CHAPTER 2

Polynomials

Level - 1

CORE SUBJECTIVE QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Mark)

1. Option (D) is correct

Explanation: Since (–6, 0) and (6, 0) are the x-intercepts.

Thus, the zeroes of the polynomial are $x = -6$ and $x = 6$.

2. Option (B) is correct

Explanation: The general form of a quadratic polynomial with zeroes α and β is:

$$p(x) = a(x-\alpha)(x-\beta)$$

$$p(x) = a\left(x+\sqrt{\frac{5}{2}}\right)\left(x-\sqrt{\frac{5}{2}}\right)$$

$$p(x) = a\left[x^2-\left(\sqrt{\frac{5}{2}}\right)^2\right] = a\left(x^2-\frac{5}{2}\right)$$

To match the polynomial to one of the options, put $a = 8$

$$p(x) = 8\left[x^2-\frac{5}{2}\right] = 8x^2-20$$

3. Option (A) is correct

Explanation: The relationship between the zeroes of two polynomials is given.

Let the zeroes of the polynomial $4x^2 - 5x - 6$ be α and β. Then, the zeroes of the polynomial $x^2 + px + q$ are 2α and 2β.

For the polynomial $4x^2 - 5x - 6$

Sum of zeroes: $\alpha + \beta = -\frac{-5}{4} = \frac{5}{4}$

Product of zeroes: $\alpha\beta = \frac{-6}{4} = -\frac{3}{2}$

For the polynomial $x^2 + px + q$

Sum of zeroes $= \frac{-p}{1} = 2\alpha + 2\beta = 2(\alpha + \beta) = 2 \times \frac{5}{4} = \frac{5}{2}$

$$\frac{-p}{1} = \frac{5}{2}$$

Therefore, $p = -\frac{5}{2}$ (since the sum of the zeroes of a quadratic polynomial $ax^2 + bx + c$ is given by $\frac{-b}{a}$).

4. Option (C) is correct

Explanation: The sum of reciprocals of the zeroes can be found using the formula:

$$\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$$

For the polynomial $5x^2 + 3x - 7$

$a = 5, b = 3, c = -7$

Sum of zeroes $\alpha + \beta = \frac{-b}{a} = \frac{-3}{5}$

Product of zeroes $\alpha\beta = \frac{c}{a} = \frac{-7}{5}$

Now, sum of reciprocals

$$\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$$

$$= \frac{-\frac{3}{5}}{\frac{-7}{5}} = \frac{3}{7}$$

5. Option (B) is correct

Explanation: Given, polynomial is

$$p(x) = 2x^2 - k\sqrt{2}x + 1$$

sum of zeroes $= \dfrac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$

$\sqrt{2} = \dfrac{-(-k\sqrt{2})}{2}$ (Given, sum of zeroes $= \sqrt{2}$)

$\therefore \quad k = 2$

6. Option (B) is correct

Explanation: To calculate $\frac{1}{\alpha}+\frac{1}{\beta}$, we use the formula:

$$\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$$

For the polynomial $6x^2 - 5x - 4$

$a = 6, b = -5, c = -4$

The sum of zeroes $\alpha + \beta = -\frac{b}{a} = -\frac{-5}{6} = \frac{5}{6}$

The product of zeroes $\alpha\beta = \frac{c}{a} = \frac{-4}{6} = -\frac{2}{3}$

Now

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\frac{5}{6}}{-\frac{2}{3}} = -\frac{5}{4}$$

7. Option (B) is correct

Explanation: We know that:

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

For the polynomial $2x^2 - 9x + 5$

The sum of zeroes $\alpha + \beta = -\frac{-9}{2} = \frac{9}{2}$

The product of zeroes $\alpha\beta = \frac{5}{2}$

Now

$$\alpha^2 + \beta^2 = \left(\frac{9}{2}\right)^2 - 2 \times \frac{5}{2} = \frac{81}{4} - \frac{10}{2}$$

$$= \frac{81}{4} - \frac{20}{4} = \frac{61}{4}$$

8. Option (B) is correct

Explanation: For a quadratic polynomial $ax^2 + bx + c$, the difference between the zeroes is:

$$a - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$$

for the polynomial $-x^2 + 8x + 9$

Sum of zeroes: $\alpha + \beta = -\frac{8}{-1} = 8$

Product of zeroes: $\alpha\beta = \frac{9}{-1} = -9$

On substituting values we get,

$$\alpha - \beta = \sqrt{8^2 - 4(-9)} = \sqrt{64+36} = \sqrt{100} = \pm 10.$$

Given, $(a > \beta)$ so $a - \beta = 10$

9. Option (D) is correct

Explanation: For the polynomial $kx^2 - 30x + 45k$

$a = k, b = -30, c = 45k$

We know the relationships for a quadratic polynomial

Sum of zeroes: $\alpha + \beta = -\frac{b}{a} = \frac{30}{k}$

Product of zeroes: $\alpha\beta = \frac{c}{a} = \frac{45k}{k} = 45$

From the given $\alpha + \beta = \alpha\beta$, we have:

$$\frac{30}{k} = 45$$

$$k = \frac{2}{3}$$

10. Option (C) is correct

Explanation: Since 15 is a zero of the polynomial,

$$f(x) = x^2 - 16x + 30,$$

therefore $(x - 15)$ is a factor of $f(x)$.

Thus, $x = 15$ and on substituting 15 the result should be 0.

Let the number to be subtracted from the polynomial be 'a'.

$$15^2 - 16 \times 15 + 30 - a = 0$$
$$\Rightarrow 225 - 240 + 30 - a = 0$$
$$\Rightarrow 255 - 240 - a = 0$$
$$\Rightarrow 255 - 240 - a = 0$$
$$\Rightarrow 15 - a = 0$$
$$\Rightarrow 15 = a$$

Thus, 15 should be subtracted.

11. Option (C) is correct

Explanation: Let $y = (x-1)^2 (x+2)$ ...(i)

At X–axis, $y = 0$

Thus, $(x-1)^2 (x+2) = 0$

$\therefore$ $x = -2, 1, 1$

At Y–axis, $x = 0$

$\Rightarrow$ $y = (0-1)^2 (0+2)$ (from(i))

$\Rightarrow$ $y = (1)(2) = 2$

So, the graph must cut X–axis at 1, –2 and Y–axis at 2.

$\therefore$ Option (c) is correct.

[Marking Scheme, APQ 2024]

12. Option (A) is correct

Explanation: Let the other zero be α.

$\therefore$ $\alpha + 4 = 0$

$\Rightarrow$ $\alpha = -4$

$\therefore$ Product of zeroes $= 4 \times -4 = -16$

Required quadratic polynomial $= x^2 -$ (sum of zeroes) x + product of zeroes.

$$= x^2 - 0.x + (-16)$$
$$= x^2 - 16$$

13. Option (B) is correct

Explanation: From the graph, the linear polynomial crosses the x–axis at $x = 3$, indicating a single zero at $x = 3$.

14. Option (B) is correct

Explanation: The graph intersects the x–axis at one point, indicating that the polynomial has one zero.

15. Option (A) is correct

Explanation: Using $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$ with sum of zeroes $\alpha + \beta = -1$ and product $\alpha\beta = -1$

we have

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-1}{-1} = 1$$

16. Option (D) is correct

Explanation: For the polynomial $x^2 - 1$, the zeroes are $\alpha = 1$ and $\beta = -1$. Thus, $\alpha + \beta = 0$

17. Option (D) is correct

Explanation: For zeroes $\pm\frac{2}{3}$, the polynomial is:

$$p(x) = \left(x - \frac{2}{3}\right)\left(x + \frac{2}{3}\right) = x^2 - \left(\frac{2}{3}\right)^2 = x^2 - \frac{4}{9}$$

Multiply by 9 gives $9x^2 - 4$.

18. Option (A) is correct

Explanation: We are given the polynomial:

$6x^2 + 37x - (k - 2)$

It is also given that one zero is the reciprocal of the other. Let the two zeros of the polynomial be α and $\frac{1}{\alpha}$.

Use the relationships from Vieta's formulas

For a quadratic equation $ax^2 + bx + c = 0$, the sum and product of the roots are given by:

Sum of the roots: $\alpha + \frac{1}{\alpha} = -\frac{b}{a}$

Product of the roots: $\alpha . \frac{1}{\alpha} = \frac{c}{a}$

For the given polynomial $6x^2 + 37x - (k - 2)$, we have:

$$a = 6$$
$$b = 37$$
$$c = -(k - 2)$$

Now,

$$\alpha . \frac{1}{\alpha} = 1$$

$$\frac{c}{a} = 1$$

Substitute the values of c and a:

$$\frac{-(k-2)}{6} = 1$$
$$-(k - 2) = 6$$
$$k - 2 = -6$$
$$k = -6 + 2 = -4$$

19. Option (B) is correct

Explanation: For the given polynomial $x^2 - ax - b$, we

have: $a = 1, b = -a, c = -b$

substitute value of $a + \beta = -a$ and $a\beta = -b$ in

$$a^2 + \beta^2 = (a + \beta)^2 - 2a\beta$$
$$= (-a)^2 - 2 \times -b$$
$$= a^2 + 2b$$

20. Option (B) is correct

Explanation: Let the zeroes be α and 2α. For a quadratic polynomial $ax^2 + bx + c$, the sum of zeroes is $\alpha + 2\alpha = 3\alpha$, and the product of zeroes is $\alpha \,.\, 2\alpha = 2\alpha^2$.

For the given polynomial $x^2 - 3kx + 4k$:

The sum of zeroes is $\frac{-b}{a} = 3k$

The product of zeroes is $\frac{c}{a} = 4k$

Equating the sum and product gives the equations:

$$3\alpha = 3k \text{ and } 2\alpha^2 = 4k$$

Solving these equations, we find $k = 2$.

21. Option (D) is correct

Explanation: Since,

Given the quadratic polynomial $ax^2 - 5x + c$ with zeroes α and β,

$$\alpha + \beta = \alpha\beta = 10$$

Use the relationships for the sum and product of the roots

Sum of roots $\alpha + \beta = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

Product of roots $\alpha\beta = \frac{\text{constant term}}{\text{coefficient of } x^2}$

$$\Rightarrow \quad \alpha + \beta = \frac{-(-5)}{a} = \frac{5}{a}$$

$$\alpha\beta = \frac{c}{a}$$

$$\alpha + \beta = 10$$
$$\alpha\beta = 10$$

$$\frac{5}{a} = 10 \Rightarrow a = \frac{5}{10} = \frac{1}{2}$$

$$\frac{c}{a} = 10 \Rightarrow \frac{c}{\frac{1}{2}} = 10 \Rightarrow c = 10 \times \frac{1}{2} = 5$$

The correct values are $a = \frac{1}{2}$ and $c = 5$.

22. Option (D) is correct

Explanation: For a quadratic polynomial $p(x) = 4x^2 - 3x - 7$ the sum and product of the roots

Sum of the root $\alpha + \beta = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

$$= -\frac{-3}{4} = \frac{3}{4}$$

Product of the roots $\alpha\beta = \frac{\text{constant term}}{\text{coefficient of } x^2} = \frac{-7}{4}$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$$

Substitute the values of $\alpha + \beta$ and $\alpha\beta$ from the above relationships:

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\frac{3}{4}}{\frac{-7}{4}} = \frac{3}{4} \times \frac{4}{-7} = \frac{3}{-7} = -\frac{3}{7}$$

Thus, $\frac{1}{\alpha} + \frac{1}{\beta} = -\frac{3}{7}$

23. Option (C) is correct

By splitting the middle term, we get,

$3x^2 + 12x - x - 4$

$(3x^2+12x) - (x + 4)$

$3x(x + 4) - 1 (x + 4)$

$(3x - 1) (x + 4)$

Thus, the factorization of the quadratic polynomial $3x^2 + 11x - 4 = (3x - 1)(x + 4)$

The zeroes of the polynomial are $x = \frac{1}{3}$ and $x = -4$.

24. Option (D) is correct

Explanation: Given, $p(x) = x^2 + 4x + 3$

By factorisation

$x^2 + 3x + x + 3$

$x(x + 3) + 1 (x + 3)$

$(x + 3) (x + 1)$

$\therefore x = -3$ and $x = -1$

$\therefore$ zeroes are $-3, -1$

25. Option (A) is correct

Explanation: If a quadratic polynomial's graph does not intersect the x–axis, it has no real zeroes. The graph intersecting the y–axis is irrelevant to the number of x–intercepts, so the number of zeroes is 0.

26. Option (B) is correct

Explanation: Use the formula $\alpha^2+\beta^2=(\alpha + \beta)^2-2\alpha\beta$

Sum of zeroes: $\alpha + \beta = \frac{-(-7)}{2} = \frac{7}{2}$

Product of zeroes: $\alpha\beta = \frac{3}{2}$

Now,

$$\alpha^2 + \beta^2 = \left(\frac{7}{2}\right)^2 - 2\left(\frac{3}{2}\right)$$

$$= \frac{49}{4} - \frac{6}{2} = \frac{49}{4} - \frac{12}{4} = \frac{37}{4}$$

ASSERTION-REASON QUESTIONS (1 Mark)

1. Option (D) is correct

Explanation: The polynomials of the form $(x + a)^2$ and $(x - a)^2$ have only equal roots, and the graphs of these polynomials cut x–axis at only one point. These polynomials are quadratic.

So, the assertion is false.

The reason is true because a polynomial of degree n can have n zeroes.

$\therefore$ Assertion (A) is false but reason (R) is true.

2. Option (B) is correct

Explanation: The degree of a zero polynomial is not defined because it has no non-zero terms, so there is no exponent to defined as its degree, which makes the assertion true.

The reason is also true, as the degree of a non–zero constant polynomial is 0 because it can be written as a term with x°.

Reason correctly defines the degree of a non-zero constant polynomial, but it is does not directly explain why the degree of the zero polynomial is not defined.

Thus, Both Assertion and Reason are true but reason is not the correct explanation of Assertion.

3. Option (A) is correct

4. Option (D) is correct

Explanation: The polynomial $p(x) = x^2 +3x+3$ does not have two real zeroes because its discriminant $(b^2 - 4ac)$ is negative $(3^2 - 4 \times 1 \times 3 = 9 - 12 = -3)$. This means it has no real zeroes, so the assertion is false.

However, the reason is true because a quadratic polynomial can indeed have at most two real zeroes.

$\therefore$ Assertion (A) is false but reason (R) is true.

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. Let α and β are zeroes of the polynomial $p(x) = 5x^2 - 6x + 1$

$a = 5, b = -6, c = 1$

The sum of zeroes $\alpha + \beta = -\frac{b}{a} = \frac{-(-6)}{5} = \frac{6}{5}$

The product of zeroes $\alpha\beta = \frac{c}{a} = \frac{1}{5}$

Now, $\alpha + \beta + \alpha\beta = \frac{6}{5} + \frac{1}{5} = \frac{7}{5}$

2. α and β are zeroes of the quadratic polynomial

$p(x) = x^2 - 5x + 4$

For the polynomial $x^2 - 5x + 4$ we know:

$$\alpha + \beta = 5$$

$$\alpha\beta = 4$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{5}{4}$$

Now,

$$\Rightarrow \frac{\alpha + \beta}{\alpha\beta} - 2\alpha\beta = \frac{5}{4} - 2(4) = \frac{5}{4} - 8$$

$$\frac{5 - 32}{4} = \frac{-27}{4}$$

$$\therefore \frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{-27}{4}$$

3. α and β are the zeroes of $x^2 - x - 2$,

Thus, Sum of roots $= \alpha + \beta = 1$

and, product of roots $= \alpha\beta = -2$

Now, form a new polynomial with zeroes $2\alpha + 1$

and $2\beta + 1$

Sum of new zeroes = $(2\alpha + 1) + (2\beta + 1) = 2(\alpha + \beta) + 2 = 2(1) + 2 = 4$

Product of new zeroes= $(2\alpha + 1)(2\beta + 1) = 4\alpha\beta + 2(\alpha + \beta) + 1 = 4(-2) + 2(1) + 1 = -8 + 2 + 1 = -5$

Thus, the required polynomial is $k(x^2 - 4x - 5)$

4. α and β are the zeroes of $f(x) = 2x^2 + 5x + k$ such that $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$

We use the formula $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$:

Sum of zeroes: $\alpha + \beta = -\frac{5}{2}$

Product of zeroes: $\alpha\beta = \frac{k}{2}$

Now, substitute these into the equation:

$$\alpha^2 + \beta^2 = \left(\frac{-5}{2}\right)^2 - 2\left(\frac{k}{2}\right) = \frac{25}{4} - k$$

Given $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, we have:

$$\frac{25}{4} - k + \frac{k}{2} = \frac{21}{4}$$

Solving this, we get $k = 2$.

5. False. A polynomial can have two zeroes but might not necessarily be quadratic. $(x + a)^2$ have one zero repeated but equation is Quadratic, a cubic or higher–degree polynomial can also have exactly two distinct real zeroes, while the remaining zeroes may be complex or repeated.

6. Let the zeroes of the polynomial be α and $\frac{1}{\alpha}$ (since one is the reciprocal of the other).

The sum and product of the zeroes for a quadratic polynomial $6x^2 + 37x - (k - 2)$ are given by:

Sum of the zeroes $= \alpha + \frac{1}{\alpha} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2} = -\frac{37}{6}$

Product of the zeroes $= \alpha \times \frac{1}{\alpha}$

$$= \frac{\text{constant term}}{\text{coefficient of } x^2} = \frac{-(k-2)}{6}$$

Since $\alpha \times \frac{1}{\alpha} = 1$, we get:

$$\frac{-(k-2)}{6} = 1 \Rightarrow -(k-2) = 6 \Rightarrow k - 2 = -6 \Rightarrow k = -4$$

Thus, the value of k is -4.

7. p and q are the zeroes of the quadratic polynomial $f(x) = 6x^2 + x - 2$

Sum of the zeroes $p + q = -\frac{1}{6}$

Product of the zeroes $pq = \frac{-2}{6} = -\frac{1}{3}$

Now,

Using the identity $\frac{1}{p} + \frac{1}{q} = \frac{p+q}{pq}$ we get:

$$\frac{1}{p} + \frac{1}{q} = \frac{p+q}{pq} = \frac{-\frac{1}{6}}{-\frac{1}{3}} = \frac{1}{2}$$

$$\frac{1}{p} + \frac{1}{q} - pq = \frac{1}{2} - \left(-\frac{1}{3}\right) = \frac{1}{2} + \frac{1}{3} = \frac{3+2}{6} = \frac{5}{6}$$

Thus, the value is $\frac{5}{6}$.

8. One zero of the polynomial $p(x) = (a^2 + 4)x^2 + 20x + 4a$ is the reciprocal of the other, Let the zeroes of the polynomial be α and $\frac{1}{\alpha}$.

For the quadratic polynomial $ax^2 + bx + c$, the sum and product of the zeroes are:

Sum $\alpha + \frac{1}{\alpha} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2} = -\frac{20}{a^2 + 4}$

Product $\alpha \times \frac{1}{\alpha} = \frac{\text{constant term } x}{\text{coefficient of } x^2} = \frac{4a}{a^2 + 4}$

Since $\alpha \times \frac{1}{\alpha} = 1$, we get:

$$\frac{4a}{a^2 + 4} = 1 \Rightarrow 4a = a^2 + 4 \Rightarrow a^2 - 4a + 4 = 0$$

$$(a - 2)^2 = 0 \Rightarrow a = 2$$

Thus, the value of a is 2 .

9. We are given the quadratic polynomial $2x^2 - 8x + 5$.

The sum and product of the zeroes α and β are

Sum $\alpha + \beta = -\frac{-8}{2} = 4$

Product $\alpha\beta = \frac{5}{2}$

Now, $\left(\alpha + \frac{1}{\beta}\right) \times \left(\beta + \frac{1}{\alpha}\right)$

$$\Rightarrow \left(\alpha + \frac{1}{\beta}\right) \times \left(\beta + \frac{1}{\alpha}\right) = \alpha\beta + \alpha \times \frac{1}{\alpha} + \beta \times \frac{1}{\beta} + \frac{1}{\alpha\beta}$$

$$\alpha\beta + 1 + 1 + \frac{1}{\alpha\beta} = \alpha\beta + 2 + \frac{1}{\alpha\beta}$$

$$= \frac{5}{2} + 2 + \frac{2}{5}$$

$$= \frac{5}{2} + \frac{4}{2} + \frac{2}{5} = \frac{9}{2} + \frac{2}{5}$$

$$= \frac{45}{10} + \frac{4}{10} = \frac{49}{10}$$

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. As, α and β are zeroes of polynomial $6x^2 - 5x + 1$

Then, $\alpha + \beta = \frac{-(-5)}{6} = \frac{5}{6}$

and, $\alpha\beta = \frac{1}{6}$

Now, quadratic polynomial whose zeroes are α^2 and β^2 is (x^2 – (sum of zeroes) x + Product of zeroes)

$= x^2 - (\alpha^2 + \beta^2)x + \alpha^2\beta^2$

$= x^2 - [(\alpha + \beta)^2 - 2\alpha\beta)]x + (\alpha\beta)^2$

$[\because \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta]$

$= x^2 - \left[\left(\frac{5}{6}\right)^2 - 2\times\frac{1}{6}\right]x + \left(\frac{1}{6}\right)^2$

$= x^2 - \left[\frac{25}{36} - \frac{1}{3}\right]x + \left(\frac{1}{36}\right) = x^2 - \left(\frac{25-12}{36}\right)x + \frac{1}{36}$

$= x^2 - \frac{13x}{36} + \frac{1}{36} = \frac{1}{36}(36x^2 - 13x + 1)$

Thus, the required Polynomial $= \frac{1}{36}(36x^2 - 13x + 1)$

2. The given polynomial is $p(x) = x^2 - 15$. To find the zeroes, set $p(x) = 0$:

$x^2 - 15 = 0$

$x^2 = 15$

$x = \pm\sqrt{15}$

So, the zeroes are $\alpha = \sqrt{15}$ and $\beta = -\sqrt{15}$.

Verification:

For a quadratic polynomial $ax^2 + bx + c$, the relationships are

Sum of the zeroes $\alpha + \beta = \frac{-b}{a}$

Product of the zeroes $\alpha\beta = \frac{c}{a}$

For $p(x) = x^2 - 15$, we have $a = 1$, $b = 0$, and $c = -15$.

Sum of the zeroes:

$\alpha + \beta = \sqrt{15} + (-\sqrt{15}) = 0$ and $-\frac{b}{a} = -\frac{0}{1} = 0$

This verifies that the sum is correct.

Product of the zeroes:

$\alpha\beta = \sqrt{15} \times (-\sqrt{15}) = -15$ and $\frac{c}{a} = \frac{-15}{1} = -15$

This verifies that the product is correct.

3. Since ,

$4x^2 + 4x - 3 = 4x^2 + 6x - 2x - 3$

$= (4x^2 + 6x) + (-2x - 3)$

$= 2x(2x + 3) - 1(2x + 3)$

$= (2x + 3)(2x - 1)$

So, the zeroes are $\alpha = \frac{1}{2}$ and $\beta = -\frac{3}{2}$

Verification:

For $p(x) = 4x^2 + 4x - 3$, we have $a = 4$, $b = 4$, and $c = -3$.

Sum of the zeroes:

$\alpha + \beta = \frac{1}{2} + \left(-\frac{3}{2}\right) = -1$ and $-\frac{b}{a} = -\frac{4}{4} = -1$

This verifies that the sum is correct.

Product of the zeroes:

$\alpha \times \beta = \frac{1}{2} \times \left(-\frac{3}{2}\right) = -\frac{3}{4}$ and $\frac{c}{a} = \frac{-3}{4} = -\frac{3}{4}$

This verifies that the product is correct.

Thus, the relationships between the zeroes and the coefficients are verified.

4. α and β are the zeroes of the polynomial $x^2 + x - 2$,

The polynomial is $p(x) = x^2 + x - 2$.

The sum and product of the zeroes are:

Sum $\alpha + \beta = -\frac{1}{1} = -1$

Product $\alpha\beta = \frac{-2}{1} = -2$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$

We know $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$.

$\alpha^2 + \beta^2 = (-1)^2 - 2(-2) = 1 + 4 = 5$

Now, substitute this into the expression:

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{5}{\alpha\beta} = \frac{5}{-2} = -\frac{5}{2}$

Thus, the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ is $-\frac{5}{2}$.

5. Since α and β are the zeroes of the quadratic polynomial $f(x) = 6x^2 + x - 2$,

Sum of the zeroes $= (\alpha + \beta) = \frac{-b}{a} = \frac{-1}{6}$

The product of the zeroes

$\alpha\beta = \frac{c}{a} = \frac{-2}{6} = \frac{-1}{3}$

Now,

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{(a^2 + \beta^2)}{a\beta}$ (by taking LCM)

$\because \quad (\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{[(\alpha + \beta)^2 - 2\alpha\beta]}{\alpha\beta}$

By substituting the values of the sum of zeroes and products of the zeroes, we will get

$$= \frac{\left[\left(\frac{-1}{6}\right)^2 - 2\left(\frac{-1}{3}\right)\right]}{\left(\frac{-1}{3}\right)}$$

$$= \frac{\left[\frac{1}{36} + \frac{2}{3}\right]}{\left(\frac{-1}{3}\right)}$$

$$= \frac{\left[\frac{1+24)}{36}\right]}{\left(\frac{-1}{3}\right)}$$

$$= \frac{\left(\frac{25}{36}\right)}{\left(\frac{-1}{3}\right)}$$

$$= \frac{-25}{12}$$

Hence, $\frac{a}{\beta} + \frac{\beta}{\alpha} = \frac{-25}{12}$

6. The polynomial is $t^2 + 4\sqrt{3}\, t - 15$, where:

$a = 1$

$b = 4\sqrt{3}$

$c = -15$

By splitting the middle terms we get,

$$t^2 + 4\sqrt{3}t - 15 = t^2 + 5\sqrt{3}t - \sqrt{3}t - 15$$

$$= \left(t^2 + 5\sqrt{3}t\right) + (-\sqrt{3}t - 15)$$

$$= t(t + 5\sqrt{3}) - \sqrt{3}(t + 5\sqrt{3})$$

$$= (t + 5\sqrt{3})(t - \sqrt{3})$$

Therefore, zeroes of the given polynomial are: $-5\sqrt{3}$ and $\sqrt{3}$.

Verification:

Sum of the zeroes: $-5\sqrt{3} + \sqrt{3} = -4\sqrt{3}$ (matches $-\frac{b}{a}$).

Product of the zeroes: $-5\sqrt{3} \times \sqrt{3} = -15$ (matches $\frac{c}{a}$). Hence, verified.

7. For a quadratic polynomial $ax^2 + bx + c$, the relationships between the zeroes and the coefficients are:

Sum of the zeroes: $\alpha + \beta = -\frac{b}{a}$

Product of the zeroes: $\alpha\beta = \frac{c}{a}$

For the polynomial $5x^2 + 5x + 1$, where $a = 5$, $b = 5$, and $c = 1$, we get:

$$\alpha + \beta = -\frac{5}{5} = -1$$

$$\alpha\beta = \frac{1}{5}$$

(i) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

Substitute the values of $\alpha + \beta = -1$ and $\alpha\beta = \frac{1}{5}$

$$\alpha^2 + \beta^2 = (-1)^2 - 2\left(\frac{1}{5}\right)$$

$$\alpha^2 + \beta^2 = 1 - \frac{2}{5} = \frac{5}{5} - \frac{2}{5} = \frac{3}{5}$$

So, $\alpha^2 + \beta^2 = \frac{3}{5}$

(ii) $\alpha^{-1} + \beta^{-1} = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta}$

Substitute the values of $\alpha + \beta = -1$ and $\alpha\beta = \frac{1}{5}$:

$$\alpha^{-1} + \beta^{-1} = \frac{-1}{\frac{1}{5}} = -1 \times 5 = -5$$

So, $\alpha^{-1} + \beta^{-1} = -5$.

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. (i) The product of the zeroes of a quadratic polynomial $g(x) = px^2 + qx + r$ is given by:

Product of zeroes $= \frac{r}{p}$

So, for $g(x) = px^2 + qx + 152$, the equation for the product of the zeroes is:

Product of zeroes $= \frac{152}{p}$

The prime factorization of $152 = 2^3 \times 19$

Since the zeroes are distinct prime numbers, we can identify the zeroes as 2 and 19.

(ii) Product of Zeroes: From the formula the product of the zeroes

$$= 2 \times 19 = 38$$

Now, using the equation for the product:

$$\frac{152}{p} = 38$$

Solving for p:

$$p = \frac{152}{38} = 4$$

Sum of zeroes $= -\frac{q}{p}$

The sum of the zeroes $\alpha + \beta = 2 + 19 = 21$. So:

$$21 = -\frac{q}{4}$$

Solving for q:

$$q = -21 \times 4 = -84$$

2. (i) Let the zeroes of $f(x)$ be the two distinct prime numbers, prime factorisation of $325 = 5^2 \times 13$.

say $\alpha = 5$ and $\beta = 13$ (you can choose any distinct primes).

(ii) Using the relationships between the zeroes and the coefficients:

Sum of the zeroes: $\alpha + \beta = -\frac{b}{a}$

Product of the zeroes: $\alpha\beta = \frac{325}{a}$

Substitude $\alpha = 5$ and $\beta = 13$:

$$\alpha + \beta = 5 + 13 = 18 \Rightarrow -\frac{b}{a} = 18$$

From the product of the zeroes:

$$\alpha\beta = 5 \times 13 = 65 \Rightarrow \frac{325}{a} = 65$$

$$\Rightarrow \quad a = \frac{325}{65} = 5$$

Now, substitutde $a = 5$ into $-\frac{b}{5} = 18$

$$b = -90$$

Thus, the values of a and b are:

$$a = 5, b = -90$$

Level - 2 ADVANCED COMPETENCY FOCUSED QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Mark)

1. Option (A) is correct

Explanation: To find the area of the rectangular garden, we multiply the length and breadth.

Given: Length $= x + 5$ meters,

Breadth $= x + 3$ meters

Area = Length × Breadth

$= (x + 5)(x + 3)$

$= x^2 + 3x + 5x + 15$

$= x^2 + 8x + 15$

2. Option (A) is correct

Explanation: We are given the cost function:

$$C(x) = 2x^2 + 3x + 5$$

We need to find the cost of producing 10 items, *i.e.*, substitute $x = 10$ into the expression.

$C(10) = 2 \times (10)^2 + 3 \times 10 + 5$

$= 2 \times 100 + 30 + 5$

$= 200 + 30 + 5 =$ ₹ 235

3. Option (B) is correct

Explanation: The constant term represents the revenue when zero pens are sold — often interpreted as fixed income or revenue independent of sales.

4. Option (A) is correct

Explanation: To find the dimensions, we need to factor the polynomial.

$x^2 + 7x + 10 = x^2 + 5x + 2x + 10$

$= x(x + 5) + 2(x + 5)$

$= (x + 5)(x + 2)$

5. Option (A) is correct

Explanation: We are given the polynomial:

$$p(x) = -x + 6x^2 - 1$$

We want to find where it intersects the x–axis, so we set $p(x) = 0$

$$6x^2 - x - 1 = 0$$

By splitting the middle term

$$6x^2 - 3x + 2x - 1 = 0$$

$\Rightarrow \quad 3x(2x - 1) + 1(2x - 1) = 0$

$\Rightarrow \quad (2x - 1)(3x + 1) = 0$

$\Rightarrow \quad x = \frac{1}{2}$ or $x = -\frac{1}{3}$

The two x–values where the polynomial intersects the x–axis are $x = \frac{1}{2}$ and $x = \frac{-1}{3}$

Since the negative x–axis corresponds to negative values of x, the point where the graph intersects the negative x–axis is $\frac{-1}{3}$.

6. Option (A) is correct

Explanation: From the graph, observe the x–intercepts at $x = 2$ and $x = -4$. The quadratic polynomial with these intercepts is $(x - 2)(x + 4)$.

7. Option (A) is correct

Explanation: We are given the polynomial:

$$p(x) = -2x + 8x^2 - 1$$

To find where the polynomial intersects the x–axis, we set $p(x) = 0$

$$0 = -2x + 8x^2 - 1$$

Rearranging the equation:

$$8x^2 - 2x - 1 = 0$$

By splitting the middle term,

$$8x^2 - 4x + 2x - 1 = 0$$

$\Rightarrow \quad 4x(2x - 1) + 1(2x - 1) = 0$

$\Rightarrow \quad (4x + 1)(2x - 1) = 0$

$\Rightarrow \quad x = \frac{1}{2}$ or $x = -\frac{1}{4}$

The two x–values where the polynomial intersects the x–axis are $x = \frac{1}{2}$ and $x = \frac{(-1)}{4}$.

Since we are looking for the intersection on the positive x–axis, the answer is:

$$x = \frac{1}{2}$$

8. Option (A) is correct

Explanation: The sum of zeroes of a quadratic polynomial $ax^2 + bx + c$ is $\frac{-b}{a}$. For the polynomial $5x^2 - kx + 7$, the sum of zeroes is:

$$-\frac{-k}{5} = 4$$

Solving this, we get:

$$k = 20$$

9. Option (C) is correct

Explanation: The sum of the zeroes $\alpha + \beta$ is $\frac{-(-4\sqrt{3})}{1} = 4\sqrt{3}$, and the product of the zeroes $\alpha\beta = 3$. The required value of:

$$\alpha + \beta - \alpha\beta = 4\sqrt{3} - 3$$

ASSERTION-REASON QUESTIONS (1 Mark)

1. Option (A) is correct
2. Option (A) is correct
3. Option (A) is correct

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. To find the number of units (values of x) for which profit is zero, we need to factorise:

$$x^2 - 5x + 6 = 0 = (x-2)(x-3) = 0$$

So, the zeroes are: $x = 2$ and $x = 3$

The profit is zero when the company sells 2 units or 3 units.

Verify the relationship between zeroes and coefficients

Let the quadratic polynomial be:

$$P(x) = ax^2 + bx + c$$
$$= x^2 - 5x + 6$$

Here, $a = 1, b = -5, c = 6$

Let the zeroes be $\alpha = 2, \beta = 3$

Sum of zeroes $= \alpha + \beta = 2 + 3 = 5$

Compare with formula:

$$-\frac{b}{a} = \frac{(5)}{1} = 5$$

Product of zeroes $= \alpha\cdot\beta = 2 \times 3 = 6$

Compare with formula:

$$\frac{c}{a} = \frac{6}{1} = 6$$

Verified

$$\text{Sum of zeroes} = -\frac{b}{a}$$

$$\text{Product of zeroes} = \frac{c}{a}$$

2. Given: Area of the rectangular garden:

$$A(x) = x^2 + 9x + 20$$

Breadth of the garden: $B(x) = x + 4$

Let length be $L(x)$

Since, Area = Length × Breadth

$$x^2 + 9x + 20 = L(x).(x+4)$$

Factorising: $x^2 + 9x + 20$

$$x^2 + 9x + 20 = (x+4)(x+5)$$

So, $L(x).(x+4) = (x+4)(x+5)$

Now cancel $x + 4$ from both sides:

$$L(x) = x + 5$$

The length of the garden is $x + 5$ meters.

3. Given: Surface area of the top of the toy box:

$$S(x) = x^2 + 10x + 24$$

Length of the box: $L = x + 4$

To Find: Breadth and zeroes of the polynomial

Since, Surface Area (S) = Length (L) × Breadth (B)

$$x^2 + 10x + 24 = (x+4).\ B$$

Factorising $x^2 + 10x + 24$

$$x^2 + 10x + 24 = (x+4)(x+6)$$

So, $(x+4)(x+6) = (x+4).\ B$

Cancel $x+4$ from both sides:

$$B = x+6$$

Therefore, breadth of the box is $x + 6$

Zeroes of the polynomial

From the factorised form:

$$x^2 + 10x + 24 = (x+4)(x+6)$$

So the zeroes are: $x = -4$ and $x = -6$

Zeroes of the polynomial $= -4$ and -6

4. Factorise $C(x) = x^2 + 4x + 3$

So,
$$C(x) = x^2 + 3x + x + 3$$
$$= x(x + 3) + 1(x + 3)$$
$$= (x + 1)(x + 3)$$

So, the zeroes are – 1 and – 3

Verify the Relationship between Zeroes and Coefficients

For a quadratic polynomial of the form $ax^2 + bx + c$, the relationships are:

$$\text{Sum of zeroes} = -\frac{b}{a}$$

$$\text{Product of zeroes} = \frac{c}{a}$$

Here, $a = 1, b = 4, c = 3$

Zeroes: $\alpha = -1, \beta = -3$

Now verify:

$$\text{Sum} = \alpha + \beta = -1+(-3) = -4 = -\frac{b}{a}$$

$$\text{Product} = \alpha\cdot\beta = (-1)(-3) = 3 = \frac{c}{a}$$

Verified

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. (i) Given area of a rectangular cardboard sheet represented by the polynomial:

$A(x) = x^2 + 11x + 30$

The breadth is given as $x+5$ cm.

Since, Area = Length × Breadth,

we get:

$x^2 + 11x + 30 = \text{Length} \times (x+5)$

Factorising the area, $x^2 + 11x + 30$,

$x^2 + 6x + 5x + 30 = x(x + 6) + 5(x + 6)$

$= (x + 6)(x + 5)$

So, $(x + 6)(x + 5) = \text{Length} \times (x + 5)$

Cancel $(x + 5)$ from both sides

Length $= x + 6$ cm

(ii) For the zeroes of the polynomial:

Factors of the area polynomial $= (x + 5)(x + 6)$

Zeroes are – 5 and – 6

(iii) For a quadratic polynomial of the form:

$ax^2 + bx + c$

$$\text{Sum of zeroes} = -\frac{b}{a}$$

$$\text{Product of zeroes} = \frac{c}{a}$$

Here, $a = 1, b = 11, c = 30$

Zeroes are -5 and -6

Now, Sum of zeroes $= -5+(-6)$

$$= -11 = -\frac{11}{1}$$

$$\text{Product of zeroes} = (-5)(-6) = 30 = \frac{30}{1}$$

Verified.

2. (i) Given polynomial: $R(x) = 5x^2 + 20x$

Substitute $x = 10$ in the expression:

$R(10) = 5(10)^2 + 20(10)$

$= 5(100) + 200$

$= 500 + 200 = ₹\ 700$

Thus, the revenue when 10 toys are sold = Rs 700

(ii) The term $20x$ is linear in x, meaning it contributes ₹ 20 per toy to the total revenue.

So, it represents the variable part of revenue that increases proportionally with the number of toys sold. In real-world terms, ₹ 20 is the revenue earned per toy due to a constant rate.

3. (i) Given, Area of the rectangular plot:

$A(x) = 2x^2 + 9x + 10$

Length of the plot: $L = 2x + 5$

Since, Area = Length × Breadth

$2x^2 + 9x + 10 = (2x + 5) \times B$

Factorising $2x^2 + 9x + 10$

$2x^2 + 4x + 5x + 10 = 2x(x + 2) + 5(x + 2)$

$= (2x + 5)(x + 2)$

So, $(2x + 5)(x + 2) = (2x + 5) \times B$

Breadth $= x + 2$ m

(ii) Length $= 2x + 5$

Breadth $= x + 2$

So, dimensions in factor form:

$(2x + 5)(x + 2)$

(iii) Since, $A(x) = (2x + 5)(x + 2)$

Set $A(x) = 0$ to find the zeroes:

$$2x + 5 = 0 \Rightarrow x = -\frac{5}{2}$$

$$x + 2 = 0 \Rightarrow x = -2$$

Zeroes are: $-\frac{5}{2}$ and -2

4. (i) Given, Area of the tablet box:

$A(x) = x^2 + 8x + 15 \text{ cm}^2$

Length $= x + 5$ cm

Area = Length × Breadth

Factorising: $x^2 + 8x + 15 = x^2 + 5x + 3x + 15$

$= x(x + 5) + 3(x + 5)$

$= (x + 5)(x + 3)$

Since, Length $= x + 5$ cm

Therefore, Breadth $= x + 3$ cm

(ii) We already factorised:

$x^2 + 8x + 15 = (x + 5)(x + 3)$

(iii) Set area to zero:

$(x + 5)(x + 3) = 0 \Rightarrow x = -5$ or $x = -3$

Zeroes of the polynomial are: -5 and -3

5. (i) Given polynomial: $C(x) = x^2 + 9x + 20$

where x is the number of units produced in hundreds.

Factorising $x^2 + 9x + 20 = x^2 + 4x + 5x + 20$

$= x(x + 4) + 5(x + 4)$

$= (x + 4)(x + 5)$

So, $(x + 4)(x + 5) = 0$

$x = -4$ and $x = -5$

Zeroes are: -4 and -5

(ii) Let zeroes be $\alpha = -4, \beta = -5$

Sum of zeroes:

$\alpha + \beta = -4 + (-5) = -9$

$$\text{Coefficient relation: } -\frac{b}{a} = \frac{9}{1} = -9$$

Product of zeroes:

$\alpha \cdot \beta = (-4)(-5) = 20$

$$\text{Coefficient relation: } \frac{c}{a} = \frac{20}{1} = 20$$

Verified.

(iii) Cost of 200 units produced:

Since x represents units in hundreds,

200 units $= 2x$

Substituting in the expression:

$C(2) = (2)^2 + 9(2) + 20$

$= 4 + 18 + 20 = 42$

Cost = ₹ 42 (in hundreds)

= ₹ 4200

CASE BASED QUESTIONS (4 Marks)

1. (i) A ball is thrown in the air so that t seconds after it is thrown, its height h (in meters) above its starting point is given by the polynomial:

$$h(t) = 25t - 5t^2$$

$$25t - 5t^2 = 0 \Rightarrow 5t\,(5 - t) = 0$$

Thus, $t = 0$ and $t = 5$ are the zeroes of the polynomial. This represents the times at which the ball is at the ground level (height = 0), i.e., at the start and when it lands.

(ii) The maximum height occurs at the vertex of the parabola. The time at which the vertex occurs is given by:

$$t = -\frac{b}{2a} = -\frac{25}{2(-5)} = \frac{25}{10}$$

$$= 2.5 \text{ seconds}$$

Now, substitute $t = 2.5$ into the height equation to find the maximum height:

$h(2.5)=25(2.5)-5(2.5)^2=62.5-31.25=31.25$ meters

(iii) (a) The time when the ball reaches 30 meters, put $h(t) = 30$:

$$30 = 25t - 5t^2$$

$$\Rightarrow 5t^2 - 25t + 30 = 0$$

$$\Rightarrow t^2 - 5t + 6 = 0$$

$$t^2 - 2t - 3t + 6 = 0$$

$$(t - 2)(t - 3) = 0$$

So, $t = 2$ or $t = 3$.

OR

(b) Put $h(t) = 20$ to find the values of t when the height is 20 meters:

$$20 = 25t - 5t^2 \Rightarrow 5t^2 - 25t + 20 = 0$$

$$\Rightarrow t^2 - 5t + 4 = 0$$

Solve the quadratic equation:

$$t^2 - 4t - t + 4 = 0$$

$$t(t - 4) - 1\,(t - 4) = 0$$

$$(t - 4)\,(t - 1) = 0$$

Thus, $t = 4$ or $t = 1$.

2. (i) The zeroes of a polynomial are the points where the graph intersects the x–axis. In the given graph, the polynomial intersects the x –axis at two distinct points (at $x = -7$ and $x = 7$). Therefore, the polynomial has 2 zeroes.

(ii) From the graph, the polynomial intersects the x–axis at the points (–7, 0) and (7, 0). Therefore, the zeroes of the polynomial are $x = -7$ and $x = 7$.

(iii) (a) We are given that the zeroes of the polynomial are 2 and –3. The sum and product of the zeroes are:

Sum of zeroes: $2 + (-3) = -1$

Product of zeroes: $2 \times (-3) = -6$

For a quadratic polynomial of the form $x^2+(a + 1)\,x + b$, the sum and product of the zeroes are:

Sum of zeroes: $-\dfrac{\text{coefficient of } x}{\text{coefficient of } x^2} = -(a+1)$

Product of zeroes: $\dfrac{\text{constant term}}{\text{coefficient of } x^2} = b$

Thus:

From the sum of zeroes: $-(a + 1) = -1$

$\Rightarrow a + 1 = 1 \Rightarrow a = 0$

From the product of zeroes: $b = -6$

OR

(b) Let the zeroes of the polynomial $x^2 + px + 45$ be α and β . We know that the square of the difference of the zeroes is given as 144, i.e.,

$$(\alpha - \beta)^2 = 144$$

We also know the relationships for a quadratic polynomial $x^2 + px + 45$

- Sum of the zeroes: $\alpha + \beta = -p$
- Product of the zeroes: $\alpha\beta = 45$

Using the identity for the square of the difference:

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\,\alpha\beta$$

Substitute the known values:

$$144 = (-p)^2 - 4\,(45) \Rightarrow 144 = p^2 - 180$$

$$p^2 = 144 + 180 = 324$$

$$\Rightarrow \quad p = \pm\sqrt{324} = \pm 18$$

Thus, the value of p is ± 18.

3. **(i)** Given polynomial:

$$A(x) = x^2 + 7x + 12$$

The degree of a polynomial is the highest power of x.

Here, the highest power is x^2

The degree is 2.

(ii) Factorising:
$$x^2 + 7x + 12 = x^2 + 3x + 4x + 12$$
$$= x(x + 3) + 4\,(x + 3)$$
$$= (x + 3)(x + 4)$$

So, the zeroes are: $x = -3$ and $x = -4$

(iii) (a) (1) Factorise the polynomial:

Already done above: $A(x) = (x + 3)(x + 4)$

(2) Since: Area = length × breadth

Given: Area = $(x + 3)(x + 4)$,

Length = $x + 4$

So, Breadth = $x + 3$

OR

(b) Since, zeroes are $\alpha = -3, \beta = -4$

Sum of zeroes:

$$\alpha + \beta = -3 + (-4) = -7$$

Product of zeroes:

$$\alpha\cdot\beta = (-3)(-4) = 12$$

Now verify with coefficient relationships:

Given: $A(x) = x^2 + 7x + 12$

$\Rightarrow \quad a = 1, b = 7, c = 12$

$$\text{Sum} = -\frac{b}{a} = -\frac{7}{1} = -7$$

$$\text{Product} = \frac{c}{a} = \frac{12}{1} = 12$$

Verified

4. (i) The degree of a polynomial is the highest power of x.

$$\text{Degree} = 2$$

Since it is a polynomial of degree 2, it is called a quadratic polynomial.

(ii) Factorising the polynomial:

$$x^2 + 6x + 8 = x^2 + 2x + 4x + 8$$
$$= x(x + 2) + 4(x + 2)$$
$$= (x + 2)(x + 4)$$

So the zeroes are $x = -2$ and $x = -4$

(iii) (a) We have the standard quadratic polynomial form:

$ax^2 + bx + c$

$$\text{Here, } a = 1, b = 6, c = 8$$
$$\text{Zeroes: } \alpha = -2, \beta = -4$$

Now verify:

$$\text{Sum of zeroes} = \alpha + \beta = -2 + (-4) = -6$$
$$\text{Product of zeroes} = \alpha \cdot \beta = (-2)(-4) = 8$$

Now using formulas:

$$\text{Sum of zeroes} = -\frac{b}{a} = -\frac{6}{1} = -6$$
$$\text{Product of zeroes} = \frac{c}{a} = \frac{8}{1} = 8$$

Verified

OR

(b) If length $= x + 2$,

breadth $= x + 4$, find area:

$$\text{Area} = \text{Length} \times \text{Breadth}$$
$$= (x + 2)(x + 4)$$

Multiply using identity:

$$(x + 2)(x + 4) = x^2 + 4x + 2x + 8$$
$$= x^2 + 6x + 8$$
$$\text{Area} = x^2 + 6x + 8$$

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. (i) To verify, we multiply the length and breadth:

$$(2x+1)(x+5) = 2x(x+5) + 1(x+5)$$
$$= 2x^2+10x + x + 5$$
$$= 2x^2 + 11x + 5$$

This matches the given area polynomial.

Yes, the farmer's claim is correct.

(ii) Factorising: $2x^2 + 11x + 5 = (2x+1)(x+5)$

$$\Rightarrow \text{Zeroes: } x = -\frac{1}{2}, -5$$

Let's verify using the relationships:

$$\text{Sum of zeroes: } -\frac{b}{a} = -\frac{11}{2}$$
$$\text{Product of zeroes: } \frac{c}{a} = \frac{5}{2}$$

From the factors:

$$\text{Sum} = -\frac{1}{2} + (-5) = -\frac{11}{2}$$
$$\text{Product} = \left(-\frac{1}{2}\right).(-5) = \frac{5}{2}$$

Verified.

(iii) The zeroes of the area polynomial represent the values of x for which the area becomes zero. They help in factorising and understanding the algebraic structure of the area. They indicate boundary values where the dimensions collapse to make the area 0 which is useful in mathematical modeling.

(iv) The perimeter P of a rectangle is:

$$P = 2(\text{Length} + \text{Breadth})$$
$$= 2[(2x + 1) + (x + 5)]$$
$$= 2(3x + 6) = 6x + 12$$
$$\text{Perimeter} = 6x + 12$$

2. (i) Multiply the suggested dimensions:

$$(x + 3)(x + 4) = x^2 + 4x + 3x + 12$$
$$= x^2 + 7x + 12$$

Yes, the dimensions match the given polynomial.

(ii) Factorising the polynomial:

$$x^2 + 7x + 12 = (x + 3)(x + 4)$$
$$\Rightarrow \text{Zeroes are: } x = -3, -4$$

(iii) Given: $a = 1, b = 7, c = 12$

From zeroes:

$$\text{Sum} = -3 + (-4) = -7 = -\frac{b}{a}$$
$$\text{Product} = (-3)(-4) = 12 = \frac{c}{a}$$

Verified.

(iv) If the gardener adds a 1 meter path all around, each dimension increases by 2 meters (1 meter on each side):

$$\text{New length} = (x + 3) + 2 = x + 5$$
$$\text{New breadth} = (x + 4) + 2 = x + 6$$

New area:

$$A'(x) = (x + 5)(x + 6)$$
$$= x^2 + 6x + 5x + 30$$
$$= x^2 + 11x + 30$$