NDA / NA
National Defence Academy / Naval Academy

MATHEMATICS
II

QUESTION PAPER
2025

Time Allowed : 2 hrs 30 min **Total Marks : 300**

Instructions

1. *This Test Booklet contains **120** items (questions). Each item is printed in **English**. Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose **ONLY ONE** response for each item.*
2. *You have to mark all your responses **ONLY** on the separate Answer Sheet provided. See directions in the Answer Sheet.*
3. ***All** items carry equal marks.*
4. *Before you proceed to mark in the Answer Sheet the response to the various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions.*
5. ***Penalty for wrong answers :***
   THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
   (i). *There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.*
   (ii). *If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.*
   (iii). *If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.*

1. If $p^x = q^y = r^z$, where x, y and z are in GP, then consider the following statements :

I. p, q and r are in AP.

II. In p, ln q and In r are in GP.

Which of the statements given above is/are correct?

(a) I only **(b)** II only

(c) Both I and II **(d)** Neither I nor II

2. If A and B are non-empty subsets of a set, and A^c and B^c represent their complements, then which of the following is/are correct?

I. $A - B = B^c - A^c$

II. $A - B^c = A^c - B$

Select the answer using the code given below.

(a) I only **(b)** II only

(c) Both I and II **(d)** Neither I nor II

3. Let $y = x!$ and $z = (2x)!$. If $(z/y) = 120$, then what is the value of $(3x)!$?

(a) 362880 **(b)** 181440 **(c)** 90720 **(d)** 45360

4. Let n be a natural number. The number of consecutive zeros at the end of the expansion of $n!$ is exactly 2. How many values of n are possible?

(a) 3 **(b)** 4

(c) 5 **(d)** More than 5

5. If $(10 + \log_{10} x)$, $(10 + \log_{10} y)$ and $(10 + \log_{10} z)$ are in AP, then consider the following statements:

I. The GM of x and z is y^2.

II. The AM of $\log_{10} x$ and $\log_{10} z$ is $\log_{10} y$.

Which of the statements given above is/are correct?

(a) I only **(b)** II only

(c) Both I and II **(d)** Neither I nor II

6. How many terms of the series $1 + 3 + 5 + 7 + \ldots$ amount to a sum equal to 12345678987654321?

(a) 11111111 **(b)** 110000011

(c) 111101111 **(d)** 111111111

7. How many terms are identical in the two APS 19, 21, 23, ... up to 110 terms and 19, 22, 25, 28,... up to 75 terms?

(a) 35 **(b)** 36 **(c)** 37 **(d)** 38

8. If

$$\alpha = \frac{-1+\sqrt{-3}}{2}$$

then what is the value of

$(1 + \alpha^{19} - \alpha^{35})^{100} - (1 - 3\alpha^{25} + \alpha^{38})^{50}$?

(a) –2 **(b)** –1 **(c)** 0 **(d)** 2

9. What is the remainder when 5^{99} is divided by 13?

(a) 10 **(b)** 9 **(c)** 8 **(d)** 6

10. What is the value of the determinant of the inverse of the matrix

$$\begin{bmatrix} -4 & -5 \\ 2 & 2 \end{bmatrix}?$$

(a) $\frac{1}{2}$ **(b)** 1 **(c)** 2 **(d)** 4

11. In a class of 45 students, 34 like to play cricket and 26 like to play football. Further, each student likes to play at least one of the two games. How many students like to play exactly one game?

(a) 45 **(b)** 30 **(c)** 25 **(d)** 15

12. The system of equations

$$2x - 3y - 5 = 0,\ 15y - 10x + 50 = 0$$

(a) has a unique solution

(b) has infinitely many solutions

(c) is inconsistent

(d) is consistent and has exactly two solutions

13. If

$$\left(\frac{1-i}{1+i}\right)^{2m}\left(\frac{1+i}{1-i}\right)^{2n} = 1$$

where $i = \sqrt{-1}$, then what is the smallest positive value of $(m - n)$?

(a) 1 **(b)** 2 **(c)** 4 **(d)** 8

14. In obtaining the solution of the system of equations $x + y + z = 7$, $x + 2y + 3z = 16$ and $x + 3y + 4z = 22$ by Cramer's rule, the value of y is obtained by dividing D by D_2, where

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{vmatrix}$$

What is the value of the determinant D_2?

(a) –13 **(b)** –3 **(c)** 3 **(d)** 13

15. Consider the following in respect of non-singular matrices A and B :

I. $(AB)^{-1} = A^{-1}B^{-1}$

II. $(BA)(AB)^{-1} = I$, where I is the identity matrix

III. $(AB)^T = A^T B^T$

How many of the above are correct?

(a) None **(b)** One

(c) Two **(d)** All three

16. The value of the determinant

$$\begin{vmatrix} a & b & c \\ l & m & n \\ p & q & r \end{vmatrix}$$

is equal to

(a) $\begin{vmatrix} a & b & c \\ p & q & r \\ l & m & n \end{vmatrix}$ **(b)** $\begin{vmatrix} l & m & n \\ a & b & c \\ p & q & r \end{vmatrix}$

(c) $\begin{vmatrix} p & q & r \\ a & b & c \\ l & m & n \end{vmatrix}$ **(d)** $\begin{vmatrix} a & p & l \\ b & q & m \\ c & r & n \end{vmatrix}$

17. Let 1, ω, ω^2 be three cube roots of unity. If $x = a + b$, $y = a\omega + b\omega^2$, $z = a\omega^2 + b\omega$, then what is $x^2 + y^2 + z^2$ equal to?

(a) $6ab$ **(b)** $3ab$ **(c)** $a^2 + b^2$ **(d)** 1

18. How many 4-digit numbers that are divisible by 4 can be formed using the digits 1, 2, 3 and 4 (repetition of digits is not allowed)?

(a) 3 **(b)** 6 **(c)** 9 **(d)** 12

19. If a, b, c are the sides of a triangle ABC and p is the perimeter of the triangle, then what is

$$\begin{vmatrix} p+c & a & b \\ c & p+a & b \\ c & a & p+b \end{vmatrix}$$

equal to?

(a) p^3 **(b)** $2p^3$ **(c)** $3p^3$ **(d)** $4p^3$

20. Which one of the following is the greatest coefficient in the expansion of $(1+x)^{100}$?

(a) The coefficient of x^{100}

(b) The coefficient of x^{99}

(c) The coefficient of x^{51}

(d) The coefficient of x^{50}

For the following two (02) items:

Let α and β be the roots of the quadratic equation

$$x^2 + (\log_{0.5}(a^2))x + (\log_{0.5}(a^2))^4 = 0$$

where $a^2 \neq 1$ and $\log_{0.5}(a^2) > 0$. Further, $\beta^2 = \alpha(\log_{a^2}(0.5))$.

21. What is β equal to?

(a) $\log_{a^2}(0.5)$ **(b)** $\log_{0.5}(a^2)$

(c) $2(\log_{a^2}(0.5))$ **(d)** $21\log_{0.5}(a^2)$

22. What is the relation between α and β?

(a) $\alpha = 2\beta$ **(b)** $2\alpha = \beta$

(c) $\alpha = -2\beta$ **(d)** $2\alpha = -\beta$

For the following two (02) items:

Let $p = \sum_{j=1}^{n} \log_{10} 2^j$ and $q = \sum_{j=1}^{n} \log_{10} 5^j$.

23. If $p + q = 66$, then which one following is correct?

(a) $n < 7$ **(b)** $7 < n < 9$

(c) $9 < n < 12$ **(d)** $n > 12$

24. If $p + q = 15$, then what is $q - p$ equal to?

(a) $\log_{10} 2.5$ **(b)** $5\log_{10} 2.5$

(c) $10\log_{10} 2.5$ **(d)** $15\log_{10} 2.5$

For the following two (02) items:

Let $\sin A + \sin B = p$ and $\cos A + \cos B = q$.

25. What is $\frac{p}{q}$ equal to?

(a) $\tan\left(\frac{A-B}{2}\right)$ **(b)** $\cot\left(\frac{A-B}{2}\right)$

(c) $\tan\left(\frac{A+B}{2}\right)$ **(d)** $\cot\left(\frac{A+B}{2}\right)$

26. What is $\frac{p^2-q^2}{p^2+q^2}$ equal to?

(a) $\cos(A + B)$ **(b)** $\cos(A - B)$

(c) $\cos\left(\frac{\pi}{2} - A - B\right)$ **(d)** $\cos(\pi - A - B)$

For the following two (02) items:

Let $p = \text{cosec } 20°$ and $q = \text{cosec } 70°$.

27. What is $\left(\frac{\sqrt{3}p}{4} - \frac{q}{4}\right)$ equal to?

(a) -1 **(b)** 0 **(c)** 1 **(d)** 2

28. What is $\frac{p^2+q^2}{p^2q^2}$ equal to?

(a) $\frac{1}{2}$ **(b)** 1 **(c)** $\frac{3}{2}$ **(d)** 2

For the following two (02) items:

Let $\cos(2x + 3y) = \frac{1}{2}$ and $\cos(3x + 2y) = \frac{\sqrt{3}}{2}$, where $-\pi < (2x + 3y) < \pi$ and $-\pi < (3x + 2y) < \pi$.

29. How many values does $(x + y)$ have?

(a) Two **(b)** Three

(c) Four **(d)** More than four

30. How many values does $(y - x)$ have?

(a) Two **(b)** Three

(c) Four **(d)** More than four

For the following two (02) items :

Consider the equation

$$abx^2 + bcx + ca = cax^2 + abx + bc$$

31. If the roots of the equation are equal, then which one of the following is correct?

(a) $ac = b^2$ **(b)** $a + c = 2b$

(c) $\frac{1}{a} + \frac{1}{c} = \frac{1}{2b}$ **(d)** $\frac{1}{a} + \frac{1}{c} = \frac{2}{b}$

32. If the roots of the equation are equal, then a, b, c are in

(a) AP **(b)** GP

(c) HP **(d)** None of the above

For the following two (02) items:

Let (6 + 10 + 14 + ... up to m terms)

= (1 + 3 + 5 + 7 + ... up to n terms)

where $m < 25$ and $n < 25$.

33. What is the relation between m and n?

(a) $n^2 = m(m + 1)$ **(b)** $n^2 = m(m + 2)$

(c) $n^2 = 2m(m + 1)$ **(d)** $n^2 = 2m(m + 2)$

34. How many values of m are possible?

(a) None **(b)** One

(c) Two **(d)** More than two

For the following two (02) items :

There are 8 points on a plane out of which 4 points are collinear.

35. How many triangles can be formed by joining these points?

(a) 56 **(b)** 54 **(c)** 53 **(d)** 52

36. How many quadrilaterals can be formed by joining these points?

(a) 70 **(b)** 69

(c) 53 **(d)** None of the above

For the following two (02) items:

Let $f(x) = ax^2 + bx + c$ be a quadratic polynomial such that $f(1) = f(4) = 2$. Further, 2 is a root of $f(x) = 0$.

37. What is the other root of $f(x) = 0$?

(a) 1

(b) 2

(c) 3

(d) Cannot be determined

38. What is $(a + b + c)$ equal to?

(a) 0

(b) 1

(c) 2

(d) Cannot be determined

For the following two (02) items:

Let

$$A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

39. What is the value of the determinant of the matrix A^4?

(a) 0 **(b)** 1

(c) $\cos 4\theta - \sin 4\theta$ **(d** $\cos^2 4\theta - \sin^2 4\theta$

40. What is $[\text{adj } A]^{-1}$ equal to?

(a) $-A$ **(b)** $-A^T$ **(c)** A **(d)** A^T

41. What is the sum of the binary numbers $(101101101)_2$ and $(100011)_2$?

(a) $(110010000)_2$ **(b)** $(110001000)_2$

(c) $(110000100)_2$ **(d)** $(100100000)_2$

42. Set X contains $3n$ elements and set Y contains $2n$ elements, and they have n elements in common elements does $(X-Y)\times(Y-X)$ have?

(a) $5n^2$ **(b)** $4n^2$ **(c)** $3n^2$ **(d)** $2n^2$

43. Let A=(–3, –2, –1, 0, 1, 2, 3} and B= {0, 1, 4, 9). How many elements does the subset of $A \times B$ corresponding to the relation $R = \{(x, y) : |x| < y\}$ have, where $x \in A$ and $y \in B$?

(a) 9 **(b)** 12 **(c)** 15 **(d)** 16

44. Consider the following statements:

Statement-I: If X is an $n \times n$ matrix, then det $(mX) = m^n$ det(X), where m is a scalar.

Statement-II: If Y is a matrix obtained from X by multiplying any row or column by a scalar m, then det$(Y) = m$det (X).

Which one of the following is correct in respect of the above statements?

(a) Both Statement-I and Statement-II are correct and Statement-II explains Statement-I

(b) Both Statement-I and Statement-II are correct but Statement-II does not explain Statement-I

(c) Statement-I is correct but Statement-II is not correct

(d) Statement-I is not correct but Statement-II is correct

45. Consider the following statements about the matrix $M=\begin{bmatrix}71 & 23 & 48\\ 57 & 28 & 29\\ 65 & 17 & 48\end{bmatrix}$:

Statement I: The inverse of M does not exist.

Statement II: M is non-singular.

Which one of the following is correct in respect of the above statements?

(a) Both Statement-I and Statement-II are correct and explains Statement-I Statement-II

(b) Both Statement-I and Statement-II are correct but Statement-II does not explain Statement-I

(c) Statement-I is correct but Statement-II is not correct

(d) Statement-I is not correct Statement-II is correct

46. What is

$$\cot^{-1}9+\text{cosec}^{-1}\left(\frac{\sqrt{41}}{4}\right)$$

equal to?

(a) $\frac{\pi}{4}$ **(b)** $\frac{\pi}{3}$ **(c)** $\frac{\pi}{2}$ **(d)** π

47. How many values of θ, where $-\pi < \theta < \pi$, satisfy both the equations $\cot\theta = -\sqrt{3}$ and $\text{cosec}\theta = -2$ simultaneously?

(a) 4 **(b)** 2 **(c)** 1 **(d)** None

48. If $x+\frac{1}{x}=2\cos\theta$, then what is $x^3+\frac{1}{x^3}$ equal to?

(a) $\cos^3\theta$ **(b)** $\cos 3\theta$

(c) $2\cos 3\theta$ **(d)** $3\cos 3\theta$

49. If $0 \le x \le \frac{\pi}{2}$, then what is the number of values of x satisfying the equation

$$\tan x + \sec x = 2\cos x?$$

(a) 0 **(b)** 1 **(c)** 2 **(d)** 3

50. What is the value of

$$\tan\left[\frac{1}{2}\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)\right]?$$

(a) $2-\sqrt{3}$ **(b)** $2+\sqrt{3}$ **(c)** $\sqrt{3}-1$ **(d)** $\sqrt{3}+1$

For the following two (02) items:

A plane P is parallel to the line having direction ratios $\langle 1, 3, 2\rangle$ and contains the line of intersection of the planes $6x + 4y - 5z = 2$ and $x - 2y + 3z = 0$.

51. Which of the following are the direction ratios of the line of intersection of the given planes?

(a) $\langle 2, 23, 16\rangle$ **(b)** $\langle 2, -23, -16\rangle$

(c) $\langle 2, 3, 2\rangle$ **(d)** $\langle -1, 3, -2\rangle$

52. What is the equation of the plane P?

(a) $2x - 20y + 29z + 2 = 0$

(b) $2x - 20y + 29z - 2 = 0$

(c) $2x + 3y + 2z - 4 = 0$

(d $x - 3y + 2z + 5 = 0$

For the following two (02) items :

Suppose S is the sphere with the smallest radius that passes through the points $A(1, 0, 0)$, $B(0, 1, 0)$ and $C(0, 0, 1)$.

53. What is the radius of S?

(a) $\sqrt{\frac{1}{3}}$ **(b)** $\sqrt{\frac{2}{3}}$ **(c)** $\frac{1}{3}$ **(d)** 1

54. On which one of the following planes does the centre of S lie?

(a) $x + y + z - 1 = 0$

(b) $x + y + z + 1 = 0$

(c) $3x + 3y + 3z - 1 = 0$

(d) $3x + 3y + 3x + 1 = 0$

For the following two (02) items:

Let $A(1, -1, 0)$, $B(-2, 1, 8)$ and $C(-1, 2, 7)$ are three consecutive vertices of a parallelogram $ABCD$.

55. What is the fourth vertex D?

(a) $(0, -2, 1)$ **(b)** $(2, 0, -1)$

(c) $(1, 0, 1)$ **(d)** $(1, 2, 0)$

56. If angle BCD is θ, then what is $\cos^2\theta$ equal to?

(a) $\frac{26}{77}$ **(b)** $\frac{27}{77}$ **(c)** $\frac{82}{237}$ **(d)** $\frac{83}{237}$

57. For different values of m, the equation $4y = mx - m + 2$ represents

(a) parallel lines

(b) concurrent lines

(c) lines at a fixed distance from the origin of coordinates

(d) the same line

58. The equation of the locus of a point equidistant from the points (a, b) and (c, d) is $(a - c)x + (b - d)y + k = 0$. What is the value of k?

(a) $a^2 - c^2 + b^2 - d^2$

(b) $c^2 + d^2 - a^2 - b^2$

(c) $(a^2 - c^2 + b^2 - d^2)/2$

(d $(c^2 + d^2 - a^2 - b^2)/2$

59. Consider the following statements in respect of the equation $x^2 + 3y = 0$:

I. The equation represents equation to parabola that opens upwards.

II. The axis of the parabola is $x = 0$.

III. The equation of the latus rectum is $4y - 3 = 0$.

How many of the statements given above are correct?

(a) None **(b)** One

(c) Two **(d)** All three

60. What is the sum of the intercepts of the line

$$\frac{x}{a^2} + \frac{y}{b^2} = \frac{2}{a^2 + b^2}$$

on the coordinate axes?

(a) 2 **(b)** 1 **(c)** $\frac{1}{2}$ **(d)** $a^2 + b^2$

61. Which one of the following is perpendicular form of the straight line $\sqrt{3}x + 2y = 7$?

(a) $y = -\frac{\sqrt{3}}{2}x + \frac{7}{2}$ **(b)** $\frac{x}{\left(\frac{7}{\sqrt{3}}\right)} + \frac{y}{\left(\frac{7}{2}\right)} = 1$

(c) $\frac{\sqrt{3}}{\sqrt{7}}x + \frac{2}{\sqrt{7}}y = \sqrt{7}$ **(d)** $\frac{\sqrt{3}}{\sqrt{7}}x + \frac{2}{\sqrt{7}}y = 7$

62. If the vertices B and D of a square $ABCD$ are (2, 3) and (4, 1) respectively, then what is the area of the square?

(a) 2 square units **(b)** 3 square units

(c) 4 square units **(d)** 8 square units

63. What is the value of $\sin\theta$ if θ is the acute angle between the lines whose equations are $px + qy = p + q$ and $p(x - y) + q(x + y) = 2q$?

(a) $\frac{\sqrt{3}}{2}$ **(b)** $\frac{3}{4}$ **(c)** $\frac{1}{2}$ **(d)** $\frac{1}{\sqrt{2}}$

64. The circle $x^2 + y^2 - 2kx - 2ky + k^2 = 0$ touches the x-axis at P and y-axis at Q.

What is PQ equal to?

(a) $\sqrt{2}k$ **(b)** $2k$ **(c)** $2\sqrt{2}k$ **(d)** $4k$

65. What is the distance between the foci of the hyperbola $x^2 - 4y^2 = 1$?

(a) $\sqrt{3}$ **(b)** $\sqrt{5}$ **(c)** $2\sqrt{3}$ **(d)** $2\sqrt{5}$

66. Let $\vec{p} = \vec{a} - \vec{b}$, $\vec{q} = \vec{a} + \vec{b}$. If $|\vec{a}| = |\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 2$, then what is the value of $|\vec{p} \times \vec{q}|$?

(a) $\sqrt{3}$ **(b)** $\sqrt{6}$ **(c)** $2\sqrt{3}$ **(d)** $4\sqrt{3}$

67. How many of the following can be a vector perpendicular to both the vectors $2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+3\hat{k}$?

I. $4\hat{i}+5\hat{j}-3\hat{k}$

II. $-8\hat{i}-10\hat{j}+6\hat{k}$

III. $\frac{1}{50}(-4\hat{i}-5\hat{j}+3\hat{k})$

Select the correct answer.

(a) None **(b)** One

(c) Two **(d)** All three

68. What is the area of the parallelogram whose sides are represented by the vectors $\hat{i}+2\hat{j}+3\hat{k}$ and $2\hat{i}+\hat{j}+2\hat{k}$?

(a) $\frac{1}{2}\sqrt{26}$ square unit **(b)** $\frac{1}{2}\sqrt{27}$ square unit

(c) $\sqrt{26}$ square unit **(d)** $\sqrt{27}$ square unit

69. The position vectors of the vertices *A*, *B*, *C* and *D* of a quadrilateral *ABCD* are given by $3\hat{i}+4\hat{j}-2\hat{k}$, $4\hat{i}-4\hat{j}-3\hat{k}$, $2\hat{i}-3\hat{j}+2\hat{k}$ and $6\hat{i}-2\hat{j}+\hat{k}$ respectively. What is the angle between the diagonals *AC* and *BD* of the quadrilateral?

(a) 90° **(b)** 75° **(c)** 60° **(d)** 45°

70. A force $\vec{F}=2\hat{i}-\lambda\hat{j}+5\hat{k}$ is applied at the point *A*(1, 2, 5). If its moment about the point *B*(–1, –2, 3) is $16\hat{i}-6\hat{j}+2\lambda\hat{k}$, then what is the value of λ?

(a) –2 **(b)** 0 **(c)** 1 **(d)** 2

For the following two (02) items:

Let

$$f(x)\begin{cases}\dfrac{1-\cos 2x}{x^2}, & x<0\\ 9, & x=0\\ \dfrac{\sqrt{x}}{\sqrt{(16)+\sqrt{x}}-4}, & x>0\end{cases}$$

71. What is $\lim_{x\to 0-} f(x)$ equal to?

(a) 2 **(b)** 4 (c) 6 **(d)** 8

72. What is $\lim_{x\to 0+} f(x)$ equal to?

(a) 6 **(b)** 7 **(c)** 8 **(d)** 9

For the following three (03) items :

Consider the function $f(x) = x|x|$.

73. What is $\lim_{x\to -1} f(x)$ equal to?

(a) –1 **(b)** 0

(c) 1 **(d)** Limit does not exist

74. What is the area bounded by the curve $f(x)$, the x-axis and the lines $x = -2$ and $x = 1$?

(a) $\frac{1}{3}$ **(b)** $\frac{2}{3}$ **(c)** $\frac{5}{2}$ **(d)** 3

75. Consider the following statements :

I. The function is increasing in the interval $(-\infty, \infty)$.

II. The function is differentiable at $x = 0$.

Which of the statements given above is/are correct?

(a) I only **(b)** II only

(c) Both I and II **(d)** Neither I nor II

For the following two (02) items :

Consider the function

$$f(x)=\frac{x}{1-x}(x>0, x\neq 1)$$

76. What is $\frac{f(x)}{f(x+1)}$ equal to?

(a) $-f(x^2)$ **(b)** $-f(\sqrt{x})$ **(c)** $f(x^2)$ **(d)** $f(x-1)$

77. What is $(1-x)f(\sqrt{x})+xf(\sqrt{x}+1)$ equal to?

(a) $-f(x)$ **(b)** $f(x)$ **(c)** x **(d)** 0

For the following three (03) items :

Let $y=f(x)=\frac{x\sin^{-1}x}{\sqrt{1-x^2}}+\ln\sqrt{1-x^2}$.

78. What is the slope of the tangent to the curve $y = f(x)$ at $x = 0.5$?

(a) $\frac{4\pi\sqrt{3}}{27}$ **(b)** $\frac{8\pi\sqrt{3}}{27}$ **(c)** 4π **(d)** 8π

79. What is $\frac{d^2y}{dx^2}$ at $x = 0$ equal to?

(a) 0 **(b)** 0.5 **(c)** 1 **(d)** 1.5

80. If $x = \sin\theta$, then what is $\frac{dy}{dx}$ equal to?

(a) $\theta\sec\theta$ **(b)** $\theta\sec^2\theta$

(c) $\theta\sec^3\theta$ **(d)** $2\tan\theta + \theta\sec^2\theta$

For the following two (02) items:

Consider the function $f(x) = 1 - \sqrt[3]{(x-1)^2}$.

81. What is the domain of the function?

(a) $(1, \infty)$ **(b)** $(-\infty, \infty)$

(c) $(0, \infty)$ **(d)** $(-\infty, \infty \backslash \{1\})$

82. The function has

(a) a minimum at $x = 1$

(b) a maximum at $x = 1$

(c) neither maximum nor minimum at $x = 1$

(d) no extremum

For the following two (02) items:

Consider the function

$$f(x) = \begin{cases} 4(5^x), & x < 0 \\ 8k + x, & x \geq 0 \end{cases}$$

83. If the function is continuous, then what is the value of k?

(a) 0.5 **(b)** 1 **(c)** 1.5 **(d)** 2

84. What is $f'(-1)$ equal to?

(a) $\frac{2}{5}\ln 5$ **(b)** $\frac{3}{5}\ln 5$ **(c)** $\frac{4}{5}\ln 5$ **(d)** $20\ln 5$

For the following two (02) items:

Let $u = \int e^x \cos x \, dx$ and $v = \int e^x \sin x \, dx$.

85. What is $u + v$ equal to?

(a) $-\frac{du}{dx}$ **(b)** $-\frac{dv}{dx}$ **(c)** $\frac{du}{dx}$ **(d)** $\frac{dv}{dx}$

86. Consider the following:

I. $\frac{du}{dx} = -v$

II. $\frac{dv}{dx} = -u$

Which of the above is/are correct?

(a) I only **(b)** II only

(c) Both I and II **(d)** Neither I nor II

For the following two (02) items:

Let the function $f(x) = |x-3| + |x-4|$ be defined on the interval $[0, 5]$.

87. What is $\frac{dy}{dx}$ at $x = 3.5$ equal to?

(a) 0 **(b)** 1 **(c)** 2 **(d)** 3.5

88. Consider the following statements:

I. The function is differentiable at $x = 3$.

II. The function is differentiable at $x = 4$.

Which of the statements given above is/are correct?

(a) I only **(b)** II only

(c) Both I and II **(d)** Neither I nor II

For the following two (02) items:

Consider the function $f(x) = \frac{10^x - 10^{-x}}{10^x + 10^{-x}}$.

89. What is $f \circ f \circ f \circ f \circ f(0)$ equal to?

(a) 0 **(b)** 1 **(c)** 5 **(d)** 10

90. What is the inverse of the function?

(a) $\log_{10}(2x-1)$ **(b)** $\frac{1}{2}\log_{10}(2x-1)$

(c) $\frac{1}{4}\log_{10}\left(\frac{2x}{2-x}\right)$ **(d)** $\frac{1}{2}\log_{10}\left(\frac{1+x}{1-x}\right)$

91. What is the degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}} = \left(\frac{dy}{dx}\right)^{\frac{5}{2}}?$$

(a) 3 **(b)** 2 **(c)** $\frac{5}{2}$ **(d)** $\frac{3}{2}$

92. What is $\int_n^{n+1} (x - [x])\, dx$, where [.] is the greatest integer function and n is natural number?

(a) $\frac{4n+1}{2}$ **(b)** $\frac{2n+1}{2}$ **(c)** $\frac{1}{2}$ **(d)** 1

93. Consider the following statements:

I. $y = xe^{2x}$ is the solution of $\frac{dy}{dx} = y\left(2 + \frac{1}{x}\right)$.

II. $y = x\ln|x| + cx$ is the solution of $\frac{dy}{dx} = \frac{x+y}{x}$.

Which of the statements given above. is/are correct?

(a) I only **(b)** II only

(c) Both I and II **(d)** Neither I nor II

94. If k is an arbitrary constant, then what is the general solution of the equation $(x + y)^2 \frac{dy}{dx} = k^2$?

(a) $y + x = \tan(x + c) + k$ **(b)** $x + y = k\tan\left(\frac{y-c}{k}\right)$

(c) $x - y = k\tan\left(\frac{y-c}{k}\right)$ **(d)** $y - x = \tan(x + c) + k$

95. What is $\int \frac{dx}{10^x + 10^{-x}}$ equal to?

(a) $\tan^{-1}(10^x) + c$ **(b)** $(\ln 10)\tan^{-1}(10^x) + c$

(c) $\frac{1}{\ln 10}\tan^{-1}(10^x) + c$ **(d)** $\ln(10^x + 10^{-x}) + c$

96. A wire of length 20 cm is to be bent into a rectangle. Which of the following statements is/are correct?

I. The rectangle of the largest area is the square.

II. It is possible to form a rectangle of an area of 27 cm^2.

Select the answer using the code given below.

(a) I only **(b)** II only

(c) Both I and II **(d)** Neither I nor II

97. If $I_1 = \int_e^{e^2} \frac{dx}{\ln x}$ and $I_2 = \int_1^2 \frac{e^x}{x} dx$, then which one of the following is correct?

(a) $I_1 - I_2 = 0$ **(b)** $I_1 + I_2 = 0$

(c) $I_1 - 2I_2 = 0$ **(d)** $2I_1 - I_2 = 0$

98. What is the area of the region bounded by $|x| \leq 2k$ and $|y| \leq k$, where k is a positive real number?

(a) $2k^2$ **(b)** $4k^2$ **(c)** $5k^2$ **(d)** $8k^2$

99. Consider the following regarding the function $f(x) = \frac{1}{x-5}$:

Statement I: $f(x)$ is decreasing on the intervals $x < 5$ and $x > 5$.

Statement II: $f'(x) > 0$ for all $x \neq 5$.

Which one of the following is correct in 6 respect of the above statements?

(a) Both Statement-I and Statement-II are correct and Statement-II explains Statement-I

(b) Both Statement-I and Statement-II are correct but Statement-II does not explain Statement-I

(c) Statement-I is correct but Statement-II is not correct

(d) Statement-I is not correct but Statement-II is correct

100. Consider the following statements:

Statement I: The function $f(x) = \frac{x^3 + 128}{x}$ has a minimum value 48 at $x = 4$.

Statement II: As x increases through 4, $f'(x)$ changes sign from positive negative.

Which one of the following is correct in respect of the above statements?

(a) Both Statement-I and Statement-II are correct and Statement-II explains Statement-I

(b) Both Statement-I and Statement-II are correct but Statement-II does not explain Statement-I

(c) Statement-I is correct but Statement-II is not correct

(d) Statement-I is not correct but Statement-II is correct

101. What is the harmonic mean of the numbers $C(10, 3)$, $C(10, 4)$, $C(10, 5)$, $C(10, 6)$ and $C(10, 7)$?

(a) $\frac{3150}{19}$ **(b)** $\frac{4000}{19}$ **(c)** 252 **(d)** 225

102. In a sample survey of a village, the probability that a farmer is in debt is 0.60. What is the probability that three randomly selected farmers are all in debt (assume independence of events)?

(a) 0.000216 **(b)** 0.064

(c) 0.216 **(d)** 0.512

103. The probability that a family owns a laptop is 0.68; that it also owns a desktop is 0.56. If the probability that it owns both is 0.48, then what is the probability that a randomly selected family owns a laptop or a desktop?

(a) 0.80 **(b)** 0.76 **(c)** 0.36 **(d)** 0-28

104. An urn contains 10 white and 5 red balls. If two balls are drawn at random, then what is the probability that both the balls are red?

(a) $\frac{2}{21}$ **(b)** $\frac{1}{7}$ **(c)** $\frac{4}{21}$ **(d)** $\frac{3}{7}$

105. An urn contains 5 white, 6 red and 4 blue balls. Three balls are drawn at random. What is the probability that a white ball, a red ball and a blue ball are drawn?

(a) $\frac{28}{91}$ **(b)** $\frac{2}{7}$ **(c)** $\frac{24}{91}$ **(d)** $\frac{23}{91}$

106. Under which of the following conditions may binomial distribution be used?

I. The number of trials is infinite and not fixed.

II. The trials are independent.

III. Each trial has two possible outcomes.

Select the correct answer using the code given below.

(a) II only **(b)** III only

(c) I and II **(d)** II and III

107. A person X speaks the truth 4 out of THE 5 times and person Y speaks the truth 5 out of 6 times. What is the probability asping that they will contradict each other in stating the fact?

(a) $\frac{3}{10}$ **(b)** $\frac{1}{15}$ **(c)** $\frac{1}{6}$ **(d)** $\frac{7}{10}$

108. The probability that a student passes Physics test is 2/3 and the probability that he passes both Physics test and English test is 11/15. The probability that he passes at least one test is 4/5. What is the probability that he passes English test?

(a) $\frac{11}{15}$ **(b)** $\frac{13}{15}$ **(c)** $\frac{14}{15}$ **(d)** 1

109. An event X can happen with probability p and event Y can happen with probability q. Further, X and Y are independent events. Which of the following statements is/are correct?

I. The probability that exactly one of the events happens is $p + q - pq$.

II. The probability that at least one of the events happens is $p + q - 2pq$.

Select the answer using the code given below.

(a) I only **(b)** II only

(c) Both I and II **(d)** Neither I nor II

110. Three faces of a die are black, two faces are white and one face is red. The die is tossed three times. What is the probability that the colours black, white and red appear in the first, second and third tosses respectively?

(a) $\frac{1}{36}$ **(b)** $\frac{1}{6}$ **(c)** $\frac{7}{36}$ **(d)** $\frac{5}{36}$

111. A fair coin is tossed 4 times. What is the probability that two heads do not occur consecutively?

(a) $\frac{1}{8}$ **(b)** $\frac{3}{8}$ **(c)** $\frac{7}{16}$ **(d)** $\frac{1}{2}$

112. In a throw of three dice, what is the probability of getting one prime number, one composite number and one number which is neither prime nor composite?

(a) $\frac{1}{2}$ **(b)** $\frac{1}{3}$ **(c)** $\frac{1}{4}$ **(d)** $\frac{1}{6}$

113. An integer is chosen at random from the first 50 integers. What is the probability that the integer is neither divisible by 5 nor 9?

(a) $\frac{7}{10}$ **(b)** $\frac{18}{25}$ **(c)** $\frac{37}{50}$ **(d)** $\frac{19}{25}$

114. Out of 50 consecutive natural numbers, two integers are chosen at random. What is the probability that their sum is odd?

(a) $\frac{1}{2}$ **(b)** $\frac{24}{49}$ **(c)** $\frac{1}{4}$ **(d)** $\frac{25}{49}$

115. The standard deviation of 100 observations is 10. If 20 is added to each observation, then what will be the new standard deviation?

(a) 10 **(b)** 15 **(c)** 20 **(d)** 25

116. Let X be a random variable following binomial distribution with parameters $n = 5$ and $p = k$. Further, $P(X = 1) = 0.4096$ and $P(X = 2) = 0.2048$. What is the value of k?

(a) 0.2 **(b)** 0.25 **(c)** 0.3 **(d)** 0.35

117. The frequency distribution of the marks obtained by students in a Science examination is given below:

Marks	5–15	15–25	25–35	35–45
Number of students	20	30	30	20

What is the arithmetic mean?

(a) 20 **(b)** 25 **(c)** 30 **(d)** 35

118. If $P(A) = 0.3$, $P(B) = 0.4$ and $P(A|B)$ 0.5, then what is the value of $P(B|A)$?

(a) 0:325 **(b)** 0.333 **(c)** 0.375 **(d)** 0-667

119. If $P(A) = 1/3$, $P(B) = 1/2$ and $P(A \cap B) = 1/4$ then, what is the value of $P(\bar{A} \cup B)$?

(a) $\frac{7}{12}$ **(b)** $\frac{2}{3}$ **(c)** $\frac{3}{4}$ **(d)** $\frac{11}{12}$

120. Consider the following statements :

I. Mean and variance have the same unit of measurement.

II. Mean deviation and standard deviation have the same unit of measurement.

Which of the statements given above is/are correct?

(a) I only **(b)** II only

(c) Both I and II **(d)** Neither I nor II

Answer Key

Q. No	Answer Key	Topic's Name	Chapter's Name
1	(b)	Geometric Progressions (G.P.)	Sequence and Series
2	(a)	Relation and Types of Relation	Set Theory and Relation
3	(a)	Factorial	Permutation and Combination
4	(c)	Factorial	Permutation and Combination
5	(b)	Basics of Logarithm & Properties of Logarithms	Logarithms and their Applications
6	(d)	Series of Natural Numbers and other Miscellaneous Series	Sequences and Series
7	(c)	Arithmetic Progressions (A.P.)	Sequences and Series
8	(c)	De Moivre's Theorem, Roots of Unity, Geometry of Complex Numbers	Complex Numbers
9	(c)	Operations on Binary Numbers	Number System
10	(a)	Inverse of a Matrix	Matrices and Determinants
11	(b)	Basics of Sets & Algebra of Sets	Set Theory and Relation
12	(c)	Systems of Linear Equations	Matrices and Determinants
13	(b)	Basics of Complex Numbers & Algebra of Complex Numbers	Complex Numbers
14	(b)	Systems of Linear Equations	Matrices and Determinants
15	(a)	Algebra of Matrices	Matrices and Determinants
16	(c)	Properties of Determinants	Matrices and Determinants
17	(a)	De Moivre's Theorem, Roots of Unity, Geometry of Complex Numbers	Complex Numbers
18	(b)	Combinations	Permutation and Combination
19	(b)	Relations Between Sides and Angles of a Triangle	Properties of Triangle
20	(d)	Middle Term and Greatest Term in Binomial Expansion	Binomial Theorem and its Applications
21	(b)	Relation between Roots and Coefficients	Quadratic Equations
22	(c)	Relation between Roots and Coefficients	Quadratic Equations
23	(c)	Basics of Logarithm & Properties of Logarithms	Logarithms and their Applications
24	(d)	Basics of Logarithm & Properties of Logarithms	Logarithms and their Applications
25	(c)	Trigonometric Functions, properties and basic trigonometric Identities	Trigonometric Ratios, Functions and Identities
26	(d)	Trigonometric Functions, properties and basic trigonometric Identities	Trigonometric Ratios, Functions and Identities
27	(c)	Trigonometric Functions, properties and basic trigonometric Identities	Trigonometric Ratios, Functions and Identities
28	(b)	Trigonometric Functions, properties and basic trigonometric Identities	Trigonometric Ratios, Functions and Identities
29	(c)	General Solution of Standard Trigonometric Equations	Trigonometric Equations
30	(c)	General Solution of Standard Trigonometric Equations	Trigonometric Equations
31	(d)	Nature of Roots	Quadratic Equations
32	(c)	Arithmetic Progressions (A.P.)	Sequences and Series

Q. No	Answer Key	Topic's Name	Chapter's Name
33	(d)	Series of Natural Numbers and other Miscellaneous Series	Sequences and Series
34	(c)	Arithmetic Progressions (A.P.)	Sequences and Series
35	(d)	Combinations	Permutation and Combination
36	(c)	Combinations	Permutation and Combination
37	(c)	Polynomial, Quadratic Equation and its Solution	Quadratic Equations
38	(c)	Polynomial, Quadratic Equation and its Solution	Quadratic Equations
39	(b)	Determinant of a Square Matrix	Matrices and Determinants
40	(c)	Adjoint of a Matrix	Matrices and Determinants
41	(a)	Operations on Binary Numbers	Number System
42	(d)	Basics of Sets & Algebra of Sets	Set Theory and Relation
43	(c)	Relation & Types of Relation	Set Theory and Relation
44	(a)	Properties of Determinants	Matrices and Determinants
45	(c)	Inverse of a Matrix	Matrices and Determinants
46	(a)	Trigonometric Functions, properties and basic trigonometric Identities	Trigonometric Ratios, Functions and Identities
47	(c)	General Solution of Standard Trigonometric Equations	Trigonometric Equations
48	(c)	Trigonometric Identities for Multiple and Sub-multiple angles	Trigonometric Ratios, Functions and Identities
49	(b)	Trigonometric Inequalities	Trigonometric Equations
50	(a)	Trigonometric Identities for Compound Angles	Trigonometric Ratios, Functions and Identities
51	(b)	Interaction between a Line and a Plane	Three-Dimensional Geometry
52	(a)	Plane and a Point	Three-Dimensional Geometry
53	(b)	Sphere	Three-Dimensional Geometry
54	(a)	Sphere	Three-Dimensional Geometry
55	(b)	Basics of 3D Geometry	Three-Dimensional Geometry
56	(b)	Lines in 3D	Three-Dimensional Geometry
57	(b)	Straight Line and its Equations	Point and Straight Line
58	(d)	Straight Line and a Point	Point and Straight Line
59	(b)	Basics of Parabola	Parabola
60	(a)	Straight Line and its Equations	Point and Straight Line
61	(c)	Straight Line and its Equations	Point and Straight Line
62	(c)	Special Points in Triangles	Point and Straight Line
63	(d)	Interaction between Two Lines	Point and Straight Line
64	(a)	Interaction between Circle and a Line	Circle
65	(b)	Basics of Hyperbola	Hyperbola
66	(d)	Scalar and Vector Products	Vector Algebra
67	(d)	Scalar and Vector Products	Vector Algebra
68	(c)	Scalar and Vector Products	Vector Algebra
69	(a)	Addition of Vectors	Vector Algebra
70	(a)	Triple Products	Vector Algebra
71	(a)	Methods of Evaluation of Limits	Limits
72	(c)	Methods of Evaluation of Limits	Limits

Q. No	Answer Key	Topic's Name	Chapter's Name
73	(a)	Standard Limits in Calculus	Limits
74	(d)	Area Bounded by Curves	Area under Curves
75	(c)	Differentiability of a Function	Continuity and Differentiability
76	(a)	Composite Functions	Functions
77	(d)	Composite Functions	Functions
78	(a)	Rules of Differentiation	Differential Coefficient
79	(c)	Higher Order Derivatives	Differential Coefficient
80	(c)	Rules of Differentiation	Differential Coefficient
81	(b)	Domain & Types of Functions	Functions
82	(d)	Maxima and Minima	Application of Derivatives
83	(a)	Continuity of a Function	Continuity and Differentiability
84	(c)	Rules of Differentiation	Differential Coefficient
85	(d)	Methods of Integration	Indefinite Integration
86	(d)	Methods of Integration	Indefinite Integration
87	(a)	Rules of Differentiation	Differential Coefficient
88	(d)	Differentiability of a Function	Continuity and Differentiability
89	(a)	Composite Functions	Functions
90	(d)	Inverse Functions	Functions
91	(a)	Formation of Differential Equations	Differential Equations
92	(c)	Properties of Definite Integrals	Definite Integration
93	(c)	Solution of Differential Equation using inspection	Differential Equations
94	(b)	Solution of Linear Differential Equations	Differential Equations
95	(c)	Integration Using Trigonometric Identities	Indefinite Integration
96	(a)	Maxima and Minima	Application of Derivatives
97	(a)	Properties of Definite Integrals	Definite Integration
98	(d)	Area Bounded by Curves	Area under Curves
99	(c)	Monotonicity	Application of Derivatives
100	(c)	Maxima and Minima	Application of Derivatives
101	(a)	Combinations	Permutation and Combination
102	(c)	Multiplication Theorems of Probability	Probability
103	(b)	Addition Theorems of Probability	Probability
104	(a)	Probability Distribution of a Random Variate, Bernoulli Trials and Binomial Distribution	Probability
105	(c)	Probability Distribution of a Random Variate, Bernoulli Trials and Binomial Distribution	Probability
106	(d)	Probability Distribution of a Random Variate, Bernoulli Trials and Binomial Distribution	Probability
107	(a)	Conditional Probability, Bayes' Theorem & Total Probability Theorem	Probability
108	(b)	Addition Theorems of Probability	Probability
109	(d)	Conditional Probability, Bayes' Theorem & Total Probability Theorem	Probability
110	(a)	Multiplication Theorems of Probability	Probability

Q. No	Answer Key	Topic's Name	Chapter's Name
111	(d)	Probability Distribution of a Random Variate, Bernoulli Trials and Binomial Distribution	Probability
112	(d)	Probability Distribution of a Random Variate, Bernoulli Trials and Binomial Distribution	Probability
113	(b)	Basics of Probability	Probability
114	(d)	Basics of Probability	Probability
115	(a)	Measures of Dispersion	Statistics
116	(a)	Probability Distribution of a Random Variate, Bernoulli Trials and Binomial Distribution	Probability
117	(b)	Measures of Central Tendency	Statistics
118	(d)	Conditional Probability, Bayes' Theorem & Total Probability Theorem	Probability
119	(d)	Conditional Probability, Bayes' Theorem & Total Probability Theorem	Probability
120	(b)	Measures of Dispersion	Statistics

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EXAM II 2025

Mathematics Solutions

ANSWERS WITH EXPLANATIONS

1. Option (b) is correct.

Explanation:

$\because$ x, y and z are in G.P.

$$\frac{y}{x} = \frac{z}{y} \quad \text{...(i)}$$

Now, $p^x = q^y = r^z = k$

$x = \log_p k$, $y = \log_q k$ and $z = \log_r k$

$$\frac{\log_q k}{\log_p k} = \frac{\log_r k}{\log_q k} \quad \text{[From (i)]}$$

$$\Rightarrow \frac{\ln k}{\ln q} \times \frac{\ln p}{\ln k} = \frac{\ln k}{\ln r} \times \frac{\ln q}{\ln k}$$

$$\Rightarrow \frac{\ln p}{\ln q} = \frac{\ln q}{\ln r}$$

$\therefore$ $\ln p$, $\ln q$ and $\ln r$ in G.P

2. Option (a) is correct.

Explanation:

Statement I:

$$\text{L.H.S.} = A - B = A \cap B^c$$

$$\text{R.H.S.} = B^c - A^c = B^c \cap (A^c)^c = B^c \cap A$$

$\because$ L.H.S. = R.H.S.

So, statement I is correct

Statement II:

$$\text{L.H.S.} = A - B^c = A \cap (B^c)^c = A \cap B$$

$$\text{R.H.S.} = A^c - B = A^c \cap B^c = \cup - (A \cup B)$$

$\because$ L.H.S. $\neq$ R.H.S.

So, statement II is incorrect

3. Option (a) is correct.

Explanation:

$$\frac{z}{y} = \frac{(2x)!}{x!} = 120$$

$$\Rightarrow \frac{2x(2x-1)....(x+1)\cdot x!}{x!} = 120$$

$$\Rightarrow 2x(2x-1).....(x+1) = 6 \times 5 \times 4$$

$$\therefore 2x = 6 \Rightarrow x = 3$$

Now, $(3x)! = 9! = 362880$

4. Option (c) is correct.

Explanation: The number of consecutive zeros at the end of a number is determined by the number of factor of 10 i.e., number of pairs of 2 and 5. in $n!$. Since number of factors of 2 is more than number of factors of 5.

So, detrmine the number of factors of 5

$$2 = \left|\frac{n}{5}\right| + \left|\frac{n}{25} + ...\right|$$

Since $\left[\frac{n}{5}\right] = 0$, for $n < 5$

$\left[\frac{n}{5}\right] = 1$ for $5 \le n < 10$

$\left[\frac{n}{5}\right] = 2$ for $10 \le n < 15$

Hence, $n = 10, 11, 12, 13, 14$

5. Option (b) is correct.

Explanation:

$\because$ $(10 + \log_{10} x)$, $(10 + \log_{10} y)$ and $(10 + \log_{10} z)$ are in A.P.

$$\Rightarrow 2(10 + \log_{10} y) = (10 + \log_{10} x) + (10 + \log_{10} z)$$

$$\Rightarrow 20 + 2\log_{10} y = 20 + \log_{10} x + \log_{10} z$$

$$\Rightarrow 2\log_{10} y = \log_{10} x + \log_{10} z$$

$\therefore$ The A.M of $\log_{10} x$ and $\log_{10} y$ is $\log_{10} y$

So, statement II is correct

Now, $\log_{10} y^2 = \log_{10} (xz)$

$$\Rightarrow y^2 = xz$$

so, GM of x and z is y

So, statement I is incorrect.

6. Option (d) is correct.

Explanation:

$$S = 1 + 3 + 5 + 7 ... \text{upto } n \text{ terms}$$

$$= \frac{n}{2}(2 + (n-1)2)$$

$$12345678987654321 = n^2$$

$$\therefore n = 111,111,111$$

7. Option (c) is correct.

Explanation: 110^{th} terms of first A.P is

$$T_{110} = 19+(110-1)2 = 237$$

75^{th} terms of second A.P is

$$T_{75} = 19+\frac{(75-1)}{3} = 241$$

Identical terms both A.P

19, 25, 31 ....

$$\therefore \quad T_n \le 237$$

$$19+(n-1)6 \le 237$$

$$\Rightarrow \quad (n-1) \le 218 \Rightarrow n \le 37.33$$

$$\therefore \quad n = 37.$$

8. Option (c) is correct.

Explanation:

$$\alpha = \frac{-1+\sqrt{-3}}{2} = \frac{-1+\sqrt{3}i}{2} = \omega$$

(Cube roots of unit y)

We have $\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$

Now,

$$(1+\alpha^{19}-\alpha^{35})^{100}-(1-3\alpha^{25}+\alpha^{38})^{50}$$

$$= (1+\omega^{19}-\omega^{35})^{100}-(1-3\omega^{25}+\omega^{38})^{50}$$

$$= (1+\omega-\omega^2)^{100}-(1-3\omega+\omega^2)^{50}$$

$$= (-2\omega^2)^{100}-(-4\omega)^{50} = 2^{100}\omega^{200}-2^{100}\omega^{50}$$

$$= 2^{100}\omega^2-2^{100}\omega^2 = 0$$

9. Option (c) is correct.

Explanation:

$$5^1 = 5 \pmod{13}$$

$$5^2 = 12 \pmod{13}$$

$$5^3 = 5\times12 = 60 \equiv 8 \pmod{13}$$

$$5^4 = 5\times8 = 40 \equiv 1 \pmod{13}$$

Now, $5^{99} = 5^{4\times24+3} = (5^4)^{24}\cdot(5)^3$

$$\equiv (1)^{24}\cdot(5)^3 \pmod{13}$$

$$1^{24}\cdot5^3 \equiv 1\times8 \pmod{13}$$

$$= 8 \pmod{13}$$

Hence remainder = 8

10. Option (a) is correct.

Explanation:

Let $A = \begin{bmatrix}-4 & -5\\ 2 & 2\end{bmatrix}$

$$\Rightarrow \quad |A| = -8+10 = 2$$

$$\because \quad |A^{-1}| = \frac{1}{|A|} = \frac{1}{2}$$

11. Option (b) is correct.

Explanation:

Here, $n(C\cup F) = 45$, $n(C) = 34$ and $n(F) = 26$

$$n(C\cap F) = n(C)+n(F)-n(C\cup F)$$

$$= 34+26-45 = 15$$

$$n(\text{only } C)+n(\text{only } F) = n(C)+n(F)-2n(C\cap F)$$

$$= 34+26-30 = 30$$

12. Option (c) is correct.

Explanation:

$$\frac{a_1}{a_2} = \frac{2}{-10} = -\frac{1}{5}$$

$$\frac{b_1}{b_2} = \frac{-3}{15} = -\frac{1}{5}$$

$$\frac{c_1}{c_2} = \frac{-5}{50} = -\frac{1}{10}$$

$$\because \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \ne \frac{c_1}{c_2}$$

$\therefore$ The system of equations is inconsistent.

13. Option (b) is correct.

Explanation:

$$\left(\frac{1-i}{1+i}\right)^{2m}\left(\frac{1+i}{1-i}\right)^{2n} = 1$$

$$\Rightarrow \left(\frac{1-i}{1+i}\times\frac{1-i}{1-i}\right)^{2m}\left(\frac{1+i}{1-i}\times\frac{1+i}{1+i}\right)^{2n} = 0$$

$$\Rightarrow \quad \left(\frac{1+i^2-2i}{1-i^2}\right)^{2m}\left(\frac{1+i^2+2i}{1-i^2}\right) = 1$$

$$\Rightarrow \quad (-i)^{2m}\cdot(i)^{2n} = 1$$

$$\Rightarrow \quad (-1)^m(-1)^n = 1$$

$$\Rightarrow \quad (-1)^{m+n} = 1$$

So both m and n are even or odd

So, possible value of m and n = (3, 1), (4, 2)

Smallest positive value of $m - n = 2$

14. Option (b) is correct.

Explanation:

$$D_2 = \begin{vmatrix}1 & 7 & 1\\ 1 & 16 & 3\\ 1 & 22 & 4\end{vmatrix} = (64-66)-7(4-3)+(21-16)$$

$$= -2-7+6 = -3$$

15. Option (a) is correct.

Explanation:

I: We have

$$(AB)^{-1} = B^{-1}A^{-1}$$

II: We have

$$(AB)(AB)^{-1} = I$$

So, statement II is incorrect

III: We have

$$(AB)^T = B^T A^T$$

So, statement III is incorrect.

16. Option (c) is correct.

Explanation:

$$\begin{vmatrix} a & b & c \\ l & m & n \\ p & q & r \end{vmatrix} = -\begin{vmatrix} a & b & c \\ p & q & r \\ l & m & n \end{vmatrix} \quad (\text{By } R_2 \Leftrightarrow R_3)$$

$$= +\begin{vmatrix} p & q & r \\ a & b & c \\ l & m & n \end{vmatrix} \quad (\text{By } R_1 \Leftrightarrow R_2)$$

17. Option (a) is correct.

Explanation:

We have $w^3 = 1$ and $1 + w + w^2 = 0$

$$\begin{aligned} x^2 + y^2 + z^2 &= (a+b)^2 + (aw + bw^2)^2 + (aw^2 + bw)^2 \\ &= a^2 + b^2 + 2ab + a^2w^2 + b^2w^4 \\ &\quad + 2abw^3 + a^2w^4 + b^2w^2 + 2abw^3 \\ &= a^2(1 + w^2 + w^4) + b^2(1 + w^2 + w^4) \\ &\quad + 6ab \\ &= a^2(1 + w^2 + w) + b^2(1 + w^2 + w) \\ &\quad + 6ab \\ &= 6ab. \end{aligned}$$

18. Option (b) is correct.

Explanation: Last two digit of number which is divisible by 4 are 12, 24, 32

No. of number whose last two digits is 12 = 2!

No. of number whose last two digits is 24 = 2!

No. of nubmer whose last two digits is 32 = 2!

Total number = 2! + 2! + 2! = 6

19. Option (b) is correct.

Explanation:

Here $p = a + b + c$

$$\text{Now, } \begin{vmatrix} p+c & a & b \\ c & p+a & b \\ c & a & p+b \end{vmatrix} \quad (\text{By } c_1 \to c_1 + c_2 + c_3)$$

$$= \begin{vmatrix} p+a+b+c & a & b \\ p+a+b+c & p+a & b \\ p+a+b+c & a & p+b \end{vmatrix}$$

$$= (p+a+b+c)\begin{vmatrix} 1 & a & b \\ 1 & p+a & b \\ 1 & a & p+b \end{vmatrix}$$

By $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= (p+a+b+c)\begin{vmatrix} 1 & a & b \\ 0 & P & 0 \\ 0 & 0 & P \end{vmatrix}$$

$$= (p+p)(p^2 - 0) = 2p^3$$

20. Option (d) is correct.

Explanation: We have middle has greatest coefficient

$$\text{So, middle term} = \frac{100+2}{2} = 51^{th} \text{ term}$$

$$T_{51} = T_{50+1} = {}^{100}C_{50}$$

$\therefore$ Greatest coefficient

$$= \text{Coefficient of } x^{50}.$$

21. Option (b) is correct.

Explanation:

Let $\log_{0.5} a^2 = t$

$\therefore$ $x^2 + tx + t^4 = 0$

$\therefore$ $\alpha + \beta = -t$ ...(i)

$\alpha\beta = t^4$ ...(ii)

$\beta^2 = \frac{\alpha}{t} \Rightarrow \alpha = \beta^2 t$ ...(iii)

From (ii) and (iii)

$$\beta^3 t = t^4 \Rightarrow \beta^3 = t^3 (\because \log_{0.5} a^2 > 0)$$

$$\beta = t = \log_{0.5} a^2$$

22. Option (c) is correct.

Explanation:

Let $\log_{0.5} a^2 = t$

$\therefore$ $x^2 + tx + t^4 = 0$

$\alpha + \beta = -t$ ...(i)

$\alpha\beta = t^4$ ...(ii)

$\beta^2 = \frac{\alpha}{t} \Rightarrow \alpha = \beta^2 t$ ...(iii)

from (ii) and (iii)

$$\beta^3 t = t^4 \Rightarrow \beta^3 = t^3 \quad (\because \log_{0.5} a^2 = 0)$$

$\Rightarrow$ $\beta = t$

Putting in (i)

$$\alpha + \beta = -\beta \Rightarrow \alpha = -2\beta.$$

23. Option (c) is correct.

Explanation:

$$p = \sum_{j=1}^{n} \log_{10} 2^j = \sum_{j=1}^{n} j \log_{10} 2$$

$$= (1+2+\ldots+n)\log_{10} 2 = \frac{n(n+1)}{2}\log_{10} 2$$

$$q = \sum_{j=1}^{n} \log_{10} 5^j = \sum_{j=1}^{n} j \log_{10} 5$$

$$= (1+2+\ldots+n)\log_{10} 5$$

$$\therefore \quad p+q = 66$$

$$\frac{n(n+1)}{2}\log_{10} 2 + \frac{n(n+1)}{2}\log_{10} 5 = 66$$

$$\Rightarrow \quad \frac{n(n+1)}{2} \cdot \log_{10} 10 = 66$$

$$\Rightarrow \quad n(n+1) = 132 = 11 \times 12$$

$$\therefore \quad n = 11 \in (9,12)$$

24. Option (d) is correct.

Explanation:

$$p = \sum_{j=1}^{n} \log_{10} 2^j = \sum_{j=1}^{n} j \log_{10} 2$$

$$= (1+2+\ldots n)\log_{10} 2 = \frac{n(n+1)}{2}\log_{10} 2$$

$$q = \sum_{j=1}^{n} \log_{10} 5^j = \sum_{j=1}^{n} j \log_{10} 5$$

$$= (1+2+\ldots+n)\log_{10} 5 = \frac{n(n+1)}{2}\log_{10} 5$$

$$\because \quad p+q = 15 \Rightarrow \frac{n(n+1)}{2}\log_{10} 10 = 15$$

$$n(n+1) = 30 = 5 \times 6 \Rightarrow n = 5$$

$$q-p = \frac{n(n+1)}{2}[\log_{10} 5 - \log_{10} 2]$$

$$= \frac{5(5+1)}{2}\log_{10} 2.5 = 15\log_{10} 2.5$$

25. Option (c) is correct.

Explanation:

$$\frac{p}{q} = \frac{\sin A + \sin B}{\cos A + \cos B} = \frac{2\sin\frac{A+B}{2}\cdot\cos\left(\frac{A-B}{2}\right)}{2\cos\frac{A+B}{2}\cdot\cos\left(\frac{A-B}{2}\right)}$$

$$= \tan\left(\frac{A+B}{2}\right)$$

26. Option (d) is correct.

Explanation:

$$\frac{p^2+q^2}{p^2+q^2} = \frac{4\sin^2\frac{A+B}{2}\cdot\cos^2\frac{A-B}{2} - 4\cos^2\frac{A+B}{2}\cdot\cos^2\frac{A-B}{2}}{4\sin^2\frac{A+B}{2}\cdot\cos^2\frac{A-B}{2} + 4\cos^2\frac{A+B}{2}\cdot\cos^2\frac{A-B}{2}}$$

$$= \frac{4\cos\left(\frac{A-B}{2}\right)\left[\sin^2\frac{A+B}{2} - \cos^2\frac{A+B}{2}\right]}{4\cos^2\frac{A-B}{2}\left[\sin^2\frac{A+B}{2} + \cos^2\frac{A+B}{2}\right]}$$

$$= -\cos(A+B) = \cos(\pi - A - B)$$

27. Option (c) is correct.

Explanation:

$$\frac{\sqrt{3}}{4}p - \frac{q}{4} = \frac{\sqrt{3}}{4}\operatorname{cosec} 20° - \frac{1}{4}\operatorname{cosec} 70°$$

$$= \frac{\sqrt{3}}{4}\operatorname{cosec} 20° - \frac{1}{4}\sec 20°$$

$$= \frac{1}{2\sin 20° \cdot \cos 20°}\left(\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°\right)$$

$$= \frac{1}{\sin 40°}[\cos 30° \cdot \cos 20° - \sin 30° \cdot \sin 20°]$$

$$= \frac{1}{\sin 40°} \times \cos 50° = \frac{\cos 50°}{\cos(90-40)°}$$

$$= \frac{\cos 50°}{\cos 50°} = 1.$$

28. Option (b) is correct.

Explanation:

$$\frac{p^2+q^2}{p^2q^2} = \frac{1}{q^2} + \frac{1}{p^2}$$

$$= \sin^2 20° + \sin^2 70°$$

$$= \sin^2 20° + \cos^2(90-70)°$$

$$= \sin^2 20° + \cos^2 20° = 1$$

29. Option (c) is correct.

Explanation:

$$\cos(2x+3y) = \frac{1}{2} = \cos\frac{\pi}{3}$$

$$\therefore \quad 2x+3y = \frac{\pi}{3} \qquad \ldots(i)$$

$$\text{or} \quad 2x+3y = \frac{-\pi}{3} \quad [\because -\pi < 2x+3y < \pi] \ldots(ii)$$

$$\cos(3x+2y) = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$$

$$\therefore \quad 3x+2y = \frac{\pi}{6} \qquad \ldots(iii)$$

$$\text{or} \quad 3x+2y = \frac{-\pi}{6} \quad [\because -\pi < 3x+2y < \pi] \ldots(iv)$$

On adding [(i) and (iii)], [(i) and (iv)], [(ii) and (iii)] and [(ii) and (iv)] we get four values of $x + y$.

30. Option (c) is correct.

Explanation:

$$\cos(2x+3y) = \frac{1}{2} = \cos\frac{\pi}{3}$$

$\therefore \quad 2x+3y = \frac{\pi}{3}$ ...(i)

or $\quad 2x+3y = -\frac{\pi}{3} \quad [\because -\pi < 2x+3y < \pi]$ ...(ii)

$\cos(3x+2y) = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$

$\therefore \quad 3x+2y = \frac{\pi}{6}$ ...(iii)

or $\quad 3x+2y = -\frac{\pi}{6} \quad [\because -\pi < 3x+2y < \pi]$ ...(iv)

On subtraction [(iii) from (i)], [(iii) from (ii)], [(iv) from (i)] and [(iv) from (ii)] we get four vaue of $y-x$.

31. Option (c) is correct.

Explanation:

$$abx^2+bcx+ca = cax^2+abx+bc$$

$$\Rightarrow (ab-ca)x^2+(bc-ab)x+(ca-bc) = 0$$

Since roots are equal

$$\therefore \quad B^2 = 4AC$$

$$\Rightarrow (bc-ab)^2 = 4(ab-ca)(ca-bc)$$

$$\Rightarrow \left(\frac{bc-ab}{abc}\right)^2 = 4\left(\frac{ab-ca}{abc}\right)\left(\frac{ca-bc}{abc}\right)$$

$$\Rightarrow \left(\frac{1}{a}-\frac{1}{c}\right)^2 = 4\left(\frac{1}{c}-\frac{1}{b}\right)\left(\frac{1}{b}-\frac{1}{a}\right)$$

$$\Rightarrow \frac{1}{a^2}+\frac{1}{c^2}+\frac{4}{b^2}+\frac{2}{ac}-\frac{4}{bc}-\frac{4}{ab} = 0$$

$$\Rightarrow \left(\frac{1}{a}+\frac{1}{c}-\frac{2}{b}\right)^2 = 0$$

$$\Rightarrow \frac{1}{a}+\frac{1}{c} = \frac{2}{b}$$

So, a, b and c are in H.P.

32. Option (d) is correct.

Explanation:

$\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in A.P. so a,b,c are in H.P.

33. Option (d) is correct.

Explanation:

(6 + 10 + 14 ... upto m terms)

$$\frac{m}{2}(12+(m-1)4) = \frac{n}{2}(2+(n-1)2)$$

$$2m(m+2) = n^2$$

34. Option (c) is correct.

Explanation:

(6 + 10 + 14 ... upto m terms)

$= (1+3+5+7...\text{upto } n \text{ terms})$

$$\frac{m}{2}(12+(m-1)4) = \frac{n}{2}(2+(n-1)2)$$

$$2m(m+2) = n^2 \quad \text{(Perfect square)}$$

So, $m = 2$ or 16 satisfy the condition.

35. Option (d) is correct.

Explanation:

Number of triangles $= 8c_3 - 4c_3$

$$= \frac{8\cdot7\cdot6}{3\cdot2\cdot1}-\frac{4\cdot3\cdot2}{3\cdot2\cdot1}$$

$= 56-4 = 52$

36. Option (c) is correct.

Explanation:

Number of quadrilateral $= 8c_4 - 4c_3 \times 4c_1 - 4c_4$

$$= \frac{8\cdot7\cdot6\cdot5}{4\cdot3\cdot2\cdot1}-\frac{4\cdot3\cdot2}{3\cdot2}\times4-1$$

$= 70-16-1 = 53$

37. Option (c) is correct.

Explanation:

$f(1) = 2 \Rightarrow a+b+c = 2$ ...(i)

$f(4) = 2 \Rightarrow 16a+4b+c = 2$ ...(ii)

$f(2) = 4a+2b+c = 0$ ...(iii)

On solving in (i), (ii) and (iii), we get

$a = 1, b = -5$ and $c = 6$

$\therefore$ Quadratic equation is

$$x^2-5x+6 = 0 \Rightarrow (x-2)(x-3) = 0$$

$$\Rightarrow n = 2,3$$

So, other roots in 3.

38. Option (c) is correct.

Explanation:

$f(1) = 2 \Rightarrow a+b+c = 2$

39. Option (b) is correct.

Explanation:

$$|A| = \begin{vmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{vmatrix} = \cos^2\theta+\sin^2\theta = 1$$

Now, $|A^4| = |A|^4 = (1)^4 = 1.$

40. Option (c) is correct.

Explanation:

$$adj\,A = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$

$$|adj\,A| = \cos^2\theta+\sin^2\theta = 1$$

$$adj(adj\,A) = \begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

$$\therefore \quad [adj\,A]^{-1} = \begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix} = A$$

41. Option (a) is correct.

Explanation:

$$\begin{array}{cccccccccc} & & \boxed{1} & \boxed{1} & & \boxed{1} & \boxed{1} & \boxed{1} & \boxed{1} & \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & \\ & & & 1 & 0 & 0 & 0 & 1 & 1 & \\ \hline 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \end{array}$$

42. Option (d) is correct.

Explanation:

$$n(X) = 3n,\ n(Y) = 2n$$

and $n(X \cap Y) = n$

$\therefore$ $n(X - Y) = 3n - n = 2n$

$n(Y - X) = 2n - n = n$

$n[(X - Y) \times (Y - X)] = 2n \times n = 2n^2$

43. Option (c) is correct.

Explanation: Here

$R = \{(x, y) : |x| < y, x \in A \text{ and } y \in B\}$

$= \{(0,1),(-3,4),(-2,4),(-1,4),(0,4),(1,4),$
$(2,4),(3,4)(-3,9),(-2,9),(-1,9),(0,9),$
$(1,9),(2,9),(3,9)\}$

$\therefore$ Number of elements in R is 15

44. Option (a) is correct.

Explanation:

We have $|KA| = K^n |A|$ where n is order of A

$\therefore$ $|mx| = m^n |x|$, where order of x is n

So, statement I is correct

Let $X = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$Y = \begin{bmatrix} ma_{11} & ma_{12} & ma_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$|Y| = \begin{vmatrix} ma_{11} & ma_{22} & ma_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$= m \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$= m|x|$

So, statement II is also correct and statement II is correct explanation of statement I.

45. Option (c) is correct.

Explanation:

$$|M| = \begin{vmatrix} 71 & 23 & 48 \\ 57 & 28 & 29 \\ 65 & 17 & 48 \end{vmatrix}$$

$R_1 \to R_1 - R_3$

$$= \begin{vmatrix} 6 & 6 & 0 \\ 57 & 28 & 29 \\ 65 & 17 & 48 \end{vmatrix}$$

$= 6(1344 - 493) - 6(2736 - 1885)$

$= 5106 - 5106 = 0$

So, M is singular matrix.

Hence inverse of M does not exist.

46. Option (a) is correct.

Explanation:

$$\cot^{-1} 9 + \operatorname{cosec}^{-1}\left(\frac{\sqrt{41}}{4}\right)$$

$$= \tan^{-1}\left(\frac{\frac{1}{9}+\frac{4}{5}}{1-\frac{1}{9}\times\frac{4}{5}}\right) = \tan^{-1}\left(\frac{\frac{5+36}{45}}{\frac{45-4}{45}}\right) = \tan^{-1} 1$$

$$= \frac{\pi}{4}.$$

47. Option (c) is correct.

Explanation:

$$\cot\theta = -\sqrt{3} = -\cot\frac{\pi}{6} = \cot\left(\pi - \frac{\pi}{6}\right) = \cot\frac{5\pi}{6}$$

$$\theta = \frac{5\pi}{6} \text{ or } \frac{-\pi}{6} \quad \text{...(i)}$$

$$\operatorname{cosec} = -2 = -\operatorname{cosec}\frac{\pi}{6} = \operatorname{cosec}\left(\frac{-\pi}{6}\right)$$

$$\theta = \frac{-\pi}{6} \text{ or } \frac{-5\pi}{6} \quad \text{...(ii)}$$

Common value $= \frac{-\pi}{6}$

48. Option (c) is correct.

Explanation:

$$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3 \cdot x \cdot \frac{1}{x}\left(x + \frac{1}{x}\right)$$

$$8\cos^3\theta = x^3 + \frac{1}{x^3} + 3(2\cos\theta)$$

$$x^3 + \frac{1}{x^3} = 8\cos^3\theta - 6\cos\theta$$

$$= 2(4\cos^3\theta - 3\cos\theta)$$

$$= 2\cos 3\theta$$

49. Option (b) is correct.

Explanation:

$$\tan x + \sec x = 2\cos x$$

$$\Rightarrow \quad \frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x$$

$$\Rightarrow \quad \sin x + 1 = 2\cos^2 x$$

$$= 2(1 - \sin^2 x)$$

$$\Rightarrow \quad \sin x + 1 = 2 - 2\sin^2 x$$

$$\Rightarrow \quad 2\sin^2 x + \sin x - 1 = 0$$

$$\Rightarrow \quad 2\sin^2 x + 2\sin x - \sin x - 1 = 0$$
$$\Rightarrow 2\sin x(\sin x + 1) - 1(\sin x + 1) = 0$$
$$= (2\sin x - 1)$$
$$(\sin x + 1) = 0$$
$$\sin + 1 \neq 0 \quad \left(\because 0 \le x \le \frac{\pi}{2}\right)$$
$$\therefore \quad 2\sin x - 1 = 0$$
$$\Rightarrow \quad \sin x = \frac{1}{2} = \sin\frac{\pi}{6}$$
$$x = \frac{\pi}{6}$$

50. Option (a) is correct.

Explanation: We know:

$$\left(\frac{\pi}{12}\right) = \tan 15°$$
$$\theta = \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) \Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$$
$$\Rightarrow \quad \theta = \frac{\pi}{6}$$
$$\tan\left(\frac{1}{2}\sec^{-1}\frac{2}{\sqrt{3}}\right) = \tan\left(\frac{1}{2}\times\frac{\pi}{6}\right)$$
$$= \tan\left(\frac{\pi}{12}\right) = \frac{\sqrt{3}-1}{\sqrt{3}+1}$$
$$= \frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$$
$$= 2-\sqrt{3}$$

51. Option (b) is correct.

Explanation: For d.r's of intersecting line

$$\vec{b} = \begin{vmatrix} \hat{i} & i & \hat{6} \\ 6 & 4 & -5 \\ 1 & -2 & 3 \end{vmatrix}$$
$$= i(12-10) - j(18+5) + \hat{k}(-12-4)$$
$$= 2\hat{i} - 23\hat{j} - 16\hat{k}$$

D.r's intersecting line are (2, –23, –16)

52. Option (a) is correct.

Explanation: Equation of intersecting plane

$$6x + 4y - 5z + \lambda(x - 2y + 3z) = 2$$
$$(6+\lambda)x + (4-2\lambda) + (-5+3\lambda)z = 2$$

D.r's of plane is $(6+\lambda),(4-2\lambda),(-5+3\lambda)$

Since plane parallel to line whose d.r's (1, 3, 2)

$$\therefore (6+\lambda) + 3(4-2\lambda) + 2(-5+3\lambda) = 0$$
$$\Rightarrow \quad 6 + \lambda + 12 - 6\lambda - 10 + 6\lambda = 0$$
$$\Rightarrow \quad \lambda = -8$$

$\therefore$ Equation of plane is

$$-2x + 20y + 29z = 2$$
$$\Rightarrow \quad 2x - 20y + 29z + 2 = 0$$

53. Option (b) is correct.

Explanation: Let centre of sphere is $O(x, y, z)$

$$\therefore \quad OA = \sqrt{(x-1)^2 + y^2 + z^2}$$
$$OB = \sqrt{x^2 + (y-1)^2 + z^2}$$
$$OC = \sqrt{x^2 + y^2 + (z-1)^2}$$
$$\because \quad OA = OB \Rightarrow x^2 - 2x + 1 + y^2 + z^2$$
$$= x^2 + y^2 - 2y + 1 + z^2$$
$$\Rightarrow \quad x = y$$
$$OB = OC \Rightarrow x^2 - 2y + 1 + y^2 + z^2$$
$$= x^2 + y^2 + z^2 - 2z + 1$$
$$\Rightarrow \quad y = z$$
$$\therefore \quad x = y = z$$

For smallest radius $x = y = z = \frac{1}{3}$

$$\therefore \text{ Radius} = \sqrt{\left(1-\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2}$$
$$= \sqrt{\frac{4}{9} + \frac{1}{3} + \frac{1}{9}} = \sqrt{\frac{2}{3}}$$

54. Option (a) is correct.

Explanation: Centre $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ satisfy the equation $x + y + z - 1 = 0$

55. Option (b) is correct.

Explanation:

Mid-point AC = Mid-point BD

$$\left(\frac{1-1}{2}, \frac{-1+2}{2}, \frac{0+7}{2}\right) = \left(\frac{-2+x}{2}, \frac{1+y}{2}, \frac{8+z}{2}\right)$$
$$\Rightarrow x = 2, y = 0 \text{ and } z = -1$$

$D(2, 0, -1)$

56. Option (b) is correct.

Explanation:

$$\overrightarrow{BC} = \hat{i} + \hat{j} - \hat{k}$$
$$\overrightarrow{CD} = 3\hat{i} - 2\hat{j} - 8\hat{k}$$
$$\cos\theta = \frac{\overrightarrow{BC}\cdot\overrightarrow{CD}}{|\overrightarrow{BC}|\,|\overrightarrow{CD}|} = \left|\frac{3-2+8}{\sqrt{1+1+1}\sqrt{9+4+64}}\right|$$
$$\cos\theta = \frac{9}{\sqrt{3}\sqrt{77}}$$
$$\cos^2\theta = \frac{81}{3\times 77} = \frac{27}{77}$$

57. Option (b) is correct.

Explanation:

$$4y = mx - m + 2$$
$$4y = m(x-1) + 2$$

$$y-\frac{1}{2} = \frac{m}{4}(x-1)$$

for any values of m the family of lines always posses through $\left(1,\frac{1}{2}\right)$ so, all lines are concurrent.

58. Option (d) is correct.

Explanation: Let lows of point be (x, y)

$$(x-a)^2+(y-b)^2 = (x-c)^2+(y-d)^2$$
$$x^2-2ax+a^2+y^2-2by+b^2 = x^2-2cx+c^2+y^2 -2dy+d^2$$
$$2(a-c)x+2(b-d)y+c^2+ = 0 \quad d^2-a^2-b^2$$
$$(a-c)x+(b-d)y+\frac{1}{2} = 0 \quad (c^2+d^2-a^2-b^2)$$
$$\therefore \quad k = \frac{(c^2+d^2-a^2-b^2)}{2}$$

59. Option (b) is correct.

Explanation:

$$x^2+3y = 0 \Rightarrow x^2=-3y$$

The equation represents the equation of parabola that opens downward

The axis of the parabola is $x = 0$

$$\because \quad a = \frac{-3}{4}$$

$\therefore$ Equation of latus reactum is

$$y = \frac{-3}{4} \Rightarrow 4y+3=0$$

So, only statement II is correct.

60. Option (a) is correct.

Explanation: Put $y = 0$, we get x intercepts

$$x = \frac{2a^2}{a^2+b^2}$$

Put $x = 0$, we get y intercepts.

$$y = \frac{2b^2}{a^2+b^2}$$

$$\text{Sum} = \frac{2a^2}{a^2+b^2}+\frac{2b^2}{a^2+b^2}=2$$

61. Option (c) is correct.

Explanation:

Perpendiclar form of $\sqrt{3}x+2y = 7$ is

$$\frac{\sqrt{3}x}{\sqrt{(\sqrt{3})^2+(2)^2}}+\frac{2y}{\sqrt{(\sqrt{3})^2+(2)^2}} = \frac{7}{\sqrt{(\sqrt{3})^2+(2)^2}}$$

$$\frac{\sqrt{3}x}{\sqrt{7}}+\frac{2y}{\sqrt{7}} = \frac{7}{\sqrt{7}}$$
$$\frac{\sqrt{3}}{\sqrt{7}}x+\frac{2}{\sqrt{7}}y = \sqrt{7}$$

62. Option (c) is correct.

Explanation:

$$d = BD = \sqrt{(4-2)^2+(1-3)^2}$$
$$= \sqrt{4+4} = 2\sqrt{2}$$
$$\text{Area of the square} = \frac{1}{2}d^2$$
$$= \frac{1}{2}(2\sqrt{2})^2 = \frac{1}{2}\times 8$$
$$= 4 \text{ square units}$$

63. Option (d) is correct.

Explanation:

$$\because \quad px+qy = p+q \Rightarrow m_1 = \frac{-p}{q}$$
$$p(x-y)+q(x+y) = 2q$$
$$\Rightarrow x(p+q)-(p-q)y = 2q$$
$$m_2 = \frac{p+q}{p-q}$$
$$\tan\theta = \left|\frac{m_2-m_1}{1+m_1\cdot m_2}\right| = \left|\frac{\frac{p+q}{p-q}+\frac{p}{q}}{1-\frac{p}{q}\left(\frac{p+q}{p-q}\right)}\right|$$
$$= \left|\frac{pq+q^2+p^2-pq}{pq-q^2-p^2-pq}\right| = 1$$
$$\tan\theta = \tan\frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{4}$$
$$\therefore \quad \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

64. Option (a) is correct.

Explanation:

$$x^2+y^2-2kx-2ky+k^2 = 0$$
$$(x-k)^2+(y-k)^2 = k^2$$

$\therefore$ Centre = (k, x) and radius = k

So circle touches the x-axis at $P(k, 0)$ and y-axis at $Q(0, k)$

$$\therefore \quad PQ = \sqrt{k^2+k^2} = \sqrt{2}k$$

65. Option (b) is correct.

Explanation:

$$x^2-4y^2 = 1 \Rightarrow \frac{x^2}{1}-\frac{y^2}{\left(\frac{1}{2}\right)^2}=1$$

$$a = 1 \text{ and } b = \frac{1}{2}$$

$$c = \sqrt{a^2+b^2} = \sqrt{1+\frac{1}{4}} = \frac{\sqrt{5}}{2}$$

Distance between the foci $= 2C = \sqrt{5}$

66. Option (d) is correct.

Explanation:

$$|\vec{p}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b} = 4+4-4 = 4$$

$$\Rightarrow \quad |\vec{p}| = 2$$

$$|\vec{q}| = |\vec{a}|^2 + |\vec{b}| + 2\vec{a}\cdot\vec{b} = 4+4+4 = 12$$

$$|\vec{q}| = 2\sqrt{3}$$

$$\vec{p}\cdot\vec{q} = |\vec{a}|^2 - |\vec{b}|^2 = 4-4 = 0$$

$$\therefore \quad \vec{p} \perp \vec{q} \Rightarrow \theta = 90°$$

$$\text{Now,} \quad |\vec{p}\times\vec{q}| = |\vec{p}|\,|\vec{q}|\sin 90°$$

$$= 2\times 2\sqrt{3} = 4\sqrt{3}$$

67. Option (d) is correct.

Explanation: Vector perpendicular to both given vectors.

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 1 & 3 \end{vmatrix} = \hat{i}(-3-1) - \hat{j}(6-1) + \hat{k}(2+1)$$

$$= -4\hat{i} - 5\hat{j} + 3\hat{k} = -(4\hat{i} + 5\hat{j} - 3\hat{k})$$

$$2\vec{n} = -8\hat{i} - 10\hat{j} + 6\hat{k}$$

$$\frac{1}{50}\vec{n} = \frac{1}{50}(-4\hat{i} - 5\hat{j} + 3\hat{k})$$

Given all three vectors perpendicualr two given vectors.

68. Option (c) is correct.

Explanation:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 1 & 2 \end{vmatrix}$$

$$= \hat{i}(4-3) - \hat{j}(2-6) + \hat{k}(1-4)$$

$$= \hat{i} + 4\hat{j} - 3\hat{k}$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{1^2 + 4^2 + (-3)^2}$$

$$= \sqrt{1+16+9} = \sqrt{26}$$

Area of parallelogram $= |\vec{b}_1 \times \vec{b}_2| = \sqrt{26}$

69. Option (a) is correct.

Explanation:

$$\overrightarrow{AC} = (2\hat{i} - 3\hat{j} + 2\hat{k}) - (3\hat{i} + 4\hat{j} - 2\hat{k})$$

$$= -\text{i} - 7\hat{j} + 4\hat{k}$$

$$|\overrightarrow{AC}| = \sqrt{(-1)^2 + (-7)^2 + 4^2} = \sqrt{1+49+16}$$

$$= \sqrt{66}$$

$$|\overrightarrow{BD}| = (6\hat{i} - 2\hat{j} + \hat{k}) - (4\hat{i} - 4\hat{j} - 3\hat{k})$$

$$= 2\hat{i} + 2\hat{j} + 4\hat{k}$$

$$|\overrightarrow{BD}| = \sqrt{(2)^2 + (2)^2 + 4^2} = \sqrt{4+4+16} = \sqrt{24}$$

$$\overrightarrow{AC}\cdot\overrightarrow{BD} = -2 - 14 + 16 = 0$$

$$\cos\theta = \frac{|\overrightarrow{AC}\cdot\overrightarrow{BD}|}{|\overrightarrow{AC}|\,|\overrightarrow{BD}|} = 0 \Rightarrow \theta = 90°$$

70. Option (a) is correct.

Explanation: The position vector $\vec{r}$ from B to A is

$$\vec{r} = \vec{A} - \vec{B}$$

$$= (1+1)\hat{i} + (2+2)\hat{j} + \hat{k}(5-3)$$

$$= 2\hat{i} + 4\hat{j} + 2\hat{k}$$

$$\text{Moment } \vec{M} = \vec{r}\times\vec{F}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 2 \\ 2 & -\lambda & 5 \end{vmatrix}$$

$$= \hat{i}(20 + 2\lambda) - \hat{j}(10-4) + \hat{k}(-2\lambda - 8)$$

$$\therefore \ 16\hat{i} - 6\hat{j} + 2\lambda\hat{k} = (20+2\lambda)\hat{i} - \hat{j}(6) + \hat{k}(-2\lambda - 8)$$

$$\Rightarrow \quad 16 = 20 + 2\lambda \Rightarrow \lambda = -2$$

$$\Rightarrow \quad 2\lambda = -2\lambda - 8 \Rightarrow \lambda = -2$$

71. Option (a) is correct.

Explanation:

$$f(x) = \begin{cases} \dfrac{1-\cos 2x}{x^2}, & x < 0 \\ 9, & x = 0 \\ \dfrac{\sqrt{x}}{\sqrt{(16+\sqrt{-4})}}, & x > 0 \end{cases}$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{1-\cos 2x}{x^2} = \lim_{x\to 0^-} \frac{2\sin^2 x}{x^2} = 2$$

72. Option (c) is correct.

Explanation:

$$\lim_{n\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\sqrt{x}}{\sqrt{(16+\sqrt{x})} - 4}$$

$$= \lim_{x\to 0^+} \frac{\sqrt{x}(\sqrt{(16+\sqrt{x})} + 4)}{16 + \sqrt{x} - 16}$$

$$= \lim_{x\to 0^+} \sqrt{(16+\sqrt{x})} + 4 = 4 + 4 = 8$$

73. Option (a) is correct.

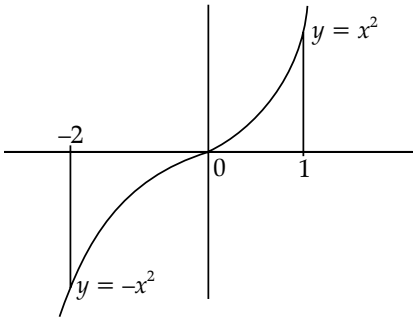
Explanation:

$$f(x) = x|x| = \begin{cases} x^2 & , \quad x \geq 0 \\ -x^2 & , \quad x < 0 \end{cases}$$

$$\lim_{x \to -1} f(x) = \lim_{x \to -1} -x^2 = -1$$

74. Option (d) is correct.

Explanation:



$$\text{Required area} = \left|-\int_{-2}^{0} x^2 dx\right| + \int_0^1 x^2 dx$$

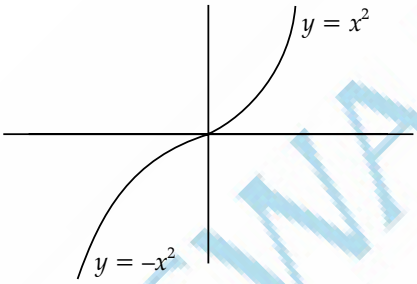
$$= -\left[\frac{x^3}{3}\right]_{-2}^{0} + \left[\frac{x^3}{3}\right]_0^1$$

$$= \left[0 + \frac{8}{3}\right] + \frac{1}{3} = \frac{8}{3} + \frac{1}{3} = \frac{9}{3} = 3$$

75. Option (c) is correct.

Explanation:

$$f(x) = \begin{cases} x^2 & , \quad x \geq 0 \\ -x^2 & , \quad x < 0 \end{cases}$$



It is clear from graph $f(x)$ is increasing in the interval $(-\infty, \infty)$

$\because$ L.H.D. $= 0 =$ R.H.D.

So, $f(x)$ is differentiable at $x = 0$

Hence both I and II are correct.

76. Option (a) is correct.

Explanation:

$$f(x) = \frac{x}{1-x} \Rightarrow f(x+1) = \frac{x+1}{1-x-1} = \frac{x+1}{-x}$$

$$\frac{f(x)}{f(x+1)} = \frac{\frac{x}{1-x}}{\frac{x+1}{-x}} = \frac{x^2}{x^2-1} = -f(x^2)$$

77. Option (d) is correct.

Explanation:

$$f(x) = \frac{x}{1-x} \Rightarrow f(\sqrt{x}) = \frac{\sqrt{x}}{1-\sqrt{x}}$$

$$f(\sqrt{x}+1) = \frac{\sqrt{x}+1}{1-\sqrt{x}-1} = \frac{\sqrt{x}+1}{-\sqrt{x}}$$

Now, $(1-x)\frac{\sqrt{x}}{1-\sqrt{x}} - \frac{x(\sqrt{x}+1)}{\sqrt{x}}$

$$\Rightarrow \frac{(1-x)x - x(1-x)}{\sqrt{x}(1-\sqrt{x})} = \frac{x - x^2 - x + x^2}{\sqrt{x}(1-\sqrt{x})} = 0$$

78. Option (a) is correct.

Explanation:

$$y = f(x) = \frac{x \sin^{-1} x}{\sqrt{1-x^2}} + \ln\sqrt{1-x^2}$$

$$f'(x) = \frac{\left(\sin^{-1} x + \frac{x}{\sqrt{1-x^2}}\right)\sqrt{1-x^2} + x\sin^{-1} x \frac{2x}{2\sqrt{1-x^2}}}{(1-x^2)} + \frac{1}{\sqrt{1-x^2}} \times \frac{1}{2\sqrt{1-x^2}} \times -2x$$

$$f'(x) = \frac{\sqrt{1-x^2}\sin^{-1} x + x + \frac{x^2 \sin^{-1} x}{\sqrt{1-x^2}}}{(1-x^2)} - \frac{x}{(1-x^2)}$$

$$= \frac{(1-x^2)\sin^{-1} x + x\sqrt{1-x^2} + x^2 \sin^{-1} x - x\sqrt{1-x^2}}{(1-x^2)\sqrt{1-x^2}}$$

$$= \frac{\sin^{-1} x}{(1-x^2)\sqrt{1-x^2}}$$

$$f'(0.5) = \frac{\sin^{-1}\frac{1}{2}}{\left(1-\frac{1}{4}\right)\left(\sqrt{1-\frac{1}{4}}\right)} = \frac{\frac{\pi}{6}}{\frac{3}{4} \times \frac{\sqrt{3}}{2}} = \frac{\pi}{6} \times \frac{8}{3\sqrt{3}}$$

$$= \frac{4\pi\sqrt{3}}{9\sqrt{3} \times \sqrt{3}} = \frac{4\pi\sqrt{3}}{27}$$

Slope of y

$$= f(x) = \frac{4\pi\sqrt{3}}{27}$$

79. Option (c) is correct.

Explanation:

$$\frac{dy}{dx} = \frac{\sin^{-1} x}{(1-x^2)^{3/2}}$$

$$\frac{dy}{dx^2} = \frac{\frac{1}{\sqrt{1-x^2}}(1-x^2)^{3/2} - \sin^{-1} x \cdot \frac{3}{2}(1-x^2)^{\frac{1}{2}}(-2x)}{(1-x^2)^3}$$

$$= \frac{\sqrt{1-x^2}+3x\sqrt{1-x^2}\sin^{-1}x}{(1-x^2)^3}$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=0} = \frac{1+0}{(1-0)^3} = 1$$

80. Option (c) is correct.

Explanation:

$$\frac{dy}{dx} = \frac{\sin^{-1}x}{(1-x^2)^{3/2}}$$

$$\left(\frac{dy}{dx}\right) = \sin\theta = \frac{\sin^{-1}(\sin\theta)}{(1-\sin^2\theta)^{3/2}}$$

$$= \frac{\theta}{\cos^3\theta} = \theta.\sec^3\theta$$

81. Option (b) is correct.

Explanation:

$$f(x) = 1-\sqrt[3]{(x-1)^2}$$

$\because (x-1)^2 \geq 0 \Rightarrow (x-1)^2$ is real number

Since, cube root of any real number is a real nubmer. So, there are no restriction on the values of x.

$\therefore$ Domain $= (-\infty, \infty)$

82. Option (d) is correct.

Explanation:

$$f'(x) = -\frac{2}{3}(x-1)^{\frac{-1}{3}} \neq 0,\ x \in R$$

So no critical point. Hence no extremum.

83. Option (a) is correct.

Explanation:

$$f(x) = \begin{cases} 4(5^x) & , \quad x<0 \\ 8k+x & , \quad x \geq 0 \end{cases}$$

$$\text{L.H.L.} = \lim_{x\to 0^-} 4(5^x) = 4(5^0) = 4$$

$$\text{R.H.L.} = \lim_{x\to 0^+} 8k+x = 8k$$

$\because f(x)$ is continuous.

$$\therefore \quad 8k = 4 \Rightarrow k = \frac{1}{2} = 0.5$$

84. Option (c) is correct.

Explanation:

$$f'(x) = \begin{cases} 4(5^x)\cdot\log 5 & , \quad x<0 \\ 1 & \quad x \geq 0 \end{cases}$$

$$f'(-1) = 4(5^{-1})\log 5 = \frac{4}{5}\log 5$$

85. Option (d) is correct.

Explanation:

$$u+v = \int e^x \cos x dx + \int e^x \sin x dx$$

$$= \int e^x(\cos x + \sin x)dx$$

$$= e^x \sin x \qquad \left[\because \frac{d\sin x}{dx} = \cos x\right]$$

$$= \frac{dv}{dx}$$

86. Option (d) is correct.

Explanation:

$$u = \int e^x \cos x dx$$

$$\frac{du}{dx} = e^x \cos x \neq -v$$

$$v = \int e^x \sin x dx$$

$$\frac{dv}{dx} = e^x \sin x \neq -u$$

So, both $e^x \sin x \neq -u$

87. Option (a) is correct.

Explanation:

$$f(x) = |x-3|+|x-4| = \begin{cases} -2x+7 & , \quad x<3 \\ 1 & , \quad 3 \leq x < 4 \\ 2x-7 & , \quad x \geq 4 \end{cases}$$

for $x = 3.5$

$$y = f(x) = 1$$

$$\frac{dy}{dx} = \frac{d(1)}{dx} = 0$$

88. Option (d) is correct.

Explanation:

at $x = 3$

L.H.D. $= -2$ and R.H.D. $= 0$

$\because$ L.H.D. $\neq$ R.H.D.

So, $f(x)$ is not differentiable at $x = 3$

at $x = 4$

L.H.D. $= 0$ and R.H.D. $= 2$

$\because$ L.H.D. $\neq$ R.H.D.

So, $f(x)$ is not differentiable at $x = 4$.

89. Option (a) is correct.

Explanation:

$$f(0) = \frac{10^0 - 10^0}{10^0 + 10^0} = 0$$

$\therefore f \circ f \circ f \circ f \circ f(0) = 0$

90. Option (d) is correct.

Explanation:

$$\frac{y}{1} = \frac{10^x - 10^{-x}}{10^x + 10^{-x}} = \frac{10^{2x}-1}{10^{2x}+1}$$

$$\frac{y+1}{y-1} = \frac{10^{2x}-1+10^{2x}+1}{10^{2x}-1-10^{2x}-1} = \frac{10^{2x}}{-1}$$

$$\frac{1+y}{1-y} = 10^{2x} \Rightarrow 2x = \log_{10}\left(\frac{1+y}{1-y}\right)$$

$$x = \frac{1}{2}\log_{10}\left(\frac{1+y}{1-y}\right)$$

$$f^{-1}(x) = \frac{1}{2}\log_{10}\left(\frac{1+x}{1-x}\right)$$

91. Option (a) is correct.

Explanation:

$$\left(\frac{d^2y}{dx^2}\right)^{3/2} = \left(\frac{dy}{dx}\right)^{5/2}$$

Squaring both sides

$$\left(\frac{d^2y}{dx^2}\right)^{3} = \left(\frac{dy}{dx}\right)^{5}$$

Degree = 3.

92. Option (c) is correct.

Explanation:

$$\int_n^{n+1}(x-[x])dx = \int_n^{n+1}(x-n)dx$$

$$= \left[\frac{x^2}{2} - nx\right]_n^{n+1}$$

$$= \frac{(n+1)^2}{2} - n(n+1) - \frac{n^2}{2} + n^2$$

$$= \frac{1-n^2}{2} + \frac{n^2}{2} = \frac{1}{2}$$

93. Option (c) is correct.

Explanation:

$$y = xe^{2x} \Rightarrow \frac{dy}{dx} = e^{2x} + 2xe^{2x}$$

$$\frac{dy}{dx} = \frac{y}{x} + 2y = y\left(2 + \frac{1}{x}\right)$$

So, statement I is correct

$$\frac{dy}{dx} = \frac{x+y}{x} = 1 + \frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx} - \frac{1}{x}\cdot y = 1$$

$$\text{I.F} = e^{\int -\frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$$

$$\therefore \quad y\cdot\frac{1}{x} = \int\frac{1}{x}dx$$

$$\frac{y}{x} = \ln|x| + c$$

$$y = x\ln|x| + cx$$

So, statement II is correct.

94. Option (b) is correct.

Explanation:

$$\frac{dy}{dx} = \frac{k^2}{(x+y)^2}$$

Let $x + y = t \Rightarrow 1 + \frac{dy}{dx} = \frac{dt}{dx}$

$$\frac{dt}{dx} - 1 = \frac{k^2}{t^2}$$

$$\Rightarrow \quad \frac{dt}{dx} = \frac{k^2}{t^2} + 1 = \frac{k^2+t^2}{t^2}$$

$$\int\frac{t^2}{k^2+t^2}dt = \int dx$$

$$\int\left(1 - \frac{k^2}{t^2+k^2}\right)dt = x$$

$$\Rightarrow \quad t - k^2\cdot\frac{1}{k}\tan^{-1}\frac{t}{k} = x + c$$

$$\Rightarrow x + y - k\tan^{-1}\left(\frac{x+y}{k}\right) = x + c$$

$$\Rightarrow \quad \frac{y-c}{k} = \tan^{-1}\left(\frac{x+y}{k}\right)$$

$$\Rightarrow \quad \frac{x+y}{k} = \tan\left(\frac{y-c}{k}\right)$$

$$\Rightarrow \quad x + y = k\tan\left(\frac{y-c}{k}\right)$$

95. Option (c) is correct.

Explanation:

$$\int\frac{dx}{10^x + 10^{-x}} = \int\frac{10^x}{(10^x)^2 + 1}dx$$

Let $10^x = t \Rightarrow 10^x\cdot\log_e 10\,dx = dt$

$$= \int\frac{1}{1+t^2}\times\log_{10} e\,dt$$

$$= \log_{10} e\tan^{-1} t + c$$

$$= \log_{10} e\tan^{-1}(10^x) + c$$

$$= \frac{1}{\log_e 10}\tan^{-1}(10^x) + c$$

96. Option (a) is correct.

Explanation:

Perimeter of rectangle $= 2(x+y) = 20$

$$x + y = 10 \Rightarrow y = 10 - x$$

$$\text{Area} = xy = x(10-x)$$

$$A = 10x - x^2$$

$$\frac{dA}{dx} = 10 - 2x = 0 \Rightarrow x = 5$$

$$\frac{d^2A}{dx} = -2 < 0$$

$\therefore$ Area of maximum at $x = 5$

$y = 10-5=5=x$

So, square has maximum area statement I is correct

Let $10x-x^2 = 27 \Rightarrow x^2-10x+27$

$D = (-10)^2-4\times27$

$= 100-108$

$= -8<0$

$\therefore$ Area 27 cm^2 is not possible

So, statement II is not correct.

97. Option (a) is correct.

Explanation:

$$I_1 = \int_e^{e^2}\frac{dx}{\ln x}$$

Let $\ln x = t \Rightarrow x = e^t$

$dx = e^y dt$

When $x = e \Rightarrow t = 1$

When $x = e^2 \Rightarrow t = 2$

$$= \int_1^2 \frac{e^t}{t}dt = I_2$$

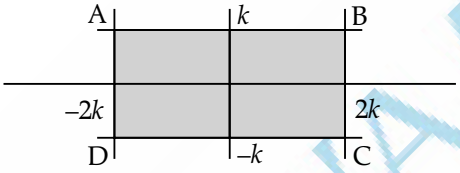
$\therefore$ $I_1 - I_2 = 0$

98. Option (d) is correct.

Explanation:

$|x| \le 2k \Rightarrow -2k \le x \le 2x$

$|y| \le k \Rightarrow -k \le y \le k$



$AB = CD = 2k-(-2k) = 4k$

$AD = BC = k-(-k) = 2k$

Area of rectangle $= 4k\times2k = 8k^2$

99. Option (c) is correct.

Explanation:

$$f(x) = \frac{1}{x-5}$$

$$f'(x) = \frac{-1}{(x-5)^2} < 0 \text{ for } x \neq 5$$

$\therefore$ $f(x)$ is decreasing on the intervals $x < 5$ and $x > 5$.

So, only statement I is correct.

100. Option (c) is correct.

Explanation:

$$f(x) = \frac{x^3+128}{x}$$

$$f'(x) = \frac{3x^2\cdot x-(x^3+128)}{x^2}$$

$$= \frac{2x^3-128}{x^2} = 0$$

$\Rightarrow$ $x = 4$

$$f''(x) = \frac{6x^2\cdot x^2-(2x^3-128)(2x)}{x^4}$$

$$f''(x) = \frac{6\times16\times16-0}{16\times16} = 6>0$$

$\therefore$ $f(x)$ is minimum at $x = 4$

Minimum value $= f(4) = \frac{64+128}{4}$

$= 48$

So, statement I is correct

$\therefore$ $f(x)$ is minimum, at $x = 4$

So, $f(x)$ is decrease in $x < 4$

$f(x)$ is increase in $x > 4$

So, sign of $f'(n)$ is changes from –ve to +ve when x increase through y.

101. Option (a) is correct.

Explanation:

$$^{10}C_3 = \frac{10!}{7!3!} = 120$$

$$^{10}C_4 = \frac{10!}{6!4!} = 210$$

$$^{10}C_5 = \frac{10!}{5!5!} = 252$$

$$^{10}C_6 = \frac{10!}{6!4!} = 210$$

$$^{10}C_7 = \frac{10!}{7!3!} = 120$$

Sum of reciprocal

$$= \frac{1}{120}+\frac{1}{210}+\frac{1}{252}+\frac{1}{210}+\frac{1}{120}$$

$$= 2\left(\frac{1}{120}+\frac{1}{210}\right)+\frac{1}{252}$$

$$= 2\left(\frac{7+4}{840}\right)+\frac{1}{252} = \frac{11}{420}+\frac{1}{252}$$

$$= \frac{33+5}{1260} = \frac{19}{630}$$

$$\text{H.M.} = \frac{n}{\sum\frac{1}{n_i}} = \frac{5}{\frac{19}{630}} = \frac{3150}{19}$$

102. Option (c) is correct.

Explanation:

$$P(\text{3 debt}) = 0.6\times0.6\times0.6$$
$$= 0.216$$

103. Option (b) is correct.

Explanation:

$$P(L) = 0.68,\ P(D) = 0.56$$
and $$P(L\cap D) = 0.48$$
$$\therefore\quad P(L\cup D) = P(L)+P(D)-P(L\cap D)$$
$$= 0.68+0.56-0.48$$
$$= 0.76$$

104. Option (a) is correct.

Explanation:

$$P(\text{Two red balls}) = \frac{5}{15}\times\frac{4}{14} = \frac{2}{21}$$

105. Option (c) is correct.

Explanation: P(One white, one red one blue)

$$= \frac{5c_1\times6c_1\times4c_1}{15c_3}$$
$$= \frac{5\times6\times4\times3\times2\times1}{15\times14\times13} = \frac{24}{91}$$

106. Option (d) is correct.

Explanation: In binomial distribution number trial is finite and fixed and independent. Each trial has two possible outcomes (success and failure).

So, statement II and III are correct.

107. Option (a) is correct.

Explanation:

$$P(X\cap Y')+P(X'\cap Y)$$
$$= P(X)\cdot[1-P(Y)]+[1-P(X)]P(Y)$$
$$= \frac{4}{5}\left[1-\frac{5}{6}\right]+\left[1-\frac{4}{5}\right]\times\frac{5}{6}$$
$$= \frac{4}{5}\times\frac{1}{6}+\frac{1}{5}\times\frac{5}{6} = \frac{9}{30} = \frac{3}{10}$$

108. Option (b) is correct.

Explanation:

$$P(P) = \frac{2}{3},\ P(P\cap E) = \frac{11}{15}$$
and $$P(P\cup E) = \frac{4}{5}$$
$$\because\quad P(P\cup E) = P(P)+P(E)-P(P\cap E)$$
$$\frac{4}{5} = \frac{2}{3}+P(E)-\frac{11}{15}$$
$$\Rightarrow\quad \frac{4}{5}-\frac{2}{3}+\frac{11}{15} = P(E)$$
$$\Rightarrow\quad P(E) = \frac{12-10+11}{15} = \frac{13}{15}$$

109. Option (d) is correct.

Explanation:

Here $P(X) = p$ and $P(Y) = q$

Also X and Y are independent

P(exactly one happens)

$$= P(X)+P(Y)-2P(X\cap Y)$$
$$= P+q-2pq$$

So statement I is incorrect

$$P(\text{at least one}) = P(X\cup Y)$$
$$= P(X)+P(Y)-P(X\cap Y)$$
$$= p+q-Pq$$

So, statement II is incorrect.

110. Option (a) is correct.

Explanation:

No. of black faces = 3

No. of white faces = 2

No. of red faces = 1

P(black, white and red appear in first, second and third resp.)

$$= \frac{3}{6}\times\frac{2}{6}\times\frac{1}{6} = \frac{1}{36}$$

111. Option (d) is correct.

Explanation:

$$n(S) = 2^4 = 16$$

Two heads consecutive (E)

$$= \{\text{HHTT, THHT, TTHH, HHHT,}$$
$$\text{HTHH, HHTH, THHH, HHHH}\}$$
$$n(E) = 8$$
So $$n(E') = 16-8 = 8$$
$$\therefore\quad P(E') = \frac{8}{16} = \frac{1}{2}$$

112. Option (d) is correct.

Explanation:

Prime number = {2, 3, 5}

Composite number = {4, 6}

Neither prime nor composite = {1}

P(One prime, one composite and one neither prime nor composite)

$$= \frac{3}{6}\times\frac{2}{6}\times\frac{1}{6}\times3!$$
$$= \frac{1}{36}\times6 = \frac{1}{6}$$

113. Option (b) is correct.

Explanation:

No. of number divisible by 5 $= 10$

No. number divisible by 9 $= 5$

No. of number divisible by 5 and 9 $= 1$

So number of number divisble by 5 or 9 $= 10+5-1=14$

$\therefore$ No. of number neither divisible by 5 nor 9 $= 50-14=36$

$\therefore$ Required probability $= \frac{36}{50}=\frac{18}{25}$

114. Option (d) is correct.

Explanation: Out of 50 consecutive natural nubmer there are 25 even and 25 odd number. We have sum of even and odd is odd.

$\therefore \quad P(\text{Sum odd}) = \frac{{}^{25}C_1 \times {}^{25}C_1}{{}^{50}C_2}$

$= \frac{25\times 25\times 2}{50\times 49}=\frac{25}{49}$

115. Option (a) is correct.

Explanation: 20 is added to each observation then standard diviation is not effected.

So new S.D is not change.

$\therefore$ New S.D. $= 10$

116. Option (a) is correct.

Explanation:

Here $n = 5,\ p = k$ and $q = 1-K$

$P(X=1) = 5c_1 K^1(1-K)^4 = 0.4096$

$P(X=2) = 5c_2 K^2(1-K)^3 = 0.2048$

$\frac{{}^5C_2K^2(1-k)^3}{{}^5C_1K(1-k)^4}=\frac{0.2048}{0.4096}$

$\frac{10.K}{5(1-K)}=\frac{1}{2}$

$4K = 1-K \Rightarrow 5K=1 \Rightarrow K=\frac{1}{5}=0.2$

117. Option (b) is correct.

Explanation:

C-I	f_1	x_i	x_if_i
5 – 15	20	10	200
15 – 25	30	20	600
25 – 35	30	30	900
35 – 45	20	40	800
Total	$\Sigma f_i = 100$		2500

$\text{Mean} = \frac{\Sigma x_i f_i}{\Sigma f_i}=\frac{2500}{100}=25$

118. Option (d) is correct.

Explanation:

Here, $P(A) = 0.3,\ P(B)=0.4$ and $P\left(\frac{A}{B}\right)=0.5$

$P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}$

$\Rightarrow \quad 0.5=\frac{P(A\cap B)}{0.4}\Rightarrow P(A\cap B)=0.2$

$\therefore \quad P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}=\frac{0.2}{0.3}=\frac{2}{3}$

$= 0.667$

119. Option (d) is correct.

Explanation:

Here $P(A)=\frac{1}{3},\ P(B)=\frac{1}{2}$ and $P(A\cap B)=\frac{1}{4}$

$P(\bar{A}\cup B) = P(\bar{A})+P(B)-P(\bar{A}\cap B)$

$= 1-P(A)+P(B)-[P(B)-P(A\cap B)]$

$= 1-\frac{1}{3}+\frac{1}{2}-\left(\frac{1}{2}-\frac{1}{4}\right)$

$= \frac{2}{3}+\frac{1}{2}-\frac{1}{4}=\frac{8+6-3}{12}=\frac{11}{12}$

120. Option (b) is correct.

Explanation: Mean and variance do not have the same unit of measurement. e.g. if data is in meters then mean is in meters but the variance is in square meters.

So statement I is incorrect.

Both mean deriation and standard deviation are measures of dispresion they are calculated in the same units as the original data.

So, statement II is correct.