CHAPTER 5

Arithmetic Progressions

Level - 1

CORE SUBJECTIVE QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Mark)

1. Option (A) is correct.

Explanation: The nth term of an arithmetic progression (A.P.) is given by:

$$a_n = 7n - 4$$

The common difference d is found by subtracting consecutive terms:

$$d = a_{n+1} - a_n = [7(n+1) - 4] - [7n - 4]$$

$$d = (7n + 7 - 4) - (7n - 4) = 7$$

Hence, the common difference, $d = 7$.

2. Option (C) is correct

Explanation:

$$Tn = 7n + 4$$
$$T_1 = 7 \times 1 + 4 = 11$$
$$T_2 = 7 \times 2 + 4 = 18$$
$$d = T_2 - T_1 = 18 - 11$$
$$d = 7$$

3. Option (C) is correct

Explanation:

Here,

$$a = 7$$
$$a_n \text{ (last term)} = 84$$

Now, $$S_n = \frac{n}{2}(a + l)$$

$$\frac{2093}{2} = \frac{n}{2}(7 + 84)$$

$$n = \frac{2093}{91} = 23$$

4. Option (C) is correct

Explanation:

$$a = -16,\ d = -2$$
$$S_{10} = ?$$
$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$S_{10} = \frac{10}{2}[2 \times -16 + 9 \times -2]$$
$$= 5(-32 - 18)$$
$$= -250$$

5. Option (A) is correct

Explanation:

$$a_{15} - a_{11} = 48$$
$$(a + 14d) - (a + 10d) = 48$$
$$4d = 48$$
$$d = 12$$

6. Option (B) is correct

Explanation: Given A.P. –29, –26, –23, ... 61

$$T_n = 16$$
$$n = ?$$

For given A.P. $a = -29$

$$d = -26 - (-29)$$
$$= 3$$

Using $$T_n = a + (n-1)d$$

$$16 = -29 + (n-1)3$$
$$45 = (n-1)3$$

$\Rightarrow$ $$n = 16$$

7. Option (B) is correct

Explanation: The common difference d of an A.P. is given by the difference between any two consecutive terms:

$$d = a_2 - a_1$$
$$d = \frac{1-4x}{2x} - \frac{1}{2x}$$
$$d = \frac{(1-4x)-1}{2x}$$
$$d = \frac{-4x}{2x}$$
$$d = -2$$

8. Option (A) is correct

Explanation: Substitute Common difference $d = 6$ into the formula:

$$S_n = \frac{n}{2}(2a + (n-1)d)$$
$$S_n = \frac{n}{2}(2a + 6n - 6)$$
$$= n(a + 3n - 3)$$

Given, $$S_n = 3n^2 + 4n$$

$$3n^2 + 4n = n(a + 3n - 3)$$

$$3n^2 + 4n = an + 3n^2 - 3n$$
$$4n + 3n = an$$
$$7n = an$$
$$\therefore \quad a = 7$$

Thus, the first term is $a = 7$.

9. Option (B) is correct

Explanation: Let the three numbers be $a - d$, a and $a + d$, where a is the middle term, and d is the common difference. Given that their sum is 30, we have:

$$(a - d) + a + (a + d) = 30$$
$$3a = 30$$
$$a = \frac{30}{3}$$
$$a = 10$$

So, the middle term a is 10.

10. Option (D) is correct

Explanation: The first three terms of an AP are $3p - 1$, $3p + 5$ and $5p + 1$ respectively

We know that if a, b and c are in AP, then:

$$b - a = c - b \Rightarrow 2b = a + c$$
$$\therefore \quad 2(3p + 5) = 3p - 1 + 5p + 1$$
$$\Rightarrow \quad 6p + 10 = 8p$$
$$\Rightarrow \quad 10 = 8p - 6p$$
$$\Rightarrow \quad 2p = 10$$
$$\Rightarrow \quad p = 5$$

11. Option (C) is correct

Explanation:

$$\sqrt{18}, \sqrt{50}, \sqrt{98}, ...$$
$$3\sqrt{2}, 5\sqrt{2}, 7\sqrt{2}$$

Now, $d = 5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}$

$$\Rightarrow \quad 4^{th} \text{ term} = 7\sqrt{2} + 2\sqrt{2}$$
$$= 9\sqrt{2}$$
$$\Rightarrow \quad 9\sqrt{2} = \sqrt{162}$$

12. Option (C) is correct

Explanation: For the n-th term:

$$49 = -8 + (n - 1) \times 3$$

$n = 20$. So, there are 20 terms.

The 7^{th} term from the end is the 14^{th} term from the beginning.

$$T_{14} = -8 + (14 - 1) \times 3 = 31$$

13. Option (B) is correct

Explanation: The sum of the first n terms of an A.P. is:

$$S_n = \frac{n}{2}.(a + l)$$
$$480 = \frac{n}{2}.(7 + 73)$$
$$480 = \frac{n}{2}.80$$
$$480 = 40n$$
$$n = \frac{480}{40} = 12$$

14. Option (D) is correct

Explanation: Odd natural numbers 1, 3, 5, ... n

$$S_n = \frac{n}{2}[2 \times 1 + (n - 1) \times 2]$$
$$= n^2$$

Even natural numbers

$$S'_n = \frac{n}{2}[2 \times 2 + (n - 1) \times 2]$$
$$= n(n + 1)$$

According to question,

$$S_n = KS'_n$$
$$n^2 = Kn(n + 1)$$
$$\Rightarrow \quad K = \frac{n}{(n + 1)}$$

15. Option (C) is correct

Explanation: Given,

Frist term of first A.P., $a_1 = -1$

First term of second A.P., $b_1 = -8$

Common difference d is same for both

n^{th} term of an A.P. is given by:

$$a_n = a + (n - 1)d$$

For the first A.P.

$$a_4 = -1 + (4 - 1)d = -1 + 3d$$

For the second A.P.

$$b_4 = -8 + (4 - 1)d = -8 + 3d$$

Difference between their 4^{th} terms

$$a_4 - b_4 = (-1 + 3d) - (-8 + 3d)$$
$$= -1 + 3d + 8 - 3d$$
$$= 7$$

16. Option (C) is correct

Explanation: From arithmetic progression

$$t_2 - t_1 = t_3 - t_2$$
$$(p + 1) - (p - 1) = (2p + 3) - (p + 1)$$
$$p + 1 - p + 1 = 2p + 3 - p - 1$$
$$2 = p + 2$$
$$\Rightarrow \quad p = 0$$

17. Option (C) is correct

Explanation: Given: a, b, c, are in A.P.

$\Rightarrow \quad d = b - a$ or $b = a + d$

also, $d = c - b \Rightarrow c = d + b$

$$= d + a + d = a + 2d$$

Now, on substituting values of b and c in $a - 2b - c$, we get,

The value of $a - 2b - c$

$$a - 2b - c = a - 2(a + d) - (a + 2d)$$
$$a - 2b - c = a - 2a - 2d - a - 2d$$
$$= a - 2a - a - 2d - 2d$$
$$= -2a - 4d$$

18. Option (B) is correct

Explanation:

Given, A.P. $\sqrt{6}, \sqrt{24}, \sqrt{54}$.....

$$a = \sqrt{6}$$
$$d = 2\sqrt{6} - \sqrt{6} = \sqrt{6}$$
$$T_n = a + (n - 1)d$$

$$T_4 = \sqrt{6}+(4-1)\sqrt{6}$$
$$= \sqrt{6}+3\sqrt{6}$$
$$T_4 = 4\sqrt{6}$$
$$= \sqrt{96}$$

19. Option (A) is correct

Explanation: Given $k + 2$, $4k - 6$ and $3k - 2$ are three consecutive terms of an A.P.

Then, $(4k - 6) - (k + 2) = (3k - 2) - (4k - 6)$
$$3k - 8 = -k + 4$$
$$k = \frac{12}{4} = 3$$

20. Option (D) is correct

Explanation: Here, $a = \sqrt{7}, a+d = \sqrt{28}$

$\therefore$ $d = \sqrt{28}-\sqrt{7} = 2\sqrt{7}-\sqrt{7}$
$$= \sqrt{7}$$

or, Next term $= \sqrt{63}+\sqrt{7}$

or, $= 3\sqrt{7}+\sqrt{7} = 4\sqrt{7}$

or $= \sqrt{7\times16}$
$$= \sqrt{112}$$

21. Option (B) is correct

Explanation: If $a_n = 3n + 7$

Then
$$a_1 = 3(1) + 7 = 10$$
$$a_2 = 3(2) + 7 = 13$$
$$a_3 = 3(3) + 7 = 16$$
$$d = \text{common difference}$$
$$= a_n - a_{n-1}$$
$$d = a_2 - a_1 = a_3 - a_2$$
$$d = 13 - 10 = 16 - 13 = 3$$

22. Option (C) is correct

Explanation: We know that $a_n = a_1 + (n-1)d$ and $a_n = a_2 + (n-2)d$ Option (A) $a_{24} = a_1 + 24d$

This is incorrect because, using the formula for the 24th term:
$$a_{24} = a_1 + (24 - 1).\ d = a_1 + 23d$$

Option (B) $a_{25} = a_2 + 24d$

This is incorrect because:
$$a_{25} = a_2 + (25 - 2).\ d = a_2 + 23d$$

Option (C) $a_{26} = a_2 + 24d$

This is correct because, for the 26th term:
$$a_{26} = a_2 + (26 - 2).\ d = a_2 + 24d$$

ASSERTION-REASON QUESTIONS (1 Mark)

1. Option (D) is correct

Explanation: In case of Assertion:

First, simplify the square roots:
$$\sqrt{24} = 2\sqrt{6}, \sqrt{54} = 3\sqrt{6}, \sqrt{96} = 4\sqrt{6}$$

So, the sequence is $\sqrt{6}, 2\sqrt{6}, 3\sqrt{6}, 4\sqrt{6}$,....

The common difference between consecutive terms:
$$2\sqrt{6}-\sqrt{6} = \sqrt{6},\ 3\sqrt{6}-2\sqrt{6} = \sqrt{6}, 4\sqrt{6}-3\sqrt{6} = \sqrt{6}$$

Hence, the common difference is $\sqrt{6}$ not $3\sqrt{6}$

Therefore, the assertion (A) is false, but the sequence does form an arithmetic progression, so the reason (R) is true.

2. Option (C) is correct

Explanation:

The sequence $-5, -\frac{5}{2}, 0, \frac{5}{2}$..... has a common difference:
$$-\frac{5}{2}-(-5) = \frac{5}{2}, 0-\left(-\frac{5}{2}\right) = \frac{5}{2}, \frac{5}{2}-0 = \frac{5}{2}$$

So, the sequence is an arithmetic progression with a common difference of $\frac{5}{2}$ meaning the assertion (A) is true.

The reason (R) is incorrect, as there is no restriction that terms in an arithmetic progression cannot be both positive and negative rational numbers. Therefore, reason (R) is false.

3. Option (B) is correct.

Explanation: The condition for three terms to be in arithmetic progression is that the middle term is the average of the other two, i.e. $b = \frac{a+c}{2}$

$\Rightarrow 2b = a + c$. Therefore, assertion (A) is true.

The reason (R) is a correct statement by itself (it is a well known result that the sum of the first n odd natural numbers is n^2), but it has no direct connection to the assertion. Hence, reason (R) is true, but it does not explain the assertion.

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. The given sequence is 20, $19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}$ ....

First term $a = 20$

Common difference, $d = 19\frac{1}{4} - 20 = \frac{77-80}{4} = -\frac{3}{4}$

Let the n^{th} term of the AP be the first negative term

So $a_n < 0$

$$\Rightarrow a + (n-1)d < 0$$
$$\Rightarrow 20 + (n-1)\left(-\frac{3}{4}\right) < 0$$
$$\Rightarrow \frac{83}{4} - \frac{3n}{4} < 0$$
$$\Rightarrow 83 - 3n < 0$$
$$\Rightarrow 3n > 83$$

$\Rightarrow \quad n > 27\frac{2}{3}$

$\Rightarrow \quad n \geq 28$

Thus, the 28^{th} term is the first negative term of the given AP

2. In the given problem, let us first find the 41st term of the given A.P.

A.P. is 8, 14, 20, 26…

Here,

First term (a) =8

Common difference of the A.P. (d) = 14 – 8 = 6

Now

$$a_n = a + (n-1)d$$

So, for 41^{st} term (n = 41)

$$\begin{aligned} a_{41} &= 8 + (41-1)(6) \\ &= 8 + 40(6) \\ &= 8 + 240 \\ &= 248 \end{aligned}$$

Let us take the term which is 72 more than the 41^{st} term as a_n

So,
$$\begin{aligned} a_n &= 72 + a_{41} \\ &= 72 + 248 \\ &= 320 \end{aligned}$$

Also $a_n = a + (n-1)d$

$$\begin{aligned} 320 &= 8 + (n-1)6 \\ \Rightarrow \quad 320 &= 8 + 6n - 6 \\ 320 &= 6n + 2 \\ 318 &= 6n \\ n &= \frac{318}{6} \\ n &= 53 \end{aligned}$$

Therefore the 53^{rd} term of the given A.P. is 72 more than the the 41^{st} term.

3. The given A.P. is 5, 15, 25, ...

Here, a = 5 and d = 15 – 5 = 10

Now, $t_{31} = a + 30d = 5 + 30 \times 10 = 5 + 300 = 305$

Let the required term be nth term.

$$\begin{aligned} \therefore \quad t_n - t_{31} &= 130 \\ \Rightarrow \quad [a + (n-1)d] - 305 &= 130 \\ \Rightarrow \quad 5 + (n-1)(10) &= 435 \\ \Rightarrow \quad n - 1 &= 43 \\ \Rightarrow \quad n &= 44 \end{aligned}$$

Thus required term = 44^{th} term

4.
$$a_1 = -30,\ a_2 = -24$$
$$d = a_2 - a_1 = -24 - (-30) = 6$$

$\therefore$ Sum of first 30 terms is

$$\begin{aligned} S_{30} &= \frac{30}{2}[2a + (30-1)d] \\ &= 15[2 \times -30 + 29 \times 6] \\ &= 15[-60 + 174] \\ &= 15 \times 114 \\ &= 1710 \end{aligned}$$

5. We know that, the n^{th} term of an A.P. is

$$a_n = S_n - S_{n-1}$$
$$a_n = n(4n+1) - (n-1)\{4(n-1) + 1\}$$
$$[\because S_n = n(4n+1)]$$

$$\begin{aligned} \Rightarrow \quad a_n &= 4n^2 + n - (n-1)(4n-3) \\ &= 4n^2 + n - 4n^2 + 3n + 4n - 3 \\ &= 8n - 3 \end{aligned}$$

Put $n = 1$

$$\begin{aligned} a_1 &= 8(1) - 3 \\ &= 5 \end{aligned}$$

Put $n = 2$

$$\begin{aligned} a_2 &= 8(2) - 3 \\ &= 16 - 3 \\ &= 13 \end{aligned}$$

Hence, the required AP is 5, 13, 21, ...

6. Sum of the first 12 two digit multiples of 6:

Here, the 2 digit numbers which are multiple of 6 are:

12, 18, 24, 30 ....

As, these number form an A.P. so.

$a = 12$ and $d = 6$ and $n = 12$

Sum formula for an A.P.:

$$S_n = \frac{n}{2} \times (2a + (n-1)d)$$

Substituting values:

$$\begin{aligned} S_{12} &= \frac{12}{2}[2 \times 12 + (12-1)6] \\ &= 6(24 + 66) \\ &= 6 \times 90 \Rightarrow 540 \end{aligned}$$

7. Here
$$a_2 = 26$$
$$a_{15} = -26$$

The n-th term of an AP is given by:

$$a_n = a_1 + (n-1) \times d$$

For a_2 (second term):

$$a_2 = a_1 + (2-1) \times d = a_1 + d \quad \text{...(i)}$$

For a_{15}

$$a_1 + 14d = -26 \quad \text{...(ii)}$$

Subtract Equation (i) from Equation (ii):

$$\begin{aligned} (a_1 + 14d) - (a_1 + d) &= -26 - 26 \\ 13d &= -52 \\ d &= \frac{-52}{13} = -4 \end{aligned}$$

Substitute $d = -4$ into Equation (i):

$$\begin{aligned} a_1 + (-4) &= 26 \\ a_1 &= 26 + 4 = 30 \end{aligned}$$

Now, $a_1 = 30$ and $d = -4$, the AP is:

30, 26, 22, 18, 14, ...

8. A.P. $-\frac{11}{2}, -3, -\frac{1}{2}, ...$ with :

First term $a_1 = -\frac{11}{2}$

Common difference $d = \frac{5}{2}$

The formula for the nth term:

$$a_n = a_1 + (n-1) \times d$$

Substitute $a_n = \frac{49}{2}$,

$$\frac{49}{2} = -\frac{11}{2} + (n-1) \times \frac{5}{2}$$

$$n = 13$$

The 13th term is $\frac{49}{2}$.

9. Given numbers a, 7, b, 23 are in A.P.

$\therefore\ 7 - a = b - 7 = 23 - b$ ....[A.P. has equal common difference]

By equating $b - 7 = 23 - b$

$\Rightarrow \quad 2b = 30$

$\Rightarrow \quad b = 15$

Now, equating $7 - a = b - 7$

$\Rightarrow \quad 7 - a = 15 - 7$

...[Putting the value of b]

$\Rightarrow \quad -a = 1$

$\Rightarrow \quad a = -1$

Hence, $a = -1$ and $b = 15$

10. Given:

n^{th} term of the A.P. 9, 7, 5... is the same as the term of the A.P. 15, 12, 9. ...

Considering 9, 7, 5 ...

$$a = 9,\ d = (7 - 9) = -2$$

n^{th} term $= 9 + (n - 1)(-2)$ $\quad [\because a_n = a + (n - 1)d]$

$= 9 - 2n + 2$

$= 11 - 2n$ ...(i)

Considering 15, 12, 9, ...

$$a = 15,\ d = (12 - 15) = -3$$

n^{th} term $= 15 + (n - 1)(-3)$ $[\because a_n = a + (n - 1)d]$

$= 15 - 3n + 3$

$= 18 - 3n$ ...(ii)

Equating (i) and (ii), we get:

$$11 - 2n = 18 - 3n$$

$\Rightarrow \quad n = 7$

Thus, 7^{th} terms of both the A.P.s are the same.

11. Given:

First term $a_1 = 10$

Sum of the first 14 terms $S_{14} = 1505$

The sum of the first n terms of an A.P. is given by:

$$S_n = \frac{n}{2} \times (2a_1 + (n-1) \times d)$$

Substituting the known values ($n = 14$, $a_1 = 10$, and $S_{14} = 1505$)

$$1505 = \frac{14}{2} \times (2 \times 10 + (14-1) \times d)$$

$$1505 = 7 \times (20 + 13d)$$

$$1505 = 140 + 91d$$

$$1505 - 140 = 91d$$

$$1365 = 91d$$

$$d = \frac{1365}{91} = 15$$

12.

$$a = 5$$

$$T_n = l = 15$$

$$S_n = 30$$

$$n = ?$$

$$S_n = \frac{n}{2}(a + l)$$

$\Rightarrow \quad 30 = \frac{n}{2}(5 + 15)$

$\Rightarrow \quad 60 = n \times 20$

$\Rightarrow \quad n = 3$

13. Given, 6^{th} term of A.P. $= 30$

or, $\quad a_6 = 30$

or, $\quad a + (6 - 1)d = 30$

or, $\quad a + 5d = 30$ ...(i)

Since, sum of n terms of A.P. is $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{11} = \frac{11}{2}[2a + (11-1)d]$

$= \frac{11}{2}[2a + 10d]$

$= \frac{11 \times 2}{2}[a + 5d]$

$= 11 \times 30$ ...[From equation (i)]

$= 330$

14. Given:

$$\frac{a_4}{a_7} = \frac{2}{3}$$

We know,

$$a_n = a + (n - 1)d$$

Where a is a first term or a_1 and d is the common difference and n is any natural number

When $n = 4$:

$\therefore \quad a_4 = a + (4 - 1)d$

$\Rightarrow \quad a_4 = a + 3d$

When $n = 6$:

$\therefore \quad a_6 = a + (6 - 1)d$

$\Rightarrow \quad a_6 = a + 5d$

When $n = 7$:

$\therefore \quad a_7 = a + (7 - 1)d$

$\Rightarrow \quad a_7 = a + 6d$

When $n = 8$:

$\therefore \quad a_8 = a + (8 - 1)d$

$\Rightarrow \quad a_8 = a + 7d$

According to the question:

$$\frac{a_4}{a_7} = \frac{2}{3}$$

$\Rightarrow \quad \frac{a+3d}{a+6d} = \frac{2}{3}$

$\Rightarrow \quad 3(a + 3d) = 2(a + 6d)$

$\Rightarrow \quad 3a + 9d = 2a + 12d$

$\Rightarrow \quad 3a - 2a = 12d - 9d$

$\Rightarrow \quad a = 3d$

Now,

$$\frac{a_6}{a_8} = \frac{a+5d}{a+7d}$$

$\Rightarrow \quad \frac{a_6}{a_8} = \frac{3d+5d}{3d+7d}$

$\Rightarrow \quad \frac{a_6}{a_8} = \frac{8d}{10d}$

$$\Rightarrow \quad \frac{a_6}{a_8} = \frac{4}{5}$$

Hence,

The ratio of $\frac{a_6}{a_8} = \frac{4}{5}$

15. Let, n^{th} term of AP be 53

Here,

$$T_n = 53$$
$$a = 293$$
$$d = 285 - 293$$
$$= -8$$
$$n = ?$$

Now,

$$T_n = a + (n-1)d$$
$$53 = 293 + (n-1)(-8)$$
$$53 - 293 = -8n + 8$$
$$-240 - 8 = -8n$$
$$-8n = -248$$
$$n = \frac{-248}{-8}$$
$$n = 31$$

Thus, there are total 31 terms in A.P.

16. Given:

$$d = 5$$
$$a_{20} = 135 \text{ (the } 20^{th} \text{ term)}$$

The n^{th} term of an AP is:

$$a_n = a + (n-1)d$$

20^{th} term (a_{20}):

$$a_{20} = a + (20-1) \times d$$
$$135 = a + 19 \times 5$$
$$135 = a + 95$$
$$a = 135 - 95 = 40$$

The sum of the first n terms of an A.P.:

$$S_n = \frac{n}{2} \times (2a + (n-1)d)$$

Substitute the known values ($n = 20$, $a = 40$, $d = 5$):

$$S_{20} = \frac{20}{2} \times (2 \times 40 + (20-1) \times 5)$$
$$S_{20} = 10 \times (80 + 95)$$
$$S_{20} = 10 \times 175 = 1750$$
$$S_{20} = 1750.$$

17. The first term $a = 7$

The common difference $d = 7$

The number of terms $n = 40$

The nth term of the A.P. is

$$a_n = a + (n-1)d$$

40th term:

$$a_{40} = 7 + (40-1)7 = 7 + 273 = 280$$

The sum of first n terms of an A.P. is

$$S_n = \frac{n}{2}(a + a_n)$$
$$S_{40} = \frac{40}{2}(7 + 280) = 20 \times 287 = 5740$$

18. The first term, $a = 12x$

The common difference, $d = 10x - 12x = -2x$

The last term, $l = -2x$

The n^{th} term of an arithmetic sequence

$$a_n = a + (n-1) \times d$$
$$-2x = 12x + (n-1) \times (-2x)$$
$$-2x = 12x - 2x(n-1)$$
$$-2x = 12x - 2x \times n + 2x$$
$$-2x = 14x - 2x \times n$$
$$-16x = -2x \times n$$
$$n = \frac{16x}{2x} = 8$$

Thus, there are 8 terms in the A.P. $12x, 10x, 8x, \ldots -2x$.

19. Let a be the first term of an A.P. and d be the common difference of an A.P.

We know that $a_n = a + (n-1)d$

Given : $a_p = \frac{1}{q}$, and $a_q = \frac{1}{p}$

$$\frac{1}{q} = a + (p-1)d \quad \text{...(i)}$$
$$\frac{1}{p} = a + (q-1)d \quad \text{...(ii)}$$

Equation (i) – (ii),

$$\Rightarrow \quad \frac{1}{q} - \frac{1}{p} = pd - d - qd + d$$
$$\Rightarrow \quad \frac{(p-q)}{pq} = d(p-q)$$
$$\Rightarrow \quad d = \frac{1}{pq}$$

Substitute the value of d in Equation (i),

$$\frac{1}{q} = a + (p-1)\frac{1}{pq}$$
$$\Rightarrow \quad \frac{1}{q} = a + \frac{1}{q} - \frac{1}{pq}$$
$$\Rightarrow \quad a = \frac{1}{pq}$$

To find $(pq)^{th}$ term

$$a_{pq} = a + (pq-1)d$$
$$= \frac{1}{pq} + (pq-1)\frac{1}{pq}$$
$$= \frac{1}{pq} + 1 - \frac{1}{pq} \Rightarrow \frac{1 + pq - 1}{pq} = \frac{pq}{pq}$$
$$= 1$$

Hence Proved.

20. Identifies the two sets of consecutive terms and finds the difference between the terms in each set by subracting a term from its next term. For example.

Second term – First term

$$= (-11 + 11a) - (-12 + 12a)$$
$$= -11 + 11a + 12 - 12a$$
$$= (1 - a)$$

Third term – Second term

$= (-10 + 10a) - (-11 + 11a)$
$= -10 + 10a + 11 - 11a$
$= (1 - a)$

On comparing the differences it is concluded that the given sequence is an arithmetic progression.

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. Given: $S_m = S_n$

$\Rightarrow \frac{m}{2}[2a + (m-1)d] = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 2am + md(m-1) = 2an + nd(n-1)$

$\Rightarrow 2am - 2an + m^2d - md - n^2d + nd = 0$

$\Rightarrow 2a(m-n) + d[(m^2 - n^2) - (m-n)] = 0$

$\Rightarrow (m-n)[2a + \{(m+n) - 1\}d] = 0$

$\Rightarrow 2a + (m+n-1)d = 0 \quad [\because m - n \neq 0]$ ...(i)

Now,

$S_{m+n} = \frac{m+n}{2}[2a + \{(m+n)-1\}d] = 0$

$\Rightarrow S_{m+n} = \frac{m+n}{2} \times 0$ [using (i)]

$\Rightarrow S_{m+n} = 0$

2. Let 3 consecutive numbers be $a - d, a, a + d$

$\therefore$ Sum $= a - d + a + a + d = 24$

$3a = 24$

$a = \frac{24}{3} = 8$

Sum of the squares = 194

$(a-d)^2 + a^2 + (a+d)^2 = 194$

$\Rightarrow a^2 + d^2 - 2ad + a^2 + a^2 + d^2 + 2ad = 194$

$\Rightarrow 3a^2 + 2d^2 = 194$

$2d^2 = 194 - 3(8)^2$

$2d^2 = 194 - 3(64)$

$2d^2 = 194 - 192$

$2d^2 = 2$

$d^2 = 1$

$d = \pm 1$

$a = 8, d = \pm 1$

Three numbers are 9, 8, 7 or 7, 8, 9

3. $S_n = \frac{n}{2}(2a + (n-1)d)$

The first 7 terms:

$S_7 = 49 \Rightarrow 2a + 6d = 14$ ...(i)

The first 17 terms:

$S_{17} = 289 \Rightarrow 2a + 16d = 34$ ...(ii)

Subtract eq. (i) from eq. (ii)

$10d = 20 \Rightarrow d = 2$

Substitute d in eq. (i)

$2a + 12 = 14 \Rightarrow 2a = 2 \Rightarrow a = 1$

Sum of the first 20 terms:

$S_{20} = 10[2(1) + 19(2)] = 10 \times 40 = 400$

4. Given,

$T_{10} : T_{30} = 1 : 3, S_6 = 42$

Let a be the first term and d be the common difference, then

$\frac{a+9d}{a+29d} = \frac{1}{3}$

$3a + 27d = a + 29d$

$3a - a = 29d - 27d$

$2a = 2d$

$\therefore a = d$

Now, $S_6 = 42$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$42 = \frac{6}{2}[2a + (6-1)d]$

$14 = 2a + 5d \quad [\because a = d]$

$14 = 7a$

$\therefore a = \frac{14}{7} = 2$

$\therefore a = d = 2$

Therefore, the first term of the AP is 2 and the common difference of A.P. is also 2.

5. Here, $S_{14} = 1050, n = 14, a = 10$

We know that $S_n = \left(\frac{n}{2}\right)[2a + (n-1)d]$

Substituting the values we have

$1050 = \left(\frac{14}{2}\right)[20 + 13d]$

$\Rightarrow 1050 = 140 + 91d$

$\Rightarrow 910 = 91d$

$\Rightarrow d = 10$

Therefore, $a_{20} = 10 + (20 - 1) \times 10 = 200$

i.e. 20^{th} term is 200

the n^{th} term

$T_n = a + (n-1)d = 10 + (n-1) \times 10$

$T_n = 10 + 10n - 10 = 10n$

6. Here,

The first term of the A.P (a) = 5

The last term of the A.P (l) = 45

Sum of all the terms $S_n = 400$

Let the common difference of the A.P. be d.

So, let us first find the number of the terms (n) using the formula,

$S_n = \frac{n}{2}(a+l)$

$400 = \left(\frac{n}{2}\right)(5+45)$

$400 = \left(\frac{n}{2}\right)(50)$

$n = \frac{400}{25}$

$n = 16$

Now, to find the common difference of the A.P. we use the following formula,

$l = a + (n-1)d$

We get

$45 = 5 + (16-1)d$

$45 = 5 + (15)d$

$45 - 5 = 15d$

$\frac{45-5}{15} = d$

$$d = \frac{40}{15}$$

$$d = \frac{8}{3}$$

Therefore, the number of terms is $n = 16$ and the common difference of the A.P. is $d = \frac{8}{3}$

7. $a = -14, a_5 = 2, a_n = 62$

$a_5 = a + (5-1)d$ $[\because a_n = a + (n-1)d]$

$\Rightarrow 2 = -14 + 4d$

$\Rightarrow 4d = 16$

$\Rightarrow d = 4$

Now, $a_n = 62$

$\Rightarrow a + (n-1)d = 62$

$\Rightarrow -14 + (n-1)4 = 62$ ...$(\because a = -14, d = 4)$

$\Rightarrow (n-1)4 = 76$

$\Rightarrow n - 1 = \frac{76}{4} = 19$

$\Rightarrow n = 19 + 1 = 20$

$\therefore$ There are 20 terms is an A.P.

8. It is given that

AP : 65, 61, 57, 53

First term $(a) = 65$ and common difference $(d) = 61 - 65 = -4$

Consider then nth term of an AP,

$a_n = a + (n-1)d,$

Here, $[a + (n-1)d] < 0$

Substituting the values

$\Rightarrow 65 + (n-1)(-4) < 0$

$\Rightarrow 65 - 4n + 4 < 0$

$\Rightarrow 4n > 69$

$\Rightarrow n > \frac{69}{4}$

$\Rightarrow n > 17\frac{1}{4}$

$\Rightarrow n = 18$

So the 18th term is the first negative term of the give AP.

9. Let a be the first term and d be the common difference of the given A.P.

Then, p^{th} term $= q \Rightarrow a + (p-1)d = q$ ...(i)

q^{th} term $= p \Rightarrow a + (q-1)d = p$ ...(ii)

Subtracting equation (ii) from equation (i), we get

$q - p = (a + pd - d) - (a + qd - d)$

$= (a + pd - d - a - qd + d)$

$q - p = (p - q)\,d$

$(p - q)d = (q - p) \Rightarrow d = -1$

Putting $d = -1$ in equation (i), we get

$a = (p + q - 1)$

n^{th} term $= a + (n-1)d$

$= (p + q - 1) + (n - 1) \times (-1)$

$= (p + q - n)$

Hence Proved.

10. The formula for the nth term of an arithmetic progression (AP) is:

$a_n = a + (n-1)d$

Given:

$a_n = 18$

$a = 50$

$d = -4$

Substitute the values into the formula for a_n:

$a_n = a + (n-1)d$

$18 = 50 - 4n + 4$

$4n = 36$

$n = 9$

Sum of first n terms of an A.P.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{9}{2}[2 \times 50 + (9-1)(-4)]$$

$$= \frac{9}{2}(100 - 32) = \frac{9}{2} \times 68 = 306$$

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. Let a and a_8 be the first and eighth terms of A.P.

Let common difference be d

$\therefore a + a_8 = 32$ (given)

$\Rightarrow a + [a + (8-1)d] = 32$

$[\because T_n = a + (n-1)d]$

$\Rightarrow a + (a + 7d) = 32$

$\Rightarrow a + 7d = 32 - a$ ...(i)

Also, $a \times a_8 = 60$ ..(given)

$\Rightarrow a \times [a + (8-1)d] = 60$

$\Rightarrow a(a + 7d) = 60$

$\Rightarrow a(32 - a) = 60$ [from eq. (i)]

$\Rightarrow 32a - a^2 = 60$

$\Rightarrow a^2 - 32a + 60 = 0$

$\Rightarrow a^2 - 30a - 2a + 60 = 0$

$\Rightarrow a(a - 30) - 2(a - 30) = 0$

$\Rightarrow (a - 30)(a - 2) = 0$

$\Rightarrow a = 2, 30$

Substitute $a = 2$ is eq.(i)

$2 + 7d = 32 - 2$

$7d = 28$

$d = 4$

Substitute $a = 30$, in eq. (i)

$30 + 7d = 32 - 30$

$7d = -28$

$d = -4$

Taking $(a, d) = (2, 4)$

$$S_{20} = \frac{20}{2}[2 \times 2 + (20-1) \times d]$$

$$\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$$

$= 10\,[4 + 19 \times 4]$

$= 10 \times 80 = 800$

Taking $(a, d) = (30, -4)$

$$S_{20} = \frac{20}{2}[30 \times 2 + (20-1)(-4)]$$

$$\left[\because S_n = \frac{n}{2}(2a + (n-1)d)\right]$$

$= 10(60 + 19(-4)]$

$= 10(60 - 76)$

$= 10 \times (-16)$

$= -160$

2. Let consider the first terms of AP is a and the common difference is d

(i) Since the sum of first n term of an AP,

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$\therefore$ Sum of first 9 terms, $S_9 = \frac{9}{2}[2a + (9-1)(d)]$

$$= 9(a + 4d)$$

Since it is given that the sum of first 9 terms is 153.

$\therefore \quad 153 = 9(a + 4d)$

$\therefore \quad 17 = a + 4d$

$\therefore \quad a + 4d = 17$ ....(i)

Next, it is given that the sum of last 6 term is 687

Since, Sum of last 6 terms = Sum of first 40 terms – Sum of first 34 terms

$\therefore \quad S_{40} - S_{34} = 687$

$\therefore \quad \frac{40}{2}[2a + (40-1)(d)] - \frac{34}{2}[2a + (34-1)(d)] = 687$

$\therefore \quad 20(2a + 39d) - 17(2a + 33d) = 687$

$\therefore \quad 40a + 780d - 34a - 561d = 687$

$\therefore \quad 6a + 219d = 687$

$\therefore \quad 2a + 73d = 229$ ...(ii)

By multiplying equation (i) by 2 and then subtracting equation (ii), we get:

$2(a + 4d) - (2a + 73d) = 2 \times 17 - 229$

$\therefore \quad 2a + 8d - 2a - 73d = 34 - 229$

$\therefore \quad -65d = -195$

$\therefore \quad d = 3$

By substituting value of d in equation (i), we get:

$a + 4d = 17$

$\therefore \quad a + 4(3) = 17$

$\therefore \quad a + 12 = 17$

$\therefore \quad a = 17 - 12$

$\therefore \quad a = 5$

Therefore, first term of given AP is 5 and common difference of AP is 3.

Now, Sum of all terms of AP

Since the sum of n terms of an AP,

$$S_n = \frac{n}{2}[2a + (n-1)d)]$$

It is given that AP has 40 terms, $a = 5$, $d = 3$

$\therefore$ Sum of 40 terms, $S_{40} = \frac{40}{2}[2(5) + (40-1)(3)]$

$\therefore \quad S_{40} = 20[10 + (39)(3)]$

$= 20(10 + 117)$

$= 20 \times 127 = 2{,}540$

Therefore, sum of its 40 terms is 2,540.

3. Given $\quad \frac{a_{11}}{a_{17}} = \frac{3}{7}$

$\Rightarrow \quad \frac{a+10d}{a+16d} = \frac{3}{4}$

[Where a is first term and d in common differentce]

$4a + 40d = 3a + 48d$

$a = 8d$ ...(i)

Now, $\quad \frac{a_5}{a_{21}} = \frac{a+4d}{a+20d}$

$= \frac{8d+4d}{8d+28d}$

$= \frac{12d}{36d}$

$= \frac{1}{3}$

$\therefore \quad a_5 : a_{21} = 3 : 7$

Now $\quad \frac{S_5}{S_{21}} = \frac{\frac{5}{2}[2a+4d]}{\frac{21}{2}[2a+20d]}$

$= \frac{5(a+2d)}{21(a+10d)}$

$= \frac{5(8d+2d)}{21(8d+10d)}$

$= \frac{5 \times 10d}{21 \times 18d}$

$\frac{S_5}{S_{21}} = \frac{25}{189}$

$\therefore \quad S_5 : S_{21} = 25 : 189$

4. Given, Total logs = 250

logs in the bottom row= 22 and then 21 logs there in the next-row, 20 in the next row and so on.

So, 22 + 21 + 20 + ...n rows = 250

$a = 22$

$d = 21 - 22 = -1$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$250 = \frac{n}{2}[2 \times 22 + (n-1)(-1)]$

$500 = n[-n + 45]$

$n^2 - 45n + 500 = 0$

$n^2 - 25n - 20n + 500 = 0$

$(n - 25)(n - 20) = 0$

$\therefore \quad n = 25$ or $n = 20$

Now, number of logs in nth row will be

$a_n = (a + (n-1)d$

$a_{25} = [22 + (25-1) - 1]$

$= [22 - 24]$

$= -2$

Similarly,

$a_{20} = [22 - (20-1) \times -1]$

$= 22 - 19$

$= 3$

As, number of logs cannot be negative.

$\therefore$ 250 logs can be placed in 20 rows and the number of logs in the top row will be 3.

5. Given:

Sum of first n terms $S_n = 5n^2 + 3n$

Now, Putting

$n = 1$; $S_1 = 5 \times 1^2 + 3 \times 1 = 8$

$n = 2$; $S_2 = 5 \times 2^2 + 3 \times 2 = 20 + 6 = 26$

$n = 3$, $S_3 = 5 \times 3^2 + 3 \times 3 = 45 + 9 = 54$

$\therefore t_2 = S_2 - S_1 = 26 - 8 = 18$

$t_3 = S_3 - S_2 = 54 - 26 = 28$

$\therefore$ First term $(a) = t_1 = 8$

$d = 28 - 18 = 18 - 8 = 10$

Now, m^{th} term = 168

$\Rightarrow a_m = [a + (m - 1)d]$

$168 = [8 + (m - 1)\ 10]$

$168 - 8 = (m - 1)\ 10$

$\frac{160}{10} = m - 1$

$16 + 1 = m$

Thus, $m = 17$

$a_{20} = 8 + (20 - 1) \times 10$

$a_{20} = 8 + 19 \times 10$

$= 8 + 190 = 198$

Thus, $m = 17$ and 20^{th} term = 198

6. 3rd and the 14th terms of the A.P. as:

$a + 2d = -9$ ....(i)

$a + 13d = 35$ ...(ii)

Where, a is the first term and d is the common difference of the A.P.

Subtracting eq. (i) from eq. (ii), we get

$11d = 44 \Rightarrow d = 4$

Substituting the value of d in eq. (i), we get

$a + 2 \times 4 = -9$

$a = -17$

Formulates the equation to find the n^{th} term which is five times the 6^{th} term as:

$a + (n - 1)d = 5(a + 5d)$

Substituting the values of a and d in the above equation

$-17 + 4(n - 1) = 5(-17 + 20)$

$\Rightarrow -17 + 4(n - 1) = 15$

$\Rightarrow 4(n - 1) = 32$

$\Rightarrow (n - 1) = 8$

$\Rightarrow n = 9$

Thus, 9^{th} term is five times the 6^{th} term.

7. (i) $a_n = a + (n - 1)d$

$a_{30} = 1000 + (30 - 1)\ 200$

$a_{30} = 1000 + 29 \times 200$

$a_{30} = ₹\ 6800$

(ii) $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{30} = \frac{30}{2}[2 \times 1000 + (30-1)200]$

$S_{30} = 15(2000 + 29 \times 200)$

$S_{30} = 15(2000 + 5800)$

$S_{30} = 15 \times 7800$

$S_{30} = ₹\ 117{,}000$

(iii) $a_{40} = 1000 + (40 - 1)\ 200$

$a_{40} = 1000 + 39 \times 200$

$a_{40} = 8800$

Thus, amount paid is the last installment = ₹ 8800.

8. (i) One section of Class 8 will plant 16 trees

$\therefore$ 3 sections of Class 8 will plant $16 \times 3 = 48$ trees

(ii) Number of trees planted by different classes 6, 12, 18, 24, ...

$\therefore$ The terms are in A.P.

Total trees planted = 6 + 12 + 18 + 24 + ...

$n = 12;\ d = 6;\ a = 6$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n = \frac{12}{2}[2 \times 6 + (12-1)6]$

$= 6(12 + 66) = 6 \times 78 = 468$

Level - 2 ADVANCED COMPETENCY FOCUSED QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Mark)

1. Option (C) is correct

Explanation: From Feb 2023 to Dec 2024 (inclusive):

Total number of months = 11 months (Feb–Dec 2023) + 12 months (Jan–Dec 2024)

= 23 months

Total amount collected: 12,000 + 23(5500)

2. Option (D) is correct

Explanation: $a = ₹54{,}000,\ d = ₹5{,}250$

$n = 23$

$S_n = \frac{n}{2}[2a + (n-1)d$

$S_{23} = \frac{23}{2}[2 \times 54000 + 22 \times 5250]$

$= \frac{23}{2}[108000 + 115500]$

$= \frac{23}{2} \times 223500$

$= 23 \times 111750$

$= ₹\ 25{,}70{,}250$

3. Option (A) is correct

Explanation: In an AP, the nth term is:

$a_n = a + (n-1)d$

where: a = first term (acres in year 1)

d = common difference (increase in acres each year)

Given: $a + 7d = 45$ (for 8th year)

$a + 4d = 30$ (for 5th year)

Subtracting the equations

$(a + 7d) - (a + 4d) = 45 - 30$

$\Rightarrow 3d = 15 \Rightarrow d = 5$

Now substituting $d = 5$ into one equation to find a:

$a + 4(5) = 30 \Rightarrow a = 10$

Acres bought in 12th year

$a_{12} = a + 11d = 10 + 11(5)$acres

4. Option (C) is correct

Explanation: The formula for the sum of first n terms of an AP is:

$S_n = \frac{n}{2}[2a + (n-1)d$

Let, $S_{10} = 4290$ and $S_6 = 1830$

So,

$$S_{10} = \frac{10}{2}[2a+9d] = 5(2a+9d)$$
$$= 4290 \quad ...(1)$$
$$S_6 = \frac{6}{2}[2a+5d] = 3(2a+5d)$$
$$= 1830 \quad ...(2)$$

From (1) : $5(2a + 9d) = 4290$

$\Rightarrow 2a + 9d = 858 \quad ...(3)$

From (2) : $3(2a + 5d) = 1830$

$\Rightarrow 2a + 5d = 610 \quad ...(4)$

Now subtract (4) from (3) :

$(2a + 9d) - (2a + 5d) = 858 - 610$

$\Rightarrow 4d = 248 \Rightarrow d = 62$

Substitute $d = 62$ in equation (4) :

$2a + 5(62) = 610 \Rightarrow 2a + 310 = 610$

$\Rightarrow 2a = 300 \Rightarrow a = 150$

10th year collection:

$a_{10} = a + 9d = 150 + 9(62)$

$= 150 + 558 = 708$

5. Option (D) is correct

Explanation: Given, $a = -76$

Sum of first 45 terms: $S_{45} = -9360$

Using the sum formula for an AP

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

Substitute the values:

$$-9360 = \frac{45}{2}[2(-76)+44d]$$

$$\Rightarrow -9360 = \frac{45}{2}[-152+44d]$$

$-18720 = 45(-152 + 44d)$

$-416 = -152 + 44d$

$\Rightarrow 44d = -416 + 152 = -264$

$$\Rightarrow d = \frac{-264}{44} = -6$$

$a_{45} = a + 44d = -76 + 44(-6)$

$= -76 - 264 = -340$

How, $-416 + 76 = -340$

ASSERTION-REASON QUESTIONS (1 Mark)

1. Option (D) is correct

Explanation: Given:

First term a = ₹5 (saving in the first week)

Common difference d = ₹2 (increase per week)

Number of terms n = 20 (20 weeks)

Formula for sum of first n terms of an AP:

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

Total savings over 20 weeks

$$S_{20} = \frac{20}{2}[2(5)+(20-1)(2)]$$

$= 10[10 + 38]$

$= 10 \times 48 =$ ₹ 480

So, the total savings is ₹480, not ₹490.

Reason is true as the formula is correct.

2. Option (A) is correct

Explanation: Given, first month stipend: ₹100 and increment per month: ₹20

So, first term $a = 100$ and common difference $d = 20$

Using the formula for the nth term of an AP:

$a_n = a + (n-1)d$

$a_8 = 100 + (8 - 1) \times 20$

$= 100 + 7 \times 20$

$= 100 + 140 =$ ₹ 240

So the student does receive ₹240 in the 8th month. Assertion is true.

Reason is true as the formula $a_n = a + (n - 1)d$ is correct for the nth term of an AP.

3. Option (A) is correct

Explanation: First term $a = 1$, common difference $d = 1$ and number of terms $n = 15$

Using the AP sum formula to verify total steps in 15 days

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

Substituting the values:

$$S_{15} = \frac{15}{2}[2(1)+(15-1)(1)] = \frac{15}{2}[2+14]$$

$$= \frac{15}{2}\times 16 = 120$$

So, the total number of steps climbed after 15 days is 120. Assertion is true.

Reason is also true because the daily step counts form an AP with first term 1 and common difference 1.

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. First term $a = 24$, common difference $d = 4$ and number of terms $n = 15$

Using the formula for the nth term of an AP:

$a_n = a + (n - 1)d$

$a_{15} = 24 + (15 - 1) \times 4$

$= 24 + 14 \times 4 = 24 + 56 = 80$

So, the last row has 80 seats.

Total number of seats

Using the sum formula of an AP:

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$S_{15} = \frac{15}{2}[2(24)+(15-1)\times 4]$$

$$= \frac{15}{2}[48+56] = \frac{15}{2}\times 104 = 780$$

Seats in the last row are 80

Total number of seats are 780.

2. First term $a = 200$, common difference $d = 250 - 200$
$= 50$ and number of months $n = 6$

Amount saved in the 6th month

$$a_n = a + (n-1)d$$
$$a_6 = 200 + (6-1) \times 50$$
$$= 200 + 250 = ₹\ 450$$

Total saving in 6 months

Using the sum formula for an AP:

$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$S_6 = \frac{6}{2}[2(200)+(6-1)\times 50] = 3[400 + 250]$$
$$= 3 \times 650 = ₹\ 1950$$

Saving in the 6th month is ₹ 450

Total saving in 6 months is ₹ 1950

3. First term $a = 10$, common difference $d = 0.5$ and number of terms $n = 15$

Height of the 15th step

Using the nth term formula:

$$a_n = a + (n-1)d$$
$$= 10 + (15-1) \times 0.5$$
$$= 10 + 14 \times 0.5 = 10 + 7 = 17 \text{ cm}$$

Total height climbed after 15 steps

Using the sum of AP formula:

$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$S_{15} = \frac{15}{2}[2(10)+14\times 0.5] = \frac{15}{2}[20+7]$$
$$= \frac{15}{2}\times 27 = 202.5 \text{ cm}$$

Height of the 15th step is 17 cm

Total height climbed after 15 steps is 202.5 cm

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. First term $a = 500$, common difference $d = 100$

(i) Water supplied on the 15th day

Using the formula for the nth term of an AP:

$$a_n = a + (n-1)d$$
$$a_{15} = 500 + (15-1) \times 100$$
$$= 500 + 1400 = 1900 \text{ litres}$$

(ii) Total water supplied in 15 days

Using the sum formula:

$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$S_{15} = \frac{15}{2}[2(500)+14\times 100]$$
$$= \frac{15}{2}[1000+1400] = \frac{15}{2}\times 2400$$
$$= 18{,}000 \text{ litres}$$

(iii) Day on which supply reach 1400 litres:

Set the nth term equal to 1400:

$$a + (n-1)d = 1400$$
$$500 + (n-1) \times 100 = 1400$$
$$(n-1) \times 100 = 900$$
$$\Rightarrow \quad n - 1 = 9 \Rightarrow n = 10$$

Supply reaches 1400 litres on 10th day.

2. First term $a = 18{,}600$ (13th month's salary)

Common difference $d = 600$

(i) Salary in the 18th month

From 13th month onward, salary increases by ₹ 600 every month:

The 18th month is the 6th term of the AP

$$a_6 = a + (6-1)d$$
$$= 18{,}600 + 5 \times 600$$
$$= 18{,}600 + 3{,}000 = ₹\ 21{,}600$$

(ii) Salary in the 24th month

The 24th month is the 12th term of the AP

$$a_{12} = 18{,}600 + (12-1) \times 600$$
$$= 18{,}600 + 6{,}600 = ₹\ 25{,}200$$

(iii) Total increment received from 13th to 24th month

We'll calculate the sum of increments from 13th to 24th month.

First term $a = 18{,}600$

Common difference $d = 600$

Number of terms $n = 12$

Using the sum formula:

$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$S_{12} = \frac{12}{2}[2a+(12-1)d]$$
$$= 6[2(18{,}600) + 11 \times 600]$$
$$= 6[37{,}200 + 6{,}600]$$
$$= 6 \times 43{,}800 = 2{,}62{,}800$$

Now, salary without increments (₹ 18,000 per month for 12 months) would have been:

$$12 \times 18{,}000 = 2{,}16{,}000$$

So, total increment $= ₹\ 2{,}62{,}800 - ₹\ 2{,}16{,}000$
$= ₹\ 46{,}800$

3. First term $a = 10$ and common difference $d = -2$

(i) Temperature on the 7th day

Using the AP formula for the nth term:

$$a_n = a + (n-1)d$$
$$a_7 = 10 + (7-1)(-2)$$
$$= 10 + (-12) = -2°C$$

(ii) Days after which temperature will become –2°C

Set $a_n = -2$

$$10 + (n-1)(-2) = -2$$

$(n-1)(-2) = -12$

$\Rightarrow \quad n - 1 = 6$

$\Rightarrow \quad n = 7$

So, the temperature becomes –2°C on the 7th day.

(iii) Total drop in temperature over 10 days

Initial temperature: 10°C

Temperature on 10th day:

$a_{10} = 10 + (10-1)(-2)$

$= 10 - 18 = -8°C$

Total drop $= 10 - (-8) = 18°C$

4. First term $a = 120$ and common difference $d = 138 - 120 = 18$

(i) Bricks laid on the 10th day

Using the formula for the nth term:

$a_n = a + (n-1)d$

$a_{10} = 120 + (10-1) \times 18$

$= 120 + 162 = 282$ bricks

(ii) Bricks laid in 10 days

Using the sum of first n terms:

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$S_{10} = \frac{10}{2}[2(120)+9\times 18]$$

$= 5[240 + 162]$

$= 5 \times 402 = 2010$ bricks

(iii) Day on which 300 bricks will be laid:

Set $a_n = 300$

$120 + (n-1) \times 18 = 300$

$\Rightarrow \quad (n-1) \times 18 = 180$

$\Rightarrow \quad n - 1 = 10$

$\Rightarrow \quad n = 11$

300 bricks are laid on the 11th day.

CASE BASED QUESTIONS (4 Marks)

1. (i) First term $a = 3$, A.P. is 3, 6, 9, 12...., 24

Common difference $d = 6 - 3 = 3$

(ii) $34 = 3 + (n-1)\,3$

$$\Rightarrow \quad n = \frac{34}{3} = 11\frac{1}{3}$$

which is not a positive integer.

Therefore, it is not possible to have 34 jars in a layer if the given pattern is continued.

(iii) (a) Expression for finding total number of jars in terms of n

$$\Rightarrow \quad S_n = \frac{n}{2}[2a+(n-1)d]$$

where $a = 3$ and $d = 3$

$$\Rightarrow \quad S_n = \frac{n}{2}[2\times 3+(n-1)3]$$

$$= \frac{n}{2}[6+3n-3]$$

$$= \frac{n}{2}[3+3n]$$

Thus, $S_n = \frac{n}{2}[3+3n]$

Putting $n = 8$ in S_n

$$S_8 = 3\times\frac{8}{2}(1+8)$$

$= 108$

OR

(b) As, A.P. 3, 6, 9 24

After adding 3 jars in each layer

AP is : 3 + 3, 6 + 3, 9 + 3......

= 6, 9, 12....

Thus, $a = 6$ and $d = 9 - 6 = 3$

So, fifth term a_5 is

$a_n = a + (n-1)d$

$a_5 = 6 + (5-1)3$

$= 6 + 12$

$= 18$

Thus, there are 18 jars in the 5th layer.

2. (i) Amount paid in the 10th installment

The amount paid in the nth installment can be calculated using the formula for the nth term of an arithmetic progression (A.P.):

$a_n = a_1 + (n-1)d$

For the 10th installment:

As $a = 2000$ and $d = 200$

$\Rightarrow \quad a_{10} = 2000 + (10-1) \times 200$

$a_{10} = 2000 + 9 \times 200$

$a_{10} = 2000 + 1800 = ₹\ 3800$

(ii) Total amount paid in the first 10 installments:

The sum formula for the first n terms of an A.P.:

$$S_n = \frac{n}{2}.(2a_1+(n-1)d)$$

For the first 10 installments:

$$S_{10} = \frac{10}{2}\times(2\times 2000+(10-1)\times 200)$$

$S_{10} = 5 \times (4000 + 9 \times 200)$

$S_{10} = 5 \times (4000 + 1800)$

$S_{10} = 5 \times 5800 = ₹\ 29{,}000$

(iii) (a) Number of installments to clear the total loan

$$S_n = \frac{n}{2}\times(2a_1+(n-1)d) = 3{,}45{,}000$$

$$\frac{n}{2}\times(4000+(n-1)\times 200) = 3{,}45{,}000$$

$n \times (4000 + (n-1) \times 200) = 6{,}90{,}000$

$4000n + 200(n^2 - n) = 6{,}90{,}000$

$200n^2 - 200n + 4000n - 6,90,000 = 0$

$\Rightarrow \quad 200n^2 + 3800n - 6,90,000 = 0$

$n^2 + 19n - 3450 = 0$

$n^2 + 69n - 50n - 3450 = 0$

$n(n + 69) - 50(n + 69) = 0$

$(n + 69)(n - 50) = 0$

$\Rightarrow \quad n = -69 \;\&\; n = 50$

Thus, he will clear his total loan in 50 installments.

OR

(b) Amount cleared in the first 45 installments

$$S_{45} = \frac{45}{2} \times (2a_1 + (45-1)d)$$

$$S_{45} = \frac{45}{2} \times (2 \times 2000 + (44) \times 200$$

$$S_{45} = \frac{45}{2} \times 12800$$

$$S_{45} = 45 \times 6400 = ₹\ 2,88,000.$$

Thus, amount he will be able to clear is ₹ 2,88,000

3. (i) As per question

Number of throws with which Sanjitha started

$= a = 40$

and, Increases by = 12 throws

$\therefore \quad d = 12$

$\Rightarrow \quad t_{11} = [a + (11-1)d]$

$= 40 + 10 \times 12$

$= 160$ throws

(ii) (a) As Sanjitha started with 7.56 m throw and was able to improve by 9 cm every week.

$\therefore \quad a = 7.56$ m, $d = 9$ cm $= 0.09$m

$n = 7$

($\because$ End of 6 weeks is given)

$a_n = [a + (n-1)d]$

$\therefore \quad a_7 = [7.56 + (7-1) \times 0.09]$

$= [7.56 + 6 \times 0.09]$

$= 7.56 + 0.54$

$= 8.1$ m

OR

(b) Here, $a = 7.56$ m, $d = 0.09$ m

$T_n = 11.16$

As, $\quad T_n = a + (n-1)d$

$11.16 = 7.56 + (n-1)\,0.09$

$3.60 = (n-1)0.09$

$40 + 1 = n$

$n = 41$

Thus, Sanjitha will be able to throw 11.16 m at the beginning of 41 week.

(iii) Here, $a = 40$, $d = 12$, $n = 15$

As, $$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{15}{2}[2 \times 40 + (15-1)12]$$

$$= \frac{15}{2}[80 + 168]$$

$$= \frac{15}{2} \times 248 = 15 \times 124$$

$= 1860$ throws

4. **(i)** First term $a = 20$ and common difference $d = 5$

Total number of seats in the 10th row

Using the formula for the nth term of an AP:

$a_n = a + (n-1)d$

$a_{10} = 20 + (10-1) \times 5$

$= 20 + 45 = 65$ seats

(ii) Expression for the number of seats in the nth row

$a_n = 20 + (n-1) \times 5$

$\Rightarrow \quad a_n = 5n + 15$

(iii) (a) Total number of seats in the first 20 rows

Using the formula for the sum of first n terms:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{20} = \frac{20}{2}[2(20) + 19 \times 5]$$

$= 10[40 + 95]$

$= 10 \times 135 = 1350$ seats

OR

(b) Row that will have exactly 95 seats:

Set $a_n = 95$

$20 + (n-1) \times 5 = 95$

$\Rightarrow \quad (n-1) \times 5 = 75$

$\Rightarrow \quad n - 1 = 15 \Rightarrow n = 16$

The 16th row has 95 seats.

5. **(i)** First term $a = 500$ and common difference $d = 100$

Amount received in the 5th month

Using the formula for the nth term of an AP:

$a_n = a + (n-1)d$

$a_5 = 500 + (5-1) \times 100$

$\Rightarrow \quad 500 + 400 = ₹\ 900$

(ii) General term for the nth month

$a_n = 500 + (n-1) \times 100$

$\Rightarrow \quad a_n = 100n + 400$

(iii) (a) Total amount in 12 months

Using sum formula:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{12} = \frac{12}{2}[2(500) + 11 \times 100]$$

$= 6[1000 + 1100]$

$= 6 \times 2100 = ₹\ 12,600$

OR

(b) Month in which the amount is ₹ 1100

$a_n = 100n + 400 = 1100$

$\Rightarrow \quad 100n = 700 \Rightarrow n = 7$

₹ 1100 is received in the 7th month.

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. First term (a) = ₹ 2000, Common difference (d) = –₹ 50 and Number of terms (n) = 15

(i) Amount received by the 15th student

Using the formula for the nth term of an AP:

$$a_n = a + (n-1)d$$
$$a_{15} = 2000 + (15-1)(-50)$$
$$= 2000 - 700 = ₹\,1300$$

(ii) Total amount distributed among all 15 students

Use the formula for the sum of n terms in an AP:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$S_{15} = \frac{15}{2}[2(2000) + 14(-50)]$$
$$= \frac{15}{2}[4000 - 700] = \frac{15}{2} \times 3300$$
$$= ₹\,24,750$$

(iii) Yes. Required: ₹ 24,750

Budget: ₹ 25,000

Yes, the school has enough funds (₹ 250 left).