CHAPTER 3

Linear Equations in Two Variables

Level - 1 CORE SUBJECTIVE QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Mark)

1. Option (B) is correct.

Explanation : The given system of equations is:

$$3x - ky = 7$$
$$6x + 10y = 3$$

For the system to be inconsistent, the lines must be parallel but not coincident. This happens when the ratio of the coefficients of x and y are equal, but the ratio of the constants is different.

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

On comparing the coefficients, we get

$$\frac{3}{6} = \frac{-k}{10}$$

Solving this gives $k = -5$

2. Option (A) is correct

Explanation: The two lines are neither parallel nor overlapping. These given lines intersect at one points, so the system has a unique solution. Therefore, the pair of these linear equations is consistent with unique solution.

3. Option (D) is correct

Explanation: Given equations are: $x + 2y + 5 = 0$

The second equation is given and as $-3x = 6y - 1$. We want to rewrite these in the form $ax + by = c$, so:

or $\quad x + 2y = -5$

and $\quad 3x + 6y = 1$

Comparing the coefficients:

$$\frac{a_1}{a_2} = \frac{1}{3}$$

$$\frac{b_1}{b_2} = \frac{2}{6} = \frac{1}{3}$$

$$\frac{c_1}{c_2} = \frac{-5}{1} = -5$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, it has no solution.

4. Option (D) is correct

Explanation: Consistent System: A system is consistent if there is at least one solution.

If the lines intersect at a point, there is one solution.

If the lines coincide, there are infinitely many solutions.

Thus, if a system is consistent, the lines can either be intersecting or coincident.

5. Option (D) is correct

Explanation: For lines to be parallel ratio is $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ on comparing each option with given equation we get,

(A) $5x - 3y = 2$ and $-15x - 9y = 5$

Here, $\frac{a_1}{a_2} = \frac{5}{-15} = \frac{1}{-3}; \frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3} \Rightarrow \frac{a_1}{a_2} \uparrow \frac{b_1}{b_2}$

Not poosible

(B) $5x - 3y = 2$ and $15x + 9y = 5$

Here, $\frac{a_1}{a_2} = \frac{5}{9} = \frac{1}{3}; \frac{b_1}{b_2} = \frac{-3}{9} = \frac{-1}{3} \Rightarrow \frac{a_1}{a_2} \uparrow \frac{b_1}{b_2}$

Not possible

(C) $5x - 3y = 2$ and $9x - 15y = 6$

Here, $\frac{a_1}{a_2} = \frac{5}{9}; \frac{b_1}{b_2} = \frac{-3}{-15} = \frac{1}{5} \Rightarrow \frac{a_1}{a_2} \uparrow \frac{b_1}{b_2}$

Not possible

(D) $5x - 3y = 2$ and $-15x + 9y = 5$

Here, $\frac{a_1}{a_2} = \frac{5}{15} = \frac{-1}{3}; \frac{b_1}{b_2} = \frac{-3}{9} = \frac{-1}{3}; \frac{c_1}{c_2} = \frac{2}{5} \Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, option (D) satisfy the condition for parallel lines.

6. Option (C) is correct

Explanation: Given

$$ax + by = a^2 - b^2 \qquad ...(i)$$
$$bx + ay = 0 \qquad ...(ii)$$

Add these two equations:

$$(ax + by) + (bx + ay) = (a^2 - b^2) + 0$$
$$a(x + y) + b(x + y) = a^2 - b^2$$
$$(a + b)(x + y) = a^2 - b^2$$

Since $a^2 - b^2 = (a + b)(a - b)$, we have:

$(a + b)(x + y) = (a + b)(a - b)$

Dividing both sides by $(a + b)$ (assuming $a + b \neq 0$):

$$x + y = a - b$$

7. Option (D) is correct

Explanation: $y = 0$ is a horizontal line on the x–axis.

$y = -7$ is a horizontal line parallel to the x–axis but shifted down by 7 units.

Since they are parallel and never intersect, there is no solution.

8. Option (A) is correct

Explanation: $3x + 4y = 5$

$6x + 8y = 7$

Checking the condition:

$$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{4}{8} = \frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{5}{7}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the lines are parallel.

9. Option (D) is correct

Explanation: Given, $6x - 3y + 10 = 0$ ...(i)

Comparing with $a_1x + b_1y + c_1 = 0$

$\therefore$ $a_1 = 6, b_1 = -3, c_1 = 10$

$2x - y + 9 = 0$ ...(ii)

Comparing with $a_2x + b_2y + c_2 = 0$

$a_2 = 2, b_2 = -1, c_2 = 9$

$$\frac{a_1}{a_2} = \frac{6}{2} = 3$$

$$\frac{b_1}{b_2} = \frac{-3}{-1} = 3$$

$$\frac{c_1}{c_2} = \frac{0}{9} = 0$$

From above: $\frac{a_1}{a_2} = \frac{b_1}{b_1} \neq \frac{c_1}{c_2}$

So, there is no solution.

$\therefore$ The lines representing the linear equations are parallel.

10. Option (C) is correct

Explanation: $x = 2a$ is a vertical line passing through $x = 2a$.

$y = 3b$ is a horizontal line passing through $y = 3b$.

Since a vertical and horizontal line will intersect at a point, the intersection is at $(2a, 3b)$.

11. Option (C) is correct

Explanation: For infinitely many solutions, the ratios of the coefficients must be equal:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$a_1 = t + 3, b_1 = -3, c_1 = t$

$a_2 = t, b_2 = t, c_2 = -12$

Applying the condition for infinitely many solutions:

$$\frac{t+3}{t} = \frac{-3}{t} = \frac{t}{-12}$$

Solving $\frac{t+3}{t} = \frac{-3}{t}$

$t + 3 = -3$

$t = -6$

12. Option (B) is correct

Explanation: Given system of equation is $kx = y + 2$ and $6x = 2y + 3$

$\Rightarrow$ $kx - y - 2 = 0$ ...(i)

$6x - 2y - 3 = 0$ ...(ii)

By comparing given equation with standard form of system

We get

$a_1 = k, b_1 = -1, c_1 = -2$

$a_2 = 6, b_2 = -2, c_2 = -3$

$\therefore$ $\frac{a_1}{a_2} = \frac{k}{6}, \frac{b_1}{b_2} = \frac{-1}{-2} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-2}{-3} = \frac{2}{3}$

As, $\frac{1}{2} \neq \frac{2}{3}$, $\therefore \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

But condition for infinitely many solution is

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, which is not possible.

Hence, the value of k does not exist.

13. Option (D) is correct

Explanation: Given,

$3x - y + 8 = 0$ ...(i)

$6x - ry + 16 = 0$ ...(ii)

By comparing given equation with standard form of system, we get

$a_1 = 3, b_1 = -1, c_1 = 8$

$a_2 = 6, b_2 = -r, c_2 = 16$

$$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{b_1}{b_1} = \frac{1}{r} = \frac{1}{r}$$

$$\frac{c_1}{c_2} = \frac{8}{16} = \frac{1}{2}$$

Since lines are coincident

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$$\frac{1}{2}=\frac{1}{r}=\frac{1}{2}$$

Thus, $\frac{1}{2}=\frac{1}{r}$

$\therefore$ $r = 2$

Q. 14. Option (D) is correct

Explanation: The equation $x = a$ represents a vertical line passing through $x = a$

The equation $y = b$ represents a horizontal line passing through $y = b$

A vertical line and a horizontal line intersect at exactly one point, which is (a, b).

15. Option (C) is correct

Explanation: We have the equations:

$$2x + 3y = 15 \quad \text{...(i)}$$

$$3x + 2y = 25 \quad \text{...(ii)}$$

On multiplying equation (i) and (ii) by 3 and 2 respectively we get,

$$6x + 9y = 45 \quad \text{...(iii)}$$

$$6x + 4y = 50 \quad \text{...(iv)}$$

Subtract equation (iv) from equation (iii):

$$(6x + 9y) - (6x + 4y) = 45 - 50$$

$$5y = -5 \Rightarrow y = -1$$

Substitute $y = -1$ into $2x + 3y = 15$:

$$2x + 3(-1) = 15 \Rightarrow 2x - 3 = 15$$

$$\Rightarrow \quad 2x = 18 \Rightarrow x = 9$$

Now, $x - y = 10$

16. Option (C) is correct

Explanation: Since,

$$2x = 5y + 6$$

$$2x - 5y = 6$$

$$15y = 6x - 18$$

$$6x - 15y = 18$$

Compare the coefficients:

$$\frac{a_1}{a_2}=\frac{2}{6}=\frac{1}{3},\ \frac{b_1}{b_2}=\frac{-5}{-15}=\frac{1}{3},\ \frac{c_1}{c_2}=\frac{6}{18}=\frac{1}{3}$$

Since the ratios are equal, the lines are coincident.

17. Option (D) is correct

Explanation: Since

$$3x - y + 8 = 0$$

$$3x - y = 24$$

or, $3x - y - 24 = 0$

Now, $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

Since $\frac{3}{3}=\frac{-1}{-1}\neq\frac{8}{24}$

$\therefore$ The two lines are parallel.

18. Option (C) is correct

Explanation: For a pair of dependent equations, one equation is a multiple of the other.

Check the options:

Multiply the first equation $-3x + 5y = 4$ by different factors to see if one of the given options matches:

Option (C): $-9x + 15y = 12$ is a multiple of $-3x + 5y = 4$ (multiplied by 3).

19. Option (B) is correct

Explanation: Non–intersecting lines imply that the system is inconsistent, which occurs when the lines are parallel. For parallel lines, the ratios of the coefficients of x and y must be equal, but the constant terms differ.

Thus, the correct condition is:

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\uparrow\frac{c_1}{c_2}$$

20. Option (A) is correct

Explanation: The given statement in mathematical form can be given as,

$$\frac{y-x}{y}=\frac{2}{3}$$

$$3(y - x) = 2y$$

$$3y - 3x = 2y$$

$$y = 3x$$

21. Option (A) is correct

Explanation: The equation of the line k_3 is

$$px + qy = 1 \quad \text{...(i)}$$

The solution of the equations represented by the lines k_1 and k_3 is $x = 3$ and $y = 0$

$\therefore$ The line k_3 represented by equation (i) passes through the point (3, 0)

$\therefore$ $(p \times 3) + (q \times 0) = 1$

$\Rightarrow$ $3p = 1$

$\Rightarrow$ $p = \frac{1}{3}$

Again the solution of the equations represented by the lines k_2 and k_3 is $x = 4$ and $y = 1$

$\therefore$ The line k_3 represented by equation (i) passes through the point (4, 1)

$\therefore$ $(p\times 4)+(q\times 1) = 1$

$$4p + q = 1$$

$$q = 1-\frac{4}{3}=\frac{3-4}{3}=\frac{-1}{3}$$

As, the equation of the line k_3 is given by

$$px + qy = 1$$

$\Rightarrow$ $\frac{x}{3}-\frac{y}{3} = 1$ $\left(\because p=\frac{1}{3}\text{ and } q=\frac{-1}{3}\right)$

$\Rightarrow$ $\frac{x-y}{3} = 1$

$\Rightarrow$ $x - y = 3$

$\therefore$ The equation of the line k_3 is $x - y = 3$

ASSERTION-REASON QUESTIONS (1 Mark)

1. Option (C) is correct

Explanation: Given equations:

$$x - 2y + 3 = 0$$
$$3x + 4y - 11 = 0$$

Here $\frac{a_1}{a_2} = \frac{1}{3}$ and $\frac{b_1}{b_2} = \frac{-2}{4} = \frac{-1}{2}$

As, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

The pair of linear equations has a unique solution.

So, Assertion (A) is true.

Coincident lines are lines that overlap each other and have infinitely many solutions. In this case, the lines are not coincident; they intersect at exactly one point.

So, Reason (R) is false.

∴ Assertion (A) is true but reason (R) is false.

2. Option (D) is correct

Given equations:

$$9x + 12y - 7 = 0$$
$$6x + 8y - 14 = 0$$

Consistency means the system has at least one solution (either a unique solution or infinitely many solutions). Let's check whether the system has a solution.

First, divide both equations by their respective constants to compare the ratios:

$$\frac{a_1}{a_2} = \frac{9}{6} = \frac{3}{2}, \frac{b_1}{b_2} = \frac{12}{8} = \frac{3}{2} \text{ and } \frac{c_1}{c_2} = \frac{-7}{-14} = \frac{1}{2}$$

Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, lines are parallel and do not intersect, meaning the system has no solution and is inconsistent.

So, Assertion (A) is false.

For Reason: A pair of linear equations $px + qy + r = 0$ and $fx + gy + h = 0$ has no solution if $\frac{p}{f} = \frac{q}{g} \uparrow \frac{r}{h}$

This is the correct condition for parallel lines (no solution).

So, Reason (R) is true.

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1.
$$7x - 2y = 5 \quad \text{...(i)}$$
$$8x + 7y = 15 \quad \text{...(ii)}$$

Multiply equation (i) by 7

$$7(7x - 2y) = 7(5) \Rightarrow 49x - 14y = 35$$

Multiply the equation (ii) by 2

$$2(8x + 7y) = 2(15) \Rightarrow 16x + 14y = 30$$

Now,

$$49x - 14y = 35 \quad \text{...(iii)}$$
$$16x + 14y = 30 \quad \text{...(iv)}$$

Add Equation (iii) and Equation (iv) to eliminate y :

$$(49x - 14y) + (16x + 14y) = 35 + 30$$
$$49x + 16x = 65$$
$$65x = 65$$
$$x = \frac{65}{65} = 1$$

Substitute $x = 1$ into the equation (i) $7x - 2y = 5$:

$$7(1) - 2y = 5$$
$$7 - 2y = 5$$
$$-2y = 5 - 7$$
$$-2y = -2$$
$$y = \frac{-2}{-2} = 1$$

Verification of answer:

Substitute the value in eq. (i)

$$7(1) - 2(1) = 5$$
$$7 - 2 = 5$$

Verified.

2. We are given the system of linear equations :

$$2x + 5y = -4 \quad \text{...(i)}$$
$$4x - 3y = 5 \quad \text{...(ii)}$$

Multiply the equation (i) by 2

$$2(2x + 5y) = 2(-4) \Rightarrow 4x + 10y = -8$$

Now,

$$4x + 10y = -8 \quad \text{...(iii)}$$
$$4x - 3y = 5 \quad \text{...(iv)}$$

Subtract the equations

$$(4x + 10y) - (4x - 3y) = -8 - 5$$
$$4x + 10y - 4x + 3y = -13$$
$$13y = -13$$
$$y = \frac{-13}{13} = -1$$

Substitute $y = -1$ into $2x + 5y = -4$:

$$2x + 5(-1) = -4$$
$$2x - 5 = -4$$
$$2x = -4 + 5$$
$$2x = 1$$
$$x = \frac{1}{2}$$

3.
$$2p + 3q = 13 \quad \text{...(i)}$$
$$5p - 4q = -2 \quad \text{...(ii)}$$

Multiply equation (i) by 5 and equation (ii) by 2:

$$10p + 15q = 65 \quad \text{...(iii)}$$
$$10p - 8q = -4 \quad \text{...(iv)}$$

Subtract

$$(10p + 15q) - (10p - 8q) = 65 - (-4)$$

$$23q = 69$$
$$q = \frac{69}{23} = 3$$

Put the value of q in *eq*. (i), we get

$$2p + 3 \times 3 = 13$$
$$2p + 9 = 13$$
$$2p = 4$$
$$p = 2$$

4. We are given the system of equations:

$$2x + y = 13 \quad \text{...(i)}$$
$$4x - y = 17 \quad \text{...(ii)}$$

Add $(2x + y = 13)$ and $(4x - y = 17)$ to eliminate y:

$$(2x + y) + (4x - y) = 13 + 17$$
$$6x = 30$$
$$x = \frac{30}{6} = 5$$

Substitute $x = 5$ into one of the original equations.

Substitute $x = 5$ into $2x + y = 13$:

$$2(5) + y = 13$$
$$10 + y = 13$$
$$y = 13 - 10 = 3$$

Now that we know $x = 5$ and $y = 3$:

$$x - y = 5 - 3 = 2$$

5. We are given two conditions:

The sum of two numbers is 105.

The difference between the two numbers is 45.

Let the two numbers be a and b.

From the given conditions, we can write two equations:

$$a + b = 105 \quad \text{...(i)}$$
$$a - b = 45 \quad \text{...(ii)}$$
$$(a + b) + (a - b) = 105 + 45$$
$$2a = 150$$
$$a = \frac{150}{2} = 75$$

Now that we know $a = 75$, substitute this into Equation (i):

$$75 + b = 105$$
$$b = 105 - 75 = 30$$

The two numbers are $a = 75$ and $b = 30$.

6. Let unit digit be x and tens digit be y

Then, number $= 10y + x$

According to question

$$x + y = 14 \quad \text{...(i)}$$

₹ $10x + y - (10y + x) = 18$ ...(ii)

$\Rightarrow$ $9x - 9y = 18$

$\Rightarrow$ $x - y = 2$ ...(ii)

On adding eq. (i) and (ii), we get

$$x + y = 14$$
$$x - y = 2$$
$$2x = 16$$

So, $x = 8$

Putting the value of x in $x + y = 14$

$$8 + y = 14$$

$\Rightarrow$ $y = 6$

Thus, required number $= 10y + x$

$$= 10 \times 6 + 8 = 68$$

7. Let the age of Rashmi be x years and the age of Nazma be y years

Three years age,

Rashmi's age $= (x - 3)$ years

Nazma's age $= (y - 3)$ years

According to question,

$$(x - 3) = 3(y - 3)$$
$$x - 3 = 3y - 9$$
$$x = 3y - 6 \quad \text{...(i)}$$

Ten years later

Rashmi's age $= (x + 10)$ years

Nazma's age $= (y + 10)$ years

According to question

$$(x + 10) = 2(y + 10)$$
$$x + 10 = 2y + 20$$
$$x = 2y + 10 \quad \text{...(ii)}$$

From eq. (i) and (ii), we get

$$3y - 6 = 2y + 10$$
$$y = 16$$

Substituting the value of y in eq. (i) we get

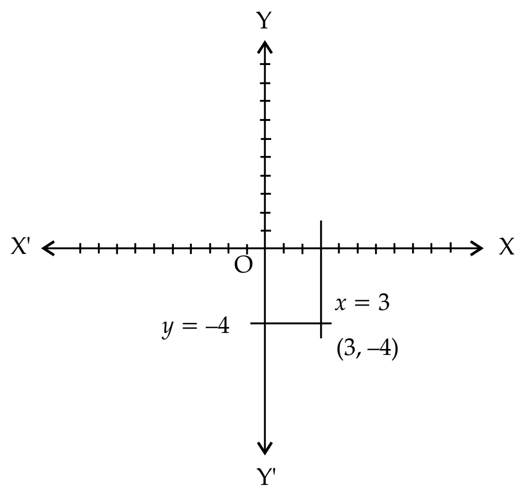
$$x = 3 \times 16 - 6$$
$$= 48 - 6 = 42$$

Thus, the age of Rashmi is 42 years and age of Nazma is 16 years.

8. Get the intersection point as $(3, -4)$

Since the lines are intersecting at $(3, -4)$, hence the solution of given equations is $(3, -4)$.

Since the lines are intersecting only at one point, therefore the given lines will have only one solution.



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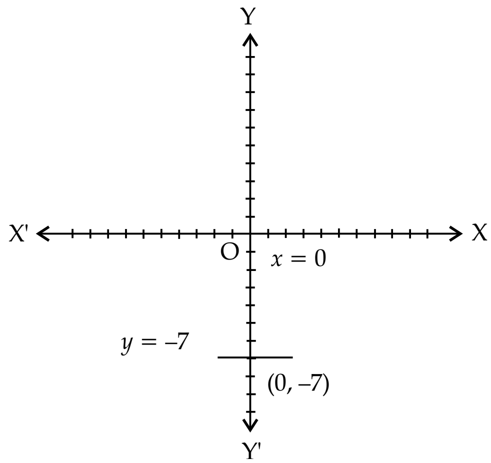


On a graph paper, plot or draw $x = 0$ and $y = -7$

In this case, line $y = -7$ is intersecting $x = 0$ at $(0, -7)$;

Since the given lines have one unique solution i.e., they are intersecting at one point $(0, -7)$.

$\therefore$ The given system of linear equations is consistent.

10. Let's solve the system of linear equations using the elimination method:

$$7x - 2y = 3 \quad \text{...(i)}$$

$$11x - \frac{3}{2}y = 8 \quad \text{...(ii)}$$

Multiply Equation (i) by 3:

$$3(7x - 2y) = 3(3) \Rightarrow 21x - 6y = 9$$

Multiply Equation (ii) by 4:

$$4\left(11x - \frac{3}{2}y\right) = 4(8) \Rightarrow 44x - 6y = 32$$

$$21x - 6y = 9 \quad \text{...(iii)}$$

$$44x - 6y = 32 \quad \text{...(iv)}$$

Subtract Equation (iii) from Equation (iv):

$$(44x - 6y) - (21x - 6y) = 32 - 9$$

$$44x - 21x = 32 - 9$$

$$23x = 23$$

$$x = 1$$

Put the value of x in eq. (i)

$$7 \times 1 - 2y = 3$$

$$-2y = -4$$

$$y = \frac{-4}{-2} = 2$$

11. The two original equations are:

$$5x - 2y = 4 \quad \text{...(i)}$$

$$3x - y = 8 \quad \text{...(ii)}$$

On Multiplying equation (ii) by 2 we get,

$$6x - 2y = 16 \quad \text{...(iii)}$$

$\therefore$ Step 1 is correct.

In step 2:

Anjali added equation (i) and (iii) which was incorrect. Instead, the correct approach is to subtract equation (i) from equation (iii) i.e.,

$$(6x - 2y) - (5x - 2y) = 16 - 4$$

On Simplifying:

$$x = 12$$

So, the correct value of x is 12.

Thus, Anjali made mistake on step 2.

12. We are given a pair of linear equations:

$$(k - 1)x + y = k + 1$$

$$(k^2 - 1)x + (k + 1)y = 1 - k^2$$

The general form of a pair of linear equations is:

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

For the given system:

1st equation:

$$(k - 1)x + y = k + 1$$

So, $a_1 = (k - 1)$, $b_1 = 1$, and $c_1 = k + 1$.

2nd equation:

$$(k^2 - 1)x + (k + 1)y = 1 - k^2$$

So, $a_2 = (k^2 - 1)$, $b_2 = (k + 1)$, and $c_2 = 1 - k^2$

We need to check the ratio of the coefficients:

$$\frac{a_1}{a_2} = \frac{k-1}{k^2-1}$$

$$= \frac{k-1}{(k-1)(k+1)} = \frac{1}{k+1}$$

Check the ratio of the b coefficients:

$$\frac{b_1}{b_2} = \frac{1}{k+1}$$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2}$, now check if $\frac{c_1}{c_2}$ also holds:

$$\frac{c_1}{c_2} = \frac{k+1}{1-k^2}$$

$$= \frac{k+1}{-(k^2-1)}$$

$$= \frac{k+1}{-(k+1)(k-1)} = \frac{1}{-(k-1)}$$

Thus, $\frac{c_1}{c_2} = -\frac{1}{k-1}$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \uparrow \frac{c_1}{c_2}$

This means the system of equations has no solution, as the lines are parallel.

13. Use the properties of parallelogram to make the following equation:

$$2x + y = 5x \quad \text{...(i)}$$

$$5x + 70° = 180° \quad \text{...(ii)}$$

Solving eq. (ii), we get

$$5x + 70° = 180°$$

$$5x = 110°$$

$$x = 22°$$

Substituting $x = 22°$, we get

$$2 \times 22° + y = 5 \times 22°$$

$$44° + y = 110°$$

$$y = 66°$$

14. Given

$$2x - y - 3 = 0 \quad ...(i)$$
$$4x - y - 5 = 0 \quad ...(ii)$$

From eq. (i), we get

$$y = 2x - 3 \quad ...(iii)$$

Substituting the value of y in eq. (ii), we get

$$4x - (2x - 3) - 5 = 0$$
$$4x - 2x + 3 - 5 = 0$$
$$2x - 2 = 0$$
$$2x = 2$$
$$x = 1$$

On substituting $x = 1$ in eqn. (iii), we get

$$y = 2 \times 1 - 3$$
$$y = 2 - 3$$
$$y = -1$$

15. For infinitely many solutions

$$\frac{a_1}{a_1} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Now, for the equations (i) and (ii), the ratio

$\frac{3}{6} \neq -\frac{2}{2} = \frac{4}{8}$ which do not satisfy the condition

For equations (ii) and (iii), the ratio is $\frac{6}{12} \neq \frac{2}{-8} = \frac{8}{16}$ do not satisfy the condition

For the third pair of equations (i) and (iii), The ratio is $\frac{3}{12} = \frac{-2}{-8} = \frac{4}{16} = \frac{1}{4}$ so the correct pair of equations having infinitely many solutions is is (i) and (iii)

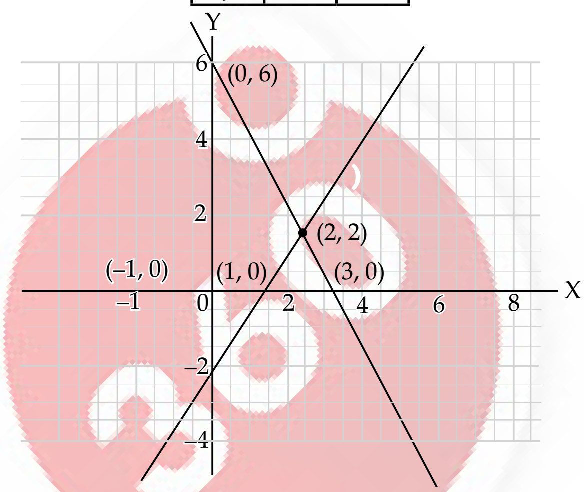
16. For given equations, we have solutions as follows:

(i) For equation $2x + y = 6$

x	0	3
y	6	0

(ii) For equation $2x = y + 2$

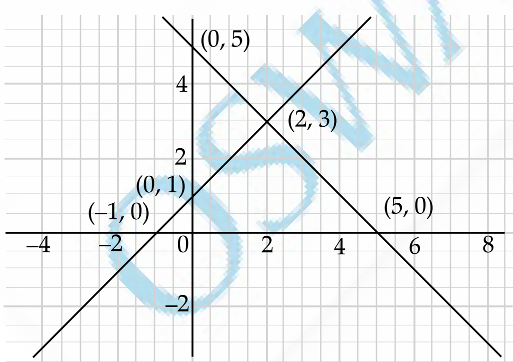
x	0	1
y	–2	0



SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. Given $x - y + 1 = 0$
and $x + y = 5$
(i) $y = x + 1$

x	0	2	–1
y	1	3	0



(ii) $y = 5 - x$

x	0	2	5
y	5	3	0

$\therefore \quad x = 2, y = 3$

So, the equations formed are as :

$$y = x + 1$$

and $\quad y = 5 - x$

From graph, it is clear the system of equations intersect at unique point (2. 3). So, system is consistent and has unique solution.

2. Let the two digit number be represented as $10x + y$, where:

x is the digit in the tens place.

y is the digit in the units place.

From the problem, we have two conditions:

$$x + y = 12 \quad ...(i)$$

Seven times the number is equal to four times the number obtained by reversing the digits:

$$7(10x + y) = 4(10y + x)$$
$$70x + 7y = 40y + 4x$$
$$70x - 4x = 40y - 7y$$
$$66x = 33y$$
$$2x = y$$
$$y = 2x$$

Substituting $y = 2x$ into the eq (i), we get

$$x + 2x = 12$$
$$3x = 12$$
$$x = 4$$

Since $y = 2x$, substitute $x = 4$:

$$y = 2(4) = 8$$

The two-digit number is $10x + y = 10(4) + 8 = 48$

So, the number is 48.

3. We are given the system of linear equations:

$$62x + 43y = 167 \quad ...(i)$$

$$43x + 62y = 148 \quad ...(ii)$$

On adding eqs (i) and (ii), we get

$$105x + 105y = 315$$

or $$x + y = 3 \quad ...(iii)$$

On subtracting eq (i) from eq (ii), we get

$$-19x + 19y = -19$$

or $$-x + y = -1 \quad ...(iv)$$

On adding eq (iii) and (iv), we get

$$2y = 2$$

or $$y = 1$$

On substituting $y = 1$ in eq (iii), we get

$$x + 1 = 3$$

$$x = 2$$

The solution is $x = 2$ and $y = 1$.

4. Let the two numbers be x (the greater number) and y (the smaller number).

From the problem, we are given two conditions:

Half of the difference between two numbers is 2:

$$\frac{x-y}{2} = 2$$

Multiply both sides by 2 to eliminate the fraction:

$$x - y = 4 \quad ...(i)$$

The sum of the greater number and twice the smaller number is 13:

$$x + 2y = 13 \quad ...(ii)$$

From Equation (i),

$$x = y + 4$$

Substitute $x = y + 4$ into Equation (ii)

$$(y + 4) + 2y = 13$$

Simplify :

$$y + 4 + 2y = 13$$

$$3y + 4 = 13$$

Now,

$$3y = 13 - 4$$

$$3y = 9$$

$$y = \frac{9}{3} = 3$$

Substitute $y = 3$ into $x - y = 4$

Now,

$$x = 3 + 4 = 7$$

The two numbers are $x = 7$ and $y = 3$.

5. Given system of equations is:

$$2x + 3y = 7 \text{ and}$$

$$2ax + (a + b)y = 28$$

The above equations can be rewritten as:

$$2x + 3y - 7 = 0 \text{ and}$$

$$2ax + (a + b)y - 28 = 0$$

Here the system must have infinite solutions. We know that, for the given system of equations to have infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

i.e.,

$$\frac{2}{2a} = \frac{3}{(a+b)} = \frac{-7}{-28}$$

$$\frac{1}{a} = \frac{3}{(a+b)} = \frac{1}{4}$$

$$\frac{1}{a} = \frac{1}{4}$$

$$a = 4$$

$$\frac{3}{(a+b)} = \frac{1}{4}$$

$$a + b = 12$$

$$4 + b = 12$$

$$b = 8$$

6. We are given the system of linear equations:

$$217x + 131y = 913 \quad ...(i)$$

$$131x + 217y = 827 \quad ...(ii)$$

On adding eqs. (i) and (ii), we get

$$348x + 348y = 1740$$

$$x + y = 5 \quad ...(iii)$$

On subtracting eq (ii) from eq (i), we get

$$86x - 86y = 86$$

$$x - y = 1 \quad ...(iv)$$

On adding eqs. (iii) and (iv), we get

$$2x = 6$$

or, $$x = 3$$

Substituting $x = 3$ in eq (iii), we get

$$3 + y = 5$$

$$y = 2$$

The solution is $x = 3$ and $y = 2$.

7. Let their speeds x and y kmph.

In Opposite direction, effective speed will gets added

Hence, effective speed $= x + y$

We know that time taken $= \frac{\text{distance}}{\text{speed}}$

Therefore, Time taken $= \frac{16}{x+y}$

It is given that during opposite direction walking, time taken is 2 hours

Therefore, $$\frac{16}{x+y} = 2$$

$$\Rightarrow \quad \frac{16}{2} = x + y$$

$$\Rightarrow \quad x + y = 8 \quad ...(i)$$

Now, in same direction, effective speed will be differential speed.

hence, effective speed $= x - y$

We know that time taken $= \dfrac{\text{distance}}{\text{speed}}$

Therefore, Time taken $= \dfrac{16}{x-y}$

It is given that during same direction walking, time taken is 8 hours

$$\text{Therefore,} \quad \frac{16}{(x-y)} = 8$$

$$\Rightarrow \quad \frac{16}{8} = x - y$$

$$\Rightarrow \quad x - y = 2 \quad \text{...(ii)}$$

By adding the equations (i) and (ii), we get:

$$(x + y) + (x - y) = 8 + 2$$

$$\Rightarrow \quad 2x = 10$$

$$\Rightarrow \quad x = 5$$

By substituting value of x in equation (i), we get:

$$x + y = 8$$

$$\Rightarrow \quad 5 + y = 8$$

$$\Rightarrow \quad y = 8 - 5$$

$$\Rightarrow \quad y = 3$$

Therefore the walking speeds are 5 km/h and 3 km/h.

8.
$$\frac{ax}{b} - \frac{by}{a} = a + b \quad \text{...(i)}$$

$$ax - by = 2ab \quad \text{...(ii)}$$

Dividing equation (ii) by 'a' on both sides

$$x - \frac{b}{a}y = 2b \quad \text{...(iii)}$$

Subtracting equation (iii) from equation (i), we get

$$\left(\frac{a}{b} - 1\right)x = a - b$$

$$\frac{(a-b)}{b}x = a - b$$

$$\Rightarrow \quad x = \frac{(a-b)b}{(a-b)}$$

$$\Rightarrow \quad x = b$$

Now,

Dividing equation (ii) by 'b' on both sides, we get

$$\frac{a}{b}x - y = 2a \quad \text{...(iv)}$$

Subtracting equation (iv) from equation (i), we get

$$\left(1 - \frac{b}{a}\right)y = b - a$$

$$\left(\frac{a-b}{a}\right)y = b - a$$

$$\Rightarrow \quad y = \frac{a(b-a)}{(a-b)}$$

$$\Rightarrow \quad y = -a$$

$$\therefore \quad x = b \text{ and } y = -a$$

9. Let the father's age be x years and the sum of ages of 2 children be y years.

As per the question,

$$x = 2y \quad \text{...(i)}$$

After 20 years,

$$x + 20 = y + 20 + 20$$

$$\Rightarrow \quad x + 20 = y + 40$$

$$\Rightarrow \quad x = y + 20 \quad \text{...(ii)}$$

Equating (i) & (ii),

$$y = 20$$

Substituting $y = 20$ in equation (i), we get

$$x = 40$$

Hence, the father's present age is 40 years.

10. Let the three consecutive odd numbers be x, $x + 2$, and $x + 4$.

Given: The sum of the squares of the first two numbers is greater than the square of the third number by 65.

Equation:

$$x^2 + (x + 2)^2 = (x + 4)^2 + 65$$

Expanding:

$$x^2 + (x^2 + 4x + 4) = (x^2 + 8x + 16) + 65$$

Simplifying:

$$2x^2 + 4x + 4 = x^2 + 8x + 81$$

Bring all terms to one side:

$$2x^2 + 4x + 4 - x^2 - 8x - 81 = 0$$

$$x^2 - 4x - 77 = 0$$

$$(x - 11)(x + 7) = 0$$

$\therefore$ $x = 11$ and -7

Thus, the three consecutive odd integers are 11, 13, and 15 or –7, –5, –3

11. (i) The pair will have infinitely many solutions.

Reasons that as there are more than one points of intersection, the pair is of coincident or overlapping lines.

(ii) Substitute the values of the point of intersection (6, 0) in the equation of a line $ax + by = c$ as:

$$6a + 0 = c$$

$$\text{or} \quad a = \frac{c}{6}$$

Substitute the values of the second point of intersection (0, 2) in the equation as:

$$2b = c$$

$$\text{or} \quad b = \frac{c}{2}$$

Rewrites the equation of a line by substituting the values of a and b in terms of c as:

$$\frac{c}{6}x + \frac{c}{2}y = c$$

Simplify the above equation by taking c =1 to get the equation of the line as $x + 3y = 6$.

12. (i) For the equations to have unique solution:

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Hence, in the given equations:

$$\frac{m}{n} \neq \frac{2}{4} \text{ or } \frac{m}{n} \uparrow \frac{1}{2}$$

Substitute a set of values for m and n in the given pair of equations which satisfies the above condition and frames a pair of equations.

For example:

$$2x - 2y = 9$$
$$4x - 6y = 9$$

(ii) For the equations to have infinitely many solutions:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Reasons that in the pair of equations provided:

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$$

while $$\frac{c_1}{c_2} = \frac{9}{9} = 1$$

Concludes that as the required condition can never be satisfied, it is not feasible to frame a pair of equations having infinitely many solutions.

(iii) For the equations to have no solution:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \uparrow \frac{c_1}{c_2}$$

In the given equations:

$$\frac{c_1}{c_2} \neq \frac{a_1}{a_2}$$

Now, substitutes a pair of values for m and n in the given equations such that:

$$\frac{m}{n} = \frac{a_1}{a_2} = \frac{1}{2}$$

For example,

$$2x - y = 9$$
$$4x - 2y = 9$$

13.

$$\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$$
$$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$$

Let $\frac{1}{\sqrt{x}} = p$ and $\frac{1}{\sqrt{y}} = q$, then the equations changes as below:

$$2p + 3q = 2 \quad \text{...(i)}$$
$$4p - 9q = -1 \quad \text{...(ii)}$$

Multiplying equation (i) by 3 , we get

$$6p + 9q = 6 \quad \text{...(iii)}$$

Adding equation (ii) and (iii), we get

$$10p = 5$$
$$p = \frac{1}{2}$$

Putting $p = \frac{1}{2}$ in equation (i), we get

$$2 \times \frac{1}{2} + 3q = 2$$
$$3q = 1$$
$$q = \frac{1}{3}$$
$$p = \frac{1}{\sqrt{x}} = \frac{1}{2}$$
$$\sqrt{x} = 2$$
$$x = 4$$

and

$$q = \frac{1}{\sqrt{y}} = \frac{1}{3}$$
$$\sqrt{y} = 3$$
$$y = 9$$

Hence, $x = 4, y = 9$

14. The difference between the two numbers is 6:

$$x - y = 6$$

The average of the two numbers is 4 . This implies:

$$\frac{(x+y)}{2} = 4$$

Multiply both sides by 2 to eliminate the fraction:

$$x + y = 8$$

So, the two linear equations are:

$$x - y = 6 \quad \text{...(i)}$$
$$x + y = 8 \quad \text{(ii)}$$

(ii) Now, solve this system of linear equations. Add equations (i) and (ii):

$$(x - y) + (x + y) = 6 + 8$$
$$2x = 14$$
$$x = 7$$

Now, substitute $x = 7$ into equation (ii):

$$7 + y = 8$$
$$y = 1$$

The two numbers are $x = 7$ and $y = 1$

15. We are given two equations:

$$2c - 3d = 7 \quad \text{...(i)}$$
$$4c + d = 1 \quad \text{...(ii)}$$

From the second equation:

$$d = 1 - 4c$$

Substitute $d = 1 - 4c$ into $2c - 3d = 7$:

$$2c - 3(1 - 4c) = 7$$
$$2c - 3 + 12c = 7$$
$$14c - 3 = 7$$

$$14c = 10 \Rightarrow c = \frac{5}{7}$$

Substitute $c = \frac{5}{7}$ into $d = 1 - 4c$:

$$d = 1 - 4 \times \frac{5}{7} = 1 - \frac{20}{7}$$

$$d = \frac{7}{7} - \frac{20}{7} = \frac{-13}{7}$$

The product of c and d is:

$$c \times d = \frac{5}{7} \times \frac{-13}{7} = \frac{-65}{49}$$

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. Given, $x + 2y = 3$

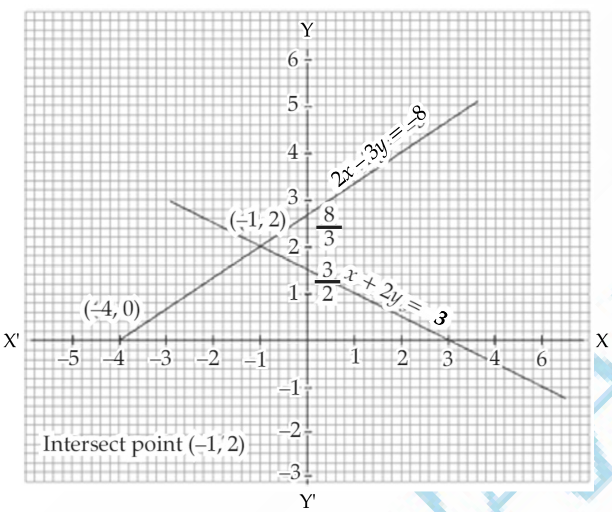
$2x - 3y + 8 = 0$

Now, $x + 2y = 3$

x	0	3	1
y	3/2	0	1

$2x - 3y = -8$

x	0	–4	–1
y	8/3	0	2



2. Let car I starts from A with speed x km/h car II starts from B with speed y km/h $(x > y)$

Case I. When cars are moving in the same direction

Distance covered by car I in 9 hours $= 9x$

Distance covered by car II in 9 hours $= 9y$

Therefore, $9(x - y) = 180$

$\Rightarrow \quad x - y = 20$ ...(i)

Case II. When cars are moving in opposite direction

Distance covered by car I in 1 hour $= x$

Distance covered by car II in 1 hours $= y$

Therefore, $x + y = 180$ ...(ii)

On solving (i) and (ii) we get, $x = 100$ km/h

$y = 80$ km/h

3. (i) Given, $3x + y + 4 = 0$ and $3x - y + 2 = 0$ on putting different values in x, we get values of y.

Eq. (i) $\quad 3x + y = -4$

$\Rightarrow \quad y = -4 - 3x$

x	0	1	–2
y	–4	–7	2

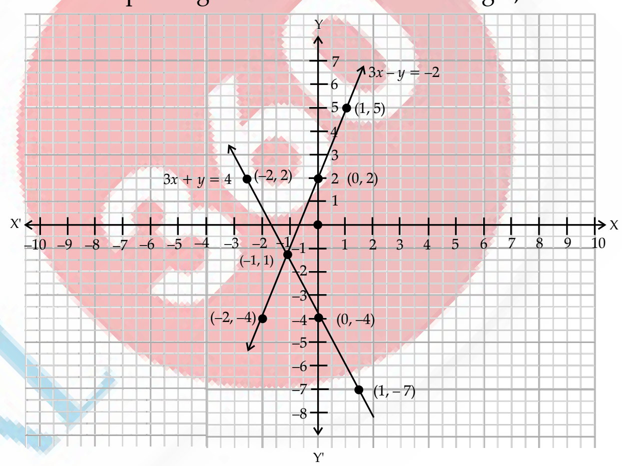
Eq. (ii) $\quad 3x - y + 2 = 0$

$\Rightarrow \quad -y = -2 - 3x$

$\Rightarrow \quad y = 2 + 3x$

x	0	1	–2
y	2	5	–4

On plotting values of both tables we get,



Intersection point = (–1, –1)

(ii) Let x be the number of right Answer

y be the number of wrong Answer

$\therefore$ According to the question,

$3x - y = 40$ ...(i)

$4x - 2y = 50$ ...(ii)

Multiply with 2 in eq. (i) and subtract the resulting equation from eq. (ii), we get

$6x - 2y = 80$ ...(iii)

$4x - 2y = 50$ ...(ii)

$-\quad +\quad -$

$2x = 30$

$x = 15$

Putting the value of x in eqn. (i)

$3 \times 15 - y = 40$

$-y = 40 - 45$

$y = 5$

Total no. of question

$= 15 + 5$

$= 20$

4. Let the greater no. be x and smaller no. be y

Given $\quad 3x = 4y + 3$ ...(i)

By using:

Dividend = Divisor × Quotient + Remainder

$$7y = 5x + 1 \quad ...(ii)$$

Multiply eqn. (i) by 5 and (ii) by 3

$$15x - 20y = 15 \quad ...(iii)$$

$$-15x + 21y = 3 \quad ...(iv)$$

On adding $y = 18$

From equation (i)

$$x = \frac{18 \times 4 + 3}{3}$$

$$= \frac{75}{3}$$

$$= 25$$

Greater number = 25

Smaller number = 18

5. Let the fixed charge of taxi be x and the charge per km be y.

For a distance of 10 km,

$$x + 10y = 105 \quad ...(i)$$

For the distance of 15 km,

$$x + 15y = 155 \quad ...(ii)$$

Now subtracting eqn. (i) from eqn. (ii)

$$5y = 50$$

$$y = 10$$

Putting value of y in eqn. (i)

$$x + 10y = 105$$

$$x + 10 \times 10 = 105$$

$$x = 5$$

So, fixed charge of taxi = ₹ 5 and charges per km = ₹ 10

(ii) Charge for travelling a distance of 25 km $= x + 25y = 5 + 25 \times 10 =$ ₹ 255

6. Let x and y be the number of students in room A and room B respectively

When 5 students shifted from Room A to Room B, we get,

$$x - 5 = y + 5$$

$$x - y = 10 \quad ...(i)$$

and from Room B to Room A

$$x + 5 = 2(y - 5)$$

$$x + 5 = 2y - 10$$

$$x - 2y = -15 \quad ...(ii)$$

Subtracting eqn. (i) from eqn. (ii)

$$y = 25$$

Putting value of y in eqn. (i)

$$x - y = 1$$

$$x - 25 = 10$$

$$x = 35$$

Number of students in Room A = 35

Number of students in Room B = 25

7. (i) Let x m be the length and y m be the breadth of Ram's plot and $2x$ m be the length, and $(y + 5)$ m be the breadth of Sham's plot

Given: $2(x + y) = 50$ and $2[2x + (y + 5)] = 100$

$$\Rightarrow \quad x + y = 25 \quad ...(i)$$

and $$2x + y + 5 = 50$$

$$2x + y = 45 \quad ...(ii)$$

(ii) Now, subtracting eqn (ii) from (i)

$$-x = -20$$

$$\Rightarrow \quad x = 20 \text{ m}$$

So dimensions of Ram's plot length = 20 m and breadth = 5 m

Dimensions of Sham's plot length = 2 × 20 = 40 m breadth = 5 + 5 = 10 m

8. Let speed of Anisha = x km/h

And, speed of bus = y km/h

As, per questions

$$\frac{3}{x} + \frac{12}{y} = 1.5 \text{ and } \frac{5}{x} + \frac{10}{y} = 2$$

Let $\frac{1}{x} = a$ and $\frac{1}{y} = b$

$3a + 12b = 1.5$ and $5a + 10b = 2$

Multiply both equations by 5 and 3 respectively

$$15a + 60b = 7.5$$

$$15a + 30b = 6$$

$$- \quad - \quad -$$

$$30\,b = 1.5$$

$$b = \frac{1.5}{30} = \frac{1}{20}$$

$$\Rightarrow \quad y = 20 \text{ km/h}$$

Substituting value of y in equation (i)

$$\frac{3}{x} + \frac{12}{20} = 1.5$$

On solving we get $x = \frac{10}{3}$ km/h

Thus, Average speed of Anisha = $\frac{10}{3}$ km/h

and Average speed of the bus = 20 km/h

Level - 2 ADVANCED COMPETENCY FOCUSED QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Mark)

1. Option (B) is correct

Explanation: The condition for parallel lines is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, $$\frac{3}{2} = \frac{2k}{5}$$

$$\therefore \quad k = \frac{15}{4}$$

2. Option (B) is correct

Explanation: If a pair of linear equations is consistent, then the lines are intersecting or coincident i.e., they will have at least one solution or infinitely many solutions.

Given linear equations in two variables is:

$a_1x + b_1y + c = 0$

$a_2x + b_2y + c = 0$

This system of linear equations will have unique solution if $\frac{a_1}{a_2} \uparrow \frac{b_1}{b_2}$

This system of linear equations will have infinitely many solutions if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = 1$

So, the system of linear equations will have solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ or $\frac{a_1}{a_2} = \frac{b_1}{b_2} = 1$

3. Option (B) is correct

Explanation: If the pair of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has unique solution,

then $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow a_1b_2 \neq a_2b_1$

4. Option (B) is correct

Explanation: For intersecting lines.

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

From the given equation only eq (ii) $2x - 3y = 12$ and *eq* (iii) $2x - 3y = 14$ sahsfy the above condition, but from these two equation the point (–3, 2) satisfies only eq (ii).

For $2x - 3y = 12$, substituting (3, – 2) gives:

$2(3) - 3(-2) = 6 + 6 = 12$

Thus, the second line is $2x - 3y = 12$.

5. Option (B) is correct

Explanation: Let the cost of one packet of popcorn be ₹x and one mango drink be ₹y

$2x + y = 330$ ...(i)

$x + 2y = 300$ ...(ii)

Multiply eq. (i) by 2, we get,

$4x + 2y = 660$ ...(iii)

subtract eq. (ii) from eq. (iii)

$3x = 360$

$\Rightarrow x =$ ₹ 120

6. Option (B) is correct

Explanation: Let the two digit number be $10x + y$, where x is the tens digit and y is the units digit

According to the given ratio

$10x + y : x + y = 7 : 1$

Simplifying the ratio, we get

$3x = 6y \Rightarrow \frac{x}{y} = \frac{6}{3} = \frac{2}{1}$

The possible pairs for x and y are:

$10x + y = 10 \times 2 + 1 = 21$

If $x = 4, y = 2 \Rightarrow 10 \times 4 + 2 = 42$

If $x = 6, y = 3 \Rightarrow 10 \times 6 + 3 = 63$

If $x = 8, y = 4 \Rightarrow 10 \times 8 + 4 = 84$

Hence, 4 two digit numbers are possible.

7. Option (A) is correct

Explanation: Let x = number of bags and y = number of baskets

Rohit sells bags at ₹ 500 each

and sells baskets at ₹ 150 each

Total earnings: ₹ 3550

Equation: $500x + 150y = 3550$

Aarav sells same number of bags and baskets as Rohit but bags at ₹ 400 each and baskets at ₹ 200 each. Total earnings: ₹ 3400

Equation: $400x + 200y = 3400$

8. Option (D) is correct

Explanation: To solve the given problem, let x = number of columns and y = number of saplings in each column

If saplings per column increase by 4, columns decrease by 1:

$$(x-1)(y+4) = xy$$

If saplings per column decrease by 5, columns increase by 2:

$$(x+2)(y-5) = xy$$

Because total number of saplings $= xy$

Since both expressions equal the total number of saplings, we equate them:

$$(x-1)(y+4) = (x+2)(y-5)$$

Expanding both sides:

LHS: $(x-1)(y+4) = xy$

$\Rightarrow xy + 4x - y - 4 = xy$

$\Rightarrow 4x - y - 4 = 0$ ...(i)

RHS: $(x+2)(y-5) = xy$

$\Rightarrow xy - 5x + 2y - 10 = xy$

$\Rightarrow -5x + 2y - 10 = 0$ ...(ii)

From eq. (i), $4x - y - 4 = 0$, we get $y = 4x - 4$

Substituting eq. (i) in eq. (ii)

$-5x + 2(4x - 4) - 10 = 0$

$-5x + 8x - 8 - 10 = 0$

$3x - 18 = 0 \Rightarrow x = 6$

Substituting $x = 6$ in $y = 4x - y$

$\Rightarrow y = 4(6) - 4 = 24 - 4 = 20$

Solution point is (6, 20)

Now checking the graphs, we're looking for the point (6, 20)— where the two lines intersect. In option (D), the lines intersect at (6,20)

9. Option (A) is correct

Explanation: We are given the graph of two lines:

$x + y = 5$

$x - y = 2$

The lines intersect at a single point. This means there is only one solution to the system of equations. This is called a unique solution.

ASSERTION-REASON QUESTIONS (1 Mark)

1. Option (D) is correct

Explanation: Let the cost of one pen = ₹ x and the cost of one pencil = ₹ y.

From the given:

$$5x + 3y = 34$$
$$2x + 4y = 28$$

Using substitution method, from the second equation:

$$2x + 4y = 28 \Rightarrow x = \frac{28}{2} - 4\frac{y}{2} = 14 - 2y$$

Now substitute this in the first equation:

$$5(14-2y) + 3y = 34$$
$$\Rightarrow 70 - 10y + 3y = 34$$
$$\Rightarrow 70 - 7y = 34$$
$$\Rightarrow -7y = 34 - 70 = -36 \Rightarrow y = \frac{36}{7} \neq 8$$

Now substitute $y = 8$ (as claimed in Assertion) into one of the original equations:

$$2x + 4(8) = 28$$
$$\Rightarrow 2x + 32 = 28$$
$$\Rightarrow 2x = -4$$
$$\Rightarrow x = -2$$

Assertion (A) is incorrect because the solution does not give ₹ 2 for a pen and ₹ 8 for a pencil.

Reason is true because the given assertion can be solved by substitution method.

2. Option (A) is correct

Explanation: To mix: 20% acid solution (volume = x) and 30% acid solution (volume = y)

To get a 25% acid solution,

Acid content in mixture: From 20% solution = $0.2x$

From 30% solution = $0.3y$

Total = $0.25(x+y)$

So the equation is: $0.2x + 0.3y = 0.25(x+y)$

So, Reason (R) is true.

On solving the equation:

$$0.2x + 0.3y = 0.25(x+y)$$
$$\Rightarrow 0.2x + 0.3y = 0.25x + 0.25y$$
$$\Rightarrow 0.2x - 0.25x + 0.3y - 0.25y = 0$$
$$\Rightarrow -0.05x + 0.05y = 0$$
$$\Rightarrow x = y$$

So, the volumes must be equal, *i.e.* $x : y = 1 : 1$

Assertion (A) is also true.

Both assertion and reason are true and reason correctly explains the assertion.

3. Option (C) is correct

Explanation: Assertion is true as linear equations are indeed useful for comparing such plans.

Reason is false because this is not true for all linear equations.

4. Option (C) is correct

Explanation: Assertion is true as total wages of a worker can be modelled by a linear equation in two variables. For example: If a worker earns a fixed amount ₹ 100 and ₹ 50 per hour, the total wage (W) is:

W = 100 + 50x, which is a linear equation in two variables (wages and hours).

Reason is false because x and y in a linear equation can be any real numbers (including fractions, negatives, or decimals).

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. Let the number of pencils be x and the number of erasers be y.

Cost of 1 pencil = ₹ 5

Cost of x pencils = $5x$

Cost of 1 eraser = ₹ 3

Cost of y erasers = $3y$

Total cost = ₹ 30

Linear equations in two variables:

$$5x + 3y = 30$$

2. Let x be number of minutes of local calls and y be number of minutes of STD calls

Cost of Local calls = ₹ 1.5 per minute

Total cost of Local cost = ₹ $1.5x$

Cost of STD calls = ₹ 2.5 per minute

Total cost of STD calls = ₹ $2.5y$

Total amount spent = ₹ 150

Linear equation:

$$1.5x + 2.5y = 150$$

3. Let x be the total cost per student and y be the cost for food per student

Given, Travel cost per student = ₹ 400

Food cost per student = ₹ 250, $y = 250$

Since total cost is the sum of travel and food costs:

$$x = 400 + y$$

This is the required linear equation in two variables.

From the equation, $x = 400 + 250 = 650$

Total cost for 50 students:

$$50 \times 650 = ₹\ 32,500$$

4. Let x be cost of one burger (in ₹) and y be cost of one cold drink (in ₹)

According to the question:

$$2x + y = 170$$

This is the required linear equation in two variables.

Choose any value of x, then solve for y.

Let's take $x = 70$

$$2(70) + y = 170$$
$$\Rightarrow 140 + y = 170$$
$$\Rightarrow y = 30$$

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. Let x be amount (in ₹) invested in Scheme A and y be amount (in ₹) invested in Scheme B

Form the equations:

Total investment: $x + y = 20000$ ...(i)

Total interest earned in a year:

$$\text{Interest from Scheme A} = \frac{8}{100} \times x = 0.08x$$

$$\text{Interest from Scheme B} = \frac{10}{100} \times y = 0.10y$$

So, $0.08x + 0.10y = 1800$ ...(ii)

Solving the pair of equations

From (i):

$$y = 20000 - x$$

Substitute the value of y in eq. (ii):

$$0.08x + 0.10(20000 - x) = 1800$$
$$0.08x + 2000 - 0.10x = 1800$$
$$-0.02x + 2000 = 1800$$
$$-0.02x = -200$$
$$x = \frac{200}{0.02} = 10000$$

Substitute the value of x in eq. (i)

$$x + y = 20000$$
$$10000 + y = 20000$$
$$y = 20000 - 10000$$
$$= 10000$$

Amount invested in Scheme A = ₹ 10,000

Amount invested in Scheme B = ₹ 10,000

2. Let the number of persons be x, and let the total cost for each package be represented as:

Package A cost: ₹ 2000 base fare + ₹ 500 per person

$$Cost_A = 2000 + 500x$$

Package B cost: ₹ 1500 base fare + ₹ 600 per person

$$Cost_B = 1500 + 600x$$

Form the equations

Let the total cost of each package be y

So, we get the pair of linear equations:

$$y = 2000 + 500x$$
$$y = 1500 + 600x$$

Equating the two costs to find when both packages cost the same

$$2000 + 500x = 1500 + 600x$$

Subtract 1500 from both sides:

$$500 + 500x = 600x$$

Now subtract $500x$ from both sides:

$$500 = 100x \Rightarrow x = 5$$

So, both packages cost the same for 5 persons.

Number of solutions

Since both equations are linear and intersect at a single point (*i.e.*, at $x = 5$), the system of equations has one unique solution.

3. Let's represent Ravi's and Neha's total savings after a certain number of months.

Let the number of months be m.

Ravi's savings after m months:

$$\text{Total savings} = 500 + mx = 2500$$

So, the linear equation becomes:

$$mx + 500 = 2500 \quad ...(i)$$

Neha's savings after m months:

$$\text{Total savings} = 1000 + my = 2500$$

So, the linear equation becomes:

$$my + 1000 = 2500 \quad ...(ii)$$

Now, the pair of linear equations is:

$$mx + 500 = 2500$$
$$my + 1000 = 2500$$

These simplify to:

$$mx = 2000$$
$$my = 1500$$

(i) Unique solution: The pair of equations will have a unique solution if the two equations are not equivalent and not parallel, *i.e.*, the ratio of coefficients of x and y is not equal:

$$\frac{m}{m} \neq \frac{2000}{1500}$$

$\Rightarrow$ $1 \neq \frac{4}{3}$, unique solution exists.

So, this system will have a unique solution.

(ii) Infinite solutions: The pair will have infinitely many solutions if both equations are identical (*i.e.*, they represent the same line):

This means: $mx = 2000$ and $my = 1500$

$$\Rightarrow \quad x = \frac{1500}{m}, y = \frac{2000}{1500}$$

If and only if $x = y$ and $2000 = 1500$, which is not true, so infinite solutions will occur only if Ravi and Neha had same initial savings and saved at same rate.

(iii) No solution: This occurs if the equations are parallel, *i.e.*, they have same coefficients but different constants.

That means: $m = m$ but $\frac{2000}{1500} \neq 1$

So if the left-hand sides are proportional, but the right-hand sides are not, the lines are parallel, *i.e.* no solution.

4. Let Price of one box of type A be ₹ x price of one box of type B be ₹ y

Day 1: $3x + 2y = 1800$ ...(i)

Day 2 : $6x + 4y = 3600$ ...(ii)

Simplifying eq. (ii)

Divide the entire equation by 2:

$$6x + 4y = 3600 \Rightarrow 3x + 2y = 1800$$

(Same as eq. (i))

Since both equations are exactly the same, the system represents the same line.

So, the system has infinitely many solutions.

Justification: Both equations represent the same line, so all points on that line satisfy both equations. This is a consistent and dependent system.

5. Let number of ₹ 200 notes be x and number of ₹ 100 notes be y

Shivani was returned 11 notes:

$$x + y = 11 \quad ...(i)$$

Total amount returned = ₹ 2000 – ₹ 500 = ₹ 1500

Total value of the notes:

$$200x + 100y = 1500 \quad ...(ii)$$

From Equation (i)

$$x + y = 11 \Rightarrow y = 11 - x$$

Substituting the value of y into eq. (ii)

$$200x + 100(11 - x) = 1500$$
$$200x + 1100 - 100x = 1500$$
$$100x = 400 \Rightarrow x = 4$$

Number of ₹ 200 notes = 4

Number of ₹ 100 notes = 11 – 4 = 7

CASE BASED QUESTIONS (4 Marks)

1. (i) Given, Hockey ₹ x per student and Cricket ₹ y per students

$\therefore$ Algebraic equations are

$$5x + 4y = 9500 \quad ...(i)$$

and

$$4x + 3y = 7370 \quad ...(ii)$$

(ii) (a) Multiply equation (i) by 3 and equation (ii) by 4

$$15x + 12y = 28500 \quad ...(iii)$$

$$16x + 12y = 29480 \quad ...(iv)$$

On subtracting equation (iii) from equation (iv), we get

$$x = 980$$

$\therefore$ Prize amount for hockey = ₹ 980

OR

(b) Now, put this value in equation (i), we get

$$5 \times 980 + 4y = 9500$$

$$\Rightarrow \quad 4y = 9500 - 4900 = 4600$$

$$\Rightarrow \quad y = 1150$$

$\therefore$ Prize amount for cricket = ₹ 1150

Difference = 1150 – 980 = ₹ 170

$\therefore$ Prize amount for cricket is ₹ 170 more than hockey.

(iii) Total prize amount for 2 students each from two games

$$= 2x + 2y$$

$$= 2(x + y)$$

$$= 2(980 + 1150)$$

$$= 2 \times 2130$$

$$= ₹\ 4260$$

2. (i) Let distance travelled be x km and total fare be ₹ y

Cab A: Fixed ₹ 50 + ₹ 10 per km

Fare $= 50 + 10x \Rightarrow y = 10x + 50$

Cab B: ₹ 12 per km only (no fixed charge)

Fare $= 12x \Rightarrow y = 12x$

(ii) If Sanjay travels 5 km:

Cab A : $y = 10(5) + 50 = 50 + 50 =$ ₹ 100

Cab B : $y = 12(5) =$ ₹ 60

Cab B is cheaper by ₹ 40.

(iii) (a) Distance at which both cabs charge the same fare:

Set the two equations equal:

$$10x + 50 = 12x$$

$$50 = 2x \Rightarrow x = 25$$

At 25 km, both cabs charge the same fare.

OR

(b) Graphical Interpretation:

The point of intersection of the lines $y = 10x + 50$ and $y = 12x$ represents:

The distance (x) and fare (y) where both cabs cost the same.

Point of intersection: (25, 300)

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. Let quantity of ₹ 60/kg groundnuts be x kg and quantity of ₹ 90/kg groundnuts be y kg

Total quantity condition: $x + y = 100$ ...(i)

Cost condition (no profit or loss)

Cost price of mixture = Selling price

Total cost = ₹ 72 × 100 = ₹ 7200

So, $60x + 90y = 7200$ ...(ii)

From eq. (i):

$$y = 100 - x \quad ...(iii)$$

Substitute the value of y in eq. (ii):

$$60x + 90(100-x) = 7200$$

$$60x + 9000-90x = 7200$$

$$-30x + 9000 = 7200$$

$$-30x = -1800$$

$$x = 60$$

Now from eq. (iii):

$$y = 100 - 60 = 40$$

The shopkeeper should mix 60 kg of ₹ 60/kg groundnuts and 40 kg of ₹ 90/kg groundnuts

This will give a 100 kg mixture costing ₹ 72/kg with no profit or loss.

2. Let amount invested in Scheme A be ₹ x and amount invested in Scheme B be ₹ y

Total Investment Condition: $x + y = 15000$ ...(i)

Interest Condition (Simple interest after 1 year)

Scheme A gives 10% interest

$$\Rightarrow \quad \text{Interest from A} = \frac{10}{100}x = 0.1x$$

Scheme B gives 12% interest

$$\Rightarrow \quad \text{Interest from B} = \frac{12}{100}y = 0.12y$$

Total interest = ₹ 1600

$$0.1x + 0.12y = 1600 \quad ...(ii)$$

From eq. (i): $y = 15000 - x$ (iii)

Substitute eq. (iii) in eq. (ii):

$$0.1x + 0.12(15000-x) = 1600$$

$$0.1x + 1800 - 0.12x = 1600$$

$$-0.02x + 1800 = 1600$$

$$-0.02x = -200$$

$$x = \frac{200}{-0.02} = 10000$$

Now from eq. (i):

$$y = 15000 - 10000$$

$$= 5000$$

Amount invested in Scheme A = ₹ 10,000

Amount invested in Scheme B = ₹ 5,000