CHAPTER 11

Areas Related to Circles

Level - 1

CORE SUBJECTIVE QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Mark)

1. Option (A) is correct

Explanation: Area of shaded region = $\frac{5}{9}$ × Area of circle

$$\frac{\theta}{360^\circ}\pi r^2 = \frac{5}{9}\neq r^2$$

$$\theta = \frac{360^\circ \times 5}{9}$$

$$\theta = 200^\circ$$

∴ Angle of unshaded region = 360° – Angle of shaded region

$$= 360^\circ - 200^\circ$$

$$= 160^\circ$$

$$\text{Length of minor arc} = \frac{\theta}{360^\circ}\times 2\pi r$$

$$= \frac{160^\circ}{360^\circ}\times 12\pi$$

$$= \frac{16\neq}{3}\text{ cm}$$

2. Option (D) is correct

Explanation: Since the pentagon is regular, each central angle θ of the sector is given by:

$$\theta = \frac{360^\circ}{5} = 72^\circ$$

The area of a sector with central angle θ in a circle of radius 5 cm is given by:

$$A_{sector} = \left(\frac{\theta}{360^\circ}\right)\times \pi r^2$$

Substituting θ = 72° and r = 5 cm, we have:

$$A_{sector} = \left(\frac{72^\circ}{360^\circ}\right)\times \pi \times 5^2$$

$$= \left(\frac{72^\circ}{360^\circ}\right)\times \pi \times 25$$

$$= \left(\frac{1}{5}\right)\times \pi \times 25$$

$$= 5\pi\text{ cm}^2$$

Since the shaded part of the circle consists of two such sectors:

$$\text{Area}_{shaded} = 2 \times 5\pi = 10\pi\text{ cm}^2$$

3. Option (B) is correct

Explanation: Diameter of the circle: d = 10 cm

Radius of the circle: $r = \frac{d}{2} = \frac{10}{2} = 5$ cm

The circumference of the full circle is:

$$C = 2\pi r = 2 \times 3.14 \times 5$$

$$= 31.4\text{ cm}$$

$$L_{arc} = \frac{1}{4}C = \frac{1}{4}\times 31.4$$

$$= 7.85\text{ cm}$$

$$OA = 5\text{ cm}$$

$$OB = 5\text{ cm}$$

Total Perimeter

$$P_{quadrant} = L_{arc} + 2 \times \text{radius}$$

$$P_{quadrant} = 7.85 + 2 \times 5$$

$$P_{quadrant} = 7.85 + 10$$

$$P_{quadrant} = 17.85\text{ cm}$$

Thus, perimeter of the quadrant of the circle is 17.85 cm.

4. Option (C) is correct

Explanation: Area of a coin = πr^2 = 3.14

$$(3.14) \times r^2 = 3.14$$

$$r^2 = \frac{3.14}{3.14} = 1$$

$$r = \sqrt{1}$$

$$r = 1\text{ cm}$$

5. Option (A) is correct

Explanation: Area of shaded region =

$$\frac{1}{4}\text{ Area of circle } - \text{Area of }\Delta OAB$$

$$= \frac{\pi r^2}{4} - \frac{1}{2}r^2$$

$$= r^2\left(\frac{\pi}{4} - \frac{1}{2}\right)$$

$$= \frac{r^2}{4}(\pi - 2)$$

6. Option (A) is correct

Explanation: Given, radius of circle = 5 cm

$$\text{Area of sector} = \frac{1}{4}\pi r^2 = \frac{25\pi}{4}$$

$$\text{Area of segment} = \frac{1}{2}r^2 \sin 90° = \frac{1}{2} \times 25$$

$$\text{Difference} = \frac{25\pi}{4} - \frac{25}{2}$$

7. Option (C) is correct

Explanation: The arc length(s) is related to the radius r and the central angle θ by the formula:

$$s = r\theta$$

$$\theta = \frac{s}{r}$$

$$r = 21 \text{ cm}$$

$$s = 22 \text{ cm}$$

$$\theta = \frac{22}{21} \text{ radians}$$

Area of the Sector:

$$A = \frac{1}{2}r^2\theta$$

$$A = \frac{1}{2}.21^2.\frac{22}{21}$$

$$A = \frac{1}{2}.21.22$$

$$A = \frac{1}{2}.462$$

$$A = 231 \text{ cm}^2$$

The area of the sector is 231 cm^2.

8. Option (C) is correct

Explanation: The radius of the circle is given as 7 cm.

The circumference of a full circle $2\pi r$

$$\text{Arc length} = \frac{1}{4} \times 2\pi r = \frac{1}{4} \times 2\pi \times 7$$

$$= \frac{7\neq}{2} \approx 11 \text{ cm}$$

The perimeter of the quadrant includes the arc length and the two radii.

Perimeter = Arc length + 2 × Radius

= 11 + 2 × 7 = 11 + 14 = 25 cm.

9. Option (C) is correct

Explanation: Radius of the wheel be r cm.

The distance covered in one revolution = $2\pi r$

The distance covered in 5000 revolutions = 5000 × $2\pi r$

$$11 = 5000 \times 2\pi r$$

$$11 = 5000 \times 2\pi r$$

$$\Rightarrow \quad 11 = 5000 \times 2 \times \frac{22}{7} \times r$$

$$\Rightarrow \quad 2r = \frac{7 \times 11}{22 \times 5000}$$

$$\Rightarrow \quad 2r = \frac{7}{2 \times 5000}$$

$$\Rightarrow \quad 2r = \frac{7}{10000} \text{ km}$$

$$\Rightarrow \quad 2r = \frac{7 \times 100000}{10000} \text{ cm}$$

$$\Rightarrow \quad 2r = 70 \qquad (\text{As, } d = 2r)$$

$$\therefore \quad d = 70 \text{ cm}$$

10. Option (A) is correct

Explanation:

Measure of arc = 90°

Radius of circle = 14 cm

$$\text{Length of arc} = \frac{\theta}{360°} \times 2\pi r$$

$$= \frac{90°}{360°} \times 2 \times \frac{22}{7} \times 14$$

$$= 22 \text{ cm}$$

11. Option (B) is correct

Explanation: The time difference between 7:20 a.m. and 7 : 55 a.m. is:

7:55 – 7:20 = 35 minutes

The hour hand of a clock completes a full circle (360°) in 12 hours.

Hence, the angle swept by the hour hand per hour is:

$$\frac{360\Upsilon}{12} = 30° \text{ per hour.}$$

Angle Swept per minute:

$$\frac{30\Upsilon}{60} = 0.5° \text{ per minute.}$$

The total angle swept by the hour hand in 35 minutes is:

$$35 \times 0.5° = 17.5°$$

$$= \left(\frac{35}{2}\right)^{\circ}$$

12. Option (B) is correct

Explanation: The length of the arc subtending an angle θ at the centre of a circle of radius r is given by $l = r\theta$ where θ is measured in radians:

$18.5 = 16\theta$

$$\Rightarrow \theta = \frac{18.5}{16} = 1.15625 \text{ radians.}$$

The area A of the sector subtending an angle θ at the centre of a circle of radius r is given by

$$A = \frac{1}{2}r^2\theta \quad (\text{where } \theta \text{ is measured in radians.})$$

$$= \left(\frac{1}{2}\right)16^2 \ (1.15625)$$

$$= 148 \text{ cm}^2$$

13. Option (A) is correct

Explanation: Radius of the circle be r.

Circumference of circle = $2\pi r$

Area of circle = πr^2

Given, circumference of the circle and the area of the circle are equal.

$$\Rightarrow \quad 2\pi r = \pi r^2$$

$2 = r$

Thus, radius of circle = 2 units.

14. Option (A) is correct

Explanation:

Area of circle = πr^2 where 'r' is the radius of the circle.

As diameters of parks are 16 m and 12 m

$\Rightarrow$ The radius of the two parks are $\frac{16}{2}$ = 8 m and $\frac{12}{2}$ = 6 m

$\Rightarrow$ Area of the new circular park = sum of the areas of two circular parks.

Let radius of the new circular park be 'R'.

$$\Rightarrow \quad \pi R^2 = \pi(8)^2 + \pi(6)^2$$
$$\Rightarrow \quad \pi R^2 = \pi(64 + 36)$$
$$\Rightarrow \quad \pi R^2 = \pi \times 100$$
$$\Rightarrow \quad R^2 = 100$$
$$\Rightarrow \quad R = \sqrt{100} = 10 \text{ m}$$

ASSERTION-REASON QUESTIONS (1 Mark)

1. Option (D) is correct

Explanation: The formula for the length of an arc is:

Length of an arc, $l = \theta \times r$

where:

θ is the central angle (in radians),

r is the radius of the circle.

Thus, The length of an arc depends on both the radius of the circle and the angle it subtends at the center.

So, for two circles of different radii (r_1 and r_2) subtending the same angle θ their arc lengths would be different unless $r_1 = r_2$

Thus, Assertion is false.

Reason is true. As per the arc length formula ($l = \theta \times r$), the length of an arc is directly proportional to the radius.

2. Option (C) is correct

Explanation :

Assertion (A): Arc lengths can be equal even if radii and angles are different (since $l = r\theta$).

Reason (R): Arc length does depend on the angle.

Assertion is true, but Reason is false.

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. Total area removed

$$= \frac{\angle A}{360°}\pi r^2 + \frac{\angle B}{360°}\pi r^2 + \frac{\angle C}{360°}\pi r^2$$
$$= \frac{\angle A + \angle B + \angle C}{360°}\pi r^2$$
$$= \frac{180°}{360°}\pi r^2$$
$$= \frac{180°}{360°} \times \frac{22}{7} \times (14)^2$$
$$= 308 \text{ cm}^2$$

2. The area of sector ABD $= \frac{60°}{360°} \times \pi \times 3^2$

$$= \frac{3\neq}{2} \text{ cm}^2$$

Now, in ΔABD, AB = AD

$\therefore$ $\angle$ABD = $\angle$ADB = x (angles opposite to equal sides)

$$\Rightarrow \quad x + x + 60° = 180°$$
$$\Rightarrow \quad 2x = 180° - 60°$$
$$\Rightarrow \quad x = 60°$$

Thus, ΔABD is an equailateral triangle.

Now, Area of an equailateral $\Delta ABD = \frac{\sqrt{3}}{4} \times 9 = \frac{9\sqrt{3}}{4}$ cm^2

Area of segment BD from sector ABD

= (area of sector ABD – area of ΔABD)

$$= \left(\frac{3}{2}\pi - \frac{9\sqrt{3}}{4}\right)$$

Similarly, area of segment BD from sector CBD

$$= \frac{3}{2}\pi - \frac{9\sqrt{3}}{4}$$

Now, area of shaded region = area of both segments

$$= 2 \times \left(\frac{3\pi}{2} - \frac{9\sqrt{3}}{4}\right)$$
$$= \left(3\pi - \frac{9\sqrt{3}}{2}\right) \text{ cm}^2$$

3. Area of the entire model is 120π cm^2

Here, θ of major arc = 360° – 60° = 300°

Now, Area of sector $= \frac{\theta}{360°}\pi r^2$

$$\frac{\theta}{360°}\pi r^2 = 120\pi$$
$$\frac{300°}{360°}\pi r^2 = 120\pi$$
$$r^2 = \frac{120 \times 6}{5}$$
$$r^2 = 144$$
$$r = 12 \text{ cm}$$

Minimum length of ribbon required = length of major arc + r + r = $\frac{\theta}{360°} \times 2\pi r$ = 300°

$\Rightarrow$ length of ribbon $= \frac{300°}{360°} \times 2 \times \pi \times 12 + 12 + 12$

$\Rightarrow (20\pi + 24)$ cm

4. As, per the given figure
Combined shaded regions forms one sector.
Total number of sectors in a circle = 8
Area of combined shaded region = 77 units^2
Therefore,

$$\frac{1}{8}\neq r^2 = 77$$

$$\Rightarrow \quad \frac{1}{8}\times\frac{22}{7}\times r^2 = 77$$

$$r^2 = \frac{77\times 7\times 8}{22}$$

$$r^2 = 7 \times 7 \times 4$$

$$r = 7 \times 2$$

$$= 14 \text{ units}$$

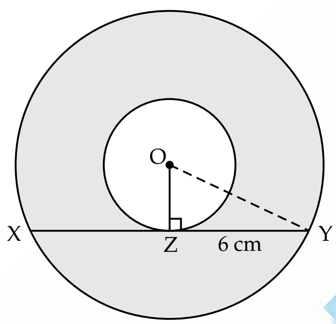
Thus, radius of the largest circle = 14 units.

5. Let OZ (radius) be r
Join OY
According to Pythagoras theorem
In right angled ΔOZY,

$$OY^2 = OZ^2 + ZY^2$$

$$R^2 = r^2 + 6^2$$

$$R = \sqrt{r^2 + 36} \quad \text{...(i)}$$



Area of shaded region = Area of large circle – Area of small circle

$$= \pi R^2 - \pi r^2$$

$$= \pi\left(\sqrt{r^2+36}\right)^2 - \pi r^2 \quad \text{(from i)}$$

$$= \pi r^2 + 36\pi - \pi r^2$$

$$= 36\pi \text{ unit}^2$$

6. Circumference of a circle is 66 m.
$\Rightarrow 2\pi r = 66$ m

$$\Rightarrow \quad r = \frac{66}{2\pi} = \frac{66}{2\times\frac{22}{7}}$$

$$= \frac{3\times 7}{2} = \frac{21}{2} = 10.5 \text{ cm}$$

The radius of the circle is r = 10.5 cm

The area of the circle is $\pi r^2 = \frac{22}{7}\times 10.5\times 10.5$

$$= 231 \times 1.5 = 346.5 \text{ cm}^2$$

7. Perimeter of propractor = 108 cm

$$\Rightarrow \quad \pi \times r + 2r = 108$$

$$r\left(\frac{22}{7}+2\right) = 108$$

$$r\left(\frac{22+14}{7}\right) = 108$$

$$r = 108 \times \frac{7}{36}$$

$$r = 3 \times 7$$

$$r = 21 \text{ cm}$$

$$d = 2r$$

$$d = 2 \times 21 = 42 \text{ cm}$$

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. Area of semi circle PQ $= \frac{1}{2}\pi\left(\frac{PQ}{2}\right)^2$

Area of semi circle QR $= \frac{1}{2}\pi\left(\frac{QR}{2}\right)^2$

Area of semi circle PR $= \frac{1}{2}\pi\left(\frac{PR}{2}\right)^2$

Given that ΔPQR is right angled triangle $\angle\theta = 90°$

$$\therefore \quad PR^2 = PQ^2 + QR^2 \quad \text{...(i)}$$

(by Phythagoras theorem)

Now, Area of semi circle PR = Area of semi circle PQ + Area of semi circle QR

$$\Rightarrow \quad \frac{1}{2}\pi\left(\frac{PR}{2}\right)^2 = \frac{1}{2}\pi\left(\frac{PQ}{2}\right)^2 + \pi\left(\frac{QR}{2}\right)^2$$

$$\frac{PR^2}{4} = \frac{PQ^2}{4} + \frac{QR^2}{4}$$

$$\Rightarrow \quad \pi PR^2 = \pi(PQ^2 + QR^2)$$

$$\Rightarrow \quad \pi PR^2 = \pi PR^2 \quad \text{[From (i)]}$$

Hence Proved.

2. Area of minor segment $= \frac{\theta}{360°}\pi r^2 - \frac{1}{2}r^2\sin\theta$

$$= \frac{60°}{360°}\times\frac{22}{7}\times 10^2 - \frac{1}{2}\times 10^2\times\sin 60°$$

$$= \frac{1100}{21} - 50\times\frac{\sqrt{3}}{2}$$

$$= \frac{1100}{21} - 25\sqrt{3}$$

$$= 52.38 - 43.3 = 9.08 \text{ cm}^2$$

3. Let sides of the two squares be x m and y m, their perimeter will be $4x$ and $4y$ respectively and their areas will be x^2 and y^2 respectively.

$$4x - 4y = 24 \quad \text{...(i)}$$

$$x - y = 6$$

$$x = y + 6$$

Also, $x^2 + y^2 = 468$ ...(ii)

$\Rightarrow (6 + y)^2 + y^2 = 468$

$\Rightarrow 36 + y^2 + 12y + y^2 = 468$

$\Rightarrow 2y^2 + 12y - 432 = 0$

$\Rightarrow y^2 + 6y - 216 = 0$

$\Rightarrow y^2 + 18y - 12y - 216 = 0$

$\Rightarrow y(y + 18) - 12(y + 18) = 0$

$\Rightarrow (y + 18)(y - 12) = 0$

$\Rightarrow y = -18, 12$

As side cannot be negative. So ignore –18.

$\therefore$ The sides of the squares are 12 m and (12 + 6) m = 18 m

4. Radius = r = 21cm

Sweeping angle = 120°

Total area cleaned by two wipers = 2 × area cleaned by one wiper

= 2 × area of sector with 120°

$$= 2 \times \frac{\theta}{360°} \times \pi r^2$$

$$= 2 \times \frac{120°}{360°} \times \frac{22}{7} \times (21)^2$$

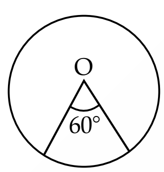
$$= 2 \times \frac{120°}{360°} \times \frac{22}{7} \times 441$$

$$= \frac{2 \times 22 \times 147}{7}$$

$$= \frac{6468}{7} = 924 \text{ cm}^2$$

So, area cleaned by both wipers is 924 cm^2

5.



Area of the minor sector making angle $\theta = \left(\frac{\theta}{360°}\right) \times \pi r^2$

Area of the sector making angle 60°

$$= \left(\frac{60°}{360°}\right) \times \pi r^2 \text{ cm}^2$$

$$= 6 \times \frac{22}{7} = \frac{132}{7} \text{cm}^2$$

$$= 6 \times \frac{22}{7} = \frac{132}{7} \text{cm}^2$$

Thus, area of minor sector = $\frac{132}{7}$ cm^2

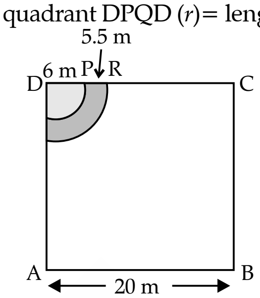
Area of major section = Area of circle – Area of minor sector.

$$= \frac{22}{7} \times 6 \times 6 - \frac{132}{7}$$

$$= \frac{792}{7} - \frac{132}{7} = \frac{660}{7} \text{cm}^2$$

6. A horse is tied with a rope of length 6 m at the corner of a square grassy lawn of side 20 m.

Radius of quadrant DPQD (r) = length of rope = 6 m



Area of sector DPQD = $\frac{\pi r^2 \theta}{360°}$

$$= \frac{3.14 \times (6)^2 \times 90°}{360°}$$

$= 0.785 \times 36 = 28.26$ m^2

Now, length of the rope is increased by 5.5 m

Total length of the rope (R) = 6 + 5.5 = 11.5 m

Area of sector DRSD = $\frac{\pi R^2 \theta}{360°}$

$$= \frac{3.14 \times (11.5)^2 \times 90°}{360°}$$

$= 0.785 \times 132.25$

$= 103.81625$ m^2

Increased area = Area of sector DRSD – Area of sector DPQD

$= 103.81625 - 28.26$

$= 75.55625$ m^2

$= 75.56$ m^2

The increase in area of the grassy lawn is 75.56 m^2

7. Radius of the circle = 14 cm

Central angle for minors sector = 90°

Area of the minor sector = $\frac{\theta}{360°} \times \pi r^2$

$$= \frac{90°}{360°} \times \frac{22}{7} \times 14^2$$

$= 22 \times 7 = 154$ cm^2

Area of minor segment = Area of sector – Area of right triangle AOB

$$= 154 - \frac{1}{2} \times 14 \times 14$$

$= 154 - 98 = 56$ cm^2

8. Given that ABC is a triangle with sides AB = 6 cm, BC = 8 cm, CA = 10 cm

We can see that,

$10^2 = 6^2 + 8^2$

(By Pythagoras theorem)

$100 = 100$

$\Rightarrow \Delta ABC$ is a right angled Δ

Thus,

Area of $\Delta ABC = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 8 \times 6$

$= 24$ cm^2

Remove the area of 3 arcs each of radius 2 cm from this area of triangle.

Now, area of three sectors

$$\Rightarrow \frac{\angle A}{360°}\pi r^2 + \frac{\angle B}{360°}\pi r^2 + \frac{\angle C}{360°}\pi r^2$$

$$\Rightarrow \frac{(\angle A+\angle B+\angle C)}{306°}\pi r^2$$

$$\Rightarrow \frac{180°}{360°}\times 3.14\times 2\times 2$$

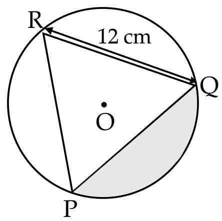
$\Rightarrow 6.28$ cm^2

Hence, the area of shaded region = area of ΔABC – combined area of 3 arcs = 24 – 6.28 = 17.72 cm^2

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. Given that PQR is equilateral triangle

Required Area of shaded region



$$= \frac{1}{3}(\text{Area of circle} - \text{Area of }\Delta)$$

$$= \frac{1}{3}\left(\pi r^2 - \frac{\sqrt{3}}{4}side^2\right)$$

$$= \frac{1}{3}\left(\pi r^2 - \frac{\sqrt{3}}{4}\times 12\times 12\right)$$

$$= \frac{1}{3}\left(\pi\left(\frac{12}{\sqrt{3}}\right)^2 - 36\sqrt{3}\right)$$

$$\left(r = \frac{\text{side}}{\sqrt{3}}\right)$$

$$= \frac{1}{3}(48\pi - 36\sqrt{3})$$

$$= \frac{3}{3}(16\pi - 12\sqrt{3})$$

$$= 16\pi - 12\sqrt{3}\text{ cm}^2$$

2. r = 14 cm and θ = 60°

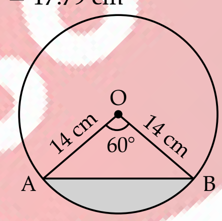
Area of minor segment

$$= \frac{\pi r^2\theta}{360°} - \frac{\sqrt{3}}{4}r^2 \quad (\because \Delta \text{ is an equalateral})$$

$$= \frac{22}{7}\times 14\times 14\times\frac{60°}{360°} - \frac{\sqrt{3}}{4}\times 14\times 14$$

$$= \frac{308}{3} - 49\sqrt{3}$$

$$= 17.79\text{ cm}^2$$



Area of the major segment

$$= \pi r^2 - 17.79$$

$$= \frac{22}{7}\times 14\times 14 - 17.79$$

$$= 616 - 17.89$$

$$= 598.21 = 598\text{ cm}^2$$

Level - 2 ADVANCED COMPETENCY FOCUSED QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Mark)

1. Option (B) is correct

Explanation: Area of sector $= \frac{\theta}{360°}\times \pi r^2$

$$= \frac{90°}{360°}\times\frac{22}{7}\times 7^2$$

$$= \frac{1}{4}\times\frac{22}{7}\times 49$$

$$= \frac{22\times 49}{28} = \frac{1078}{28}$$

$$\approx 38.5\text{ cm}^2$$

Since $\angle$ROP = 90°, triangle OPR is a right-angled triangle with both sides = 7 cm:

Area of triangle OPR $= \frac{1}{2}\times 7\times 7 = 24.5$ cm^2

Area of segment = Area of sector – Area of triangle

= 38.5 – 24.5 = 14 cm^2

2. Option (D) is correct

Explanation: Area of sector $= \frac{\theta}{360°}\times \pi r^2$

Area of outer sector (radius 8 cm)

$$= \frac{60°}{360°}\times \pi\times 8^2$$

$$= \frac{1}{6}\times\pi\times 64 = \frac{64\pi}{6}$$

Area of inner sector (radius 5 cm)

$$= \frac{60°}{360°} \times \pi \times 5^2$$

$$= \frac{1}{6} \times \pi \times 25 = \frac{25\pi}{6}$$

Area of shaded region

$$= \frac{64\pi}{6} - \frac{25\pi}{6}$$

$$= \frac{(64-25)\pi}{6} = \frac{39\pi}{6}$$

$$= \frac{13\pi}{2} \text{ cm}^2$$

3. Option (B) is correct

Explanation: Since the central angle is 45° < 180°, the arc is a minor arc.

$$\text{Arc Length} = \frac{\theta}{360°} \times 2\pi r$$

$$\text{Arc Length} = \frac{45°}{360°} \times 2 \times \frac{22}{7} \times 14$$

$$= \frac{1}{8} \times 2 \times \frac{22}{7} \times 14$$

$$= \frac{1}{8} \times \frac{616}{7} = \frac{1}{8} \times 88$$

$$= 11 \text{ cm}$$

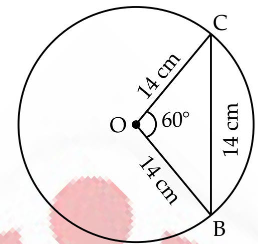
The arc is a minor arc and its is 11 cm.

4. Option (C) is correct

Explanation: Given, diameter of circle = 28 cm, and a chord BC = 14 cm. So, radius of the circle is 14 cm.

So, chord BC is equal to the radius. In a triangle with OB = OC = 14 cm and BC = 14 cm, all sides are equal

$\Rightarrow$ Triangle OBC is an equilateral triangle with angle = 60°



Length of Minor Arc BC,

$$\text{Arc length} = \frac{\theta}{360°} \times 2\pi r$$

$$= \frac{60°}{360°} \times 2 \times \frac{22}{7} \times 14 = \frac{1}{6} \times 88 = 14.7 \text{ cm}$$

Major Arc Length

Major arc = Total Circumference – Minor arc

$$= 2\pi r - 14.7 = \frac{2 \times 22}{7 \times 14}$$

$$= 88 - 14.7 = 73.3 \text{ cm}$$

ASSERTION-REASON QUESTIONS (1 Mark)

1. Option (D) is correct

Explanation: Area of the path = Area of bigger circle – Area of smaller circle

$$= \pi(12^2) - \pi(10^2)$$

$$= \pi(144 - 100)$$

$$= 44\pi \text{ m}^2$$

So, assertion is false.

Reason is true because this is the correct formula to find the area of the path.

2. Option (D) is correct

Explanation: Area of the border $= \pi(R^2 - r^2)$

$$= \pi(1.5^2 - 1.4^2)$$

$$= \pi(2.25 - 1.96)$$

$$= \pi(0.29)$$

$$\approx 3.14 \times 0.29$$

$$\approx 0.91 \text{ m}^2$$

Thus, assertion is false.

Reason is true because this is the correct formula to find the area of the border.

3. Option (C) is correct

Explanation: Circumference $= 2\pi r$

$$\Rightarrow \quad 220 = \frac{2 \times 22}{7 \times r}$$

$$\Rightarrow \quad 220 = \frac{44}{7 \times r}$$

$$\Rightarrow \quad r = \frac{220 \times 7}{44} = 35 \text{ m}$$

Thus, assertion is true.

Reason is false because circumference $= 2\pi r$, and not $2\pi r^2$.

4. Option (D) is correct

Explanation: Area of sector covered by minute hand in

$$15 \text{ minutes} = \frac{90°}{360°} \times \pi \times 14^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 196$$

$$= \frac{1}{4} \times \frac{4312}{7} = \frac{4312}{28}$$

$$\approx 154 \text{ cm}^2$$

So, the actual area $= 154 \text{ cm}^2$, not 77 cm^2

Thus, assertion is false.

Reason is true because in 15 minutes, the hand sweeps:

$$\frac{15}{60°} \times 360° = 90°$$

CASE BASED QUESTIONS (4 Marks)

1. (i) Given,

Radius of the circle $r = 7$ cm

Distance AD = 3 cm and BC = 3 cm (so the total base AB = 6 cm)

Rate of silver plating =20 per cm^2

The quadrant ODCO is one-fourth of the area of the circle.

The formula for the area of a circle is:

Area of the circle = πr^2

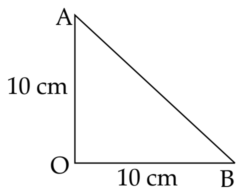
For the quadrant:

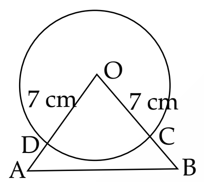
$$\text{Area of the quadrant} = \frac{1}{4}\times\pi r^2 = \frac{1}{4}\times\pi\times 7^2$$

$$= \frac{49\neq}{4}\text{cm}^2$$

Area of the quadrant = 38.5 cm^2

(ii) The ΔAOB is an isosceles right angle triangle where ∠O = 90°





Now, OB = OC + CB = 7 + 3 = 10 cm

OA = OD + DA = 7 + 3 = 10 cm

Let, OB = Base and OA = Height.

So, In right angled ΔAOB (Area) = $\frac{1}{2}\times$ B × H

$$= \frac{1}{2}\times 10\times 10$$

$$= 50 \text{ cm}^2$$

(iii) (a) The shaded area ABCD is the difference between the area of the triangle AOB and the area of the quadrant ODCO.

∴ Shaded area ABCD = Area of ΔAOB – Area of quadrant ODCO

Shaded area = 50 cm^2 – 38.5 cm^2 = 11.5 cm^2

Now, The total cost of silver plating is:

Cost = Shaded area × rate = 11.5 ×20 = ₹ 230

OR

(iii) (b) The length of an arc is given by:

$$\text{Length of the arc} = \frac{\theta}{360^\circ}\times 2\pi r$$

For a quadrant, θ = 90°, so:

$$\text{Length of the arc} = \frac{90^\circ}{360^\circ}\times 2\pi\times 7 = 11 \text{ cm}$$

2. (i)

$$\text{Area} = \pi r^2 = \frac{22}{7}\times 38^2 = \frac{22}{7}\times 1444$$

$$= \frac{31768}{7} = 4538.29\,\text{m}^2$$

(ii) Area of cycling track is the area between the outer circle (r = 38 m) and the park (r = 35 m)

$$\text{Area}_{\text{inner}} = \pi r^2 = \frac{22}{7}\times 35\times 35$$

$$= \frac{22}{7}\times 1225 = 3850 \text{ m}^2$$

So, area of cycling track = (4538.29 – 3850)

= 688.29 m^2

(iii) (a) Area of the grass-covered region (excluding fountain and track) is the area of the inner park (r = 35 m) minus the fountain (r = 7 m).

$$\text{Area} = \frac{22}{7}\times(35^2 - 7^2)$$

$$= \frac{22}{7}\times(1225 - 49)$$

$$= \frac{22}{7}\times 1176 = \frac{25872}{7}$$

= 3696 m^2

OR

(b) Cost of tiling the track = 688.29 × 450

= ₹ 309,730.50

3. (i) Area of circular garden = $\pi r^2 = \frac{22}{7}\times 441$

$$= \frac{22}{7}\times 21^2$$

$$= \frac{9702}{7}$$

= 1386 m^2

(ii) Total area covered by the three flower beds

Total angle = 180°, radius = 21 m

$$A = \frac{\theta}{360^\circ}\pi r^2 = \frac{180^\circ}{360^\circ}\times\frac{22}{7}360^\circ$$

$$= \frac{1}{2}\times\frac{22}{7}\times 441$$

$$= \frac{22\times 441}{14} = \frac{9702}{14}$$

= 693 m^2

(iii) (a) Grassy area = Total garden area – Area of flower beds = 1386 – 693 = 693 m^2

OR

(b) Area of walking path surrounding the garden is the area between the outer circle (radius = 23.5 m) and the garden (radius = 21 m)

$$A = \frac{22}{7}\times(23.5^2 - 21^2)$$

$$= \frac{22}{7}\times(552.25 - 441)$$

$$= \frac{22}{7}\times 111.25 = \frac{2447.5}{7}$$

= 349.64 m^2

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. Area of outer circle $= \pi R^2$

$$= \left(\frac{22}{7}\right) \times 70^2$$

$$= \left(\frac{22}{7}\right) \times 4900$$

$= 22 \times 700 = 15{,}400 \text{ m}^2$

Area of inner circle $= \pi r^2$

$$= \left(\frac{22}{7}\right) \times 63^2$$

$$= \left(\frac{22}{7}\right) \times 3969$$

$= 22 \times 567 = 12{,}474 \text{ m}^2$

Area of the track = Area of outer circle – Area of inner circle

$= 15{,}400 - 12{,}474 = 2{,}926 \text{ m}^2$

The area of the surface of the track is $2{,}926 \text{ m}^2$.

2. Area of sector $= \dfrac{\theta}{360°} \times \pi r^2$

Substituting the values:

$$\text{Area} = \frac{120°}{360°} \times 3.14 \times 12^2$$

$$= \frac{1}{3} \times 3.14 \times 144$$

$= 3.14 \times 48 = 150.72 \text{m}^2$

The area watered in that session is 150.72 m^2.

3. Total area of the pizza:

$$\pi r^2 = \frac{22}{7} \times 14 \times 14 = \frac{22}{7} \times 196$$

$= 22 \times 28 = 616 \text{ cm}^2$

Area of one slice $= \dfrac{\text{area}}{8} = \dfrac{616}{8} = 77 \text{ cm}^2$

Area of remaining pizza $= 7 \times 77 = 539 \text{ cm}^2$

4. Area of square $= \text{side}^2 = 20^2 = 400 \text{ m}^2$

Area of circular flower bed $= \pi r^2 = \dfrac{22}{7} \times 7 \times 7$

$= 154 \text{ m}^2$

Area of playground left = area of square – area of circular flower bed $= 400 - 154 = 246 \text{ m}^2$.

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. Area of lawn $= \pi r^2 = \dfrac{22}{7} \times 21 \times 21$

$= 1386 \text{ m}^2$

Area of the outer circle (lawn + path)

Area of outer circle $= \pi R^2 = \dfrac{22}{7} \times 24 \times 24$

$= 1804.57 \text{m}^2$

Area of path = Area of outer circle – Area of lawn

$= 1804.57 - 1386$

$= 418.57 \text{m}^2$

Cost of laying grass = ₹ 4 × 1386 = ₹ 5,544

Cost of constructing path = ₹ 5 × 418.57 = ₹ 2,092.85

2. Area of full length cloth including lace:

$$\text{Area}_{outer} = \pi R^2 = \frac{22}{7} \times 1.75 \times 1.75$$

$$= \frac{22}{7} \times 3.0625$$

$\approx 9.625 \text{ m}^2$

Area of inner circle (only table cloth without lace)

$$\text{Area}_{inner} = \pi r^2 = \frac{22}{7} \times 1.4 \times 1.4$$

$$= \frac{22}{7} \times 1.96$$

$\approx 6.16 \text{ m}^2$

Area of lace:

$\text{Area}_{lace} = \text{Area}_{outer} - \text{Area}_{inner}$

$= 9.625 - 6.16$

$= 3.465 \text{ m}^2$

Thus, the area of lace is 3.465 m^2 and the total area of cloth including lace is 9.625 m^2.

3. Area of outer circle (fountain + path)

$\text{Area}_{outer} = \pi R^2 = 3.14 \times (7.5)^2$

$= 3.14 \times 56.25$

$= 176.625 \text{m}^2$

Area of inner circle (only fountain)

$\text{Area}_{inner} = \pi r^2 = 3.14 \times 52$

$= 3.14 \times 25 = 78.5 \text{m}^2$

Area of the path

$\text{Area}_{path} = \text{Area}_{outer} - \text{Area}_{inner}$

$= 176.625 - 78.5$

$= 98.125 \text{m}^2$

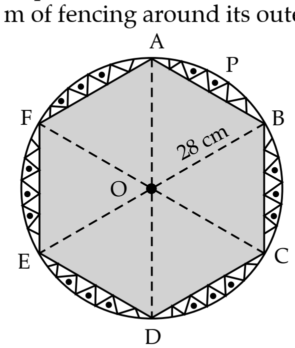
Length of fencing around outer edge

$\text{Circumference}_{outer} = 2\pi R = 2 \times 3.14 \times 7.5$

$= 47.1 \text{m}$

Therefore, the path has an area of 98.125 m^2 and will require 47.1 m of fencing around its outer edge.

4.



Consider segment APB. Chord AB is a side of the hexagon. Each chord will substitute $\frac{360°}{6} = 60°$ at the centre of the circle.

In ΔOAB,

$$\angle OAB = \angle OBA \text{ (As OA = OB)}$$
$$\angle AOB = 60°$$
$$\angle OAB + \angle OBA + \angle AOB = 180°$$
$$2\angle OAB = 180° - 60° = 120°$$
$$\angle OAB = 60°$$

Therefore, ΔOAB is an equilateral triangle.

$$\text{Area of } \Delta OAB = \frac{\sqrt{3}}{4} \times (\text{side})^2$$
$$= \frac{\sqrt{3}}{4} \times (28)^2 = 196\sqrt{3}$$
$$= 196 \times 1.7 = 333.2 \text{ cm}^2$$
$$\text{Area of sector OAPB} = \frac{60°}{360°} \times \pi r^2$$
$$= \frac{1}{6} \times \frac{22}{7} \times 28 \times 28$$
$$= \frac{1232}{3} \text{ cm}^2$$

Area of segment APB = Area of sector OAPB – Area of ΔOAB

$$= \left(\frac{1232}{3} - 333.2\right) \text{ cm}^2$$

$$\text{Therefore, area of designs} = 6 \times \left(\frac{1232}{3} - 333.2\right) \text{ cm}^2$$
$$= (2464 - 1999.2) \text{ cm}^2$$
$$= 464.8 \text{ cm}^2$$

Cost of making 1 cm^2 designs = ₹ 0.35

Cost of making 464.8 cm^2 designs = 464.8 × 0.35 = ₹ 162.68

Therefore, the cost of making such designs is ₹ 162.68.

5. (i) Total length of wire required will be the length of 5 diameters and the circumference of the brooch.

Radius of circle $= \frac{35}{2}$ mm

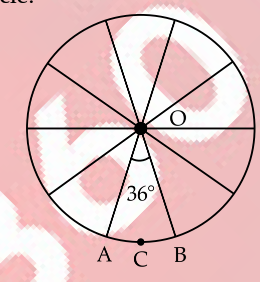
Circumference of brooch = $2\pi r$

$$= 2 \times \frac{22}{7} \times \left(\frac{35}{2}\right)$$
$$= 110 \text{ mm}$$

Length of wire required = 110 + 5 × 35

= 110 + 175 = 285 mm

(ii) It can be observed from the figure that each of 10 sectors of the circle is subtending 36° at the centre of the circle.



$$\text{Therefore, area of each sector} = \frac{36°}{360°} \times \pi r^2$$
$$= \frac{1}{10} \times \frac{22}{7} \times \left(\frac{35}{2}\right) \times \left(\frac{35}{2}\right)$$
$$= \frac{385}{4} \text{ mm}^2$$

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. Let the width of the path be x m

Then, the outer radius (field + path) = $(70 + x)$ m

Area of the field

$$A_{field} = \pi r^2 = \pi (70)^2 \text{ m}^2$$

Area of the field + path (larger circle)

$$A_{outer} = \pi(70 + x)^2$$

Area of path = Area of field ⇒

$$A_{outer} - A_{field} = A_{field} \Rightarrow \pi(70 + x)^2 - \pi(70)^2 = \pi(70)^2$$

Now cancel π from all terms:

$$(70 + x)^2 - 70^2 = 70^2$$
$$\Rightarrow (70 + x)^2 = 2 \times 70^2$$
$$\Rightarrow (70 + x)^2 = 9800 \Rightarrow 70 + x = \sqrt{9800}$$
$$\left[\sqrt{9800} = \sqrt{100 \times 98} = 10\sqrt{98}\right.$$
$$\left.= 10 \times 9.899 = 98.99 = 99\right]$$
$$\Rightarrow 70 + x = 99 \Rightarrow x = 99 - 70 = 29 \text{ m}$$

The width of the path is 29 meters.

2. When a sector is folded into a cone, the arc length of the sector becomes the circumference of the base of the cone and the radius of the sector becomes the slant height (l) of the cone.

$$\text{Arc length} = \frac{\theta}{360°} \times 2\pi r = \frac{120°}{360°} \times 2 \times \frac{22}{7} \times 21$$
$$= \frac{1}{3} \times 2 \times \frac{22}{7} \times 21$$
$$= \frac{1}{3} \times \frac{924}{7} = \frac{1}{3} \times 132 = 44 \text{ cm}$$

So, the circumference of the cone base = 44 cm

Using circumference of the cone base to find the radius of the cone

Circumference of cone base = $2\pi R$

$\Rightarrow \quad 44 = 2 \times \frac{22}{7} \times R$

$\Rightarrow \quad 44 = \frac{44}{7} \times R$

$\Rightarrow \quad R = 7 \text{ cm}$

Using Pythagoras theorem to find the height of the cone

$l^2 = R^2 + h^2$

$\Rightarrow \quad h^2 = l^2 - R^2$

$\Rightarrow \quad h^2 = 21^2 - 7^2$

$\Rightarrow \quad h^2 = 441 - 49$

$\Rightarrow \quad h^2 = 392$

$\Rightarrow \quad h = \sqrt{392}$

$\left[\sqrt{392} = \sqrt{49 \times 8} = 7\sqrt{8} = 7 \times 2.828\right]$

$\therefore \quad h = 19.8 \text{ cm (approx)}$

3. (i) Two semicircles together form 1 full circle of radius 35 m

Circumference of full circle $= 2\pi r = \frac{2 \times 22}{7} \times 35$

$= 220 \text{ m}$

Length of two straight paths = 2 × 100 = 200 m

So, total length of one round:

= 220 + 200 = 420 m

Total distance for 8 rounds:

= 8 × 420 = 3360 m

(ii) Area enclosed by the track is the area of a rectangle in the middle:

Length = 100 m,

Width = diameter of semicircle

= 2 × 35 = 70 m

$\text{Area}_{\text{rect}} = 100 \times 70 = 7000 \text{ m}^2$

Two semicircles = 1 full circle of radius 35 m

$\text{Area}_{\text{circle}} = \pi r^2 = \frac{22}{7} \times 35^2$

$= \frac{22}{7} \times 1225$

$= 3850 \text{ m}^2$

Total enclosed area:

$= 7000 + 3850 = 10850 \text{ m}^2$

(iii) Cost to lay synthetic turf

Rate = ₹ 120 per m^2

Total area = 10850 m^2

Cost = 10850 × 120 = ₹ 13,02,000