4 CHAPTER

Quadratic Equations

Level - 1 CORE SUBJECTIVE QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Mark)

1. Option (A) is correct

Explanation: To find the nature of the roots, find the discriminant D from the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For the quadratic equation $x^2 + x - 1 = 0$, the coefficients are $a = 1$, $b = 1$ and $c = -1$

Discriminant (D) $= b^2 - 4ac$

$$D = 1^2 - 4 \times 1 \times (-1) = 1 + 4$$
$$= 5$$

Now, $x = \frac{-1+\sqrt{5}}{2}$ and $\frac{-1-\sqrt{5}}{2}$

Therefore, the roots are irrational and distinct.

2. Option (D) is correct

Explanation: The quadratic equation $x^2 + x + 1 = 0$ has not real roots.

This can be determined by using the discriminant formula $D = b^2 - 4ac$. For the given equation $x^2 + x + 1 = 0$, we have:

$$a = 1, b = 1 \text{ and } c = 1$$

Now, calculating the discriminant :

$$D = 1^2 - 4(1)(1) = 1 - 4 = -3$$

Since the discriminant is negative ($D<0$), the equation has two complex (not real) roots.

3. Option (B) is correct

Explanation:

Sum of the roots $= -\frac{b}{a} = -\frac{(-6)}{5} = \frac{6}{5}$

Product of the roots $= \frac{c}{a} = \frac{21}{5}$

Ratio $\frac{6}{5} : \frac{21}{5} = 6 : 21 = 2 : 7$

The ratio of the sum and product of the roots of the quadratic equation $5x^2 - 6x + 21 = 0$ is 2 : 7.

4. Option (C) is correct

$$D = b^2 - 4ac = 0, \text{ equal root}$$
$$b^2 - 4ac = 0$$
$$b^2 = 4ac$$
$$\Rightarrow \quad c = \frac{b^2}{4a}$$

5. Option (B) is correct

Explanation: The value of k, we substitute $x = 5$ into the quadratic equation:

$$2x^2 + (k - 1)x + 10 = 0$$

Substitute $x = 5$:

$$2(5)^2 + (k - 1)(5) + 10 = 0$$
$$2(25) + 5(k - 1) + 10 = 0$$
$$50 + 5k - 5 + 10 = 0$$
$$55 + 5k = 0$$
$$5k = -55$$
$$k = -11$$

6. Option (D) is correct

Explanation:

$$3x^2 = 6x$$
$$\Rightarrow \quad 3x^2 - 6x = 0$$
$$\Rightarrow \quad 3x(x - 2) = 0$$
$$\Rightarrow \quad 3x = 0, x - 2 = 0$$
$$\Rightarrow \quad x = 0 \text{ and } 2$$

7. Option (B) is correct

Explanation: The quadratic equation will have real root only when its discriminant will be greater than or equal to zero.

$$D \geq 0$$
$$b^2 - 4ac \geq 0$$
$$(-8)^2 - 4 \times 1 \times k \geq 0$$
$$64 - 4k \geq 0$$
$$4k \leq 64$$
$$k \leq 16$$

8. Option (B) is correct

Explanation: Given, $2x^2 + kx - 4 = 0$

Here $a = 2$, $b = k$ and $c = -4$

Discriminant, $D = b^2 - 4ac$

$$= (k)^2 - 4 \times 2 \times -4$$
$$= k^2 - (-32)$$
$$D = k^2 + 32$$

Also, as per quadratic formula

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

a and b are already rational numbers

So in order to have rational roots $\sqrt{D}$ must be a rational number i.e., D must be a perfect square.

$$D = k^2 + 32$$

At $\quad k = 2$

$$D = (2)^2 + 32 = 4 + 32 = 36$$

(a perfect squre)

$\therefore$ Least value of $k = 2$

9. Option (B) is correct

Explanation:

Option (A): $2x^2 - 4x + 8$

Here, $a = 2$ and $b = -4$.

Sum of roots $= -\frac{(-4)}{2} = \frac{4}{2} = 2$

Option (B): $-x^2 + 4x + 4 = 0$

Here, $a = -1$ and $b = 4$.

Sum of roots $= -\frac{4}{(-1)} = 4$

Option (C): $\sqrt{2}x^2 - \frac{4}{\sqrt{2}}x + 1 = 0$

Here, $a = \sqrt{2}$ and $b = -\frac{4}{\sqrt{2}}$

Sum of roots $= -\frac{-\frac{4}{\sqrt{2}}}{\sqrt{2}} = \frac{4}{\sqrt{2} \times \sqrt{2}} = \frac{4}{2}$

Option (D): $4x^2 - 4x + 4 = p$

Rearranging gives $4x^2 - 4x + (4 - p) = 0$.

Here, $a = 4$ and $b = -4$.

Sum of roots $= -\frac{-4}{4} = \frac{4}{4} = 1$

10. Option (A) is correct

Explanation:

$$x^2 + 3x - 10 = 0$$

$\Rightarrow \quad x^2 + 5x - 2x - 10 = 0$

$\Rightarrow \quad x(x + 5) - 2(x + 5) = 0$

$\Rightarrow \quad (x - 2)(x + 5) = 0$

$\therefore \quad x = 2$ or $x = -5$

11. Option (A) is correct

Explanation:

Roots of quadratic equation are $2 + \sqrt{3}$ and $2 - \sqrt{3}$

$\therefore$ Required quadratic equation is

$$\left(x - (2 + \sqrt{3})\right) + \left(x - (2 - \sqrt{3})\right) = 0$$

$\Rightarrow \quad \left((x - 2) - \sqrt{3}\right)\left((x - 2) + \sqrt{3}\right) = 0$

$\Rightarrow \quad (x - 2)^2 - (\sqrt{3})^2 = 0$

[$(a - b)(a + b) = a^2 - b^2$]

$\Rightarrow \quad x^2 - 4x + 4 - 3 = 0$

[$(a - b)^2 = a^2 - 2ab + b^2$]

$\Rightarrow \quad x^2 - 4x + 1 = 0$

12. Option (C) is correct

Explanation:

If roots are real and equal then

$$b^2 - 4ac = 0$$

$$b^2 = 4ac$$

$$\frac{b^2}{4} = ac$$

13. Option (B) is correct

Explanation:

Given equation $4x^2 + kx + 9 = 0$ is in the form of

$$ax^2 + bx + c = 0$$

where $a = 4$, $b = k$ and $c = 9$

For the equation to have real and equal roots, the condition is

$$D = b^2 - 4ac = 0$$

$\Rightarrow \quad k^2 - 4 \times 4 \times 9 = 0$

$\Rightarrow \quad k^2 - 144 = 0$

$\Rightarrow \quad k^2 = 144$

$\Rightarrow \quad k = \underline{+}12$

14. Option (A) is correct

Explanation: Given equation $2x^2 - 10x + k = 0$ is in the form $ax^2 + bx + c = 0$, where $a = 2$, $b = -10$ and $c = k$.

For the equation to have real and equal roots,

$$D = b^2 - 4ac = 0$$

$\Rightarrow \quad (-10)^2 - 4 \times 2 \times k = 0$

$\Rightarrow \quad 100 - 8k = 0$

$\Rightarrow \quad 8k = 100$

$\Rightarrow \quad k = \frac{25}{2}$

15. Option (C)is correct

Explanation: The given quadratic equation:

$$9x^2 - 6x - 2 = 0$$

Here, $a = 9$, $b = -6$ and $c = -2$

$\therefore$ Discriminant, $D = b^2 - 4ac$

$= (-6)^2 - 4(9)(-2)$

$= 36 - 36(-2)$

$= 36 - (-72)$

$= 36 + 72$

$= 108$

$\because$ $D = 108 > 0$, the roots are real and unequal.

Hence given quadratic equation has 2 distinct real roots is true.

16. Option (C) is correct

Explanation: For a quadratic equation to have equal roots, the discriminant must be zero, i.e., D = 0.

Check the discriminant for each equation:

(A) $\quad 3x^2 + 9x + 3 = 0$

$a = 3$, $b = 9$, $c = 3$

$D = 9^2 - 4(3)(3) = 81 - 36 = 45$

$D \neq 0$, so the roots are not equal.

(B) $x^2 - x + 1 = 0$

$a = 1, b = -1, c = 1$

$D = (-1)^2 - 4(1)(1) = 1 - 4 = -3$

$D \neq 0$, so the roots are not equal.

(C) $x^2 + 2x + 1 = 0$

$a = 1, b = 2, c = 1$

$D = 2^2 - 4(1)(1) = 4 - 4 = 0$

$D = 0$, so the roots are equal.

(D) $4x^2 + 8x - 4 = 0$

$a = 4, b = 8, c = -4$

$D = 8^2 - 4(4)(-4) = 64 + 64$

$= 128$

$D \neq 0$, so the roots are not equal.

$\therefore$ The correct answer is:

$x^2 + 2x + 1 = 0$

17. Option (A) is correct

Explanation: The quadratic equation $x^2 - 7x + 10 = 0$ can be factored by finding two numbers that multiply to give 10 (the constant term) and add to give -7 (the coefficient of x).

These numbers are -5 and -2, since:

$-5 \times -2 = 10$ and $-5 + (-2) = -7$

So, the quadratic equation can be factored as:

$(x - 5)(x - 2) = 0$

18. Option (C) is correct

Explanation: This can be written as:

$$z + \frac{1}{z} = 4$$

To eliminate the fraction, multiply both sides of the equation by z:

$$z^2 + 1 = 4z$$

19. Option (B) is correct

Explanation: To determine which equation has no real roots, we will examine the discriminant D for each quadratic equation. A quadratic equation has no real roots if the discriminant D is negative.

The discriminant D is given by:

$$D = b^2 - 4ac$$

Check each equation.

(i) $2x^2 - bx - b^2 = 0$

Coefficients: $a = 2, b = -b, c = -b^2$

$D = (-b)^2 - 4(2)(-b^2) = b^2 + 8b^2$

$= 9b^2 > 0$

(ii) $a^2x - ax + 2 = 0$

Coefficients: $a = a^2, b = -a, c = 2$

$D = (-a)^2 - 4(a^2)(2)$

$= a^2 - 8a^2 = -7a^2 < 0$

(iii) $x^2 + ax - b = 0$

Coefficients: $a = 1, b = a, c = -b$

$D = a^2 - 4(1)(-b) = a^2 + 4b$, may be + ve or – ve

20. Option (D) is correct

Explanation: The method that is not a way of solving a quadratic equation is:

Identifying the nature of the root.

ASSERTION-REASON QUESTIONS (1 Mark)

1. Option (B) is correct

Explanation: In case of assertion:

Given, -7 is a root of the quadratic equation

$z^2 - kz - 28 = 0$

$\Rightarrow (-7)^2 - k(-7) - 28 = 0$

$\Rightarrow 49 + 7k - 28 = 0$

$\Rightarrow 21 + 7k = 0$

$\Rightarrow k = -3$

Therefore, assertion is true.

In case of reason:

$z^2 + 3z - 28 = 0$

$\Rightarrow z^2 + 7z - 4z - 28 = 0$

$\Rightarrow z(z + 7) - 4(z + 7) = 0$

$\Rightarrow z = 4, z = -7$

Therefore, the other solution is 4.

But reason does not explain the value of k.

Hence, both assertion and reason are true but reason is not the correct explanation of assertion.

2. Option (B) is correct

Explanation: In case of assertion:

$h(t) = v(t) - 2t^2 + d$

Now according to the questions,

$0 = 10(t) - 2t^2 + 48$

$\Rightarrow 2t^2 - 10t - 48 = 0$

$\Rightarrow t^2 - 5t - 24 = 0$

$\Rightarrow t^2 - 8t + 3t - 24 = 0$

$\Rightarrow t(t - 8) + 3(t - 8) = 0$

$\Rightarrow (t - 8)(t + 3) = 0$

$\Rightarrow t = -3, 8$ seconds

$\therefore$ Time taken = 8 seconds ($\because$ Time cannot be negative)

Thus, assertion is true.

Reason is also true but it does not explain the assertion.

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. $2x^2 - 9x + 4 = 0$

$$\text{Sum of roots} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$= \frac{-(-9)}{2} = \frac{9}{2}$$

$$\text{Product of roots} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

$= \frac{4}{2} = 2$

2. $4x^2 - 5 = 0$

Here $a = 4$, $b = 0$ and $c = -5$

Discriminant (D) $= b^2 - 4ac$

$= 0 - 4 \times 4 \times -5$

$= 80$

$\because \quad 80 > 0$

Thus, $\quad D > 0$

$\therefore$ Roots are real and distinct.

3. Let the cost price of the article be x.

$\therefore$ Gain percent $x\%$

According to the condition,

$$₹\, x + \left(\frac{x}{100} \times x\right) = ₹\, 75$$

$\Rightarrow \quad x^2 + 100x = 7500$

$\Rightarrow \quad x^2 + 100x - 7500 = 0$

$\Rightarrow \quad x^2 + 150x - 50x - 7500 = 0$

$\Rightarrow \quad x(x + 150) - 50(x + 150) = 0$

$\Rightarrow \quad (x + 150)(x - 50) = 0$

$\Rightarrow \quad x = 50$, or $x = -150$

$\therefore \quad x = ₹\, 50$

($\because$ cost price cannot be negative)

4. $(m - 1)x^2 + 2(m - 1)x + 1 = 0$

For real and equal roots,

$b^2 - 4ac = 0$

$\Rightarrow \quad 4(m - 1)^2 - 4(m - 1) = 0$

$\Rightarrow \quad 4(m - 1)(m - 2) = 0$

$\Rightarrow \quad (m - 1)(m - 2) = 0$

$\Rightarrow \quad m = 1$ or $m = 2$

$\therefore \quad m = 2$ but $m \neq 1$

[If $m = 1$ then the given equation will not be quadratic equation]

5. $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

$\sqrt{3}x^2 + 7x + 3x + 7\sqrt{3} = 0$

$x(\sqrt{3}x + 7) + \sqrt{3}(\sqrt{3}x + 7) = 0$

$(x + \sqrt{3})(\sqrt{3}x + 7) = 0$

$x + \sqrt{3} = 0, \sqrt{3}x + 7 = 0$

$x = -\sqrt{3}, -\frac{7}{\sqrt{3}}$

6. To find the value of k for which the quadratic equation $kx^2 - 5x + k = 0$ has real roots, we will use the discriminant $D = b^2 - 4ac = 0$

According to question, a and c are both equal to k and b is equal to -5.

Substituting the values, we get

$(-5)^2 - 4k \times k = 0$

$25 - 4k^2 = 0$

$4k^2 = 25$

$$k = \pm\sqrt{\frac{25}{4}} = \pm\frac{5}{2}$$

7. Given,

$(x + 4) = 0$

$\Rightarrow \quad x = -4$

To find value of k in the equation $x^2 + kx + 8 = 0$ we substitute $x = -4$ into the equation:

$(-4)^2 + k(-4) + 8 = 0$

$16 + k(-4) + 8 = 0$

$16 - 4k + 8 = 0$

$24 - 4k = 0$

$k = 6$

Let α and β be two roots of given equation.

Thus, product of roots $= \alpha\beta = \frac{c}{a}$

$\Rightarrow \quad 4\beta = \frac{8}{1} \quad [\because$ Given $\alpha = 4]$

$\Rightarrow \quad \beta = \frac{8}{4} = 2$

Thus, other root = 2.

8. Let the number of standard toys be x.

The number of premium toys is then $16 - x$, since the customer buys a total of 16 toys.

The problem states that the product of the number of standard toys and premium toys is 28 . This gives the equation:

$x(16 - x) = 28$

$16x - x^2 = 28$

$x^2 - 16x + 28 = 0$

We now solve $x^2 - 16x + 28 = 0$ using the quadratic formula:

$$x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(1)(28)}}{2(1)}$$

$$x = \frac{16 \pm \sqrt{256 - 112}}{2}$$

$$x = \frac{16 \pm \sqrt{144}}{2}$$

$$x = \frac{16 + 12}{2} = 14 \text{ or } x = \frac{16 - 12}{2} = 2$$

The two possible solutions are $x = 14$ and $x = 2$.

$\therefore$ Standard toys are 14 and preminium toys are 2 or vice versa.

9. Let the two consecutive even numbers be x and $x + 2$.

The sum of their squares is given as:

$x^2 + (x + 2)^2 = 340$

$x^2 + (x^2 + 4x + 4) = 340$

$2x^2 + 4x + 4 = 340$

$2x^2 + 4x - 336 = 0$

$x^2 + 2x - 168 = 0$

$x^2 + 14x - 12x - 168 = 0$

$x(x + 14) - 12(x + 14) = 0$

$(x - 12)(x + 14) = 0$

$x - 12 = 0$ or $x + 14 = 0$

$x = 12$ or $x = -14$

Thus, the two consecutive even numbers are either 12 and 14 , or -14 and -12.

10. The given equation is:

$px^2 + 4x + 4 = 0$

For the equation to be a quadratic equation, the coefficient of x^2 (i.e., p) must be non – zero. If $p = 0$, the term involving x^2 disappears, and the equation reduces to:

$$4x + 4 = 0$$

The equation will not be a quadratic equation when $p = 0$, because a quadratic equation must have a non – zero coefficient for the x^2 term.

11. Given the quadratic polynomial:

$$2x^2 - 5x + m$$

The product of the zeroes of a quadratic polynomial $ax^2 + bx + c$ is given by:

$$\text{Product of the zeroes} = \frac{c}{a}$$

Here, $a = 2$, $b = -5$, and $c = m$. We are told that the product of the zeroes is 4 . So, we can set up the equation:

$$\frac{m}{2} = 4$$

$\therefore$ $m = 8$

12. The quadratic equation $6x^2 + 6 = 4kx$ can be written in standard form as

$$6x^2 - 4kx + 6 = 0$$

For roots to be real and equal,

$$D = 0$$

$\because$ Discriminant $D = b^2 - 4ac$

$\therefore$ $(-4k)^2 - 4 \times 6 \times 6 = 0$

$$16k^2 - 144 = 0$$

$$k^2 = \frac{144}{16}$$

$$k = \sqrt{9}$$

$$k = \pm 3$$

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. Let x be the digit at unit place and y be the digit at ten's place.

Hence, the number is $10y + x$

Given, $y = x + 5$ ...(i)

Product of two digits is 36

$\therefore$ $xy = 36$ ...(ii)

Substituting the value of y in eq. (ii), we get

$$x \times (x + 5) = 36$$
$$x^2 + 5x - 36 = 0$$
$$x^2 + 9x - 4x - 36 = 0$$
$$x(x + 9) - 4(x + 9) = 0$$

$\therefore$ $x = 4$ and -9

x cannot be negative

$\therefore$ $x = 4$

From eq. (i), we get

$$y = x + 5$$
$$= 4 + 5 = 9$$

$\therefore$ The number is $10y + x = 10 \times 9 + 4$
$= 94$

2. The given quadratic equation is:

$$px(x - 2) + 6 = 0$$

First, expand the equation:

$$p(x^2 - 2x) + 6 = 0$$
$$px^2 - 2px + 6 = 0$$

This is a quadratic equation of the form:

$$ax^2 + bx + c = 0$$

where $a = p$, $b = -2p$, and $c = 6$

For the equation to have two equal real roots, the discriminant must be zero. The discriminant (D) of a quadratic equation $ax^2 + bx + c = 0$ is given by:

$$D = b^2 - 4ac$$

Substitute the values of a, b, and c:

$$D = (-2p)^2 - 4(p)(6)$$
$$D = 4p^2 - 24p$$

For two equal real roots, $D = 0$:

$$4p^2 - 24p = 0$$

Factor the equation:

$$4p(p - 6) = 0$$

Thus, $p = 0$ or $p = 6$.

$p = 0$ is rejected;

$\therefore$ $p = 6$

3. Given quadratic equation is

$$p(x - 4)(x - 2) + (x - 1)^2 = 0$$

$\Rightarrow p(x^2 - 4x - 2x + 8) + (x^2 - 2x + 1) = 0$

$\Rightarrow$ $px^2 - 6px + 8p + x^2 - 2x + 1 = 0$

$\Rightarrow x^2(p + 1) - 2x(3p + 1) + (8p + 1) = 0$

Comparing the above equation with $ax^2 + bx + c = 0$ we get

$a = p + 1$, $b = -2(3p + 1)$, $c = 8p + 1$

For real and equal roots

$$D = 0$$

i.e., $b^2 - 4ac = 0$

$\therefore$ $[-2(3p + 1)]^2 - 4(p + 1)(8p + 1) = 0$

$\Rightarrow$ $4(3p + 1)^2 - 4(8p^2 + 9p + 1) = 0$

$\Rightarrow$ $4(9p^2 + 1 + 6p) - 32p^2 - 36p - 4 = 0$

$\Rightarrow$ $36p^2 + 4 + 24p - 32p^2 - 36p - 4 = 0$

$\Rightarrow$ $4p^2 - 12p = 0$

$\Rightarrow$ $4p(p - 3) = 0$

$\Rightarrow$ $p = 0$ or 3

4. Let the actual marks be x

$$7(x + 8) = x^2 - 4$$

$\Rightarrow$ $7x + 56 = x^2 - 4$

$\Rightarrow$ $x^2 - 7x - 60 = 0$

$\Rightarrow$ $x^2 - 12x + 5x - 60 = 0$

$\Rightarrow$ $x(x - 12) + 5(x - 12) = 0$

$\Rightarrow$ $(x - 12)(x + 5) = 0$

$\therefore$ $x = 12$ or -5

$\because$ Marks cannot be negative

$\therefore$ Aarush scored 12 marks in Mathematics

5. Given quadratic equation is

$$2kx^2 - 40x + 25 = 0$$

On comparing the given equation with

$ax^2 + bx + c = 0$ we get

$a = 2k$, $b = -40$, $c = 25$

For real and equal roots,

$$D = 0$$

i.e., $b^2 - 4ac = 0$

$\Rightarrow \quad (-40)^2 - 4 \times 2k \times 25 = 0$

$\Rightarrow \quad 1600 - 200k = 0$

$\Rightarrow \quad 200k = 1600$

$\Rightarrow \quad k = 8$

6. Given quadratic equation is:

$$x^2 + 2\sqrt{2}x - 6 = 0$$

$$\Rightarrow x^2 + 3\sqrt{2}x - \sqrt{2}x - 6 = 0$$

$$\Rightarrow x(x+3\sqrt{2}) - \sqrt{2}(x+3\sqrt{2}) = 0$$

$$\Rightarrow (x-\sqrt{2})(x+3\sqrt{2}) = 0$$

$x = \sqrt{2}$ and $-3\sqrt{2}$

7. Given quadratic equation is:

$$ky^2 - 11y + (k-23) = 0$$

Let the roots of the given equation be α and β

Now sum of roots

$$\alpha + \beta = \frac{-(-11)}{k} = \frac{11}{k} \quad ...(i)$$

and Product of roots

$$\alpha\beta = \frac{k-23}{k} \quad ...(ii)$$

According to question,

$$\alpha + \beta = \alpha\beta + \frac{13}{21}$$

$$\frac{11}{k} = \frac{k-23}{k} + \frac{13}{21}$$

(from eqns. (i) and (ii))

$$\Rightarrow \frac{11}{k} - \frac{(k-23)}{k} = \frac{13}{21}$$

$$\Rightarrow \frac{11-k+23}{k} = \frac{13}{21}$$

$\Rightarrow \quad 21(34-k) = 13k$

$\Rightarrow \quad 34k = 714$

$k = 21$

8. Given quadratic equations are:

$$ax^2 + x - 3a = 0 \quad ...(i)$$

$$x^2 + bx + b = 0 \quad ...(ii)$$

Since, given $x = -2$ is the common solution of the above quadratic equation

∴ from eq. (i)

$a(-2)^2 + (-2) - 3a = 0$

$\Rightarrow \quad 4a - 2 - 3a = 0$

$\Rightarrow \quad a = 2$

From eq. (ii),

$(-2)^2 + b(-2) + b = 0$

$\Rightarrow \quad 4 - 2b + b = 0$

$\Rightarrow \quad -b = -4$

$\Rightarrow \quad b = 4$

Now, $\quad a^2b = (2)^2 \times 4$

$= 4 \times 4 = 16$

9. $x^2 - 2ax + (a^2 - b^2) = 0$

$\Rightarrow \quad (x^2 - 2ax + a^2) - b^2 = 0$

$\Rightarrow \quad (x-a)^2 - b^2 = 0$

$\Rightarrow \quad (x-a+b)(x-a-b) = 0$

$\Rightarrow \quad x - a + b = 0$

or $\quad x - a - b = 0$

$\Rightarrow \quad x = -(-a+b)$

$\Rightarrow \quad x = -(-a-b)$ or $x = a - b$ or $a + b$

10. Let the greater number be x and smaller number be y.

$$\therefore \quad 2x - 16 = \frac{1}{2}y$$

$$\Rightarrow \quad 4x - y = 32 \quad ...(i)$$

and $$\frac{1}{2}x - 1 = \frac{1}{2}y$$

$$\Rightarrow \quad x - y = 2 \quad ...(ii)$$

Solving, we get $x = 10$ and $y = 8$

Hence the two numbers are 10 and 8.

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. Let the original speed be x km/h

New speed = $(x + 15)$ km/h

As, per the given information

$$\frac{90}{x} - \frac{90}{x+15} = \frac{1}{2}$$

$\Rightarrow \quad x^2 + 15x - 2700 = 0$

$\Rightarrow \quad (x+60)(x-45) = 0$

$\Rightarrow \quad x = -60, x = 45$

∵ Speed cannot be negative

∴ The original speed of the train = 45 km/h

2. Given equadratic equation is

$(c+1)x^2 - 6(c+1)x + 3(c+9) = 0$

Where $a = c + 1$

$b = -6(c+1)$

and $c = 3(c+9)$

Now, for roots to be equal and real

$D = 0$

$\Rightarrow \quad b^2 - 4ac = 0$

$\Rightarrow \quad [-6(c+1)]^2 - 4(c+1)\,3(c+9) = 0$

$\Rightarrow \quad 36(c+1)^2 - 12(c+1)(c+9) = 0$

$\Rightarrow \quad 12(c+1)[3(c+1) - (c+9)] = 0$

$\Rightarrow \quad 12(c+1)(3c+3-c-9) = 0$

$\Rightarrow \quad 12(c+1)(2c-6) = 0$

$\Rightarrow \quad (c+1)(2c-6) = 0$

$\Rightarrow \quad c = -1$ & $2c = 6$ or $c = 3$

As $c \neq -1$ so $c = 3$

3. Let the required no. be $10x + y$

Here $\quad xy = 18 \quad ...(i)$

$(10x + y) - 63 = 10y + x$

$9x - 9y = 63$

or $x - y = 7$ ....(ii)

$$x = 7 + y$$

Subsituting value of x is eqn.(i) we get

$$(7 + y)\, y = 18$$
$$7y + y^2 = 18$$
$$y^2 + 7y - 18 = 0$$
$$y^2 + 9y - 2y - 18 = 0$$
$$y(y + 9) - 2\,(y + 9) = 0$$
$$(y + 9)\,(y - 2) = 0$$
$$y = -9 \text{ and } 2$$

Number cannot be negative

$\therefore$ $y = 2$

Now, substitute value of y in eqn. (ii)

$$x - 2 = 7$$
$$x = 9$$

$x = 9$ and $y = 2$

Hence, required number is 92.

4. Let the original speed of the plane be x km/h.

If the speed of the plane is reduced by 100 km/h, then reduced speed of the train = $(x - 100)$ km/h

Time taken by the plane to reach its destination at original speed $t_1 = \frac{2800}{x}$ hr.

Given, Time taken by the plane to reach its destination at reduced speed – time taken by the plane to reach its destination at original speed = 30 minutes.

$$t_2 - t_1 = \frac{1}{2} \text{ hr}$$

$$\Rightarrow \quad \frac{2800}{x - 100} - \frac{2800}{x} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{2800x - 2800(x - 100)}{x(x - 100)} = \frac{1}{2}$$

$$\Rightarrow \quad (2800x - 2800x + 280000) \times 2 = x^2 - 100x$$

$$\Rightarrow \quad x^2 - 100x - 560000 = 0$$

$$\Rightarrow \quad x\,(x - 800) + 700\,(x - 800) = 0$$

$$\Rightarrow \quad (x - 800)\,(x + 700) = 0$$

$\therefore$ $x = 800$ or -700

$\therefore$ $x = 800$ [$\because$ speed cannot be negative]

$\therefore$ Original duration of the flight $= \frac{2800}{800} = \frac{7}{2}$

$$= 3\frac{1}{2} \text{hr}$$

or 3 hours 30 minutes.

5. Let the numerator be A.

Then by given 1st condition denominator = 2A + 1

Hence, the fraction is $\frac{A}{2A+1}$

and given 2nd condition:

$$\frac{A}{2A+1} + \frac{2A+1}{A} = 2\frac{16}{21}$$

$$\Rightarrow \quad \frac{A^2 + (2A+1)^2}{A(2A+1)} = \frac{58}{21}$$

$\therefore$ $21[A^2 + (4A^2 + 4A + 1) = 58A(2A + 1)$

$\therefore$ $105A^2 + 84A + 21 = 116A^2 + 58A$

$\therefore$ $11A^2 - 26A - 21 = 0$

$\therefore$ $11A^2 - 33A + 7A - 21 = 0$

$\therefore$ $11A(A - 3) + 7(A - 3) = 0$

$\therefore$ $(A - 3)\,(11A + 7) = 0$

$\therefore$ A = 3 and $\frac{-7}{11}$

Here A = $\frac{-7}{11}$ is rejected because it is negative

$\therefore$ A = 3

Hence, numerator is 3 and donominator = 2A +1

$= 3 \times 2 + 1 = 7$

$\therefore$ Fraction is $\frac{3}{7}$

6. Given equadratic equation is:

$$(k + 1)x^2 - 6(k + 1)x + 3(k + 9) = 0$$

where $a = k + 1$

$b = -6(k + 1)$

and $c = 3(k + 9)$

Now, for real and equal roots.

$D = b^2 - 4ac = 0$

$\Rightarrow [-6(k + 1)]^2 - 4(k + 1)\, 3(k + 9) = 0$

$\Rightarrow 36(k + 1)^2 - 12(k + 1)\,(k + 9) = 0$

$\Rightarrow 12(k + 1)\,[3(k + 1) - (k + 9)] = 0$

$\Rightarrow 12\,(k + 1)\,(2k - 6) = 0$

$\Rightarrow (k + 1)\,(2k - 6) = 0$

$\Rightarrow k + 1 = 0$ & $2k - 6 = 0$

$\Rightarrow k = -1$ & $2k = 6$

$k = 3$

As $k \neq -1$ so $k = 3$

7. Let the present age of the son $= x$ years then, the present age of the man $= 2x^2$ years.

8 years hence, the age of son will be $= (x + 8)$ years and the age of man $= (2x^2 + 8)$ years

According to the question,

$$2x^2 + 8 = 3(x + 8) + 4$$

$$\Rightarrow \quad 2x^2 + 8 = 3x + 24 + 4$$

$$\Rightarrow \quad 2x^2 - 3x - 24 - 4 + 8 = 0$$

$$\Rightarrow \quad 2x^2 - 3x - 20 = 0$$

$$\Rightarrow \quad 2x^2 - 8x + 5x - 20 = 0$$

$$\Rightarrow \quad 2x(x - 4) + 5(x - 4) = 0$$

$$\Rightarrow \quad (x - 4)\,(2x + 5) = 0$$

Either $x - 4 = 0$

then $x = 4$

$2x + 5 = 0$

or $x = \frac{-5}{2}$

(But, it is not possible)

Present age of the son = 4 years and present age of the man = $2x^2 = 2(4)^2 = 32$ years .

8. Let the speed of the stream be x km/h

Speed of the boat upstream = Speed of boat in still water – Speed of the stream

$\therefore$ Speed of the boat upstream = $(18 - x)$ km/h

Speed of the boat downstream = Speed of the boat still water + Speed of the stream

$\therefore$ Speed of the boat downstream = $(18 + x)$ km/h

Time of upstream journey = Time for downstream journey + 1 hr

$$\therefore \frac{\text{Distance covered upstream}}{\text{Speed of the boat upstream}} - \frac{\text{Distance covered downstream}}{\text{Speed of the boat downstream}} = 1$$

$$\Rightarrow \frac{24}{18-x} - \frac{24}{18+x} = 1$$

$$\Rightarrow \frac{432+24x-432+24x}{(18-x)(18+x)}$$

$$\Rightarrow 48x = 324 - x^2$$

$$\Rightarrow x^2 + 48x - 324 = 0$$

$$x^2 + 54x - 6x - 324 = 0$$

$$\Rightarrow x(x + 54) - 6(x + 54) = 0$$

$$\Rightarrow (x + 54)(x - 6) = 0$$

$$\Rightarrow x + 54 = 0 \text{ or } x - 6 = 0$$

$$\Rightarrow x = -54 \text{ or } x = 6$$

$\therefore x = 6$ ($\because$ Speed of the stream cannot be negative)

$\therefore$ Speed of the stream is 6 km/h.

9. Let one of the number be x then other number is $(18 - x)$.

Then according to question,

$$\frac{1}{x} + \frac{1}{18-x} = \frac{1}{4}$$

$$\Rightarrow \frac{18-x+x}{x(18-x)} = \frac{1}{4}$$

$$\Rightarrow 18 \times 4 = 18x - x^2$$

$$\Rightarrow 72 = 18x - x^2$$

$$\Rightarrow x^2 - 18x + 72 = 0$$

$$\Rightarrow x^2 - 12x - 6x + 72 = 0$$

$$\Rightarrow x(x - 12) - 6(x - 12) = 0$$

$$\Rightarrow (x - 6)(x - 12) = 0$$

$$\Rightarrow x - 6 = 0$$

$$\Rightarrow x = 6$$

Or

$$\Rightarrow x - 12 = 0$$

$$\Rightarrow x = 12$$

Since, x being a number,

Therefore,

When $x = 12$ then another number will be

$$18 - x = 18 - 12 = 6$$

And when $x = 6$ then another number will be

$$18 - x = 18 - 6 = 12$$

Thus, the two numbers are 6 and 12.

10. $$\frac{4}{x} - \frac{5}{2x+3} = 3$$

Multiply by the common denominator $x(2x + 3)$:

$$4(2x + 3) - 5x = 3x(2x + 3)$$

$$8x + 12 - 5x = 6x^2 + 9x$$

$$0 = 6x^2 + 6x - 12$$

$$0 = x^2 + x - 2$$

$$0 = (x + 2)(x - 1)$$

$$x = -2 \text{ and } x = 1$$

The solutions are $x = -2$ and $x = 1$.

11. Given, the quadratic equation $3x^2 + 14x + p = 0$ with –3 as a root:

Substituting $x = -3$:

$$3(-3)^2 + 14(-3) + p = 0$$

$$\Rightarrow 27 - 42 + p = 0$$

$$\Rightarrow p = 15$$

For the equation $x^2 + k(4x + k - 4) + 15 = 0$

$$x^2 + (4k)x + (k^2 - 4k + 15) = 0$$

For roots to be equal

$$D = 0$$

$$\Rightarrow b^2 - 4ac = 0$$

$$\Rightarrow (4k)^2 - 4(1)(k^2 - 4k + 15) = 0$$

$$\Rightarrow 16k^2 - 4(k^2 - 4k + 15) = 0$$

$$12k^2 + 16k - 60 = 0$$

$$\Rightarrow 3k^2 + 4k - 15 = 0$$

$$k = \frac{-4 \pm \sqrt{16+180}}{6}$$

$$= \frac{-4 \pm 14}{6}$$

$$k = \frac{10}{6} = \frac{5}{3}$$

or, $$k = \frac{-18}{6} = -3$$

12. Let the three consecutive natural numbers be x, $x + 1$ and $x + 2$

According to the given condition

$$\therefore (x + 1)^2 - [(x + 2)^2 - x^2] = 60$$

$$(x^2+1+2x)-(x^2+4+4x-x^2) = 60$$

$$\Rightarrow x^2+1+2x-4-4x = 60$$

$$\Rightarrow x^2 - 2x - 63 = 0$$

$$\Rightarrow x^2 - 9x + 7x - 63 = 0$$

$$x(x - 9) + 7(x - 9) = 0$$

$$(x + 7)(x - 9) = 0$$

$$\therefore x = 9 \text{ or } -7$$

$$\therefore x = 9 \text{ (neglect } x = -7)$$

$\therefore$ Numbers are 9, 10, 11

13. Distance = 54 km

Let the average speed be x km/h

Distance = 63 km

The average speed to cover this distance = $(x + 6)$ km/h

$$\frac{D}{S} = t$$

Therefore, $$\frac{54}{x} + \frac{63}{x+6} = 3$$

$$54(x + 6) + 63x = 3x(x + 6)$$
$$54x + 324 + 63x = 3x^2 + 18x$$
$$3x^2 - 99x - 324 = 0$$
$$x^2 - 33x - 108 = 0$$
$$x^2 - 36x + 3x - 108 = 0$$
$$x(x - 36) + 3(x - 36) = 0$$

Thus, $x = -3$ and 36.

As speed cannot be negative.

Hence, $x = 36$ km/h

∴ The first average speed of the train is 36 km/h

14. Let the smaller pipe takes x hours to fill tank

Then, the larger one takes $(x - 2)$ hours to fill the tank

Tank filled in 1 hour by smaller pipe = $\frac{1}{x}$

Tank filled in 1 hour by larger pipe = $\frac{1}{x-2}$

Tank filled in 1 hours by both the pipes = $\frac{8}{15}$

$$\frac{1}{x} + \frac{1}{x-2} = \frac{1}{\frac{15}{8}} = \frac{8}{15}$$

$$\frac{1}{x} + \frac{1}{x-2} = \frac{8}{15}$$

$$\frac{(x-2)+x}{x(x-2)} = \frac{8}{15}$$

$$\frac{2x-2}{x(x-2)} = \frac{8}{15}$$

$$15(2x - 2) = 8x(x - 2)$$
$$30x - 30 = 8x^2 - 16x$$
$$8x^2 - 46x + 30 = 0$$

Using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $a = 8$, $b = -46$, and $c = 30$:

$$b^2 - 4ac = (-46)^2 - 4\times8\times30$$
$$= 2116 - 960 = 1156$$

Calculating $\sqrt{1156} = 34$

$$x = \frac{46 \pm 34}{16}$$

$$x = \frac{80}{16} = 5 \text{ (valid)}$$

$$x = \frac{12}{16} = \frac{3}{4} \text{ (not valid)}$$

The smaller pipe takes 5 hours to fill the tank

For the larger pipe:

$$x - 2 = 5 - 2 = 3 \text{ hours}$$

Level - 2 ADVANCED COMPETENCY FOCUSED QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Mark)

1. Option (A) is correct

Explanation: Original share per person = $\frac{4000}{x}$

New share per person (with 10 more people) = $\frac{4000}{x+10}$

According to the question,

$$\frac{4000}{x} - \frac{4000}{x+10} = 80$$

Multiply both sides by $x(x+10)$ to eliminate denominators:

$$4000(x + 10) - 4000x = 80x(x + 10)$$
$$4000x + 40000 - 4000x = 80x^2 + 800x$$
$$40000 = 80x^2 + 800x$$

Divide the entire equation by 8 to simplify:

$$5000 = 10x^2 + 100x$$
$$\Rightarrow 10x^2 + 100x - 5000 = 0$$
$$\Rightarrow x^2 + 10x - 500 = 0$$

2. Option (B) is correct

Explanation: Given, $x^2 - 9x + 18 + k = 0$

Simplifying: $x^2 - 9x + (18 + k) = 0$

We want the roots to be factored easily, i.e., we want two integers whose product is $18 + k$ and whose sum is 9.

So we are looking for two integers a and b such that:

$$a + b = 9$$
$$ab = 18 + k$$

If we take, $a = 4, b = 5$

$$a + b = 4 + 5 = 9$$
$$ab = 4 \times 5 = 20$$
$$\Rightarrow k = 2$$

So, the smallest positive k such that it can be factored is 2

3. Option (B) is correct

Explanation: Given the quadratic equation:

$$3x^2 - 10x - 8 = 0$$

To split the middle term, we need two numbers m and n such that:

$$m + n = -10$$

(sum equals the middle coefficient), and

$$m.n = 3.(-8) = -24$$

(product equals the product of the first and last coefficients).

We want two numbers whose Sum = –10 and Product = –24

Check factor pairs of –24: $-12 + 2 = -10$

So: $m = -12$, $n = 2$ (or vice versa)

4. Option (B) is correct

Explanation: Given Equation: $3x^2 - 11x - 20 = 0$

Step 1: $3x^2 - 11x - 20 = 0$

We need to split the middle term into two numbers that:

Add to –11 and multiply to $3 \times (-20) = -60$

Look for two numbers whose sum = –11 and product = –60

These numbers are –15 and +4

Step 2: $3x^2 - 15x + 4x - 20 = 0$

This is correct splitting. So Step 2 is correct.

Step 3: $3(x-5) + 4(x-5) = 0$

Let's factor the expression from Step 2 correctly:

$$(3x^2 - 15x) + (4x - 20) = 3x(x-5) + 4(x-5)$$

This gives: $3x(x-5) + 4(x-5)$

Now factor: $(3x + 4)(x - 5) = 0$

But Vijender wrote in Step 3:

$$3(x-5) + 4(x-5) = 0$$

This is incorrect because it doesn't follow from grouping correctly.

So the first error is in Step 3.

5. Option (C) is correct

Explanation: Given equation:

$$\frac{x}{x+1} + \frac{x+1}{x} = \frac{34}{15}$$

Combining the left hand side:

$$\frac{x}{x+1} + \frac{x+1}{x}$$

Take LCM of $x(x + 1)$:

$$= \frac{x^2 + (x+1)^2}{x(x+1)} = \frac{x^2 + x^2 + 2x + 1}{x(x+1)}$$

$$= \frac{2x^2 + 2x + 1}{x(x+1)}$$

Now equate to the RHS:

$$\frac{2x^2 + 2x + 1}{x(x+1)} = \frac{34}{15}$$

On cross multiplying:

$$15(2x^2 + 2x + 1) = 34x(x + 1)$$

Expanding both sides:

LHS: $30x^2 + 30x + 15$

RHS: $34x^2 + 34x$

Now equate:

$$30x^2 + 30x + 15 = 34x^2 + 34x$$

Bringing all terms to one side:

$$30x^2 + 30x + 15 - 34x^2 - 34x = 0$$

$$-4x^2 - 4x + 15 = 0$$

$$4x^2 + 4x - 15 = 0$$

Using quadratic formula:

$$x = \frac{-4 \pm \sqrt{4^2 - 4(4)(-15)}}{2(4)}$$

$$= \frac{-4 \pm \sqrt{16 + 240}}{8}$$

$$= \frac{-4 \pm \sqrt{256}}{8} = \frac{-4 \pm 16}{8}$$

So,

$$x = \frac{-4 + 16}{8} = \frac{12}{8} = \frac{3}{2}$$

and

$$x = \frac{-4 - 16}{8} = \frac{-20}{8} = \frac{-5}{2}$$

Both Ravi and Ankit are correct.

6. Option (A) is correct

Explanation: To verify whether $x = 2$ and $x = 3$ are roots of the quadratic equation:

$$x^2 - 5x + 6 = 0$$

Substitute $x = 2$ into the left-hand side:

$$2^2 - 5(2) + 6 = 4 - 10 + 6 = 0$$

Substitute $x = 3$:

$$3^2 - 5(3) + 6 = 9 - 15 + 6 = 0$$

Since both give 0, they are roots of the equation.

ASSERTION-REASON QUESTIONS (1 Mark)

1. Option (A) is correct

Explanation: Assertion is true. Let's denote the width as x m.

Then the length = $x + 7$ m.

Area = $x(x + 7) = 180$, which gives a quadratic equation.

Reason is also true. Expanding $x(x + 7) = 180$ gives $x^2 + 7x - 180 = 0$, which is indeed a quadratic equation.

Both assertion and reason are true and reason correctly explains the assertion.

2. Option (A) is correct

Explanation: Assertion is true. Let the speed for the second part (80 km) be x km/h.

Then the speed for the first part (60 km) = $x - 20$ km/h.

$$\text{Using time} = \frac{\text{distance}}{\text{speed}}$$

$$\text{Time for 60 km} = \frac{60}{x-20}$$

$$\text{Time for 80 km} = \frac{80}{x}$$

Total time = 4 hours:

$$\frac{60}{x-20} + \frac{80}{x} = 4$$

This is clearly a rational equation, which becomes quadratic when simplified.

Reason is also true as using rational expressions for time and setting total time leads to a quadratic equation when simplified.

Both assertion and reason are true and reason is the correct explanation of assertion.

3. Option (D) is correct

Explanation: Let one share be ₹ x, so the other is ₹ $(1000 - x)$.

Given: $x(1000 - x) = 240000 \Rightarrow x^2 - 1000x + 240000 = 0$

This is a quadratic equation, not linear. So, assertion is false.

Reason is true because the condition $x(1000 - x) = 240000$ leads to a quadratic equation.

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. Let speed of boat in still water be x km/h

$$\text{Downstream speed} = x + 2 \text{ km/h}$$

$$\text{Upstream speed} = x - 2 \text{ km/h}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Time to row 8 km downstream: $\frac{8}{x+2}$

Time to row 6 km upstream: $\frac{6}{x-2}$

Total time = 4 hours:

$$\frac{8}{x+2} + \frac{6}{x-2} = 4$$

Multiply through by $(x + 2)(x - 2)$ to eliminate denominators:

$$8(x - 2) + 6(x + 2) = 4(x^2 - 4)$$

Simplifying both sides:

So, $14x - 4 = 4x^2 - 16$

$$4x^2 - 14x - 12 = 0$$

$$2x^2 - 7x - 6 = 0$$

Using the quadratic formula:

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-6)}}{2(2)}$$

$$= \frac{7 \pm \sqrt{49 + 48}}{4} = \frac{7 \pm \sqrt{97}}{4}$$

$$\sqrt{97} \approx 9.85$$

$$x \approx \frac{7 + 9.85}{4} = \frac{16.85}{4} = 4.21$$

or $x = \frac{7 - 9.85}{4} = \frac{-2.85}{4} \approx -0.71$

(reject, speed cannot be negative)

The speed of the boat in still water is approximately 4.21 km/h

2. Let the original side be x meters

Then the new side becomes $x + 3$ meters.

So, the new area is $(x + 3)^2 = 196$

$$(x + 3)^2 = 196$$

$$\Rightarrow x^2 + 6x + 9 = 196$$

$$\Rightarrow x^2 + 6x - 187 = 0$$

Using the quadratic formula:

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-187)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{36 + 748}}{2}$$

$$= \frac{-6 \pm \sqrt{784}}{2} = \frac{-6 \pm 28}{2}$$

So,

- $x = \frac{-6 + 28}{2} = \frac{22}{2} = 11$
- $x = \frac{-6 - 28}{2} = \frac{-34}{2} = -17$

(reject, length cannot be negative)

The original side length of the garden is 11 meters.

3. Let the first integer be x

Then the next consecutive integer is $x + 1$

So their product is $x(x + 1) = 182$

$$x^2 + x = 182$$

$$\Rightarrow x^2 + x - 182 = 0$$

Using the quadratic formula:

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-182)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1 + 728}}{2}$$

$$= \frac{-1 \pm \sqrt{729}}{2} = \frac{-1 \pm 27}{2}$$

So,

$$x = \frac{-1 + 27}{2} = \frac{26}{2} = 13$$

$$x = \frac{-1 - 27}{2} = \frac{-28}{2} = -14$$

(reject, only positive integers allowed)

The two consecutive positive integers are 13 and 14.

4. Let the increase in price be ₹ x

New price per ticket = ₹ $100 + x$

New number of tickets sold = $50 - x$

So, new revenue = $(100 + x)(50 - x)$

We want this new revenue to be equal to the original revenue of ₹ 5000:

$$(100 + x)(50 - x) = 5000$$

Forming the quadratic equation

$$(100 + x)(50 - x) = 5000$$

$$\Rightarrow 5000 - 100x + 50x - x^2 = 5000$$

$$\Rightarrow -50x - x^2 + 5000 = 5000$$

$$-x^2 - 50x = 0$$

$$x^2 + 50x = 0$$
$$\Rightarrow \quad x(x + 50) = 0$$
$$x = 0 \text{ or } x = -50$$

Since price increase x must be non-negative, we reject

So, $x = -50$

$x = 0$

This means no price increase will result in the same revenue. So, any increase in price will reduce revenue below ₹ 5000.

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. Let the breadth be x meters

Then the length $= x + 7$ meters

Area of a rectangle = Length × Breadth:

$$x(x + 7) = 330$$
$$x^2 + 7x = 330$$
$$\Rightarrow \quad x^2 + 7x - 330 = 0$$

Using the quadratic formula:

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(-330)}}{2(1)}$$
$$= \frac{-7 \pm \sqrt{49 + 1320}}{2}$$
$$= \frac{-7 \pm \sqrt{1369}}{2} = \frac{-7 \pm 37}{2}$$

So:

- $x = \frac{-7 + 37}{2} = \frac{30}{2} = 15$
- $x = \frac{-7 - 37}{2} = \frac{-44}{2} = -22$

(reject, breadth can't be negative)

Breadth $= x = 15$ meters

Length $= x + 7 = 22$ meters

2. Let the amount invested at 10% be ₹ $100 \times x$

So, amount invested at 12% = ₹ $100 \times (300 - x)$, because total is ₹ 30,000 = 100×300

Interest from: 10% investment $= \frac{10}{100} \times 100x = 10x$

12% investment $= \frac{12}{100} \times 100(300 - x)$

$= 12(300 - x)$

Total interest: $10x + 12(300 - x) = 3400$

$$10x + 3600 - 12x = 3400$$
$$\Rightarrow \quad -2x + 3600 = 3400$$
$$\Rightarrow \quad -2x = -200$$
$$\Rightarrow \quad x = 100$$

₹ $100 \times x = 100x \rightarrow$ ₹ 10,000 invested at 10%

₹ 30,000 – ₹ 10,000 = ₹ 20,000 invested at 12%

3. Let the number of ₹ 10 increase in ticket price be x.

Then, new price per ticket = ₹ $100 + 10x$

Number of people attending $= 300 - 10x$

Revenue = (price) × (number of people)

$= (100 + 10x)(300 - 10x)$

$R(x) = (100 + 10x)(300 - 10x)$

$R(x) = 100 \times 300 - 100 \times 10x + 10x \times 300 - 10x \times 10x$

$= 30{,}000 + 2000x - 100x^2$

$R(x) = -100x^2 + 2000x + 30{,}000$

This is a quadratic equation in x, representing revenue.

Maximise revenue

Since the coefficient of x^2 is negative, the parabola opens downward, and the maximum occurs at the vertex:

$$x = \frac{-b}{2a} = \frac{-2000}{2(-100)}$$
$$= \frac{-2000}{-200} = 10$$

Ticket price $= 100 + 10 \times 10$

= ₹ 200

People attending $= 300 - 10 \times 10$

$= 200$

Maximum revenue: ₹ 200 × 200 = ₹ 40,000

4. Let the width of the path be x meters

Then, new length including the path $= 30 + 2x$

New breadth including the path $= 20 + 2x$

Total area $= (30 + 2x)(20 + 2x)$

Area of garden $= 30 \times 20 = 600 \text{ m}^2$

Area of path = Total area – Garden area

$= (30 + 2x)(20 + 2x) - 600$

Given, $(30 + 2x)(20 + 2x) - 600 = 264$

$(30 + 2x)(20 + 2x) = 864$

Expanding LHS:

$$(30 + 2x)(20 + 2x) = 600 + 60x + 40x + 4x^2$$
$$= 4x^2 + 100x + 600$$

Set equal to 864:

$$4x^2 + 100x + 600 = 864$$
$$\Rightarrow \quad 4x^2 + 100x + 600 - 864 = 0$$
$$\Rightarrow \quad 4x^2 + 100x - 264 = 0$$
$$\Rightarrow \quad x^2 + 25x - 66 = 0$$

Using the quadratic formula:

$$x = \frac{-25 \pm \sqrt{25^2 - 4(1)(-66)}}{2(1)}$$
$$= \frac{-25 \pm \sqrt{625 + 264}}{2}$$
$$= \frac{-25 \pm \sqrt{889}}{2}$$
$$\sqrt{889} \approx 29.83$$
$$x = \frac{-25 + 29.83}{2}$$
$$\approx \frac{4.83}{2} \approx 2.42$$

(Reject the negative root)

Width of the path is approximately 2.42 meters.

CASE BASED QUESTIONS (4 Marks)

1. Original no. of tiles = 200.

Original length of 1 tile = x

(i) $200x^2 = 128(x+1)^2$

(ii) $200x^2 = 128(x^2+1+2x)$

$\Rightarrow 200x^2 = 128x^2 + 128 + 256x.$

$\Rightarrow 72x^2 - 256x - 128 = 0$

$\Rightarrow 9x^2 - 32x - 16 = 0$

(iii) (a) $9x^2 - 32x - 16 = 0$

$9x^2 - 36x + 4x - 16 = 0$

$9x(x-4) + 4(x-4) = 0$

$(x-4)(9x+4) = 0 \Rightarrow x = 4, -\frac{4}{9}$

$\therefore \quad x = 4$

($\because$ length cannot be negative)

OR

(b) $$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-32 \pm \sqrt{(-32)^2 - 4\times 9\times 16}}{2\times 9}$$

$$= \frac{32 \pm \sqrt{1024+576}}{2\times 9}$$

$$= \frac{32\pm 40}{18} = \frac{72}{18}, \frac{-8}{18}$$

$$= 4, -\frac{4}{9}$$

$x = 4$ ($\because x$ cannot be negative)

2. (i) Revenue = (Price per ticket) × (Number of tickets sold)

$R(x) = x.(100 - 2(x-10))$

So, $R(x) = x(120 - 2x) = 120x - 2x^2$

$R(x) = -2x^2 + 120x$

(ii) This is a quadratic equation of the form:

$R(x) = -2x^2 + 120x$

Maximum occurs at:

$$x = \frac{-b}{2a} = \frac{-120}{2(-2)} = \frac{-120}{-4} = 30$$

The ticket price for maximum revenue is ₹ 30

(iii) (a) Substitute $x = 30$ into the revenue equation:

$R(30) = -2(30)^2 + 120(30)$

$= -1800 + 3600$

$=$ ₹ 1800

Maximum revenue is ₹ 1800

OR

(b) Number of tickets sold $= 100 - 2(x-10)$

$= 120 - 2x$

Set this to 0:

$120 - 2x = 0$

$\Rightarrow 2x = 120$

$\Rightarrow x = 60$

No tickets would be sold if the price is ₹ 60.

3. (i) Let breadth be x meters, then length $= x + 5$ meters

Perimeter of a rectangle is

2(length + breadth) = 80

Substitute the expressions: $2(x + x + 5) = 80$

$\Rightarrow 2(2x+5) = 80$

(Which is the expression for perimeter)

$\Rightarrow 4x + 10 = 80$

$\Rightarrow 4x = 70$

$\Rightarrow x = 17.5$ m

So the breadth = 17.5 m, and the length = 22.5 m

Area in terms of x:

$A(x) = x(x+5) = x^2 + 5x,$

is the required question equation

(ii) Breadth = 17.5 m and Length = 17.5 + 5 = 22.5 m

(iii) (a) Maximum area that can be enclosed

Area $= x(x+5) = 17.5 \times 22.5$

$= 393.75 \text{ m}^2$

OR

(b) We already formed:

$A(x) = x^2 + 5x$

To verify if the roots are real, set $A(x) = 0$

$x^2 + 5x = 0$

$\Rightarrow x(x+5) = 0$

$\Rightarrow x = 0$

or $x = -5$

Both are real roots.

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. (i) Let flat road speed $= x$ km/h

Then uphill speed $= x - 10$ km/h

Time for 40 km flat road $= \frac{40}{x}$

Time for 25 km uphill $= \frac{25}{x-10}$

Now, we are told that if Vitthal were going at flat road speed on the slope too, the time would have been shorter by 0.5 hr.

So, $$\frac{25}{x-10} = \frac{25}{x} + 0.5$$

$$\frac{25}{x-10} - \frac{25}{x} = 0.5$$

$$25\left(\frac{1}{x-10} - \frac{1}{x}\right) = 0.5$$

$$25\left(\frac{x-(x-10)}{x(x-10)}\right) = 0.5$$

$$\Rightarrow \quad 25.\frac{10}{x(x-10)} = 0.5$$

$$\frac{250}{x(x-10)} = 0.5$$

$$\Rightarrow \quad 500 = x(x-100)$$

$$\Rightarrow \quad x^2 - 10x - 500 = 0$$

Using quadratic formula:

$$x = \frac{10 \pm \sqrt{(-10)^2 - 4(1)-(500)}}{2}$$

$$= \frac{10 \pm \sqrt{100+2000}}{2}$$

$$= \frac{10 \pm \sqrt{2100}}{2}$$

$$\sqrt{2100} \approx 45.83$$

$$\Rightarrow \quad x \approx \frac{10+45.83}{2} = \frac{55.83}{2}$$

$$\approx 27.92 \text{ km/h}$$

[We leave – *ve* value of x as speed can't be –*ve*]

So, flat road speed = 27.92 km/h

Uphill speed = 17.92 km/h

Time taken to reach the hill:

$$\text{Time} = \frac{40}{27.92} + \frac{25}{17.92}$$

$$\approx 1.43 + 1.395$$

$$\approx 2.825 \text{ hrs } (\approx 2 \text{ hrs } 50 \text{ min})$$

(ii) Let downhill speed be y km/h

$$\frac{40}{27.92} + \frac{25}{y} = 2.5 \Rightarrow 1.43 + \frac{25}{y} = 2.5$$

$$\Rightarrow \quad \frac{25}{y} = 2.5 - 1.43 = 1.07$$

$$\Rightarrow \quad y = \frac{25}{1.07} = 23.36 \text{ km/h}$$

2. (i) Let the distance between gates 1 and 3 be 'x' m.

Then, the distance between gates 2 and 3 is $(x + 1)$, m.

Applying Pythagoras theorem to the triangle formed by gates 1, 2 and 3 and frames a quadratic equation as:

$$x^2 + (x+1)^2 = 29^2$$

or $$x^2 + x - 420 = 0$$

$$x^2 + 21x - 20x - 420 = 0$$

$$x(x+21) - 20(x+21) = 0$$

$$(x+21)(x-20) = 0$$

$$x = -21$$

and $$x = 20$$

(since the width cannot be negative)

The width of the park is 20 meters.

(ii) Let the former number of rows and columns of saplings be x

Then the number of saplings = $x^2 + 24$

Let the latter number of rows and columns of saplings be $(x + 1)$

Then, the number of saplings $= (x+1)^2 - 25$

$$x^2 + 24 = x^2 + 1 + 2x - 25$$

$$2x = 48$$

$$x = 24$$

Number of saplings available with the caretaker

$$= 24^2 + 24$$

$$= 576 + 24$$

$$= 600$$