10

CHAPTER

Heights and Distances

Level - 1 CORE SUBJECTIVE QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Mark)

1. Option (B) is correct

Explanation: To find horizontal distance between rod and the base of the crane's arms various trigonometric ratio's used are;

(a) $\tan\theta = \frac{P}{B} = \frac{y+43}{\text{Horizontal Distance}}$

$\Rightarrow \text{Horizontal Distance} = \frac{y+43}{\tan\theta}$

$\therefore$ Ananya is incorrect.

(b) $\cos\theta = \frac{B}{H} = \frac{\text{Horizontal Distance}}{x}$

$\Rightarrow x\cos\theta = \text{Horizontal Distance}$

$\therefore$ Suman is correct

(c) $\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$

$= \frac{x}{y+43}$

Here, horizontal distance is not used in formula

$\therefore$ Dipti is incorrect.

Hence, option (B) i.e., only suman is correct.

2. Option (C) is correct

Explanation: The rods form two right-angled triangles with the beam's height h as one side and half of the distance AB (the horizontal distance between the two points where the rods meet the ground) as the other side.

The angle between the rods is $2x$, so each of the triangles has an angle of x. The height h is adjacent side to the angle x, and half the distance between the points where the rods touch the ground (which is half of AB) is the opposite side.

From basic trigonometry:

$$\tan(x) = \frac{\text{opposite}}{\text{adjacent side}}$$

$$= \frac{\frac{AB}{2}}{h} = \frac{AB}{2h}$$

Solving for AB:

$$AB = 2h\tan(x)$$

3. Option (B) is correct

Explanation: The height of the pyramid $h = 21.6$ m.

The base is a square with an edge of 34 m.

The midpoint of one edge of the base is half of the edge's length, which is $\frac{34}{2} = 17$m away from the center of the base.

The angle of elevation is formed between the line of sight from the midpoint of the base edge to the top of the pyramid and the horizontal.

To find the angle of elevation θ, we use the tangent function:

$$\tan(\theta) = \frac{\text{opposite (height of the pyramid)}}{\text{adjacent (distance from midpoint to the center of base)}}$$

$$\tan(\theta) = \frac{21.6}{17}$$

$$\tan(\theta) = 1.26$$

Now, we find the angle using the arctan function:

$$\theta \approx \tan^{-1}(1.26) \approx 50°$$

So, the closest angle of elevation is (B) 50°

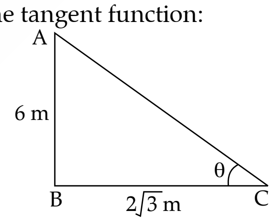
4. Option (A) is correct

Explanation: The height of the pole (h) is the opposite side, given as 6 meters.

The length of the shadow (s) is the adjacent side, given as $2\sqrt{3}$ meters.

The angle of elevation of the sun θ, is the angle we need to find.

Using the tangent function:



$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{h}{s}$$

Substitute, $h = 6$ and $s = 2\sqrt{3}$:

$$\tan\theta = \frac{6}{2\sqrt{3}}$$

Simplify this expression:

$$\tan\theta = \frac{6}{2\sqrt{3}}$$

$$= \frac{3}{\sqrt{3}} = \sqrt{3}$$

Now, we know that:

$$\tan 60° = \sqrt{3}$$

Therefore:

$$\theta = 60°$$

5. Option (C) is correct

Explanation: The height of the pole = 10 m

The shadow of the pole = 5 m

The shadow of the tower = 12.5 m

The height of the tower = H m

Since the triangles are similar:

$$\frac{\text{Height of pole}}{\text{Shadow of pole}} = \frac{\text{Height of tower}}{\text{Shadow of tower}}$$

Substitute the known values:

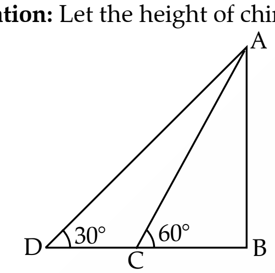
$$\frac{10}{5} = \frac{H}{12.5}$$

$$2 = \frac{H}{12.5}$$

$$H = 2 \times 12.5 = 25 \text{ m}$$

6. Option (B) is correct

Explanation: Let the height of chimney be AB (h).



Given that: angle of elevation changes from angle $\angle D = 30°$ to $\angle C = 60°$.

And, the distance CD = 20 m and we assume BC = y m

In right angle triangle ABC, $\angle B = 90°$

$$\Rightarrow \tan C = \frac{AB}{BC}$$

$$\Rightarrow \tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{h}{y}$$

$$\Rightarrow y = \frac{h}{\sqrt{3}} \quad \text{...(i)}$$

Again in right angle triangle ABD, $\angle B = 90°$

$$\Rightarrow \tan D = \frac{AB}{BC + CD}$$

$$\Rightarrow \tan 30° = \frac{h}{y + 20}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{y+20}$$

$$\Rightarrow \sqrt{3}h = y + 20$$

$$\Rightarrow \sqrt{3}h = \frac{h}{\sqrt{3}} + 20 \quad \text{[from (i)]}$$

$$\Rightarrow h\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = 20$$

$$\Rightarrow h = \frac{20}{\sqrt{3} - \frac{1}{\sqrt{3}}}$$

$$\Rightarrow h = \frac{20 \times \sqrt{3}}{3 - 1}$$

$$\Rightarrow h = 10\sqrt{3} \text{ m}$$

7. Option (A) is correct

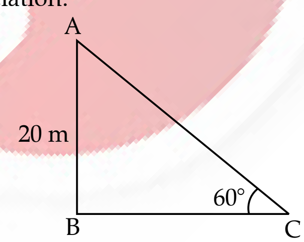
Explanation: Given:

Height of the tower (h) = 20 m

Angle of elevation (θ) = 60°

Length of the shadow = l

Using the relation:



$$\tan \theta = \frac{\text{Height of the tower}}{\text{Length of the shadow}}$$

$$= \frac{AB}{BC}$$

$$\Rightarrow \tan 60° = \frac{20}{l}$$

$$\Rightarrow \sqrt{3} = \frac{20}{l}$$

$$\Rightarrow l = \frac{20}{\sqrt{3}} \text{ m}$$

Thus, the length of the shadow is $\frac{20}{\sqrt{3}}$ meters.

ASSERTION-REASON QUESTIONS (1 Mark)

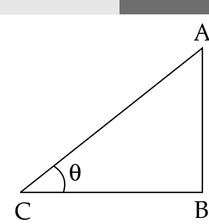
1. Option (B) is correct

Explanation: In case of assertion,

Let AB be a vertical rod and BC be its shadow.

From the figure, $\angle ACB = \theta$

In right angled ΔABC, $\angle B = 90°$



$$\tan\theta = \frac{AB}{BC}$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}}$$

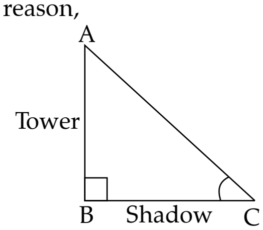
$$\left(\because \frac{AB}{BC} = \frac{1}{\sqrt{3}} \text{(Given)}\right)$$

$$\Rightarrow \quad \tan\theta = \tan 30° \quad \left(\because \tan 30° = \frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \quad \theta = 30°$$

∴ Assertion is true.

In case of reason,



Let the height of the tower be AB and its shadow be BC.

$$\therefore \quad \frac{AB}{BC} = \tan\theta$$

$$= \frac{\sqrt{3}}{1} = \tan 60°$$

Hence the angle of elevation of sun is 60°

∴ Reason is true.

Thus, both assertion and reason are true but reason is not the correct explanation of A.

2. Option (D) is correct.

Explanation: In case of assertion

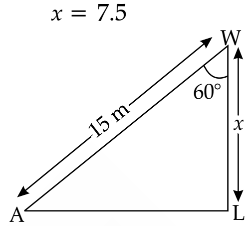
Let AW be the ladder and WL = x be the height of wall.

In right angled ΔAWL, ∠L = 90°

$$\cos 60° = \frac{x}{15}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{x}{15}$$

$$\Rightarrow \quad x = 7.5$$

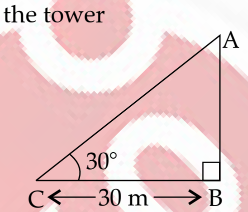


Hence, the height of the wall is 7.5 m.

∴ Assertion is false.

In case of reason,

Let AB be the tower



In right angled ΔABC, ∠B = 90°

$$\tan 30° = \frac{AB}{30}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{30}$$

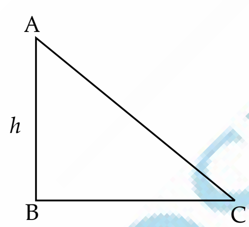
$$\Rightarrow \quad AB = 10\sqrt{3}$$

Hence, the required height of the tower is $10\sqrt{3}$ m

∴ Reason is true.

VERY SHORT ANSWER TYPE QUESTIONS (1 Mark)

1.



Consider the height of tower be AB, h

∴ Height of shadow, BC= $\sqrt{3} \times h$

In right angle ΔABC, ∠B = 90°

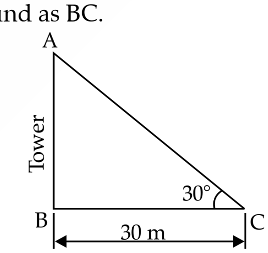
$$\tan \angle ACB = \frac{AB}{BC} = \frac{h}{\sqrt{3} \times h}$$

$$\Rightarrow \quad \tan \angle ACB = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \angle ACB = 30° \quad \left(\because \tan 30° = \frac{1}{\sqrt{3}}\right)$$

Therefore, angle of elevation is 30°

2. Let us consider the height of the tower as AB, the distance between the foot of the tower to the point on the ground as BC.



In right angle ΔABC, trigonometric ratio involving AB, BC and ∠C is tan θ.

$$\tan C = \frac{AB}{BC}$$

$$\tan 30° = \frac{AB}{30}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{30}$$

$$AB = \frac{30}{\sqrt{3}}$$

On Rationalizing the denaminator we get,

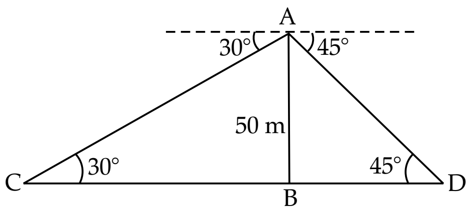
$$AB = \frac{(30\times\sqrt{3})}{(\sqrt{3}\times\sqrt{3})}$$

$$= \frac{(30\times\sqrt{3})}{3}$$

$$= 10\sqrt{3} \text{ m}$$

Thus, height of tower AB $= 10\sqrt{3}$ m.

3.



Clearly $\angle ADB = 45°$ and $\angle ACB = 30°$

[∵ Angle of depression = Angle of elevation]

Now, in right angle ΔABD, we have

$$\tan 45° = \frac{AB}{BD}$$

$$\Rightarrow \quad 1 = \frac{50}{BD}$$

$$\Rightarrow \quad BD = 50 \text{ m} \quad \text{...(i)}$$

Also, in right angle ΔABC, we have

$$\tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{50}{BC}$$

$$\Rightarrow \quad BC = 50\sqrt{3} \text{ m} \quad \text{...(ii)}$$

From equations (i) and (ii), we get

$$CD = BC + BD$$

$$= (50\sqrt{3} + 50) \text{ m}$$

$$= 50(\sqrt{3}+1) \text{ m}$$

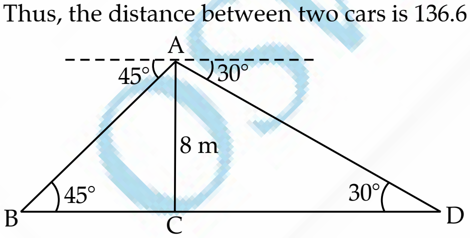
$$= 50(1.732 + 1)$$

$$= 50 \times 2.732$$

$$= 136.6 \text{ m}$$

Thus, the distance between two cars is 136.6 m.

4.



Clearly, $\angle ABC = 45°$ and $\angle ADC = 30°$

[∵ Angle of depression = Angle of elevation]

Now, in right angle ΔABC, $\angle C = 90°$ we have

$$\tan 45° = \frac{AC}{BC}$$

$$\Rightarrow \quad 1 = \frac{8}{BC}$$

$$\Rightarrow \quad BC = 8 \text{ m} \quad \text{...(i)}$$

Also, In right angle ΔACD, we have

$$\tan 30° = \frac{AC}{DC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{8}{DC}$$

$$\Rightarrow \quad DC = 8\sqrt{3} \quad \text{...(ii)}$$

From equations (i) and (ii), we get

$$BD = BC + DC$$

$$= 8 + 8\sqrt{3}$$

$$= 8(\sqrt{3}+1)$$

$$= 8(1.732 + 1)$$

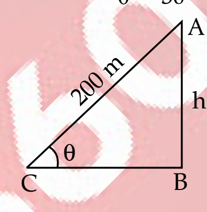
$$= 8 \times 2.732$$

$$= 21.856 \text{ m}$$

5. Given:

Distance of point to top of the tower = 200 m

Angle, $\theta = 30°$



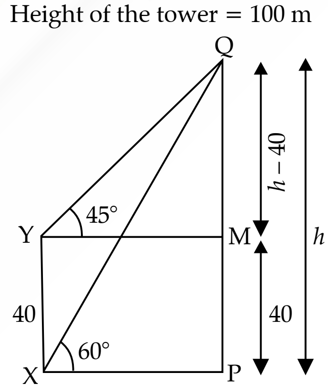
$$\text{Since } \sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 30° = \frac{\text{height of the tower}}{200}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{\text{height of the tower}}{200}$$

Height of the tower = 100 m

6.



We have

$$XY = 40 \text{ m}, \angle PXQ = 60°$$

and $\quad \angle MYQ = 45°$

Let $\quad PQ = h$

Also, $\quad MP = XY = 40$ m,

$$MQ = PQ - MP = h - 40$$

In right angle ΔMYQ, $\angle M = 90°$

$$\tan 45° = \frac{MQ}{MY}$$

$\Rightarrow \quad 1 = \dfrac{h-40}{MY}$

$\Rightarrow \quad MY = h - 40$

$\Rightarrow \quad PX = MY = h - 40 \quad ...(1)$

Now, in right anlge ΔPXQ, $\angle P = 90°$

$$\tan 60° = \frac{PQ}{PX}$$

$\Rightarrow \quad \sqrt{3} = \dfrac{h}{h-40} \quad$ [From (i)]

$\Rightarrow \quad h\sqrt{3} - 40\sqrt{3} = h$

$\Rightarrow \quad h\sqrt{3} - h = 40\sqrt{3}$

$\Rightarrow \quad h(\sqrt{3}-1) = 40\sqrt{3}$

$\Rightarrow \quad h = \dfrac{40\sqrt{3}}{(\sqrt{3}-1)}$

Rationalise the denominatore

$\Rightarrow \quad h = \dfrac{40\sqrt{3}}{(\sqrt{3}-1)} \times \dfrac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$

$\Rightarrow \quad h = \dfrac{40\sqrt{3}(\sqrt{3}+1)}{(3-1)}$

$\Rightarrow \quad h = \dfrac{40\sqrt{3}(\sqrt{3}+1)}{2}$

$\Rightarrow \quad h = 20\sqrt{3}(\sqrt{3}+1)$

$\Rightarrow \quad h = 60 + 20\sqrt{3}$

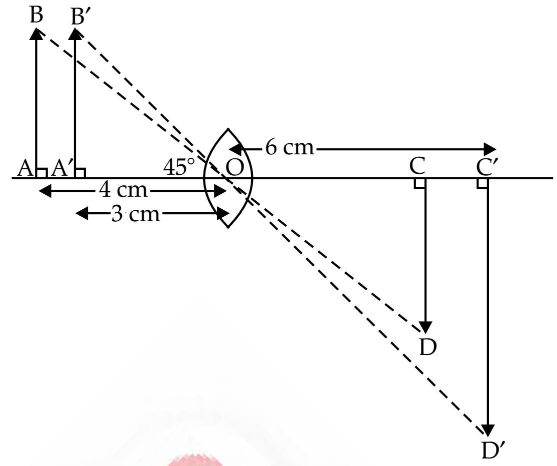
$\Rightarrow \quad h = 60 + 20 \times 1.732$

$\Rightarrow \quad h = 60 + 34.64$

$\therefore \quad h = 94.64$ m

So, the height of the tower PQ is 94. 64 m.

7.



(i) From the figure, In right angle ΔBOA, $\angle A = 90°$

$\dfrac{AB}{4} = \tan 45° \quad (\text{As } \tan 45° = 1)$

$\therefore$ The height of the object AB is 4 cm.

(ii) Let the $\angle B'OA' = \theta$

$\tan\theta = \dfrac{4}{3}$ and hence $\angle C'OD' = \angle B'OA'$

(vertical opposite angles are equal)

$\Rightarrow$ In right angle Δ C'OD', $\tan\theta = \dfrac{4}{3} = \dfrac{C'D'}{6}$

Thus, the height of image C′D′ as 8 cm.

8. For a slide to be safe, the angle θ it makes with the ground should be less than 45°.

We are provided with $\sin\theta = \dfrac{2.5}{4} = 0.625$.

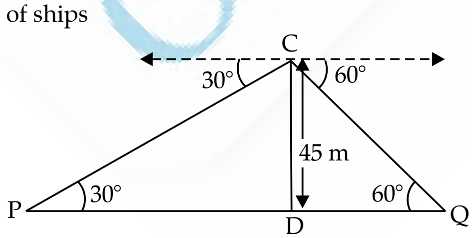
We are also given sin 45° and can use $\sqrt{2} \approx 1.4$, so:

$$\sin 45° = \frac{1}{\sqrt{2}} \oplus \frac{1}{1.4} \oplus 0.7$$

Comparing sin θ and sin 45°:
Since sin θ = 0.625 and sin 45° = 0.7,
we have that 0.625 < 0.7.
Since, $\sin\theta < \sin 45°$ implies $\theta < 45°$
Thus, the slide meets the safety requirement. Therefore, the slide is as safe for Ajay to use.

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. Let CD be the light house and P and Q be positions of ships



In right angle ΔCPD, $\angle D = 90°$

$$\tan \angle CPD = \tan 30° = \frac{CD}{PD}$$

$\therefore \quad \dfrac{1}{\sqrt{3}} = \dfrac{45}{PD}$

$\therefore \quad PD = 45\sqrt{3}$ m

In right angle ΔCQD, $\angle D = 90°$

$$\tan \angle CQD = \tan 60° = \frac{CD}{QD}$$

$\therefore \quad \sqrt{3} = \dfrac{45}{QD}$

$\therefore \quad QD = \dfrac{45}{\sqrt{3}} = 15\sqrt{3}$ m

From the diagram, PQ = PD + QD

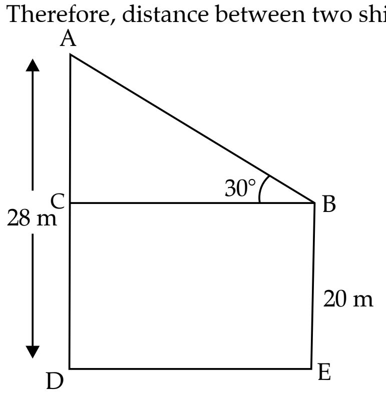
$\therefore \quad PQ = 45\sqrt{3} + 15\sqrt{3}$

$= 60\sqrt{3}$ m

$\therefore \quad PQ = 60 \times 1.732 = 103.92$ m

Therefore, distance between two ships is 103.92 m.

2.



Given, that wire makes an angle ∠B = 30°

Now, AC = (28 – 20) m = 8 m

In a right angle ΔABC, ∠C = 90°

$\Rightarrow \quad \sin B = \frac{AC}{AB}$

$\Rightarrow \quad \sin 30° = \frac{8}{AB}$

$\Rightarrow \quad \frac{1}{2} = \frac{8}{AB}$

$\Rightarrow \quad AB = 16$ m

Now, the distance between the poles is BC

In right angle ΔABC

$\tan 30° = \frac{AC}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{8}{BC}$

$\Rightarrow \quad BC = 8\sqrt{3}$ m

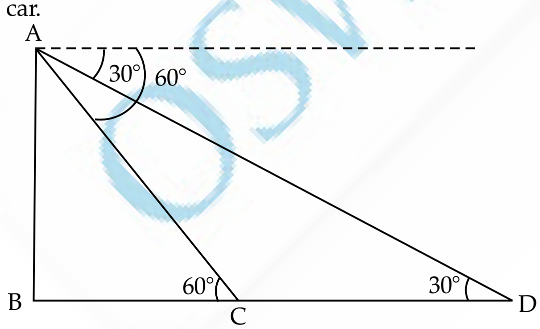
Thus, distance between the the poles = $8\sqrt{3}$ m

3. Let AB be the tower.

D is the initial and C is the final position of the car respectively.

Angles of depression are measured from A.

BC is the distance from the foot of the tower to the car.



According to question,

In right angle ΔABC, ∠B = 90°

$\tan 60° = \frac{AB}{BC}$

$\Rightarrow \quad \sqrt{3} = \frac{AB}{BC}$

$\Rightarrow \quad BC = \frac{AB}{\sqrt{3}}$ ...(i)

Also,

In right angle ΔABD, ∠B = 90°

$\tan 30° = \frac{AB}{BD}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{(BC + CD)}$

$\Rightarrow \quad AB\sqrt{3} = BC + CD$

$\Rightarrow \quad AB\sqrt{3} = \frac{AB}{\sqrt{3}} + CD$ [from eq (i)]

$\Rightarrow \quad CD = AB\sqrt{3} - \frac{AB}{\sqrt{3}}$

$\Rightarrow \quad CD = AB\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$

$\Rightarrow \quad CD = \frac{2AB}{\sqrt{3}}$

$\Rightarrow \quad AB = \frac{\sqrt{3}CD}{2}$ ...(ii)

Substitute this AB value in (i)

$\Rightarrow \quad BC = \frac{\sqrt{3}CD}{2\sqrt{3}}$

$\Rightarrow \quad BC = \frac{CD}{2}$

Here, the distance of BC is half of CD. Thus, the time taken is also half.

Time taken by car to travel distance CD = 10 sec.

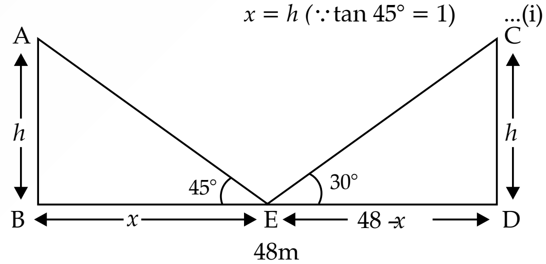
Time taken by car to travel BC = $\frac{10}{2}$ = 5 sec.

Hence, the time taken by car to reach the foot of the tower from the given point is 5 sec.

4. In right angle ΔABE ∠B = 90°

$\tan 45° = \frac{h}{x}$

$x = h$ ($\because \tan 45° = 1$) ...(i)



In right angle ΔEDC

$\tan 30° = \frac{h}{48 - x}$

$\frac{1}{\sqrt{3}} = \frac{h}{48 - x}$

$48 - x = h\sqrt{3}$

$48 = x\sqrt{3} + x$ ($\therefore h = x$)

$48 = x(1 + \sqrt{3})$

$$x = \frac{48(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})}$$

$$x = \frac{48(\sqrt{3}-1)}{2}$$

$$x = 24(\sqrt{3}-1)$$

$$x = 24 \times 0.732$$

$$x = 17.57$$

So, $h = BE = 17.57$ m

$$ED = 48 - 17.57$$
$$= 30.43\text{m}$$

Thus, height of the poles = 17.57 m

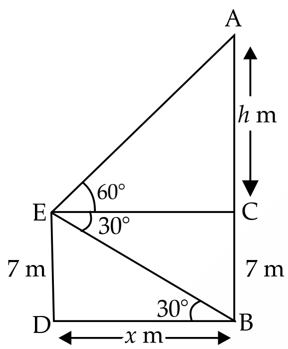
and distance of the point from the pole AB = 17.57 m

and pole CD = 30.43 m

5. Given, height of building = 7 m

Let $AC = h$ m

and $BD = x$ m



In right angle ΔBDE,

$$\tan 30° = \frac{ED}{BD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{7}{x}$$

$$\Rightarrow \quad x = 7\sqrt{3}\text{m}$$

In right angle ΔACE, ∠C = 90°

$$\tan 60° = \frac{AC}{CE}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x} \quad \text{... [∵ CE = BD]}$$

$$\Rightarrow \quad h = x\sqrt{3}$$
$$= 7\sqrt{3} \times \sqrt{3}$$
$$= 7 \times 3$$
$$= 21 \text{ m}$$

∴ Height of the tower = AB = AC + CB
= 21 + 7
= 28 m.

6. Let AB be the deck and CD be the hill

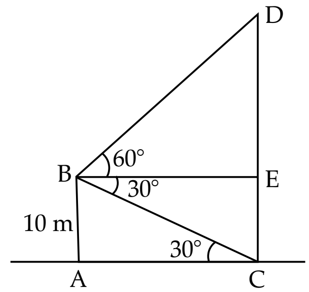
Let the man be at B.

Then, AB = 10 m

Let BE ⊥ CD and AC ⊥ CD

Then, ∠EBD = 60°

and ∠EBC = 30°



∴ ∠ACB = ∠EBC = 30°

(angle of elevation
= angle of depression)

Let $CD = h$ metres

Then, CE = AB = 10 m

and $ED = (h - 10)$ m

From right angle ΔCAB, ∠A = 90° we have

$$\Rightarrow \quad \frac{AC}{AB} = \cot 30°$$

$$\Rightarrow \quad \frac{AC}{10} = \sqrt{3}$$

$$\Rightarrow \quad AC = 10\sqrt{3}\text{ m}$$

As, BE = AC

$$\therefore \quad BE = 10\sqrt{3}\text{ m}$$

Now, from right angle ΔBED, ∠E = 90° we have

$$\Rightarrow \quad \frac{DE}{BE} = \tan 60°$$

$$\Rightarrow \quad \frac{h-10}{10\sqrt{3}} = \sqrt{3} \quad \text{[using (i)]}$$

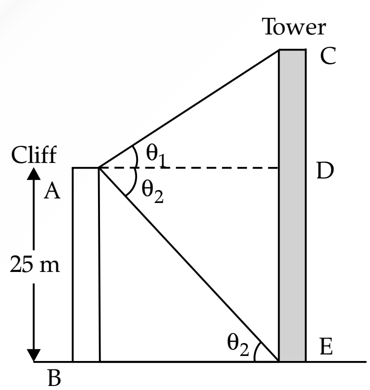
$$\Rightarrow \quad h - 10 = 30$$

$$\Rightarrow \quad h = 40 \text{ m}$$

Thus, height of the hill = 40 m

7. Given : $\theta_1 = \theta_2$

AB = 25 m



In right angle triangle ABE, ∠B = 90°

$$\tan \theta_2 = \frac{AB}{BE} = \frac{25}{BE}$$

In right angle triangle ADC,

$$\tan \theta_1 = \frac{CD}{AD}$$

We know $\theta_1 = \theta_2$

$$\Rightarrow \quad \tan \theta_1 = \tan \theta_2$$

$$\frac{CD}{AD} = \frac{25}{BE}$$

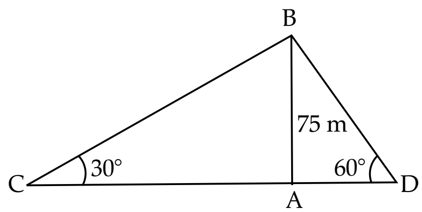
Since, $BE = AD$

$\Rightarrow$ $CD = 25$ m

$\Rightarrow$ Height of tower = CD + DE (AB)

$= 25 + 25 = 50$ m

8.



In right angled ΔBAD,

$$\tan 60° = \frac{AB}{AD}$$

$$\Rightarrow \quad \sqrt{3} = \frac{75}{AD}$$

$$\Rightarrow \quad AD = \frac{75}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{75\sqrt{3}}{3}$$

$$= 25\sqrt{3} \text{ m} \quad \text{...(i)}$$

Now, in right angled ΔBAC, ∠A = 90°

$$\tan 30° = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{75}{AC}$$

$$\Rightarrow \quad AC = 75\sqrt{3} \text{ m} \quad \text{...(ii)}$$

From equations (i) and (ii), we have

$$DC = AC + AD$$

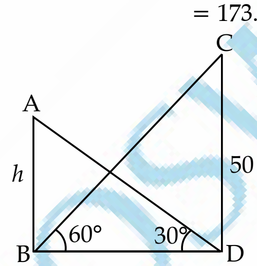
$$= 75\sqrt{3} + 25\sqrt{3}$$

$$= 100\sqrt{3}$$

$$= 100 \times 1.732$$

$$= 173.2 \text{ m}$$

9.



Given, height of tower CD = 50 m

Let the height of the building,

$$AB = h$$

In right angled triangle BDC, ∠D = 90°

$$\tan 60° = \frac{CD}{BD}$$

$$\sqrt{3} = \frac{50}{BD}$$

$$BD = \frac{50}{\sqrt{3}} = \frac{50\sqrt{3}}{3} \text{ m}$$

Now, in right angled triangle ABD, ∠B = 90°

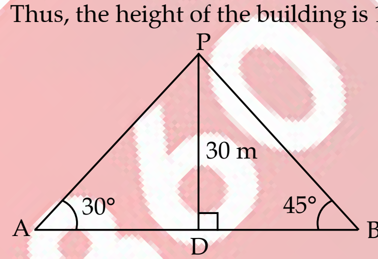
$$\tan 30° = \frac{AB}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{\frac{50\sqrt{3}}{3}}$$

$$\therefore \quad h = 16.67 \text{ m}$$

Thus, the height of the building is 16.67 m.

10.



In right angled ΔAPD, we have, ∠D = 90°

$$\tan 30° = \frac{PD}{AD}$$

$$AD = \frac{PD}{\tan 30Υ}$$

$$AD = 30\sqrt{3} \text{ m}$$

In ΔPDB, we have, ∠D = 90°

$$\tan 45° = \frac{PD}{DB}$$

$$DB = PD$$

$$DB = 30 \text{ m}$$

Now $\quad AB = AD + DB$

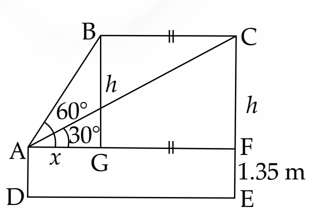
$$= 30 + 30\sqrt{3}$$

$$AB = 30\left(\sqrt{3}+1\right)\text{m}$$

Therefore, width of the river is $30\left(\sqrt{3}+1\right)$m

LONG ANSWER TYPE QUESTIONS (5 Marks)

1.



Let A be the eye level & B, C are positions of the the balloon

Distance covered by balloon in 12 sec = 3 × 12 = 36 m

BC = GF = 36 m

$$\tan 60° = \frac{h}{x}$$

$$\sqrt{3} = \frac{h}{x}$$

$\Rightarrow \quad h = x\sqrt{3}$ ..(i)

$\tan 30° = \dfrac{h}{x+36}$

$\dfrac{1}{\sqrt{3}} = \dfrac{h}{x+36}$

$\tan 30° = \dfrac{1}{\sqrt{3}} = \dfrac{h}{x+36}$

$\Rightarrow \quad h = \dfrac{x+36}{\sqrt{3}}$ ...(ii)

Substitute value of 'h' in eqn. (ii) we get

$x\sqrt{3} = \dfrac{x+36}{\sqrt{3}}$

$\Rightarrow \quad 3x - x = 36 \Rightarrow x = 18$

Now, substitute value of 'x' in eqn. (i) we get

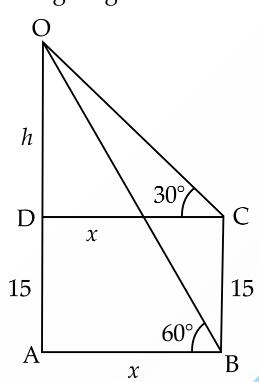
$h = 18\sqrt{3}$

$= 31.18$ m

Thus, Height of balloon from ground = 1.35 + 31.14 = 32.53 m

2. In Let OD = h and AO be the tower. DC = x and $\angle$OCD = 30°, $\angle$OBA = 60°, AD = 15 m

The corresponding diagram is as follows



In a right angled triangle ODC, $\angle$D = 90°

$\Rightarrow \quad \tan C = \dfrac{OD}{DC}$

$\Rightarrow \quad \tan 30° = \dfrac{OD}{DC}$

$\Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{h}{x}$

$\Rightarrow \quad x = \sqrt{3}h$ ...(i)

Again, in a right angled triangle OAB

$\Rightarrow \quad \tan B = \dfrac{AD+DO}{AB}$

$\Rightarrow \quad \tan 60° = \dfrac{h+15}{x}$

$\Rightarrow \quad \sqrt{3} = \dfrac{h+15}{x}$

$\Rightarrow \quad \sqrt{3} = \dfrac{h+15}{\sqrt{3}h}$ [from eq (i)]

$\Rightarrow \quad 3h = h + 15$

$\Rightarrow \quad 2h = 15$

$\Rightarrow \quad h = 7.5$ ...(ii)

$\Rightarrow \quad x = h\sqrt{3}$

$\Rightarrow \quad x = 7.5 \times 1.732$

$\Rightarrow \quad x = 12.98$

So height of the tower is:

OA = h + 15

$\Rightarrow$ OA = 7.5 + 15

$\Rightarrow$ OA = 22.5

Hence the required height is 22.5 meter and distance is 12.98 meter.

3. Let AB be the building of height 60 m and CD be the lamp post of height h, an angle of depression of the top and bottom of vertical lamp post is 30° and 60° respectively.

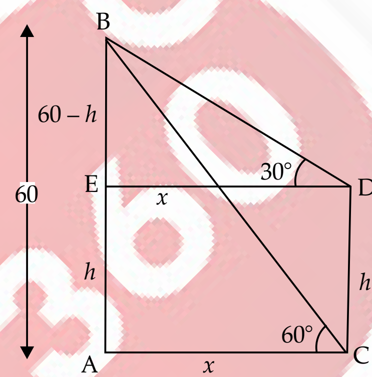
Let AE = h, AC = x, and AC = ED.

It is also given AB = 60 m.

Then BE = 60 – h

And $\angle$ACB = 60°, $\angle$BDE = 30°

We have the corresponding figure as follows



(i) In right angled ΔABC, $\angle$A = 90°

$\Rightarrow \quad \tan 60° = \dfrac{AB}{AC}$

$\Rightarrow \quad \sqrt{3} = \dfrac{60}{x}$

$\Rightarrow \quad x = \dfrac{60}{\sqrt{3}} = 20\sqrt{3}$ m

$\Rightarrow \quad AC = 20\sqrt{3}\, m = 3464$ m

Hence the horizontal distance between the building and the lamp post is 34.64 m.

(ii) In right angle ΔBED, $\angle$E = 90°

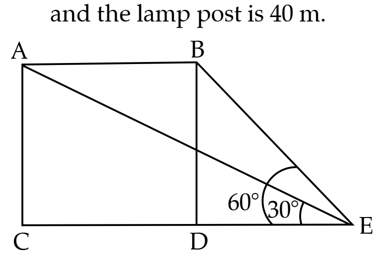
$\cos 30° = \dfrac{x}{BD}$

$\Rightarrow \quad \dfrac{\sqrt{3}}{2} = \dfrac{20\sqrt{3}}{BD}$

$\Rightarrow \quad BD = \dfrac{2 \times 20\sqrt{3}}{\sqrt{3}} = 40$ m

$\therefore$ Distance between the tops of the building and the lamp post is 40 m.

4.



Let DE = x and CD = y

Now, in right angled ΔBDE, ∠D = 90°

we have

$$\tan 60° = \frac{BD}{DE}$$

$$\Rightarrow \quad \sqrt{3} = \frac{3500\sqrt{3}}{x}$$

$$\Rightarrow \quad x = 3500 \text{ m}$$

In right angled ΔACE, ∠C = 90° we have

$$\tan 30° = \frac{AC}{CE}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{3500\sqrt{3}}{CE}$$

$$\Rightarrow \quad CE = 10500 \text{ m}$$

$$\Rightarrow \quad CD + DE = 10500$$

$$\Rightarrow \quad y + x = 10500$$

$$\Rightarrow \quad y + 3500 = 10500$$

$$\Rightarrow \quad y = 10500 - 3500$$

$$\Rightarrow \quad y = 7000$$

$$\because \quad AB = CD = 7000$$

We know that,

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\Rightarrow \quad \text{Speed} = \frac{7000}{30}$$

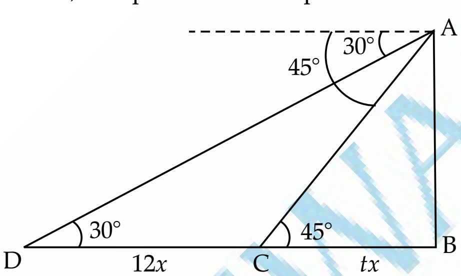
$$= 233.34 \text{ m/s}$$

$$= 233.34 \times \frac{18}{5}$$

$$= 840 \text{ km/h}$$

Hence, the speed of the aeroplane is 840 km/h

5.



Here, ∠ACB = 30°

and ∠ADB = 45°.

Let C denote the initial position of the car and D be its position after 12 minutes.

Let the speed of the car be x meter/minute, then

CD = 12 x meters (∵ Distance = speed × Time)

Let the car take t minutes to reach the tower from D.

Then, DB = tx meters

Now in the right-angled ΔACB, ∠B = 90°

$$\tan 30° = \frac{AB}{CB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{CD + DB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{12x + tx}$$

$$\Rightarrow \quad AB = \frac{12x + tx}{\sqrt{3}} \quad \text{...(i)}$$

Also, in the right-angled ΔADB, ∠B = 90°

$$\tan 45° = \frac{AB}{DB}$$

$$\Rightarrow \quad 1 = \frac{AB}{DB}$$

$$\Rightarrow \quad AB = DB = tx \quad \text{...(ii)}$$

From (i) and (ii), we have

$$t = \frac{12}{\sqrt{3} - 1}$$

$$= 12\frac{\sqrt{3} + 1}{2}$$

$$t = 6(\sqrt{3} + 1)$$

$$t = 16.38$$

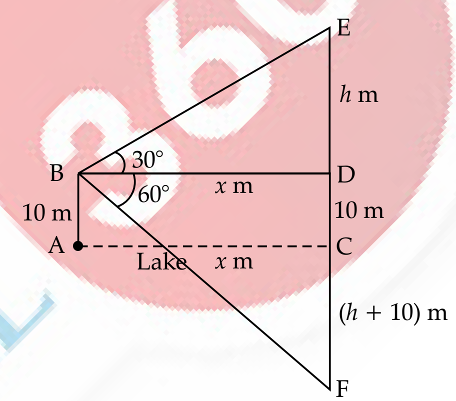
$$\therefore \quad \text{Time} = 16.38 \text{ minutes}$$

$$\text{Time} = 16 \text{ minutes } 38 \text{ seconds.}$$

$$\Rightarrow \quad 17 \text{ minutes}$$

Thus, the car will reach the tower in 17 minutes.

6.



Let the position of the cloud be E and F be the image of the cloud in the lake

Let ED = h m,

BD = AC = x m

In right angled ΔBDE, $\frac{h}{x} = \tan 30° = \frac{1}{\sqrt{3}}$

$$\Rightarrow \quad x = h\sqrt{3} \quad \text{...(i)}$$

In right angled ΔBDF,

$$\tan 60° = \frac{FD}{BD} = \frac{10 + (h + 10)}{x}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h + 20}{\sqrt{3}h} \quad \text{[using (i)]}$$

$$\Rightarrow \quad 3h = h + 20$$

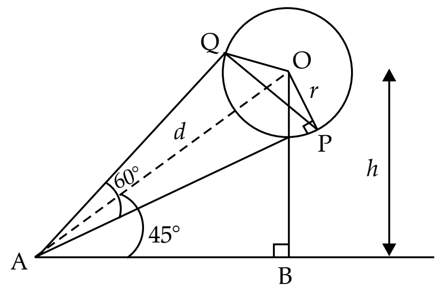
$$\therefore \quad h = 10 \text{ m}$$

So, the height of the cloud from the surface of the lake = (10+10)m = 20 m

7. Let d be the distance between observer's eye and the centre of balloon.

$$\therefore \quad OB = h$$

and $OA = d$



In ΔOAQ and ΔOAP,

$\angle$OQA = $\angle$OPA = 90° (Tangent at any point on the circumference of a circle makes 90° with the centre of the circle.)

$$OA = OA \quad \text{(Common)}$$
$$OQ = OP \quad \text{(Radius)}$$
$$\therefore \quad \Delta OAQ \cong \Delta OAP$$
(By RHS Congruence Rule)
$$\Rightarrow \quad \angle OAQ = \angle OAP \quad \text{(CPCT)}$$
$$\Rightarrow \quad \angle OAP = \frac{\angle QAP}{2} = \frac{60°}{2}$$
$$= 30° \quad \text{(By CPCT)}$$

In right angled ΔOAP $\angle$P = 90°

$$\sin \angle OAP = \frac{OP}{OA}$$
$$\Rightarrow \quad \sin 30° = \frac{r}{d}$$
$$\Rightarrow \quad \frac{1}{2} = \frac{r}{d}$$
$$\Rightarrow \quad d = 2r \quad ...(i)$$

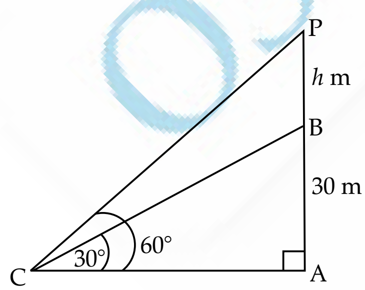
Now, in right angled ΔOAB, $\angle$B = 90°

$$\sin 45° = \frac{OB}{OA}$$
$$\Rightarrow \quad \sin 45° = \frac{h}{d}$$
$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{h}{d}$$
$$\Rightarrow \quad d = h\sqrt{2}$$

From (i), $\quad h\sqrt{2} = 2r$

$$\Rightarrow \quad h = r\sqrt{2}$$ **Proved.**

8.



Given : Height of building AB = 30 m

BP transmission tower = h(say)

$$\angle ACB = 30°, \angle ACP = 60°$$

In right angled ΔABC, $\tan 30° = \frac{AB}{AC}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{30}{AC}$$
$$\Rightarrow \quad AC = 30\sqrt{3} \quad ...(i)$$

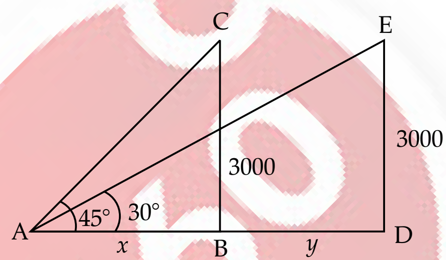
In right angled ΔAPC, $\tan 60° = \frac{AP}{AC}$

$$\sqrt{3} = \frac{30+h}{30\sqrt{3}} \quad (\because AC = 30\sqrt{3})$$
$$\Rightarrow \quad 30\sqrt{3} \times \sqrt{3} = 30 + h$$
$$\Rightarrow \quad 90 - 30 = h$$
$$\Rightarrow \quad h = 60 \text{ m}$$

$\therefore$ Height of transmission tower = 60 m

9. Let DE be the height of aeroplane which is 3000 meters above the ground.

Let AB = x, BD = y, $\angle$CAB = 45° and $\angle$EAD = 30°



In right angled ΔABC, $\angle$B = 90°

$$\Rightarrow \quad \tan A = \frac{BC}{AB}$$
$$\Rightarrow \quad \tan 45° = \frac{3000}{x}$$
$$\Rightarrow \quad 1 = \frac{3000}{x}$$
$$\Rightarrow \quad x = 3000 \text{ m} \quad ...(i)$$

Again in right angled ΔADE, $\angle$D = 90°

$$\Rightarrow \quad \tan A = \frac{DE}{AB + BD}$$
$$\Rightarrow \quad \tan 30° = \frac{3000}{x+y} \quad \text{(From i)}$$
$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{3000}{3000+y}$$
$$\Rightarrow \quad 3000 + y = 3000\sqrt{3}$$
$$\Rightarrow \quad y = 3000\sqrt{3} - 3000$$
$$\Rightarrow \quad y = 3000(\sqrt{3} - 1)$$
$$\Rightarrow \quad y = 2196 \text{ m}$$

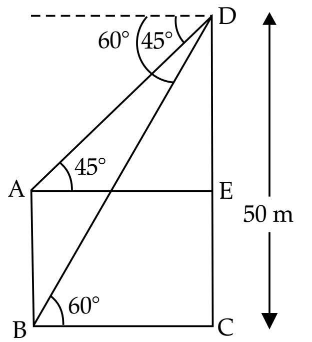
Since distance covered in

$$15 \text{ sec} = 2196 \text{ m}$$
$$\Rightarrow \quad \text{Speed} = \frac{2196}{15} = 146.4 \text{ m/s}$$
$$= \frac{146.4 \times 3600}{1000}$$
$$= 527.04 \text{ km/h}$$

Hence the speed of aeroplane is 527.04 km/h

OSWAAL 360

10.



Here CD is the tower = 50 m and AB is the pole

In right angled ΔCDB, $\angle C = 90°$

$$\tan 60° = \frac{CD}{BC}$$

$$\sqrt{3} = \frac{50}{BC}$$

$$BC = \frac{50}{\sqrt{3}}$$

$$AE = BC = \frac{50}{\sqrt{3}}$$

In right angled triangle ADE, $\angle E = 90°$

$$\tan 45° = \frac{DE}{AE}$$

$$1 = \frac{DE}{\frac{50}{\sqrt{3}}}$$

$$DE = \frac{50}{\sqrt{3}}$$

$$= 50 \times 0.577 = 28.85 \text{ m}$$

Now, $AB = EC$

and $EC = DC - DE$

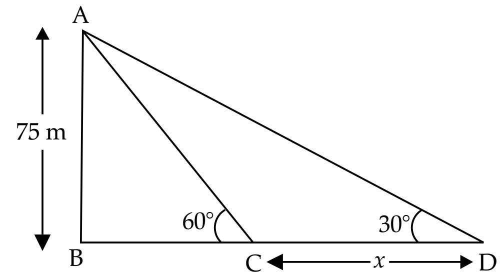
$= 50 - 28.85$

$= 21.15$ m

Thus, the height of pole = 21.15 m

11. Given, height of tower = AB = 75 m

Let distance between the cars = CD = x m



In right angled ΔABC, $\angle B = 90°$

$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{75}{BC} \quad (\because \tan 60° = \sqrt{3})$$

$$\Rightarrow \quad BC = \frac{75}{\sqrt{3}}$$

$$= \frac{75\sqrt{3}}{3}$$

$$= 25\sqrt{3} \text{ m}$$

In right angled ΔABD, $\angle B = 90°$

$$\tan 30° = \frac{AB}{BD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{75}{BC + CD}$$

$$\left(\because \tan 30° = \frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{75}{25\sqrt{3} + CD}$$

$$\Rightarrow \quad 25\sqrt{3} + CD = 75\sqrt{3}$$

$$\Rightarrow \quad CD = 75\sqrt{3} - 25\sqrt{3}$$

$$= 50\sqrt{3}$$

$$= 50 \times 1.73$$

$$= 86.5 \text{ m}$$

Thus, the distance between two cars is 86.5 m

Level - 2 ADVANCED COMPETENCY FOCUSED QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Mark)

1. Option (B) is correct

Explanation: $\angle LJM$ is the angle of depression, meaning it's the angle formed between the horizontal line (JM) and the line of sight (JL) down to the object. The line of sight refers to the direct line from the observer's eye (at point J) to the object (at point L).

2. Option (A) is correct

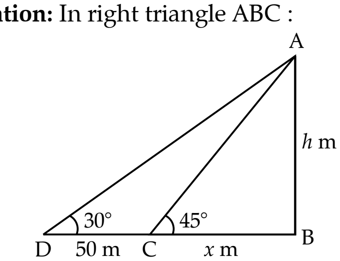
Explanation: The angle of elevation is the angle between the horizontal line from the observer's eye and the line of sight to the object above the horizontal level.

Applying to the diagram: Line of sight = AR, Horizontal from R = RC

Therefore, $\angle ARC$ is the angle between horizontal (RC) and line of sight (AR)

3. Option (B) is correct

Explanation: In right triangle ABC :



$$\tan 45° = \frac{h}{x} \Rightarrow 1 = \frac{h}{x} \Rightarrow h = x \quad \text{...(i)}$$

In right triangle ABD :

$$\tan 30° = \frac{h}{x+50} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+50} \quad ...(ii)$$

Substituting (i) into (ii):

$$\frac{1}{\sqrt{3}} = \frac{x}{x+50}$$

$$x + 50 = x\sqrt{3} \Rightarrow x\sqrt{3} - x = 50$$

$$\Rightarrow \quad x(\sqrt{3}-1) = 50 \Rightarrow x = \frac{50}{\sqrt{3}-1}$$

Now rationalize the denominator:

$$x = \frac{50}{\sqrt{3}-1}.\frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{50(\sqrt{3}+1)}{(\sqrt{3})^2-1^2}$$

$$= \frac{50(\sqrt{3}+1)}{3-1} = \frac{50(\sqrt{3}+1)}{2}$$

$$= 25(\sqrt{3}+1)$$

Since $h = x$, the height of the building is $25(\sqrt{3}+1)$ m

4. Option (A) is correct

Explanation: Let, Point L = Top of lighthouse, Point B = Foot of lighthouse, Point C = Position of the nearer ship (angle of depression = 60°), and Point D = Position of the farther ship (angle of depression = 45°)

Then triangle LCB and LDB are right-angled triangles with LB = 42 m (common vertical height)

Let, x = distance from base to ship at C, y = distance from base to ship at D

Distance between ships = $y - x$

From triangle LCB:

$$\tan 60° = \frac{42}{x} \Rightarrow \sqrt{3} = \frac{42}{x}$$

$$\Rightarrow \quad x = \frac{42}{\sqrt{3}} = \frac{42}{1.73} \approx 24.28 \text{ m}$$

From triangle LDB:

$$\tan 45° = \frac{42}{y} \Rightarrow 1 = \frac{42}{y} \Rightarrow y = 24 \text{ m}$$

Distance between the two ships:

$$y - x = 42 - 24.28$$
$$= 17.72 \text{ m}$$

Rounded to one decimal place: 17.8 m

ASSERTION-REASON QUESTIONS (1 Mark)

1. Option (A) is correct

Explanation: Let θ be the angle of elevation from the base of the building to the top of the pole

Using the formula:

$$\tan\theta = \frac{\text{height}}{\text{base}} = \frac{10}{17.3} \approx 0.578$$

Now check the angle whose tangent is approximately 0.578.

$$\tan(30°) = \frac{1}{\sqrt{3}}$$

So, Assertion is true.

Reason is also true because this is the correct definition of tangent in a right-angled triangle $\left(\frac{\text{opposite}}{\text{adjacent}}\right)$.

Both assertion and reason are true and the reason correctly explains the assertion.

2. Option (D) is correct

Explanation: Given, Angle with ground = 60° and height on the wall (opposite side) = 8 m

Hypotenuse (length of ladder) = 16 m

$$\sin(60°) = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{8}{16} = 0.5$$

But, $\sin(60°) \approx 0.866$approx

So the actual height should have been:

$$\text{Height} = 16.\sin(60°) \approx 16 \times 0.866$$
$$= 13.86 \text{ m (approx)}$$

But the given height is only 8 m, which is too small. So, the Assertion is false.

Reason is true because in a right-angled triangle,

$$\cos(60°) = \frac{1}{2}$$

3. Option (A) is correct

Explanation: Applying tangent formula to the assertion:

$$\tan(30°) = \frac{1}{\sqrt{3}} = \frac{\text{height}}{\text{shadow}}$$

$$\tan(30°) = \frac{20}{20\sqrt{3}} = \frac{1}{\sqrt{3}}$$

So, assertion is true.

Reason is also true because this is the correct definition of tangent in a right angled triangle.

Both assertion and reason are true and reason is the correct explanation of the assertion.

CASE BASED QUESTIONS (4 Marks)

1. (i) In right angled ΔOAP, ∠A = 90°

$$\frac{OP}{12\sqrt{3}} = \text{cosec } 60°$$

$$\frac{OP}{12\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$OP = \frac{2\times 12\sqrt{3}}{\sqrt{3}} = 24 \text{ m}$$

∴ Length of ladder is 24 m.

(ii) In right angled ΔOAP, ∠A = 90°

$$\frac{OA}{12\sqrt{3}} = \cot 60°$$

$$\Rightarrow \quad \frac{OA}{12\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad OA = \frac{12\sqrt{3}}{\sqrt{3}} = 12 \text{ m}$$

$\therefore$ Distance of the building Y from point O, *i.e.*, OA is 12 m.

(iii) (a) OP = OR = 24 m

$\therefore$ In right angled ΔOCR, $\angle C = 90°$

$$\frac{OC}{24} = \cos 45°$$

$$\Rightarrow \quad \frac{OC}{24} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad OC = \frac{24}{\sqrt{2}} = 12\sqrt{2} \text{ m}$$

$\therefore$ Distance between two buildings = OA + OC

$= (12 + 12\sqrt{2})$ m or $12(1+\sqrt{2})$ m

OR

(b) OP = OR = 24 m

$\therefore$ In right angled ΔOCR, $\angle C = 90°$

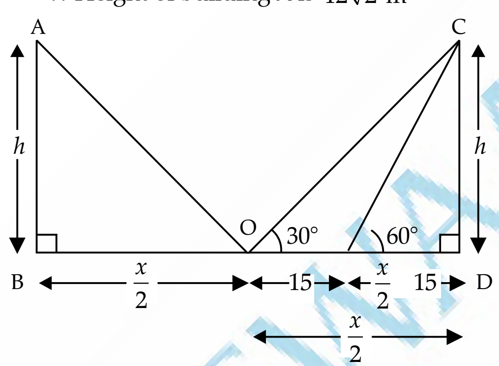
$$\frac{RC}{24} = \sin 45°$$

$$\Rightarrow \quad \frac{RC}{24} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad RC = \frac{24}{\sqrt{2}} = 12\sqrt{2} \text{ m}$$

$\therefore$ Height of building X is $12\sqrt{2}$ m

2.



(i) In right ΔABO, $\frac{h}{x/2} = \tan 30° = \frac{1}{\sqrt{3}}$

$$x = 2\sqrt{3}\,h \quad \text{...(i)}$$

which is the required relation.

(ii) In right ΔCDE, $\frac{h}{\frac{x}{2} - 15} = \tan 60°$

$$\frac{2h}{x-30} = \sqrt{3}$$

$$\Rightarrow \quad 2h = \sqrt{3}(x-30) \quad \text{...(ii)}$$

which is the required relation.

(iii) (a) $2h = \sqrt{3}\left(2\sqrt{3}h - 30\right)$ [from eq (i) & (ii)]

$$\Rightarrow \quad 2h = 6h - 30\sqrt{3}$$

$$\Rightarrow \quad 4h = 30\sqrt{3}$$

$$\Rightarrow \quad h = \frac{15\sqrt{3}}{2}$$

$\therefore$ Height of each lamp post is $\frac{15\sqrt{3}}{2}$ m.

OR

(b) $x = 2\sqrt{3}h$ [from (i)]

$$\Rightarrow \quad = \frac{x}{\sqrt{3}} = 2h$$

and $2h = \sqrt{3}(x-30)$ [from (ii)]

equating both, we get

$$x = \sqrt{3} \times \sqrt{3}(x-30)$$

$$= 3x - 90$$

$$\Rightarrow \quad 2x = 90$$

$$\Rightarrow \quad x = 45 \text{ m}$$

$\therefore$ Distance between the two lamp posts is 45 m.

3. (i) As, distance of Kaushik from the foot of the tree = BC.

So, in right angled ΔABC, $\angle B = 90°$

$$\tan C = \frac{AB}{BC}$$

$$\Rightarrow \quad \tan 45° = \frac{80}{BC} \quad (\because \tan 45° = 1)$$

$$\Rightarrow \quad 1 = \frac{80}{BC}$$

$$\Rightarrow \quad BC = 80 \text{ m}$$

(ii) (a) Distance covered by bird = AD

AD = BE

and BE = CE – CB ...(i)

In right angle ΔDCE, $\angle E = 90°$

$$\tan 30° = \frac{DE}{CE}$$

$$\left(\because \tan 30° = \frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{80}{CE}$$

$$\Rightarrow \quad CE = 80\sqrt{3}$$

Substituing value of CE in equ.(i) we get

$$BE = 80\sqrt{3} - 80$$

(As, CB = 80 m from (i))

$$BE = 80(\sqrt{3}-1)\text{m}$$

OR

(b) Distance travelled by ball after hitting the tree = AF

Here, AF = GB

and GB = CB – CG ...(i)

Now, In right angled ΔFGC, $\angle G = 90°$

$$\tan 60° = \frac{FG}{CG}$$

$$\Rightarrow \quad \sqrt{3} = \frac{80}{CG}$$

$$(\because \tan 60° = \sqrt{3})$$

$$\Rightarrow \quad CG = \frac{80}{\sqrt{3}}$$

On substituting value of CG in equn (i) we get

$$GB = 80 - \frac{80}{\sqrt{3}}$$

[As, CB = 80 m from (i)]

$$= 80\left(1 - \frac{1}{\sqrt{3}}\right) \text{m}$$

(iii) Speed of bird $= \dfrac{\text{Distance covered}}{\text{Time taken}}$

$$= \frac{20(\sqrt{3}+1)}{2} \text{ m/sec}$$

$$= 10(\sqrt{3}+1) \times 60 \text{ m/min}$$

$$= 600(\sqrt{3}+1) \text{ m/min}$$

4. (i) Let the vertical distance between the top of the tree and the drone be 'h'

Current height of a tree = 100 – 65
= 35 m

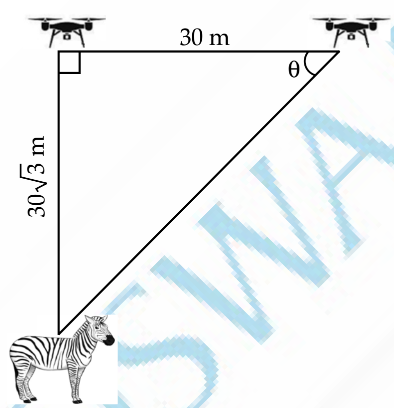
Now, $\tan 30° = \dfrac{h}{5\sqrt{3}} \Rightarrow h = 5\sqrt{3}\ \tan 30°$

$$\Rightarrow \quad h = 5\sqrt{3} \times \frac{1}{\sqrt{3}} = 5 \text{ m}$$

$$\left(\because \tan 30° = \frac{1}{\sqrt{3}}\right)$$

Thus, height of the tree ⇒ 35 – 5
= 30 m

(ii) Diagram to represent the situation is



Now, $\tan\theta = \dfrac{30\sqrt{3}}{30} = \sqrt{3}$

⇒ The value of θ as 60°. $(\because \tan 60° = \sqrt{3})$

(iii) (a) Let the horizontal distance between the remote and the drone = x

$$\Rightarrow \tan 60° = \frac{50\sqrt{3}}{x} \Rightarrow x = \frac{50\sqrt{3}}{\sqrt{3}} = 50 \text{ m}$$

Now, Distance covered by jeep in 2 min = 10 × 120 = 1200 m.

Thus, Horizontal distance covered by drone = 1200 + 50 = 1250 m

$$\Rightarrow \text{Speed of drone} = \frac{1250}{120} = 10.42 \text{ m/s}$$

OR

(b) Let the horizontal distance between the drone and the tiger to be x when the angle of depression was 30°

So, $\tan 30° = \dfrac{x}{54\sqrt{3}}$

$$x = 54\sqrt{3} \times \tan 30°$$

$$= 54\sqrt{3} \times \frac{1}{\sqrt{3}}$$

$$= 54 \text{ m}$$

Let the horizontal distance between the drone and the tiger after 3 seconds be y

$$\tan 45° = \frac{y}{54\sqrt{3}}$$

$$y = 54\sqrt{3} \times \tan 45°$$

$$= 54\sqrt{3} \text{ m}$$

Thus, the distance covered by the tiger in 3 seconds is: $y - x$

$$54\sqrt{3} - 54 = 39.42 \text{ m}$$

and, average speed of the tiger during that time is:

$$\frac{39.42}{3} = 13.14 \text{ m/s}$$

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. Given, Height of the camera from ground = 6 m (vertical side) and distance from the camera to point O = $4\sqrt{3}$ m (hypotenuse)

To find the angle of depression, which is the angle between the horizontal line at the height of the camera and the line of sight to point O.

This forms a right triangle, where:

Opposite side = 6 m (vertical)

Hypotenuse = $4\sqrt{3}$ m

Using trigonometry:

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{6}{4\sqrt{3}}$$

$$\sin\theta = \frac{6}{4.\sqrt{3}} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 60°$$

The angle of depression of point O from the camera is 60°.

2. Given: Vertical height from point A to point B = 260 m

Horizontal distance from point B to C = 75 m

Angle of depression from A to lioness at point D = 30°

Using triangle ABD (right-angled at B):

$$AB = 260 \text{ m (vertical)}$$

$$\angle ADB = 30° \text{ (angle of depression)}$$

We need to find the horizontal distance BD using:

$$\tan(30°) = \frac{AB}{BD}$$

$$\tan(30°) = \frac{260}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{260}{BD}$$

$$\Rightarrow \quad BD = 260 \times \sqrt{3}$$

$$\Rightarrow \quad BD = 260 \times 1.73 = 449.8 \text{ m}$$

We know:

$$BD = BC + CD$$

$$\Rightarrow \quad CD = BD - BC = 449.8 - 75$$

$$= 374.8 \text{ m}$$

$$= 375 \text{ m}$$

(rounded to nearest whole number).

3. Given, Height of the vertical pole = 6 m and horizontal distance of the boy from the foot of the pole = 6 m

In the right triangle formed:

Opposite side = height of the pole = 6 m

Adjacent side = distance from the pole = 6 m

Using the tangent ratio:

$$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{6}{6} = 1$$

$$\because \quad \tan 45° = 1 \Rightarrow \theta = 45°$$

The angle of elevation is 45°.

4. Given, distance of the foot of the ladder from the wall (base) = 5 m and angle between the ladder and the ground = 60°

In the right triangle formed, the ladder is the hypotenuse and the height up the wall is the opposite side to the angle. The base is the adjacent side = 5 m

Using tan function

$$\tan(60°) = \frac{\text{height}}{5}$$

$$\tan(60°) = \sqrt{3} = 1.73$$

$$1.73 = \frac{\text{height}}{5}$$

$$\Rightarrow \quad \text{height} = 1.73 \times 5 = 8.65 \text{ m}$$

The ladder reaches approximately 8.65 metres up the wall.

5. Given, Height of the kite = 40 m (opposite side) and length of the string = 80 m (hypotenuse)

Using the sine trigonometric ratio:

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{40}{80} = \frac{1}{2}$$

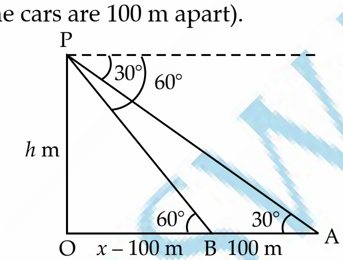
$$\because \quad \sin 30° = \frac{1}{2} \Rightarrow \theta = 30°$$

The angle of elevation of the kite is 30°.

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. Let the height of the tower be h m and the horizontal distance of car A from the foot of the tower be x m.

Then, the distance of car B (closer car) = $x - 100$ m (since the cars are 100 m apart).



From triangle formed with car A (angle = 30°):

$$\tan(30°) = \frac{h}{x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow h = \frac{x}{\sqrt{3}} \quad \text{...(i)}$$

From triangle formed with car B (angle = 60°):

$$\tan(60°) = \frac{h}{x-100} \Rightarrow \sqrt{3} = \frac{h}{x-100} \quad \text{...(ii)}$$

Substituting equation (i) into equation (ii)

$$\sqrt{3} = \frac{x}{\sqrt{3}(x-100)}$$

$$3(x-100) = x \Rightarrow 3x - 300 = x$$

$$\Rightarrow \quad 2x = 300 \Rightarrow x = 150 \text{ m}$$

From equation (i)

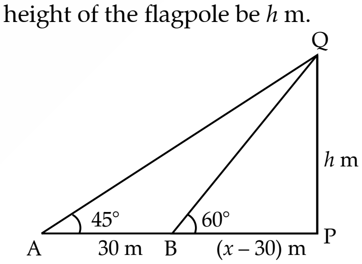
$$h = \frac{x}{\sqrt{3}} = \frac{150}{1.73} \approx 86.71 \text{ m}$$

The height of the tower is approx. 87 m (nearest to the whole number).

2. Let the initial distance from the pole be x m.

After walking 30 m closer, the new distance is $x - 30$ m.

Let the height of the flagpole be h m.



In right ΔQPA,

$$\tan(45°) = \frac{h}{x}$$

$$\Rightarrow \quad 1 = \frac{h}{x} \Rightarrow h = x \quad \text{...(i)}$$

In right ΔQPB,

$$\tan(60°) = \frac{h}{x-30} \Rightarrow \sqrt{3} = \frac{h}{x-30}$$

$$\Rightarrow \quad h = \sqrt{3}(x-30) \quad \text{...(ii)}$$

Substituting (i) into (ii):

$$x = 1.73(x-30) \Rightarrow x = 1.73x - 51.9$$

$$\Rightarrow \quad 1.73x - x = 51.9$$

$$\Rightarrow \quad 0.73x = 51.9 \Rightarrow x = \frac{51.90}{0.73} \approx 71.1$$

Now using equation (i) to find height:

$$h = x \approx 71\,\text{m}$$

The height of the flagpole is approximately 71 m.

3. Given, a 100 m high light pole (vertical segment AB, where A is the top and B is the base).
Shantanu is at point S, with $\angle SAB = 45°$
Mayank is at point M, with $\angle MAB = 30°$.
We are to find the shortest distance between S and M (i.e. straight line segment SM).
In right ΔSAB,
For Shantanu:

$$\tan(45°) = \frac{AB}{SB} = \frac{100}{SB}$$

$$\Rightarrow \quad 1 = \frac{100}{SB} \Rightarrow SB = 100 \text{ m}$$

In right ΔAMB,
For Mayank:

$$\tan(30°) = \frac{100}{MB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{MB}$$

$$\Rightarrow \quad MB = 100.\sqrt{3} = 100 \times 1.73 = 173 \text{ m}$$

Using Pythagoras Theorem to find distance SM
In triangle ΔSBM, we know:

$$SB = 100\,\text{m},\ MB = 173\,\text{m}$$

$$\angle SBM = 90°$$

(since one is standing north and one east, forming right triangle)
So,

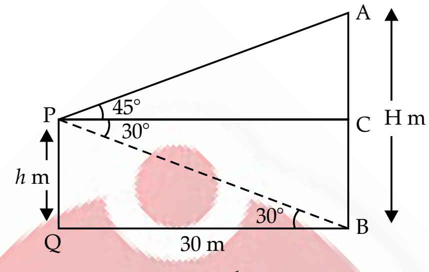
$$SM^2 = SB^2 + MB^2 = 100^2 + 173^2$$

$$= 10000 + 29929$$

$$= 39929$$

$$\Rightarrow \quad SM = \sqrt{39929} \approx 199.8 \text{ approx.}$$

4. Let the height of the shorter building be h meters and the height of the taller building be H meters.
In right ΔPQB,



$$\tan(30°) = \frac{h}{30} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{30}$$

$$\Rightarrow \quad h = \frac{30}{\sqrt{3}} = \frac{30}{1.73} = 17.34 \text{ m}$$

In right ΔPAC,
Let the difference in height be $H - h$.

$$\tan(45°) = \frac{H-h}{30} \Rightarrow 1 = \frac{H-h}{30}$$

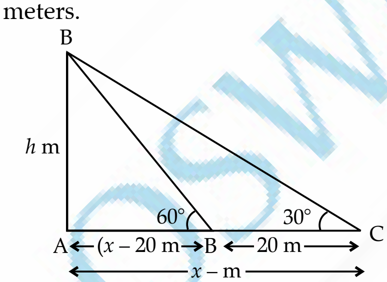
$$\Rightarrow \quad H - h = 30 \Rightarrow H = h + 30$$

$$= 17.34 + 30 = 47.34 \text{ m}$$

Height of the shorter building is 17.34 m and height of the taller building is 47.34 m.

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. **(i)** Let the original distance from the point to the tower be x meters, and the height of the tower be h meters.



In right ΔABC,

$$\tan(30°) = \frac{h}{x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow \quad h = \frac{x}{\sqrt{3}} \quad \text{...(i)}$$

In right ΔBAD,

$$\tan(60°) = \frac{h}{x-20} \Rightarrow \sqrt{3} = \frac{h}{x-20}$$

$$\Rightarrow \quad h = \sqrt{3}(x-20) \quad \text{...(ii)}$$

Equating (i) and (ii)

$$\frac{x}{\sqrt{3}} = \sqrt{3}(x-20) \Rightarrow x = 3(x-20)$$

$$\Rightarrow \quad x = 3x - 60 \Rightarrow 2x = 60 \Rightarrow x = 30$$

Substitute in (i):

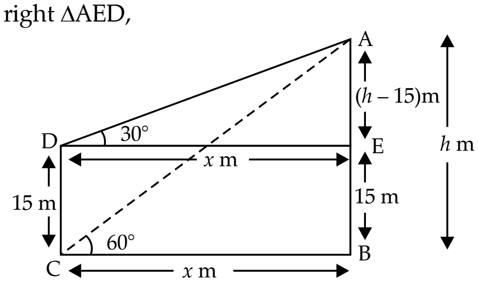
$$h = \frac{30}{\sqrt{3}} = \frac{30}{1.73} \approx 17.34 \text{ m}$$

Thus,

(i) Height of the tower ≈ 17.34 m

(ii) Original distance = 30 m

2. Let h be the height of the building and x be the horizontal distance between the tower and the building.
In right ΔAED,



$$\tan(30°) = \frac{h-15}{x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h-15}{x}$$

$$\Rightarrow \quad x = \sqrt{3}(h-15) \quad \text{...(i)}$$

In right ΔABC,

$$\tan(45°) = \frac{h}{x} \Rightarrow 1 = \frac{h}{x} \Rightarrow x = h \quad \text{...(ii)}$$

Substituting (ii) into (i)

$$h = \sqrt{3}(h-15) \Rightarrow h = 1.73(h-15)$$

$$\Rightarrow \quad h = 1.73h - 25.95$$

$$\Rightarrow \quad 1.73h - h = 25.95 \Rightarrow 0.73h = 25.95$$

$$\Rightarrow \quad h = \frac{25.95}{0.73} = 35.55$$

Therefore, height of the building is 35.55 m

From equation (ii), $h = x$

Therefore, the horizontal distance between the tower and the building is 35.55 m.

OSWAAL 360