CHAPTER 1

Real Numbers

Level - 1 CORE SUBJECTIVE QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Mark)

1. Option (D) is correct

Explanation: Given, $p = 18a^2b^4$ and $q = 20\ a^3b^2$

$$\text{LCM}\ (p, q) = \text{LCM}\ (18\ a^2\ b^4, 20\ a^3b^2) = 180\ a^3b^4$$

2. Option (A) is correct

Explanation: Given, HCF = 40, LCM = $252 \times k$

We know that

$$\text{LCM} \times \text{HCF} = \text{product of two number}$$

$$\Rightarrow \quad 40 \times 252 \times k = 2520 \times 6600$$

$$\Rightarrow \quad k = \frac{2520 \times 6600}{40 \times 252}$$

$$\therefore \quad k = 1650$$

3. Option (C) is correct

Explanation: Given, $a = 2^2 \times 3^x$, $b = 2^2 \times 3 \times 5$, $c = 2^2 \times 3 \times 7$

$$\text{LCM}\ (a, b, c) = 3780$$

Now $\text{LCM}\ (a, b, c) = 2^2 \times 3^x \times 5 \times 7$

$$3780 = 4 \times 5 \times 3^x \times 7$$

$$2^2 \times 3^3 \times 5 \times 7 = 2^2 \times 5 \times 3^x \times 7$$

$$3^x = 3^3$$

$$x = 3$$

4. Option (B) is correct

Explanation: Given, HCF (65, 104) = 13

$$\text{LCM}\ (65, 104) = 40 \times x$$

Now, Product of numbers = LCM × HCF

$$65 \times 104 = 13 \times (40 \times x)$$

$$\frac{5 \times 104}{40} = x$$

$$x = 13$$

5. Option (C) is correct

Explanation: The greatest number that divides

(281 – 5= 276)

and (1249 – 7= 1242)

exactly = HCF (276, 1242).

As,

HCF of 276 and 1242= 138

So, the greatest number is 138.

6. Option (C) is correct

Explanation: Given, $m = p^5q^2$, $n = p^3q^4$

Now, LCM $(m, n) = p^5q^4$

7. Option (C) is correct

Explanation: Since $3825 = 3^x \times 5^y \times 17^z$

$$3^2 \times 5^2 \times 17^1 = 3^x \times 5^y \times 17^z$$

$$\therefore x = 2, y = 2, z = 1$$

Now $x + y - 2z = 2 + 2 - 2 = 2$

8. Option (A) is correct

Explanation: The product of two co-prime numbers is 553, their HCF must be 1, because co-prime number has no common factor except 1.

9. Option (A) is correct

Explanation: Given, $2520 = 2^3 \times 3^a \times b \times 7$

$$2^3 \times 3^2 \times 5 \times 7 = 2^3 \times 3^a \times b \times 7$$

$$\therefore \quad a = 2 \text{ and } b = 5$$

Now, $a + 2b = 2 + 5 \times 2 = 12$

10. Option (D) is correct

Explanation: LCM (28, 44, 132) = ?

$$28 = 2^2 \times 7$$

$$44 = 2^2 \times 11$$

$$132 = 2^2 \times 3 \times 11$$

$$\text{LCM}\ (28, 44, 132) = 2^2 \times 3 \times 7 \times 11 = 924$$

11. Option (D) is correct

Explanation: Smallest prime number = 2

Smallest odd composite number = 9

LCM (2, 9) = 2 × 9 = 18

12. Option (C) is correct

Explanation: Since p is a multiple of q, then $p = kq$ for some natural number k.

The HCF of two numbers is the largest number that divides both of them.

Here, q divides both p (since $p = kq$) and q, so the HCF is q.

13. Option (A) is correct

Explanation: The least composite number is 4.

The least prime number is 2 .

HCF of 4 and 2 = 2

The least common multiple (LCM) of 4 and 2 is 4.

Now, the ratio of HCF to LCM is:

$$\frac{\text{HCF}}{\text{LCM}} = \frac{2}{4} = \frac{1}{2} = 1:2$$

14. Option (C) is correct

Explanation: We are given

$$p^2 = \frac{32}{50} = \frac{16}{25}$$

Now, take the square root of both sides:

$$p = \pm\frac{4}{5}$$

Since $p = \pm\frac{4}{5}$, which is a fraction (a ratio of two integers), p is a rational number.

15. Option (C) is correct

Explanation: The smallest 2-digit number is 10.

The smallest composite number is 4.

Now, LCM (Least Common Multiple) of 10 and 4:

Prime factorization:

$$10 = 2 \times 5$$
$$4 = 2^2$$

The LCM is found by taking the highest powers of all the prime factors involved:

The highest power of 2 is 2^2.

The highest power of 5 is 1.

So, the LCM is:

$$\text{LCM}(10, 4) = 2^2 \times 5 = 4 \times 5 = 20$$

16. Option (B) is correct

Explanation: Given:

$$a = x^3y^2$$
$$b = xy^3$$

For x, the highest power is x^3

For y, the highest power is y^3.

Thus the LCM $(a, b) = x^3y^3$

Now, we calculate the product of a and b:

$$a \times b = (x^3y^2) \times (xy^3) = x^4y^5$$

Next, we divide the product of a and b by the LCM (a, b):

$$\frac{a \times b}{\text{LCM}(a, b)} = \frac{x^4y^5}{x^3y^3} = xy^2$$

17. Option (A) is correct

Explanation: We are given that

The time for car A is 30 minutes.

The time for car B is p minutes.

The cars meet after 90 minutes.

The Highest Common Factor (HCF) of 30 and p is 15.

We can use the relationship between LCM and HCF:

$$\text{LCM}(30, p) = \frac{30 \times p}{\text{HCF}(30, p)} = 90$$

$$\frac{30 \times p}{15} = 90$$

$$2p = 90$$

$$p = \frac{90}{2} = 45 \text{ minutes}$$

18. Option (B) is correct

Explanation: The prime factorization of 3750

$$3750 = 2 \times 5^4 \times 3$$

The exponent of 5 in the prime factorization is 4.

19. Option (D) is correct

Explanation: Let the greatest number be x

$1251 - 1 = 1250$ is divisible by x

$9377 - 2 = 9375$ is divisible by x

$15628 - 3 = 15625$ is divisible by x

Thus, x must be a divisor of the numbers 1250, 9375, and 15625.

$$1250 = 2 \times 5^4$$
$$9375 = 3 \times 5^5$$
$$15625 = 5^6$$

Thus, the HCF of 1250, 9375, and 15625

$$\text{HCF} = 5^4 = 625$$

20. Option (D) is correct

Explanation: Expression:

$$2(5^n + 6^n)$$

The last digit of 5^n is always 5 for any natural number n because powers of 5 always end in 5 .

The last digit of 6^n is always 6 for any natural number n because powers of 6 always end in 6 .

The last digits of 5^n and 6^n are 5 and 6 , respectively. So, the last digit of their sum $5^n + 6^n$ is :

$$5+6 = 11$$

The last digit of 11 is 1 .

Now, multiply the last digit of $5^n + 6^n$ (which is 1) by 2:

$$2 \times 1 = 2$$

Thus, the last digit of $2(5^n + 6^n)$ is always 2 .

21. Option (C) is correct

Explanation: Given:

$$\text{LCM} = 2400$$

Let's denote HCF as h. Then we have:

$$h \times 2400 = a \times b$$

Where a and b are the two numbers.

Now, since h must be a factor of a and b, it must also divide the LCM (2400). We will check each option to see if it divides 2400 .

The option that cannot be their HCF is 500.

22. Option (C) is correct

Explanation: LCM (20, 25, 30)

the LCM take the highest power of each prime factor

$$20 = 2^2 \times 5$$
$$30 = 2^1 \times 3^1 \times 5^1$$
$$25 = 5^2$$

So,

$$\text{LCM} = 2^2 \times 3^1 \times 5^2 = 4 \times 3 \times 25 = 300$$

To convert 300 minutes into hours and minutes:

- 300 minutes =5 hours

  Starting from 12 : 00 pm (noon):

- 12 : 00 pm + 5 hours =5 : 00 pm

ASSERTION-REASON QUESTIONS (1 Mark)

1. Option (B) is correct

Explanation: Two consecutive even natural numbers can be represented as $2n$ and $2(n + 1)$ where n is an integer. Thus, difference between these two numbers is 2.

Hence, 2 is the greatest number that divides both $2n$ and $2(n + 1)$ exactly.

Thus, 2 is HCF of any two consecutive even natural numbers.

∴ Assertion is true.

In case of Reason: Even natural numbers are divisible by 2 is also true.

But it is not correct explanation of Assertion.

∴ Both Assertion (A) and Reason (R) are true but (R) is not correct explanation of (A).

2. Option (D) is correct

Explanation: Suppose b is 2

then $3^2 \times 7^2 \times b = 9 \times 49 \times 2 = 18 \times 49$

Hence, q is not definitely an odd number.

∴ Assertion is false.

Reason : $3^2 \times 7^2 = 9 \times 49 = 441$

It is an odd number

∴ Reason is true.

3. Option (C) is correct

Explanation:

Assertion: For 5^n to end with 0 it should have 2 and 5 as its factor

But 5^n contains only 5 as its factor.

So 5^n cannot end with digit 0.

Hence Assertion is true

Reason: Since 5 is a prime number, it's only Prime factor is 5.

1 is not a prime number so it is not included in Prime factorisation.

∴ Reason is false.

∴Assertion (A) is true but Reason (R) is false

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. Prime Factorization of $480 = 2^5 \times 3 \times 5$

$$720 = 2^4 \times 3^2 \times 5$$
$$\text{LCM}(480, 720) = 2^5 \times 3^2 \times 5 = 1440$$
$$\text{HCF}(480, 720) = 2^4 \times 3 \times 5 = 240$$

2. Prime Factorization of $85 = 5 \times 17$, $238 = 2 \times 7 \times 17$

$$\text{HCF}(85, 238) = 17$$
$$17 = 85 \times m - 238$$
$$m = 3$$

3. Given,

$7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$

By taking 13 as common, we get

$7 \times 11 \times 13 + 13 = 13(7 \times 11 + 1) = 13 \times 78$

Since, it has more than 2 factors so it is a composite number.

By taking 5 as common, we get:

$$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$$
$$= 5(7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1) = 5 \times 1009$$

Since, it has more than 2 factors so it is also a composite number.

4. No, because for a number to end with 0 , it must be divisible by 10, which means it should have both 2 and 5 as factors. Since 8^n only contains the factor 2, it cannot end with 0.

5. Least Common Multiple (LCM) of 9, 12, and 15:

Prime factorization:

$$9 = 3^2$$
$$12 = 2^2 \times 3$$
$$15 = 3 \times 5$$
$$\text{LCM} = 2^2 \times 3^2 \times 5 = 180$$

Thus, they will toll together after 180 minutes, or 3 hours.

6. Let $5 - 2\sqrt{3}$ be a rational number.

$$\therefore \; 5 - 2\sqrt{3} = \frac{p}{q}$$

(where p and q are integers and $q \neq 0$)

Then, $2\sqrt{3} = 5 - \frac{p}{q}$

$$\Rightarrow 2\sqrt{3} = \frac{(5q - p)}{q}$$

which implies $2\sqrt{3}$ is rational, a contradiction to $\sqrt{3}$ is irrational. Therefore, $5 - 2\sqrt{3}$ is irrational.

7. By taking 11 as common, we get

$5 \times 11 \times 17 + 3 \times 11 = 11(5 \times 17 + 3) = 11 \times 88$

It means the number can be expressed as a product of two factors other than 1. So, it is a composite number.

8. No, because for a number to end with 0 , it must be divisible by 10, which is 2×5. But $15^n = (3 \times 5)^n$, which lacks a factor 2.

9. Let $7 - 3\sqrt{5}$ be a rational number

Then, $7-3\sqrt{5}=\frac{p}{q}$ where p and q are integers and $q \neq 0$

$$\Rightarrow \quad 3\sqrt{5} = \frac{(7q-p)}{q}$$

which implies $3\sqrt{5}$ is rational, a contradiction to $\sqrt{5}$ is irrational.

Hence $7-3\sqrt{5}$ is irrational.

10. The statement is false.

Consider the composite number 4:

$\sqrt{4}=2$, which is a rational number.

Now, Let the composite number 6:

$\sqrt{6}$ cannot be expressed as a ratio of two integers and is therefore, irrational.

Thus, the square root of some composite numbers is rational, while for others, it is irrational. Hence, the given statement is false.

11. Since,

HCF of 66, 88 and 110.

Prime factorization of 66 = 2 × 3 × 11

Prime factorization of 88 = 2^3 × 11

Prime factorization of 110 = 2 × 5 × 11

HCF = 2 × 11 = 22.

Thus, the minimum number of rows required is:

$$\frac{66}{22}+\frac{88}{22}+\frac{110}{22} = 3 + 4 + 5 = 12 \text{ rows}$$

Therefore, the forester needs a minimum of 12 rows.

12. Let the numbers be $2x$ and $3x$.

The LCM of $2x$ and $3x$ is $6x$.

Given that $6x = 180$,

we get $x = 30$.

Thus, the numbers are 60 and 90, As, LCM × HCF = Product of numbers

$$180 \times \text{HCF} = 60 \times 90$$

$$\text{HCF} = \frac{(60\times 90)}{180} = 30$$

Thus, HCF = 30

13. Prime factorization:

$$72 = 2^3 \times 3^2$$
$$120 = 2^3 \times 3 \times 5$$

HCF = 2^3 × 3 = 24,

LCM = $2^3 \times 3^2 \times 5$ = 360

14. The greatest number which divides 85 and 72, leaving remainders 1 and 2 respectively. We need to find the HCF of 85 – 1 = 84 and 72 – 2 = 70.

Prime factorization:

$$84 = 2^2 \times 3 \times 7$$
$$70 = 2 \times 5 \times 7$$

HCF (84 and 70) = 14.

Thus, 14 is the required number.

15. LCM of 12, 16, and 24:

LCM = 48.

So, the least number is 48 + 7 = 55.

16. No, because $6^n = (2 \times 3)^n$. Since it lacks a factor 5, 6^n cannot end with 0.

17. Assume $2+\sqrt{3}$ is rational.

Then, $2+\sqrt{3}=\frac{p}{q}$ where p and q are integers and $q \neq 0$

$$\Rightarrow \quad \sqrt{3} = \frac{p}{q}-2$$

$$\Rightarrow \quad \sqrt{3} = \frac{p-2q}{q}$$

which implies $\sqrt{3}$ is rational, a contradiction to $\sqrt{3}$ is irrational.

Hence $2+\sqrt{3}$ is irrational number.

18. Prime factorization:

$$96 = 2^5 \times 3$$
$$120 = 2^3 \times 3 \times 5$$
$$\text{HCF} = 2^3 \times 3 = 24$$
$$\text{LCM} = 2^5 \times 3 \times 5 = 480$$

19. Let $5-2\sqrt{2}=\frac{p}{q}$, where p and q are integers and $q \neq 0$

Then,

$$2\sqrt{2} = 5-\frac{p}{q}$$

$$\Rightarrow \quad \sqrt{2} = \frac{5q-p}{2q}$$

Since $\frac{5q-p}{2q}$ is rational, $\sqrt{2}$ is also rational. This contradicts the fact that $\sqrt{2}$ is irrational. Therefore, our assumption is wrong, and $5-2\sqrt{2}$ must be irrational.

20. We need to find the HCF of 3.2, 2.4 and 1.6 , which is the length of the longest ruler that can measure the three dimensions exactly.

Convert the dimensions to centimeters:

Length = 320 cm

Breadth = 240 cm

Height = 160 cm

The HCF of 320, 240 and 160

$$320 = 2^6 \times 5$$

$$240 = 2^4 \times 3 \times 5$$
$$160 = 2^5 \times 5$$

The common prime factors are $2^4 \times 5 = 80$

Thus, the longest ruler that can measure the dimensions exactly is 80 cm or, 0.8 m

21. Prime factorization of $450 = 2 \times 3^2 \times 5^2$

Prime factorization of $490 = 2 \times 5 \times 7^2$

Since the HCF of m and 450 is 25, m must be divisible by 25.

Since the HCF of m and 490 is 35, m must be divisible by 35.

Now,

$$25 = 5^2$$
$$35 = 5 \times 7$$

Therefore, m must be divisible by 5 . The highest common factor between 450 and 490 that involves 5 is 5.

Hence, HCF of m, 450, and 490 = 5.

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. Let $\sqrt{3}$ be a rational number.

$\therefore \sqrt{3} = \frac{p}{q}$, where $q \neq 0$ and let p & q be co-prime.

$3q^2 = p^2 \Rightarrow p^2$ is divisible by $3 \Rightarrow p$ is divisible by 3 ...(i)

$\Rightarrow p = 3a$, where 'a' is some integer

$9a^2 = 3q^2 \Rightarrow q^2 = 3a^2 \Rightarrow q^2$ is divisible by $3 \Rightarrow q$ is divisible by 3 ...(ii)

(i) and (ii) leads to contradiction as 'p' and 'q' are co-prime.

Hence, $\sqrt{3}$ is an irrational number.

2. $(\sqrt{2}+\sqrt{3})^2 = 2 + 3 + 2\sqrt{6} = 5+2\sqrt{6}$

Let us assume, to the contrary, that $5+2\sqrt{6}$ is rational

$\therefore 5+2\sqrt{6} = \frac{a}{b}$; a, b are integers, $b \neq 0$

$$\therefore \quad \sqrt{6} = \frac{a-5b}{2b}$$

RHS is rational number, whereas LHS is an irrational number.

$\therefore$ Our assumption is wrong.

$\Rightarrow 5+2\sqrt{6} = (\sqrt{2}+\sqrt{3})^2$ is an irrational number.

3. Minimum number of rooms required means there should be maximum number of teachers in a room. We have to find HCF of 48, 80 and 144.

$$48 = 2^4 \times 3$$
$$80 = 2^4 \times 5$$
$$144 = 2^4 \times 3^2$$

HCF (48, 80, 144) = $2^4 = 16$

Therefore, total number of rooms required = $\frac{48}{16}+\frac{80}{16}+\frac{144}{16} = 3 + 5 + 9 = 17$

$\therefore$ Minimum no. of rooms = 17

4. Let $\sqrt{5}$ be a rational number.

$\therefore \sqrt{5} = \frac{p}{q}$, where $q \neq 0$ and let p & q be co-prime.

$5q^2 = p^2 \Rightarrow p^2$ is divisible by $5 \Rightarrow p$ is divisible by 5 ...(i)

$\Rightarrow p = 5a$, where 'a' is some integer

$25a^2 = 5q^2 \Rightarrow q^2 = 5a^2 \Rightarrow q^2$ is divisible by $5 \Rightarrow q$ is divisible by 5 ...(ii)

(i) and (ii) leads to contradiction as 'p' and 'q' are co-prime.

$\therefore \sqrt{5}$ is an irrational number.

5. Assuming $\frac{2-\sqrt{3}}{5}$ to be a rational number.

$\Rightarrow \frac{2-\sqrt{3}}{5} = \frac{p}{q}$, where p and q are integers & $q \neq 0$

$$\Rightarrow \sqrt{3} = \frac{2q-5p}{q}$$

Here RHS is rational but LHS is irrational.

Therefore, our assumption is wrong.

Hence, $\frac{2-\sqrt{3}}{5}$ is an irrational number.

6. Finds the HCF and LCM of A, B and C from the prime factorisation as:

$$\text{HCF} = 2^p \times 3^p \times 5^p$$
$$\text{LCM} = 2^r \times 3^r \times 5^q$$

From the given information, infers that HCF of A, B and C is 30 and equates it to the HCF obtained in step 1 to get the value of p as:

$$2^p \times 3^p \times 5^p = 30$$
$$\Rightarrow \quad (2 \times 3 \times 5)^p = (2 \times 3 \times 5)^1$$
$$\Rightarrow \quad p = 1$$

From the given information, infers that LCM of A, B and C is 5402 – 2 = 5400 ...(i)

Equates it to the LCM obtained in step r to get the values of q and r as:

$$2^r \times 3^r \times 5^q = 5400 \quad ...(i)$$
$$\Rightarrow \quad (2 \times 3)^r \times (5)^q = (2 \times 3)^3 \times (5)^2$$
$$\Rightarrow \quad q = 2 \text{ and } r = 3$$

Substitutes the values of p, q and r to find the values of A, B and C as :

$$A = 2^3 \times 3^1 \times 5^2 = 600$$
$$B = 2^1 \times 3^3 \times 5^1 = 270$$
$$C = 2^2 \times 3^2 \times 5^1 = 180$$

7. Let us assume that $5+6\sqrt{7}$ is rational.

$\Rightarrow\ 5+6\sqrt{7} = \frac{p}{q}$; $q \neq 0$ and p, q are integers

$\Rightarrow\ \sqrt{7} = \frac{p-5q}{6q}$

$\because$ p and q are integers,

$\therefore$ $p - 5q$ is an integer

$\Rightarrow \frac{p-5q}{6q}$ is a rational number.

$\Rightarrow \sqrt{7}$ is a rational number which is a contradiction.

So, our assumption that $5+6\sqrt{7}$ is a rational number is wrong.

Hence, $5+6\sqrt{7}$ is an irrational number.

8. Given that $\sqrt{2}$ is irrational number.

Let $\sqrt{2} = m$

Suppose $5-2\sqrt{2}$ is a rational number.

So, $5-2\sqrt{2} = \frac{a}{b}$ $(a \neq b, b \neq 0)$

$$2\sqrt{2} = -\frac{a}{b}+5$$

$$2\sqrt{2} = \frac{-a+5b}{b}$$

or $$\sqrt{2} = \frac{-a+5b}{2b}$$

So, $\frac{-a+5b}{2b} = m$

But $\frac{-a+5b}{2b}$ is rational number, so m is rational number which contradicts the fact that $\sqrt{2}$ is irrational number.

So, our supposition is wrong.

Hence, $5-2\sqrt{2}$ is also irrational.

9. Take HCF of 156, 208 and 260

HCF of 156, 208

Prime Factorization of 156 = 2 × 2 × 3 × 13

Prime Factorization of 208 = 2 × 2 × 2 × 2 × 13

Therefore HCF of 156, 208 = 2 × 2 × 13 = 52

Now, HCF of 52 and 260

52 = 2 × 2 × 13

260 = 2 × 2 × 5 × 13

HCF = 2 × 2 × 13 = 52

Max. number of students per bus = 52

Total number of students = 156 + 208 + 260 = 624 students.

No. of buses = $\frac{624}{52}$ = 12 buses

10. The lights will change together again at the least common multiple (LCM) of the intervals.

LCM of 48, 72 and 108:

Prime factorisation of 48 = $2^4 \times 3$

Prime factorisation of 72 = $2^3 \times 3^2$

Prime factorisation of 108 = $2^2 \times 3^3$

LCM = $2^4 \times 3^3$ = 432 seconds.

432 seconds = 7 minutes 12 seconds.

Thus, the lights will change together again at 7 : 07 : 12 a.m.

11. Prime factorization of 26 = 2 × 13

Prime factorization of 65 = 5 × 13

Prime factorization of 117 = $3^2 \times 13$

The common factor is 13, so HCF = 13.

LCM = $2 \times 3^2 \times 5 \times 13$ = 1170

12. Let the number $\sqrt{2}$ be rational.

This means $\sqrt{2} = \frac{p}{q}$, where p and q are integers, with $q \neq 0$ and gcd $(p, q) = 1$.

On squaring both sides,

$$2 = \frac{p^2}{q^2} \Rightarrow p^2 = 2q^2$$

This implies that p^2 is even, so p must be even.

Let $p = 2k$ for some integer k. Substituting into the equation:

$$(2k)^2 = 2q^2 \Rightarrow p^2 = 2q^2$$

This implies that q^2 is even, so q must also be even.

But this contradicts the assumption that gcd $(p, q) = 1$.

Therefore, $\sqrt{2}$ is an irrational number.

13. Prime factorization of 18180 = 2 × 2 × 3 × 3 × 5 × 101

Prime factorization of 7575 = 3 × 5 × 5 × 101

Common factors =3 × 5 × 101 = 1515

HCF = 1515

LCM = $2^2 \times 3^2 \times 5^2 \times 101$ = 90900

14. Prime factorization of 6, 12 and 18

6 = 2 × 3

12 = $2^2 \times 3$

18 = 2×3^2

The LCM is found by taking the highest powers of all the prime factors:

LCM (6, 12, 18) = $2^2 \times 3^2 = 4 \times 9 = 36$

So, the bells will ring together again after 36 minutes.

Time after 6 a.m.

6 : 00 a.m. + 36 minutes = 6 : 36 a.m.

15. Since,

60 = $2^2 \times 3 \times 5$

84 = $2^2 \times 3 \times 7$

108 = $2^2 \times 3^3$

HCF (60, 84, 108)= $2^2 \times 3 = 4 \times 3 = 12$

So, there will be 12 students per group.

The number of groups in each art form

For music: $\frac{60}{12} = 5$ groups.

For dance: $\frac{84}{12} = 7$ groups.

For handicrafts: $\frac{108}{12} = 9$ groups.

Each group needs a room, so the total number of rooms required is:

5(music) + 7(dance) + 9(handicrafts) = 21 rooms

16. Aadya starts with 143 stamps.

She gives away 11 stamps, so the remaining stamps are:

$$143 - 11 = 132$$

Let the number of groups be G, so the number of stamps in each group for Aadya is $\frac{132}{G}$.

Sumit starts with 220 stamps.

He gives away 11 stamps, so the remaining stamps are:

$$220 - 11 = 209$$

Let the number of groups be G, so the number of stamps in each group for Sumit is $\frac{209}{G}$.

Given that both Aadya and Sumit end up with the same number of groups, G, the number of groups should be a common divisor of both 132 and 209.

Now, HCF (132, 209)

HCF (132, 209) = 11

So, the number of groups is G=11.

Now, let's find the number of stamps in each group for Aadya and Sumit:

Aadya's stamps per group:

$$\frac{132}{11} = 12$$

Sumit's stamps per group:

$$\frac{209}{11} = 19$$

(i) The number of groups is 11.

(ii) Aadya has 12 stamps in each group, and Sumit has 19 stamps in each group.

17. Prime factorization of $455 = 5 \times 7 \times 13$

Prime factorization of $210 = 2 \times 3 \times 5 \times 7$

The common factors between 455 and 210 are 5 and 7.

HCF (455, 210)=$5 \times 7 = 35$

The greatest number of groups Bhargav can distribute is 35.

Each group will have:

$$\frac{455}{35} = 13 \text{ erasers}$$

$$\frac{210}{35} = 6 \text{ pencils}$$

18. Yellow lights flicker every 3 seconds.

Red lights flicker every 4 seconds.

Green lights flicker every 5 seconds.

All three lights flicker together, we need to calculate the Least Common Multiple (LCM) of 3, 4 and 5.

Prime factorization of 3, 4, and 5:

$$3 = 3$$
$$4 = 2^2$$
$$5 = 5$$

LCM (3, 4, 5) = $2^2 \times 3 \times 5 = 60$ seconds

This means all the lights will flicker together every 60 seconds (or 1 minute).

Thus, in 30 minutes, the number of times they flicker together is:

$$\frac{30}{1} = 30 \text{ times}$$

19. Let the two integers be a and b, and we are given:

$$a + b = 91$$

and the HCF of a and b is 13.

Since the HCF is 13, both numbers must be divisible by 13. Let's express a and b as:

$a = 13x$ and $b = 13y$

where x and y are integers.

Substituting into the sum equation:

$$13x + 13y = 91$$

$$x + y = \frac{91}{13} = 7$$

Now, we find all pairs of positive integers x and y that satisfy $x + y = 7$:

$x = 1, y = 6$

$x = 2, y = 5$

$x = 3, y = 4$

$x = 4, y = 3$

$x = 5, y = 2$

$x = 6, y = 1$

The corresponding values of a and b are:

$(a, b) = (13 \times 1, 13 \times 6) = (13, 78)$

$(a, b) = (13 \times 2, 13 \times 5) = (26, 65)$

$(a, b) = (13 \times 3, 13 \times 4) = (39, 52)$

So, the pairs of positive integers are (13, 78), (26, 65) and (39, 52).

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. Given:

$a = 2 + \sqrt{5}$ and $b = 3 - \sqrt{7}$

(i) Calculate the sum $(a + b)$

$a + b = (2+\sqrt{5})+(3-\sqrt{7})$

Simplify by combining the constants:

$a + b = (2 + 3) + (\sqrt{5} - \sqrt{7}) = 5 + (\sqrt{5} - \sqrt{7})$

So, the sum is:

$$a + b = 5 + (\sqrt{5} - \sqrt{7})$$

(ii) Calculate the product ab

$$ab = (2+\sqrt{5})(3-\sqrt{7})$$

$$ab = 2(3-\sqrt{7}) + \sqrt{5}(3-\sqrt{7})$$

$$= 6 - 2\sqrt{7} + 3\sqrt{5} - \sqrt{35}$$

So, the product is:

$$ab = 6 + 3\sqrt{5} - 2\sqrt{7} - \sqrt{35}$$

(iii) The additive inverse of $a = 2 + \sqrt{5}$ is the number that, when added to a, gives zero. This is:

$$-a = -(2+\sqrt{5}) = -2 - \sqrt{5}$$

(iv) We are asked to rationalize $\frac{1}{b}$, where $b = 3 - \sqrt{7}$

To do this, we multiply the numerator and denominator by the conjugate of b, which is $3 + \sqrt{7}$:

$$\frac{1}{b} = \frac{1}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}} = \frac{3+\sqrt{7}}{(3-\sqrt{7})(3+\sqrt{7})}$$

Hence, $(3-\sqrt{7})(3+\sqrt{7}) = 3^2 - (\sqrt{7})^2 = 9 - 7 = 2$

$$\frac{1}{b} = \frac{3+\sqrt{7}}{2}$$

(v)
- $a = 2 + \sqrt{5}$: Since $\sqrt{5}$ is an irrational number, and adding a rational number (2) to an irrational number still results in an irrational number, a is irrational.
- $b = 3 - \sqrt{7}$: Since $\sqrt{7}$ is also irrational and subtracting it from a rational number (3) results in an irrational number. Therefore, b is irrational.

2. Prime Factorization

$$78 = 2 \times 3 \times 13$$
$$91 = 7 \times 13$$
$$195 = 3 \times 5 \times 13$$

The HCF is the product of the common prime factors with the lowest powers. The only common factor is 13.

So,

HCF (78, 91, 195) = 13

The LCM is the product of all prime factors, considering the highest powers of each factor.

LCM (78, 91, 195) = $2 \times 3 \times 5 \times 7 \times 13 = 2730$

Thus, the HCF of 78, 91 and 195 is 13 and the LCM is 2730 .

(ii) We have already found that:

LCM (78, 91, 195) = 2730

HCF (78, 91, 195) = 13

Now,

1. Product of LCM and HCF:

LCM(78, 91, 195)×HCF (78, 91, 195) = 2730 ×13 = 35490

2. Product of a, b, and c (where $a = 78$, $b = 91$, and $c = 195$) :

$78 \times 91 \times 195$

$7098 \times 195 = 1384110$

Clearly,

LCM (a, b, c) × HCF (a, b, c) $\neq a \times b \times c$

Thus, the equation LCM (a, b, c) × HCF (a, b, c) = $a \times b \times c$ does not hold true in general.

3. To solve this problem, we need to find the Highest Common Factor (HCF) of 30 and 54 . The HCF will give us the maximum number of books that can be placed in each stack for both subjects, ensuring that each stack has the same number of books on a single subject.

Prime factorization

$$30 = 2 \times 3 \times 5$$
$$54 = 2 \times 3^3$$

HCF the common prime factors = $2 \times 3 = 6$.

So, the HCF of 30 and 54 is 6 . This means that each stack can have 6 books.

Calculate the number of stacks

For the 30 English books : $\frac{30}{6} = 5$ stacks.

For the 54 mathematics books : $\frac{54}{6} = 9$ stacks.

Thus, the minimum number of stacks possible is : 5 stacks for English books and 9 stacks for mathematics books.

Level - 2 ADVANCED COMPETENCY FOCUSED QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Mark)

1. Option (B) is correct

Explanation: To find the length of each piece, we need to find HCF of 40 and 84.

Prime Factorisation: $40 = 2^3 \times 5$

$84 = 2^2 \times 3 \times 7$

Common factors $= 2^2 = 4$

So, length of each piece is 4 cm as it is HCF of 40 and 84

2. Option (D) is correct

Explanation: To find the time when all three bulbs will flash together again, we need to find the least common multiple (LCM) of the intervals at which the bulbs flash.

The intervals at which the bulbs flash are 80 seconds, 90 seconds and 110 seconds.

The prime factorisation of 80 is $2^4 \times 5$

The prime factorisation of 90 is $2 \times 3^2 \times 5$

The prime factorisation of 110 is $2 \times 5 \times 11$

The LCM of these numbers is the product of the highest powers of all the prime factors:

$$\text{LCM} = 2^4 \times 3^2 \times 5 \times 11 = 7920 \text{ seconds}$$

Converting 7920 seconds into minutes $= \dfrac{7920}{60}$

= 132 minutes

= 2 hours 12 minutes

So, 8:00 AM + 2:12 = 10:12 am

3. Option (B) is correct

Explanation: Bangles with stones = 37 pairs

Bangles without stones = 33 pairs

Total pairs of bangles = 37 + 33 = 70 pairs

Dividing into Two Sets:

Set 1 contains 40 pairs.

Set 2 contains 30 pairs.

Let the number of pairs with stones in Set 1 be x.

Number of pairs without stones in Set 1 = $40 - x$

From the total, we have:

Number of pairs with stones in Set 2 = $37 - x$

Number of pairs without stones in Set 2: $33 - (40 - x) = 33 - 40 + x = x - 7$:

We need to find the difference:

"Difference" $= x - (x - 7) = 7$

4. Option (C) is correct

Explanation: (i) k is a multiple of 31. This is true since k must be a multiple of the HCF.

(ii) k is a multiple of 93. This is false. If k were a multiple of 93, then the HCF would be 93, not 31.

(iii) k is an even number. This can be true, as k could be 62 (which is a multiple of 31 and even).

(iv) k is an odd number. This can also be true, as could be 31 (which is a multiple of 31 and odd).

The valid statements for k are (i), (iii), and (iv).

5. Option (C) is correct

Explanation: Number of days after which both teams will meet = LCM of the practice schedules of both teams.

Now, PF of 2 = 2

PF of 4 = 2^2. Therefore, LCM = $2 \times 2 = 4$. This means that cricket and football teams will meet after 4 days, *i.e.*, they will not meet for 3 days.

Total days in March, after 9 March are 31 – 9 = 22 days. Since, both the teams meet every 4th day, therefore, they meet 5 times till 31 March.

6. Option (D) is correct

Explanation: To find the maximum number of plants in each row, we need to find the HCF.

Prime factorisation of 120 (rose bushes) $= 2^3 \times 3 \times 5$

Prime factorisation of 180 (marigold plants) $= 2^2 \times 3^2 \times 5$

HCF (120, 180) $= 2^2 \times 3 \times 5 = 60$

7. Option (B) is correct

Explanation: To find the greatest possible length of each piece such that both wooden planks (84 cm and 108 cm) can be cut into equal pieces with no leftover, we need to find the HCF of 84 and 108.

Prime factorisation of 84 $= 2^2 \times 3 \times 7$

Prime factorisation of 108 $= 2^2 \times 3^3$

HCF (84, 108) $= 4 \times 3 = 12$

Therefore, the greatest possible length of each piece is 12 cm.

8. Option (B) is correct

Explanation: To find the maximum number of identical bouquets the florist can make using all the flowers, we need to find the HCF of the number of red and white roses.

Prime factorisation of 36 $= 2^2 \times 3^2$

Prime factorisation of 60 $= 2^2 \times 3 \times 5$

HCF (36, 60) $= 2^2 \times 3 = 12$

9. Option (C) is correct

Explanation: To find the largest possible tile size that can cover the floor completely without cutting, we need to find the HCF of the floor's length and width.

Prime factorisation of 30 $= 2 \times 3 \times 5$

Prime factorisation of 45 $= 3^2 \times 5$

HCF (30, 45) $= 3 \times 5 = 15$

Therefore, the largest possible tile size is 15 cm.

ASSERTION-REASON QUESTIONS (1 Mark)

1. Option (D) is correct

Explanation: Assertion is false as the formula HCF × LCM = product of numbers only holds for two numbers, not for three.

Reason is true. For two number a and b, HCF $(a, b) \times$ LCM $(a, b) = a \times b$

2. Option (A) is correct

3. Option (A) is correct

4. Option (C) is correct

Explanation:

Assertion: Number of parts $= \dfrac{\text{Total length of rod}}{\text{Length of each part}}$

$$= \frac{2.5}{0.8} = 3.125$$

which is not a whole number.Thus, rod cannot be divided into equal parts of 0.8 m.

Hence, Assertion is true.

Reason: As, $\sqrt{3} \times \sqrt{3} = 3$

This shows that rational number can be multiple of an irrational numbers.

Hence, Reason is false.

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. Given: $M = p^5q^3r^2$ and $N = p^7q^5r$

Where p, q and r are prime numbers.

To find HCF (M, N), take the lowest powers of common prime factors

$$\text{HCF (M, N)} = p^5q^3r$$

To find LCM (M, N), take the highest powers of all prime factors

$$\text{LCM (M, N)} = p^7q^5r^2$$

2. Meera made an error in step 3.

If p and q are integers, $\left(\frac{p}{q} - 3\right)$ cannot be an integer since p and q are co-primes.

3. To find the longest possible equal-length strip that can divide both 96 m and 150 m exactly, we need to find the HCF of 96 and 150.

Prime factorisation of 96 $= 2^5 \times 3$

Prime factorisation of 150 $= 2 \times 3 \times 5^2$

HCF (96, 150) $= 2 \times 3 = 6$

Therefore, the longest possible equal length strip will be 6 m.

4. Maximum number of students who can receive an equal number of both notebooks and pens, = HCF of 252 and 198.

Prime factorisation of 252 $= 2^2 \times 3^2 \times 7$

Prime factorisation of 198 $= 2 \times 3^2 \times 11$

HCF (252, 198) $= 2 \times 3^2 = 18$

Thus, 18 students can receive an equal number of both notebooks and pens.

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. Maximum number of fruits in each box = Highest Common Factor (HCF) of 420 and 130.

Prime factorisation of 420 $= 2^2 \times 3 \times 5 \times 7$

Prime factorisation of 130 $= 2 \times 5 \times 13$

HCF (420, 130) $= 2 \times 5 = 10$

Therefore, number of fruits in each box $= 10$

Number of boxes required for apples $= \frac{420}{10} = 42$

Number of boxes required for mangoes $= \frac{130}{10} = 13$

Total number of boxes required $= 42 + 13 = 55$

2. Maximum capacity of a container used to measure petrol = Highest Common Factor (HCF) of 850 and 680.

Prime factorisation of 850 $= 2 \times 5^2 \times 17$

Prime factorisation of 680 $= 2^3 \times 5 \times 17$

HCF (850, 680) $= 2 \times 5 \times 17 = 170$

Maximum capacity of a container $= 170$ litres

No. of containers required for 850 litres $= \frac{850}{170} = 5$

No. of containers required for 680 litres $= \frac{680}{170} = 4$

Total containers required $= 5 + 4 = 9$

3. (i) Maximum number of rows = Highest Common Factor (HCF) of 360 and 432.

Prime factorisation of 360 $= 2^3 \times 3^2 \times 5$

Prime factorisation of 432 $= 2^4 \times 3^3$

HCF (360, 432) $= 2^3 \times 3^2 = 72$

Therefore, number of rows $= 72$

(ii) Number of red bulbs in each row $= \frac{360}{72} = 5$

Number of blue bulbs in each row $= \frac{432}{72} = 6$

4. (i) Maximum number fo equal groups = HCF (Highest Common Factor) of the two numbers.

Prime factorisation of 168 $= 2^3 \times 3 \times 7$

Prime factorisation of 252 $= 2^2 \times 3^2 \times 7$

HCF (168, 252) $= 2^2 \times 3 \times 7 = 84$

Therefore, number of groups are 84

(ii) Boys in each group $= \frac{168}{84} = 2$

Girls in each group $= \frac{252}{84} = 3$

5. (i) Time when they will meet again at the starting point = Least Common Multiple (LCM) of 18 and 24.

Prime factorisation of 18 $= 2 \times 3^2$

Prime factorisation of 24 $= 2^3 \times 3$

LCM (18, 24) $= 2^3 \times 3^2 = 72$ minutes

They will meet again at the starting point after 72 minutes.

(ii) Rounds completed by Rahul $= \frac{72}{18} = 4$

Rounds compleed by Sameer $= \frac{72}{24} = 3$

CASE BASED QUESTIONS (4 Marks)

1. Given

The initial number is 2

The last student ends up with 173250.

Prime Factorisation of 173250

$173250 = 2^1 \times 3^2 \times 5^3 \times 7^1 \times 11^1$

(i) The least prime number used by students is 3 as 2 was given by teacher.

(ii) (a) Each multiplication by a prime number results in a new number. The initial number 2 is multiplied by a prime number to reach 173250 .

This gives a total of 2 + 3+1+1=7 multiplications.

Therefore, there are 7 students (since the first student starts with 2 × 3 and the last multiplication was done by the 7^{th} student).

OR

(b) The highest prime number used is 11.

(iii) From the factorisation, 5 appears 3 times, which is the maximum frequency.

2. **(i)** Given:

History = 96

Science = 240

Mathematics = 336

Number of books arranged in each stack

= HCF of (96,240,336)

Prime factorisation of

$96 = 2^5 \times 3^1$

Prime factorisation of

$240 = 2^4 \times 3^1 \times 5^1$

Prime factorisation of

$336 = 2^4 \times 3^1 \times 7^1$

$\text{HCF} = 2^4 \times 3^1 = 16 \times 3 = 48$

48 books are arranged in each stack.

(ii) Number of stacks

$$= \frac{\text{Total Mathematics books}}{\text{Books per stack}} = \frac{336}{48}$$

$= 7$

7 stacks are used to arrange all the Mathematics books.

(iii) (a)

History Stacks:

Number of stacks $= \frac{96}{48} = 2$

Science Stacks:

Number of stacks $= \frac{240}{48} = 5$

Mathematics Stacks:

Number of stacks $= \frac{336}{48} = 7$

Total number of stacks:

$2 + 5 + 7 = 14$

OR

(b) Number of books in each stack = 48

Thickness of each History book =1.8 cm

Thickness of each Science book =2.2 cm

Thickness of each Mathematics book =2.5 cm

Height of each stack = number of books per stack × thickness of each book.

Height of each History stack:

48 × 1.8 cm = 86.4 cm

Height of each Science stack:

48 × 2.2 cm = 105.6 cm

Height of each Mathematics stack:

48 × 2.5 cm = 120 cm

3. **(i)** Prime factorization of 78 = 2 × 3 × 13

Prime factorization of 117 = $3^2 \times 13$

Prime factorization of 130 =2 × 5 × 13

The common factor across all three numbers is 13.

Thus, the HCF of 78, 117 and 130 is 13.

∴ The maximum number of students that can be seated in one room = 13

(ii) Number of rooms for C.A.V. Public School:

$\frac{78}{13} = 6$ rooms

Number of rooms for Bal Vidya Bhawan:

$\frac{117}{13} = 9$ rooms

Number of rooms for Bombay Public School:

$\frac{130}{13} = 10$ rooms

The minimum number of rooms required is:

6 + 9 +10 = 25 rooms

(iii) LCM of 78, 117, 130 $= 2^1 \times 3^2 \times 5^1 \times 13^1 = 2 \times 9 \times 5 \times 13 = 1170$

Thus, minimum number of chocolates that each school can be provided, ensuring equal distribution, is 1170 chocolates.

4. **(i)** Maximum number of students in each team = HCF of 108 and 135

Prime factorisation of 108 $= 2^2 \times 3^3$

Prime factorisation of 135 $= 3^3 \times 5$

HCF (108, 135) $= 3^3 = 27$

So, the maximum number of participants in each team is 27.

(ii) Teams of boys $= \frac{108}{27} = 4$

Teams of girls $= \frac{135}{27} = 5$

(iii) (a) HCF (108, 135) = 27 (Solved above)

LCM (108, 135) $= 2^2 \times 3^3 \times 5$

$= 4 \times 27 \times 5 = 540$

OR

(b) HCF × LCM = 27 × 540 = 14580

Product of 108 and 135 = 108 × 135 = 14580

Hence verified.

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. (i) Maximum number of boxes = HCF of (180, 150)

Prime Factorisation:

$$180 = 2^2 \times 3^2 \times 5$$
$$150 = 2 \times 3 \times 5^2$$

Common prime factors:

$$2 \times 3 \times 5 = 30$$
$$\text{HCF} = 30$$

So, the maximum number of boxes = 30

(ii) Chocolate muffins per box = 180 ÷ 30 = 6

Banana muffins per box = 150 ÷ 30 = 5

2. (i) Greatest length of each piece that can be cut = HCF of 40 and 72.

Prime Factorisation:

$$40 = 2^3 \times 5$$
$$72 = 2^3 \times 3^2$$

Common prime factor:

$$2^3 = 8$$
$$\text{HCF} = 8$$

So, the greatest length of each piece = 8 metres

(ii) For the 40 m rope: 40 ÷ 8 = 5 pieces

For the 72 m rope: 72 ÷ 8 = 9 pieces