

$(2-\sqrt{3})^x = \frac{1}{t}$

$t+\frac{1}{t} = 2$

$t^2+1 = 2t$

$t^2-2t+1 = 0$

$(t-1)^2 = 0$

$t = 1$

$\therefore \quad (2+\sqrt{3})^x = 1$

$\Rightarrow \quad x = 0$

$\therefore (2+\sqrt{3})^x-(2-\sqrt{3})^x$

$= (2+\sqrt{3})^0-(2-\sqrt{3})^0$

$= 1-1$

$= 0$

57. Option (b) is correct.

Explanation:

$$\frac{1}{a}+\frac{1}{b} = \frac{5}{6} \quad ...(i)$$

$$\frac{1}{a^2}+\frac{1}{b^2} = \frac{13}{36} \quad ...(ii)$$

Squaring equation (i) we get,

$$\left(\frac{1}{a}+\frac{1}{b}\right)^2 = \left(\frac{5}{6}\right)^2$$

$$\frac{1}{a^2}+\frac{1}{b^2}+\frac{2}{ab} = \frac{25}{36}$$

$$\frac{13}{36}+\frac{2}{ab} = \frac{25}{36}$$

$$\frac{2}{ab} = \frac{25}{36}-\frac{13}{36}$$

$$\frac{2}{ab} = \frac{12}{36}$$

$$\frac{1}{ab} = \frac{6}{36}$$

$$\frac{1}{a^3}+\frac{1}{b^3} = \left(\frac{1}{a}+\frac{1}{b}\right)\left(\frac{1}{a^2}+\frac{1}{b^2}-\frac{1}{ab}\right)$$

$$= \frac{5}{6}\left(\frac{13}{36}-\frac{6}{36}\right)$$

$$= \frac{5}{6}\left(\frac{7}{36}\right)$$

$$= \frac{35}{216}$$

SHORTCUT: Use $a = 2$ and $b = 3$

$$\frac{1}{a}+\frac{1}{b} = \frac{1}{2}+\frac{1}{3} = \frac{5}{6}$$

$$\frac{1}{a^2}+\frac{1}{b^2} = \frac{1}{2^2}+\frac{1}{3^2} = \frac{13}{36}$$

Satisfy the equation calculate $\frac{1}{a^3}+\frac{1}{b^3}$:

$$\frac{1}{a^3}+\frac{1}{b^3} = \frac{1}{2^3}+\frac{1}{3^3} = \frac{1}{8}+\frac{1}{27}$$

$$\frac{1}{a^3}+\frac{1}{b^3} = \frac{27}{216}+\frac{8}{216} = \frac{35}{216}$$

58. Option (a) is correct.

Explanation:

Let $p = x^6+1-1$

$p = (x^2)^3+1-1$

$p = (x^2+1)(x^4-x^2+1)-1$

$\therefore$ If p is divided by $x^2 + 1$, then remainder is –1

59. Option (a) is correct.

Explanation:

Let $p(x) = x^5-4x^4-3x^3+8x^2-14x+12$

$p(-2) = -32-64+24+32+28+12$

$= 0$

$\therefore$ $(x + 2)$ is a factor of option (a).

60. Option (c) is correct.

Explanation:

Let $N = 60^{60}$

$\log_{10}N = \log_{10}(60)^{60}$

$\log_{10}N = 60\log_{10}60$

$= 60(\log_{10}10+\log_{10}2+\log_{10}3)$

$= 60(1+0.301+0.477)$

$\log_{10}N = 106.68$

$\therefore$ $N = (10)^{106.68}$

$N = (10)^{106}\times(10)^{0.68}$

$\because 1 < 10^{0.68} < 9$

$\therefore$ Number of digits

$= 106+1$

$= 107$

61. Option (a) is correct.

Explanation:

$$\frac{(a+b)^2}{(c-a)(c+a+b)}+\frac{(a+b)c}{c^2+bc-a^2-ab}-\frac{(a+2b+c)}{2(c-a)},$$

$a \neq b, b \neq c, c \neq a$

$$= \frac{(a+b)^2}{(c-a)(c+a+b)}+\frac{(a+b)c}{(c+a)(c-a)+b(c-a)}-\frac{a+2b+c}{2(c-a)}$$

$$= \frac{1}{c-a}\left\{\frac{(a+b)^2}{a+b+c}+\frac{(a+b)c}{a+b+c}-\frac{a+2b+c}{2}\right\}$$

$$= \frac{1}{c-a}\left\{\frac{(a+b)^2+(a+b)c}{a+b+c}-\frac{a+2b+c}{2}\right\}$$

$$= \frac{1}{c-a}\left\{\frac{(a+b)(a+b+c)}{(a+b+c)}-\left(\frac{a+2b+c}{2}\right)\right\}$$

$$= \frac{1}{c-a}\left\{a+b-\left(\frac{a+2b+c}{2}\right)\right\}$$

$$= \frac{1}{c-a}\left\{\frac{2a+2b-a-2b-c}{2}\right\}$$

$$= \frac{1}{c-a}\left\{\frac{a-c}{2}\right\}$$

$$= -\frac{1}{2}$$

SHORTCUT:

$$\frac{(a+b)^2}{(c-a)(c+a+b)}+\frac{(a+b)c}{c^2+bc-a^2-ab}-\frac{(a+2b+c)}{2(c-a)}$$

Substitute $a = 1$, $b = 2$, and $c = 3$:

$$\frac{(1+2)^2}{(3-1)(3+1+2)}+\frac{(1+2)\cdot 3}{3^2+2\cdot 3-1^2-1\cdot 2}-\frac{(1+2\cdot 2+3)}{2(3-1)}$$

$$\frac{3}{4}+\frac{3}{4}-2=-\frac{1}{2}$$

62. Option (a) is correct.

Explanation:

$$a^b=b^a \Rightarrow a=(b)^{\frac{a}{b}} \quad ...(i)$$

$$\frac{a\times\left(\frac{a}{b}\right)^{\frac{a}{b}}}{a^{\left(\frac{a}{b}\right)}}$$

$$=\frac{a\times(a)^{\frac{a}{b}}}{(a)^{\frac{a}{b}}\times(b)^{\frac{a}{b}}}$$

$$=\frac{a}{(b)^{\frac{a}{b}}}$$

$= 1$ {using equation (i)}

63. Option (c) is correct.

Explanation:

$$x = 2+2^{\frac{1}{2}}+2^{\frac{3}{2}}$$

$$x-2 = \sqrt{2}+2\sqrt{2}$$

$$x-2 = 3\sqrt{2}$$

Squaring on both sides we get,

$$(x-2)^2 = (3\sqrt{2})^2$$

$$x^2-4x+4 = 18$$

$$x^2-4x+4-14 = 18-14$$

$$x^2-4x-10 = 4$$

64. Option (d) is correct.

Explanation:

$$\frac{\sqrt{p+x}+\sqrt{p-x}}{\sqrt{p+x}-\sqrt{p-x}} = p$$

$$\frac{\sqrt{p+x}+\sqrt{p-x}}{\sqrt{p+x}-\sqrt{p-x}} = \frac{p}{1}$$

Apply componendo and dividendo theorem

$$\frac{A}{B} = \frac{C}{D}\Rightarrow\frac{A+B}{A-B}=\frac{C+D}{C-D}$$

$$\therefore \quad \frac{2\sqrt{p+x}}{2\sqrt{p-x}} = \frac{p+1}{p-1}$$

Squaring on both sides we get,

$$\frac{p+x}{p-x} = \frac{(p+1)^2}{(p-1)^2}$$

$$\frac{p+x}{p-x} = \frac{p^2+2p+1}{p^2-2p+1}$$

Again apply Componendo and dividendo theorem

$$\frac{2p}{2x} = \frac{2p^2+2}{4p}$$

$$4p^2 = 2(p^2+1)x$$

$$x = \frac{2p^2}{p^2+1}$$

65. Option (b) is correct.

Explanation:

$$\left(\frac{a-b}{2}\right)x^2-\left(\frac{a+b}{2}\right)x+b=0$$

Let $$p(x)=\left(\frac{a-b}{2}\right)x^2-\left(\frac{a+b}{2}\right)x+b$$

$$P(1)=\frac{a-b}{2}-\left(\frac{a+b}{2}\right)+b$$

$$P(1) = 0$$

$\therefore$ 1 is a root of equation

$$\text{product of roots} = \frac{2b}{a-b}$$

Hence, other root is $\frac{2b}{a-b}$

66. Option (b) is correct.

Explanation:

$$x-\frac{1}{x} = 2, x>0$$

$$\left(x-\frac{1}{x}\right)^2 = 4$$

$$x^2+\frac{1}{x^2}-2 = 4$$

$$x^2+\frac{1}{x^2}-2+4 = 4+4$$

$$\left(x+\frac{1}{x}\right)^2 = 8$$

$$\therefore \quad x+\frac{1}{x} = 2\sqrt{2}$$

Hence, $x^2-\frac{1}{x^2} = 4\sqrt{2}$

67. Option (a) is correct.

Explanation:

$$(a-b)^2+(b-c)^2+(c-a)^2 = 6 \quad \text{...(i)}$$

$$a^2+b^2+c^2 = 29 \quad \text{...(ii)}$$

from equation (i),

$$a^2+b^2-2ab+b^2+c^2-2bc+c^2+$$
$$a^2-2ac = 6$$
$$2(a^2+b^2+c^2-ab-bc-ca) = 6$$
$$a^2+b^2+c^2-ab-bc-ca = 3 \quad \text{...(iii)}$$

from equation (ii) and (iii) we get,

$$29-ab-bc-ca = 3$$
$$ab+bc+ca = 26$$
$$a^2+b^2+c^2+2ab+2bc+2ca = 29+2\times 26$$
$$(a+b+c)^2 = 29+52$$
$$(a+b+c)^2 = 81$$
$$\therefore a+b+c = \pm 9$$

68. Option (c) is correct.

Explanation:

$$p = \frac{\sqrt{5}-2}{\sqrt{5}+2} \text{ and } q = \frac{\sqrt{5}+2}{\sqrt{5}-2},$$
$$p\times q = \frac{\sqrt{5}-2}{\sqrt{5}+2}\times\frac{\sqrt{5}+2}{\sqrt{5}-2}$$
$$p\times q = 1$$
$$\frac{p}{q}+\frac{q}{p} = \frac{p^2+q^2}{pq}$$
$$= p^2+q^2$$
$$= \left(\frac{\sqrt{5}-2}{\sqrt{5}+2}\right)^2+\left(\frac{\sqrt{5}+2}{\sqrt{5}-2}\right)^2$$
$$= \frac{5+4-4\sqrt{5}}{5+4+4\sqrt{5}}+\frac{5+4+4\sqrt{5}}{5+4-4\sqrt{5}}$$
$$= \frac{9-4\sqrt{5}}{9+4\sqrt{5}}+\frac{9+4\sqrt{5}}{9-4\sqrt{5}}$$
$$= \frac{(9-4\sqrt{5})^2+(9+4\sqrt{5})^2}{(9+4\sqrt{5})(9-4\sqrt{5})}$$
$$= \frac{2\{9^2+(4\sqrt{5})^2\}}{9^2-(4\sqrt{5})^2}$$
$$= \frac{2\{81+80\}}{81-80}$$
$$= 322$$

69. Option (d) is correct.

Explanation:

Let $N = (25)^{10}$

$$N = (625)^5$$
$$N = (620+5)^5$$
$$N = (620)^5+25\times(620)^4+10\times(620)^3\times 5^2$$
$$+10\times(620)^2\times 5^3+5^5\times(620)+5^5$$
$$N = (620)^5+25\times(620)^4+10\times(620)^3\times 5^2$$
$$+10\times(620)^2\times 5^3+1937500+3125$$

$\therefore$ digit at hundreds place = 5+1
= 6

70. Option (d) is correct.

Explanation: Number divisible by 4, 6, 7, 9

$$= 4\times 7\times 9$$
$$= 252$$

Least four digit number $= 252\times 4+3$
$= 1011$

71. Option (a) is correct.

Explanation:

$$N = (17)^{25}+(19)^{25}$$
$$(17)^{25} = (18-1)^{25}$$

$$= {}^{25}C_0(18)^{25} - {}^{25}C_1(18)^{24} + \cdots + {}^{25}C_{24}18 - 1$$

$$(19)^{25} = (18+1)^{25}$$

$$= {}^{25}C_0(18)^{25} + {}^{25}C_1(18)^{24} + \cdots + {}^{25}C_{24}18 + 1$$

$$\therefore \quad (17)^{25} + (19)^{25} = 2\times\{(18)^{25} + {}^{25}C_2(18)^{23} + \cdots + 25\times 18\}$$

Therefore, remainder is zero.

SHORTCUT:

$$\frac{117^{25}+19^{25}}{18} = (-1)^{25} + (1)^{25}$$

25 is an odd number, $(-1)^2 5 = -1$.

$$\frac{117^{25}+19^{25}}{18} = 0$$

72. Option (c) is correct.

Explanation:

$$p = \frac{x^3+y^3}{x^2-xy+y^2}$$

$$p = \frac{(x+y)(x^2-xy+y^2)}{x^2-xy+y^2}$$

$$p = x+y$$

$$q = \frac{x^3-y^3}{x^2+xy+y^2}$$

$$q = \frac{(x-y)(x^2+xy+y^2)}{x^2+xy+y^2}$$

$$q = x-y$$

HCF of $x, y = H$

Let $x = a\times H$

$y = b\times H$

$\therefore \quad p = a\times H + b\times H$

$= H\times(a+b)$

$q = H\times(a-b)$

HCF of p and q canbe H or 2H.

73. Option (b) is correct.

Explanation:

$$x^4 = x^2 + 1$$

$$x^4 - x^2 - 1 = 0$$

$$x^2 = \frac{1\pm\sqrt{1+4}}{2}$$

$\therefore \quad x^2 > 0$

$\therefore \quad x^2 = \frac{1+\sqrt{5}}{2}$

$$x^4 = \frac{1+5+2\sqrt{5}}{4}$$

$$2x^4 = \frac{6+2\sqrt{5}}{2}$$

$$2x^4 = 3+\sqrt{5}$$

74. Option (c) is correct.

Explanation:

$$\frac{p+q}{q+r} = \frac{r+s}{s+p}$$

$$ps+qs+p^2+pq = qr+r^2+qs+rs$$

$$ps+p^2+pq = qr+r^2+rs$$

$$p^2+pq+ps-qr-r^2-rs = 0$$

$$(p+r)(p-r)+q(p-r)+s(p-r) = 0$$

$$(p-r)(p+q+r+s) = 0$$

either $p = r$ or $p+q+r+s = 0$

75. Option (a) is correct.

Explanation:

$n \le 6$

Let $a = 2(6n+1)$

$b = (8n+1)$

If $n=1$, $\left.\begin{matrix} a=2\times 7 \\ b=9 \end{matrix}\right\}$ relatively prime

If $n=2$, $\left.\begin{matrix} a=2\times 13 \\ b=17 \end{matrix}\right\}$ relatively prime

If $n=3$, $\left.\begin{matrix} a=2\times 19 \\ b=25 \end{matrix}\right\}$ relatively prime

If $n=4$, $\left.\begin{matrix} a=2\times 25 \\ b=33 \end{matrix}\right\}$ relatively prime

If $n=5$, $\left.\begin{matrix} a=2\times 31 \\ b=41 \end{matrix}\right\}$ relatively prime

If $n=6$, $\left.\begin{matrix} a=2\times 37 \\ b=49 \end{matrix}\right\}$ relatively prime

76. Option (c) is correct.

Explanation:

Let $a = x^3+y^3+3xy-1$

$b = (x+y)^4 - 1$

$a = x^3+y^3+(-1)^3-3(x)(y)(-1)$

$= (x+y-1)(x^2+y^2+1-xy+x+y)$

$b = ((x+y)^2+1)((x+y)^2-1)$

$b = ((x+y)^2+1)(x+y+1)(x+y-1)$

$\therefore \text{HCF}(a,b) = x+y-1$

SHORTCUT :

$$\frac{117^{25}+19^{25}}{18} = (-1)^{25}+(1)^{25}$$

25 is an odd number, $(-1)^2 5 = -1$.

$$\frac{117^{25}+19^{25}}{18} = 0$$

77. Option (b) is correct.

Explanation:

$$x = n(n+1)(n+2)$$
$$n = 2a,\ a \in N$$
$$\therefore \quad x = 2a(2a+1)(2a+2)$$
$$x = 4a(a+1)(2a+1)$$

If $a = 1$

$$x = 4\times 2\times 3$$

x is not divisible by 48 .

$$x^2 = 16a^2(a+1)^2(2a+1)^2$$

$\because a(a+1)(2a+1)$ is always divisible by 6.

$\therefore x^2$ is always divisible by 16×6^2

$$= 4\times 144$$

78. Option (b) is correct.

Explanation:

Let $a = x^4+x^2y^2+y^4$

$b = x^3y+y^4$

$c = x^4y^2-x^3y^3$

Now, $b = y(x^3+y^3)$

$b = y(x+y)(x^2+y^2-xy)$

$c = x^3y^2(x-y)$

$a = x^4+2x^2y^2+y^4-x^2y^2$

$= (x^2+y^2)^2-(xy)^2$

$= (x^2+y^2+xy)(x^2+y^2-xy)$

LCM of $(a,b,c) = x^3y^2(x-y)(x+y)$

$(x^4+x^2y^2+y^4)$

$= x^3y^2(x^2-y^2)(x^4+x^2y^2+y^4)$

$= x^3y^2(x^6-y^6)$

79. Option (a) is correct.

Explanation:

$XYZ = 100x+10y+z$

$ZYX = 100z+10y+x$

$D = |xyz-zyx|$

$D = |100x+10y+z-100z-10y-x|$

$= 99|x-z|$

D is divisible by 99.

80. Option (c) is correct.

Explanation: $(p+q)^{p+q}$ is divisible by 512

If $p+q = 4 \Rightarrow (p+q)^{p+q} = 4^4$

$= 256$

Not possible

$p+q = 6$ is also not possible

If $(p+q) = 8$, then

$(p+q)^{p+q} = 8^8$

8^8 is divisible by 512.

(Q – 81 – 82)

$p\sin^2\alpha + q\cos^2\alpha = m$ ...(i)

$q\sin^2\beta + p\cos^2\beta = n$ ...(ii)

81. Option (c) is correct.

Explanation: Divide by $\cos^2\alpha$ in equation (i),

$p\tan^2\alpha + q = m\sec^2\alpha$

$p\tan^2\alpha + q = m(1+\tan^2\alpha)$

$p\tan^2\alpha - m\tan^2\alpha = m-q$

$$\tan^2\alpha = \frac{m-q}{p-m} \quad ...(iii)$$

Divide by $\cos^2\beta$ in equation (ii),

$q\tan^2\beta + p = n\sec^2\beta$

$q\tan^2\beta + p = n(1+\tan^2\beta)$

$q\tan^2\beta - n\tan^2\beta = n-p$

$$\tan^2\beta = \frac{n-p}{q-n} \quad ...(iv)$$

From equations (iii) and (iv) we get,

$$\left(\frac{\tan\alpha}{\tan\beta}\right)^2 = \left(\frac{m-q}{p-m}\right)\left(\frac{q-n}{n-p}\right)$$
$$= \left(\frac{m-q}{m-p}\right)\left(\frac{n-q}{n-p}\right)$$

82. Option (d) is correct.

Explanation:

$$\alpha+\beta = \frac{\pi}{2}$$
$$\alpha = \frac{\pi}{2}-\beta$$
$$\tan\alpha = \tan\left(\frac{\pi}{2}-\beta\right)$$
$$\tan\alpha = \cot\beta$$
$$\tan\alpha = \frac{1}{\tan\beta}$$
$$\tan\alpha\ \tan\beta = 1 \quad ...(v)$$

From equations (i), (ii) and (v) we get,

$$\frac{m-q}{p-m}\times\frac{n-p}{q-n} = 1$$

$$\frac{mn-qn-pm+pq}{pq-mq-np+mn}=1$$

$$mn-qn-pm+pq=pq-mq-np+mn$$

$$-qn-pm=-mq-np$$

$$mq-qn+np-pm=0$$

$$q(m-n)+p(n-m)=0$$

$$(m-n)(q-p)=0$$

Either $m-n=0$ or $q-p=0$

(Q 83 – 84)

$$\text{cosec}\theta-\sin\theta=p$$

$$\frac{1}{\sin\theta}-\sin\theta=p$$

$$1-\sin^2\theta=p\sin\theta$$

$$\cos^2\theta=p\sin\theta \quad ...(i)$$

$$\sec\theta-\cos\theta=q$$

$$\frac{1}{\cos\theta}-\cos\theta=q$$

$$1-\cos^2\theta=q\cos\theta$$

$$\sin^2\theta=q\cos\theta \quad ...(ii)$$

83. Option (c) is correct.

Explanation: From equations (i) and (ii),

$$\cos^2\theta+\sin^2\theta=p\sin\theta+q\cos\theta$$

$$p\sin\theta+q\cos\theta=1$$

84. Option (b) is correct.

Explanation:

$$p\sin\theta=\cos^2\theta \quad ...(i)$$

$$q\cos\theta=\sin^2\theta \quad ...(ii)$$

$$p^2=\frac{\cos^4\theta}{\sin^2\theta}$$

$$q^2=\frac{\sin^4\theta}{\cos^2\theta}$$

$$p^2+q^2+3=\frac{\cos^4\theta}{\sin^2\theta}+\frac{\sin^4\theta}{\cos^2\theta}+3$$

$$=\frac{\cos^6\theta+\sin^6\theta+3\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta}$$

$$=\frac{(\cos^2\theta+\sin^2\theta)(\cos^4\theta+\sin^4\theta-\sin^2\theta\cos^2\theta)+3\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta}$$

$$=\frac{\cos^4\theta+\sin^4\theta-\sin^2\theta\cos^2\theta+3\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta}$$

$$=\frac{\cos^4\theta+\sin^4\theta+2\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta}$$

$$=\frac{(\cos^2\theta+\sin^2\theta)^2}{\sin^2\theta\cos^2\theta}$$

$$=\frac{1}{\sin^2\theta\cos^2\theta}$$

$$p^2+q^2+3=\frac{1}{p^2q^2}$$

$$p^2q^2(p^2+q^2+3)=1$$

SHORTCUT:

Put $\theta=45°$:

Given:

- $p=\csc\theta-\sin\theta=\sqrt{2}-\frac{\sqrt{2}}{2}=\frac{\sqrt{2}}{2}$
- $q=\sec\theta-\cos\theta=\sqrt{2}-\frac{\sqrt{2}}{2}=\frac{\sqrt{2}}{2}$

Q. 83. $p\sin\theta+q\cos\theta$

$$p\sin\theta+q\cos\theta=\left(\frac{\sqrt{2}}{2}\right)\cdot\frac{\sqrt{2}}{2}+\left(\frac{\sqrt{2}}{2}\right)\cdot\frac{\sqrt{2}}{2}=1$$

Q. 84. $p^2q^2(p^2+q^2+3)$

$$p^2=q^2=\frac{1}{2},\ p^2q^2=\frac{1}{4}$$

$$p^2q^2(p^2+q^2+3)=\frac{1}{4}\cdot(1+3)=1$$

(Q 85 – 86)

$$\frac{\sin\alpha}{\sin\beta}=\frac{4\sqrt{2}}{3}$$

$$3\sin\alpha=4\sqrt{2}\sin\beta \quad ...(i)$$

$$\frac{\cos\alpha}{\cos\beta}=\frac{2\sqrt{3}}{9}$$

$$9\cos\alpha=2\sqrt{2}\cos\beta \quad ...(ii)$$

85. Option (a) is correct.

Explanation: From equations (i) and (ii) we get,

$$9\sin^2\alpha=32\sin^2\beta \quad ...(iii)$$

$$81\cos^2\alpha=12\cos^2\beta \quad ...(iv)$$

$$\sin^2\beta+\cos^2\beta=\frac{9}{32}\sin^2\alpha+\frac{81}{12}\cos^2\alpha$$

$$1=\frac{9}{32}\sin^2\alpha+\frac{27}{4}\cos^2\alpha$$

Divide by $\cos^2\alpha$,

$$\sec^2\alpha=\frac{9}{32}\tan^2\alpha+\frac{27}{4}$$

$$1+\tan^2\alpha=\frac{9}{32}\tan^2\alpha+\frac{27}{4}$$

$$\tan^2\alpha-\frac{9}{32}\tan^2\alpha=\frac{27}{4}-1$$

$$\frac{23}{32}\tan^2\alpha=\frac{23}{4}$$

$$\tan^2\alpha=8$$

86. Option (c) is correct.

Explanation: From equations (i) and (ii) we get,

$$9\sin^2\alpha = 32\sin^2\beta \quad ...(iii)$$

$$81\cos^2\alpha = 12\cos^2\beta \quad ...(iv)$$

$$\sin^2\alpha+\cos^2\alpha = \frac{32}{9}\sin^2\beta+\frac{12}{81}\cos^2\beta$$

$$1 = \frac{32}{9}\sin^2\beta+\frac{4}{27}\cos^2\beta$$

Divide by $\cos^2\beta$,

$$\sec^2\beta = \frac{32}{9}\tan^2\beta+\frac{4}{27}$$

$$1+\tan^2\beta = \frac{32}{9}\tan^2\beta+\frac{4}{27}$$

$$1-\frac{4}{27} = \frac{32}{9}\tan^2\beta-\tan^2\beta$$

$$\frac{23}{27} = \frac{23}{9}\tan^2\beta$$

$$\tan^2\beta = \frac{1}{3}$$

(Q 87 - 88)

$$\frac{1+\sin\theta}{\cos\theta} = p+\sqrt{p^2+1}$$

$$\sec\theta+\tan\theta = p+\sqrt{p^2+1} \quad ...(i)$$

$$\frac{(\sec\theta-\tan\theta)(\sec\theta+\tan\theta)}{\sec\theta-\tan\theta} = p+\sqrt{p^2+1}$$

$$\frac{\sec^2\theta-\tan^2\theta}{\sec\theta-\tan\theta} = p+\sqrt{p^2+1}$$

$$\frac{1}{\sec\theta-\tan\theta} = p+\sqrt{p^2+1}$$

$$\sec\theta-\tan\theta = \frac{1}{\sqrt{p^2+1}+p}$$

$$\sec\theta-\tan\theta = \frac{\sqrt{p^2+1}-p}{(\sqrt{p^2+1}+p)(\sqrt{p^2+1}-p)}$$

$$= \frac{\sqrt{p^2+1}-p}{p^2+1-p^2}$$

$$\sec\theta-\tan\theta = \sqrt{p^2+1}-p \quad ...(ii)$$

87. Option (b) is correct.

Explanation: Adding equations (i) and (ii) we get,

$$2\sec\theta = 2\sqrt{p^2+1}$$

$$\sec\theta = \sqrt{p^2+1}$$

88. Option (a) is correct.

Explanation:

$$\sec\theta = \sqrt{p^2+1}$$

$$\sec^2\theta = 1+p^2$$

$$1+\tan^2\theta = 1+p^2$$

$$\tan^2\theta = p^2$$

$$\tan\theta = p$$

(Q 89 – 90)

$$\sin\theta+\cos\theta = p \quad ...(i)$$

$$\sec\theta+\operatorname{cosec}\theta = q$$

$$\frac{1}{\cos\theta}+\frac{1}{\sin\theta} = q$$

$$\frac{\sin\theta+\cos\theta}{\sin\theta\cos\theta} = q \quad ...(ii)$$

89. Option (b) is correct.

Explanation: From first given equation

$$\sin\theta+\cos\theta = p$$

Squaring on both sides,

$$\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta = p^2$$

$$1+2\sin\theta\cos\theta = p^2$$

$$2\sin\theta\cos\theta = p^2-1$$

Now from second given equation

$$\sec\theta+\operatorname{cosec}\theta = q$$

$$\frac{\sin\theta+\cos\theta}{\sin\theta\cos\theta} = q$$

$$\frac{2p}{p^2-1} = q$$

$$2p = q(p^2-1)$$

90. Option (b) is correct.

Explanation:

$$q = \frac{\sin\theta+\cos\theta}{\sin\theta\cos\theta}$$

$$q = \frac{p}{\sin\theta\cos\theta}$$

$$\frac{q}{p} = \frac{1}{\sin\theta\cos\theta}$$

$$\frac{q}{p} = \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$$

$$\frac{q}{p} = \frac{\sin^2\theta}{\sin\theta\cos\theta}+\frac{\cos^2\theta}{\sin\theta\cos\theta}$$

$$\frac{q}{p} = \tan\theta+\cot\theta$$

91. Option (d) is correct.

Explanation:

$$\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\ldots}}}} = \operatorname{cosec}\theta$$

$$\operatorname{cosec}\theta = \sqrt{2+\operatorname{cosec}\theta}$$

$$\operatorname{cosec}^2\theta = 2+\operatorname{cosec}\theta$$

$$\frac{1}{\sin^2\theta} = 2+\frac{1}{\sin\theta}$$

$$1 = 2\sin^2\theta+\sin\theta$$

$$2\sin^2\theta+\sin\theta-1 = 0$$

$$(2\sin\theta-1)(\sin\theta+1) = 0$$

$$\sin\theta = -1 \text{ or } \frac{1}{2}$$

$\because \quad \operatorname{cosec}\theta > 0$

$\therefore \quad \sin\theta = \frac{1}{2}$

92. Option (c) is correct.

Explanation:

$$8\sin\theta - \cos\theta = 4$$

$$8\sin\theta - 4 = \cos\theta$$

Squaring on both sides we get,

$$64\sin^2\theta + 16 - 64\sin\theta = \cos^2\theta$$

$$64\sin^2\theta - 64\sin\theta + 16 = 1 - \sin^2\theta$$

$$65\sin^2\theta - 64\sin\theta + 15 = 0$$

$$65\sin^2\theta - 39\sin\theta - 25\sin\theta + 15 = 0$$

$$13\sin\theta(5\sin\theta - 3) - 5(5\sin\theta - 3) = 0$$

$$(5\sin\theta - 3)(13\sin\theta - 5) = 0$$

$$\sin\theta = \frac{3}{5} \text{ or } \frac{5}{13}$$

$$\operatorname{cosec}\theta = \frac{5}{3} \text{ or } \frac{13}{5}$$

93. Option (a) is correct.

Explanation:

$$2\tan\theta = \sec^2\theta - 2$$

$$2\tan\theta = 1+\tan^2\theta - 2$$

$$\tan^2\theta - 2\tan\theta - 1 = 0$$

Let $x = \tan\theta$

$$x = \frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-1)}}{2(1)}$$

$$x = \frac{2\pm\sqrt{4+4}}{2} = \frac{2\pm\sqrt{8}}{2}$$

$$= \frac{2\pm 2\sqrt{2}}{2}$$

$$x = 1\pm\sqrt{2}$$

$$(\tan\theta - 1) = \sqrt{2} \qquad \left(\because 0 < \theta < \frac{\pi}{2}\right)$$

$$\tan\theta = \sqrt{2}+1$$

$$\cot\theta = \frac{1}{\sqrt{2}+1}$$

$$= \frac{1}{\sqrt{2}+1}\left(\frac{\sqrt{2}-1}{\sqrt{2}-1}\right)$$

$$= \frac{\sqrt{2}-1}{2-1}$$

$$= \sqrt{2}-1$$

94. Option (a) is correct.

Explanation:

$$(\sec\theta - \tan\theta) - \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$

$$= \sec\theta - \tan\theta - \sqrt{\left(\frac{1-\sin\theta}{1+\sin\theta}\right)\left(\frac{1-\sin\theta}{1-\sin\theta}\right)}$$

$$= \sec\theta - \tan\theta - \frac{1-\sin\theta}{\sqrt{1-\sin^2\theta}}$$

$$= \sec\theta - \tan\theta - \frac{1-\sin\theta}{\sqrt{\cos^2\theta}}$$

$$= \sec\theta - \tan\theta - \frac{1-\sin\theta}{\cos\theta}$$

$$= \sec\theta - \tan\theta - \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)$$

$$= \sec\theta - \tan\theta - (\sec\theta - \tan\theta)$$

$$= 0$$

95. Option (d) is correct.

Explanation:

$$\cot\theta = \sqrt{7}$$

$$\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$$

$$= \frac{\frac{1}{\sin^2\theta} - \frac{1}{\cos^2\theta}}{\frac{1}{\sin^2\theta} + \frac{1}{\cos^2\theta}}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta + \cos^2\theta}$$

$$= \cos^2\theta - \sin^2\theta$$

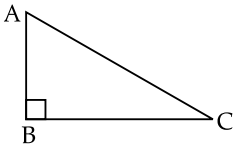
$$= \sin^2\theta\left(\frac{\cos^2\theta}{\sin^2\theta} - 1\right)$$

$$= \frac{1}{\operatorname{cosec}^2\theta}(\cot^2\theta - 1)$$

$$= \frac{\cot^2\theta - 1}{\cot^2\theta + 1}$$
$$= \frac{7-1}{7+1}$$
$$= \frac{6}{8}$$
$$= \frac{3}{4}$$

96. Option (c) is correct.

Explanation:



Let $\angle A > \angle C$

$$\angle A - \angle C = \frac{\pi}{12}$$
$$\angle A + \angle C = \frac{\pi}{2}$$
$$\therefore \quad 2\angle A = \frac{\pi}{12} + \frac{\pi}{2}$$
$$\angle A = \frac{1}{2}\left(\frac{\pi + 6\pi}{12}\right)$$
$$\angle A = \frac{7\pi}{24}$$
$$= 52.5°$$

97. Option (c) is correct.

Explanation:

$$\log_{10}[998 + \sqrt{x^2 - 18x + 76}] = 3$$
$$998 + \sqrt{x^2 - 18x + 76} = 10^3$$
$$\sqrt{x^2 - 18x + 76} = 1000 - 998$$
$$\sqrt{x^2 - 18x + 76} = 2$$
$$x^2 - 18x + 76 = 4$$
$$x^2 - 18x + 72 = 0$$
$$(x - 12)(x - 6) = 0$$
$$x = 12, 6$$
$$(\alpha - \beta)^2 = (12 - 6)^2$$
$$= 36$$

98. Option (c) is correct.

Explanation:

$$x^4 + y^4 = 14x^2y^2$$
$$x^4 + y^4 + 2x^2y^2 = 14x^2y^2 + 2x^2y^2$$
$$(x^2 + y^2)^2 = 16x^2y^2$$

taking log on both side,

$$\log_{10}(x^2 + y^2) = \log_{10}16x^2y^2$$
$$2\log_{10}(x^2 + y^2) = 2\log_{10}(4xy)$$
$$\log_{10}(x^2 + y^2) = \log_{10}4 + \log_{10}x + \log_{10}y$$
$$= 2\log_{10}2 + \log_{10}x + \log_{10}y$$

Statement I is correct.

Now,

$$x^4 + y^4 = 14x^2y^2$$
$$x^4 + y^4 - 2x^2y^2 = 14x^2y^2 - 2x^2y^2$$
$$(x^2 - y^2)^2 = 12x^2y^2$$
$$(x^2 - y^2)^2 = (2\sqrt{3}\ xy)^2$$
$$(x^2 - y^2) = 2\sqrt{3}\ xy$$

taking log on both side,

$$\log_{10}(x^2 - y^2) = \log_{10}(2\sqrt{3}\ xy)$$
$$= \log_{10}2 + \log_{10}\sqrt{3} + \log_{10}x + \log_{10}y$$
$$= \log_{10}2 + 0.5\log_{10}3 + \log_{10}x + \log_{10}y$$

Statement II is correct.

99. Option (c) is correct.

Explanation:

$(3x + y)^2 + (3x + y)(x + 5y) - 20(x + 5y)^2$

Let $3x + y = a$

$x + 5y = b$

$$= a^2 + ab - 20ab$$
$$= (a + 5b)(a - 4b)$$

$a + 5b = 3x + y + 5(x + 5y)$

$$= 8x + 26y = 2(4x + 13y)$$

$a - 4b = 3x + y - 4(x + 5y)$

$$= -x - 19y$$
$$= -(x + 19y)$$

Both $(x + 19y)$ and $(4x + 13y)$ are factors of given expression.

100. Option (b) is correct.

Explanation: Let

$$\frac{\frac{x}{x-y} + \frac{y}{y-z} + \frac{z}{z-x}}{\frac{x+y}{x-y} + \frac{y+z}{y-z} + \frac{z+x}{z-x} + 3} = \frac{p}{q}$$

$$p = \frac{x}{x-y} + \frac{y}{y-z} + \frac{z}{z-x}$$
$$q = \frac{x+y}{x-y} + \frac{y+z}{y-z} + \frac{z+x}{z-x} + 3$$
$$q = \frac{x+y}{x-y} + 1 + \frac{y+z}{y-z} + 1 + \frac{z+x}{z-x} + 1$$
$$= \frac{x+y+x-y}{x-y} + \frac{y+z+y-z}{y-z} + \frac{z+x+z-x}{z-x}$$
$$= \frac{2x}{x-y} + \frac{2y}{y-z} + \frac{2z}{z-x}$$
$$q = 2p$$
$$\therefore \quad \frac{p}{q} = \frac{1}{2}$$