CHAPTER 6

Coordinate Geometry

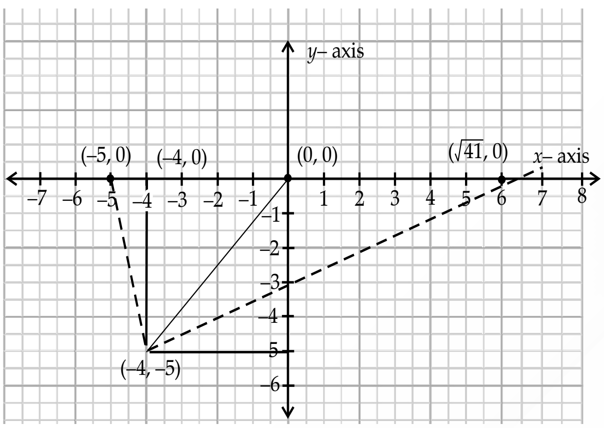
Level - 1

CORE SUBJECTIVE QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Mark)

1. Option (B) is correct

Explanation:



As, the perpendicular (⊥) distance drawn from a point to the line is the shortest distance.

∴ (–4, 0) is the nearest point the x-axis from point (–4, –5).

2. Option (C) is correct

Explanation: A line segment is divided by a point P(x, 0) in the ratio $m : n$:

The coordinates of P are given by:

$$P\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

Here, A(2, –3) and B(5, 6) and the ratio is 1 : 2.

Then the x-coordinates are:

$$x = \frac{1\times5+2\times2}{1+2} = \frac{5+4}{3} = \frac{9}{3}$$

$$= 3$$

The y-coordinates are:

$$y = \frac{1\times6+2\times(-3)}{1+2} = \frac{6-6}{3}$$

$$= \frac{0}{3} = 0$$

Thus, the point is (3, 0).

3. Option (B) is correct

Explanation: Using distance formula:

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$15^2 = (x - 3)^2 + (-5 + 5)^2$$

$$\Rightarrow (x - 3)^2 = 15^2$$

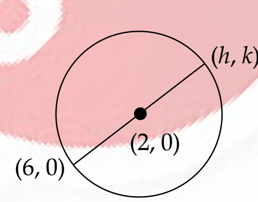
$$\Rightarrow x - 3 = \pm 15$$

$$\Rightarrow x = 18$$

$$\text{or} \quad x = -12$$

4. Option (C) is correct

Explanation: Let one end of diameter of circle be (h, k)



$$\Rightarrow 2 = \frac{6+h}{2}$$

$$\text{and} \quad \frac{0+k}{2} = 0$$

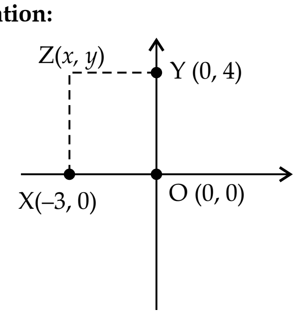
$$\therefore h = -2$$

$$\text{and} \quad k = 0$$

Hence, other end is (–2, 0).

5. Option (A) is correct

Explanation:



Since diagonals of rectangle are equal in length,

So, $OZ = XY$

$$XY = \sqrt{(0+3)^2+(4-0)^2} = \sqrt{9+16} = 5$$

Thus, length of diagonals

$$XY = OZ = 5 \text{ units}$$

6. Option (C) is correct

Explanation: A(4, –5) and B (1, 2) point P divides AB in the ratio 5 : 2.

Then, the coodinates of P are:

$$x = \frac{5\times1+2\times4}{5+2} = \frac{13}{7}$$

$$y = \frac{5\times2+2\times-5}{5+2} = 0$$

Hence, coordinates of P are $\left(\frac{13}{7}, 0\right)$

7. Option (B) is correct

Explanation:

$$D = \sqrt{(a\sin\theta - a\cos\theta)^2+(a\cos\theta+a\sin\theta)^2}$$

$$D = \sqrt{a^2\sin^2\theta + a^2\cos^2\theta - 2a^2\sin\theta\cos\theta + a^2\cos^2\theta + a^2\sin^2\theta + 2a^2\sin\theta\cos\theta}$$

$$D = \sqrt{2a^2\sin^2\theta + 2a^2\cos^2\theta}$$

$$D = a\sqrt{2(\sin^2\theta+\cos^2\theta)}$$

$$D = a\sqrt{2\times1}$$

$$D = a\sqrt{2} \text{ units}$$

8. Option (B) is correct

Explanation: Let the fourth vertex of the parallelogram, D = (x_4, y_4) and L, M be the middle points of AC and BD, respectively,

Then, $L \equiv \left(\frac{-2+8}{2}, \frac{3+3}{2}\right) = (3, 3)$

and $M \equiv \left(\frac{6+x_4}{2}, \frac{7+y_4}{2}\right)$

$\left[\text{Since mid-point of a line segment having points } (x_1, y_1) \text{ and } (x_2, y_2) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\right]$

Since, ABCD is a parallelogram, therefore diagonals AC and BD will bisect each other.

Hence, L and M are the same points.

$\therefore$ $3 = \frac{6+x_4}{2}$ and $3 = \frac{7+y_4}{2}$

$\Rightarrow$ $6 = 6 + x_4$ and $6 = 7 + y_4$

$\Rightarrow$ $x_4 = 0$ and $y_4 = 6 - 7$

$\therefore$ $x_4 = 0$ and $y_4 = -1$

Hence, the fourth vertex of the parallelogram D = (x_4, y_4) = (0, –1)

9. Option (D) is correct

Explanation: Since PQ is a diameter, the coordinates of Q can be found by using the fact that the diameter is twice the radius. Let's use the midpoint formula to find the coordinates of Q:

$$\text{Midpoint of PQ} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

where (x_1, y_1) = (–4, 5) (coordinates of P) and $(x_2, y_2) = (x, y)$ (coordinates of Q)

Since O(2, –4) is the midpoint of PQ, we can set up the equation:

$$\left(\frac{x-4}{2}, \frac{y+5}{2}\right) = (2, -4)$$

Solving for x and y, we get :

$$x - 4 = 4 \Rightarrow x = 8$$

$$y + 5 = -8 \Rightarrow y = -13$$

So, the coordinates of point Q are (8, –13).

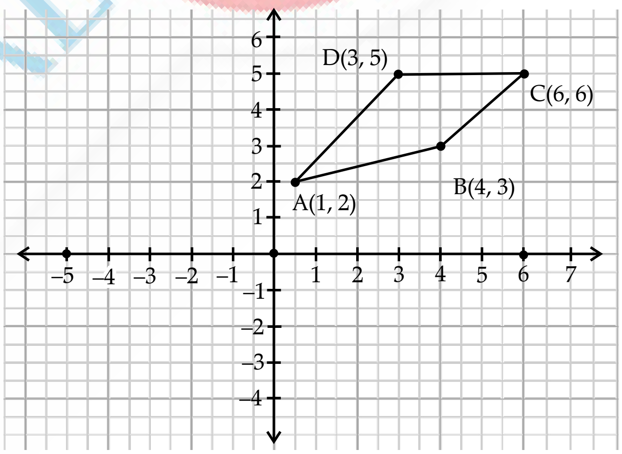
10. Option (B) is correct

Explanation:

$$\text{Radius of circle} = \sqrt{(-2.4+4)^2+(1.8-3)^2} = \sqrt{1.6^2+(-1.2)^2} = \sqrt{2.56+1.44} = \sqrt{4.00} = 2 \text{ units}$$

11. Option (D) is correct

Explanation:



Let the coordinates of fourth vertex D be (h, k)

Diagonals of a parallelogram bisects each other.

$$\Rightarrow \left(\frac{1+6}{2}, \frac{2+6}{2}\right) = \left(\frac{4+h}{2}, \frac{3+k}{2}\right)$$

$\Rightarrow$ $h = 3$

and $k = 5$

Hence, the coordinates of vertex D are (3, 5)

12. Option (B) is correct

Explanation: Let the coordinates of point B be (h, k)

ATQ,

$$\sqrt{10^2+7^2} = \sqrt{(h+4)^2+(k-3)^2}$$

$\Rightarrow$ $10^2 + 7^2 = (h + 4)^2 + (k - 3)^2$

Since point B lies in first quadranrt,

Case II: $h + 4 = 10$

and $k - 3 = 7$

$h = 6$

$k = 10$

$\therefore$ Coordinates of point B is (6, 10)

Case II: $h + 4 = 7$

and $k - 3 = 10$

$h = 3$

and $k = 13$

$\therefore$ Coordinates of point B is (3, 13).

So, possible X-coordinates of point B are 3 and 6.

Both 3 and 6 are multiples of 3

13. Option (A) is correct

Explanation: Let the line joining points A (2, –3) and B (5, 6) be divided by point P(x, 0) in the ratio $k : 1$.

$$y = \frac{ky_2+y_1}{k+1}$$

$$0 = \frac{k\times 6+1\times(-3)}{k+1}$$

$$0 = 6k - 3$$

$$k = \frac{1}{2}$$

Thus, the required ratio is 1 : 2.

14. Option (D) is correct

Explanation: Let the x-axis divideds the line segment joining the points A (3, 6) and B (–12, –3) in the ratio $k : 1$

Let the point of intersection at x-axis be (x, 0).

$$0 = \frac{k\times -3+1\times 6}{k+1}$$

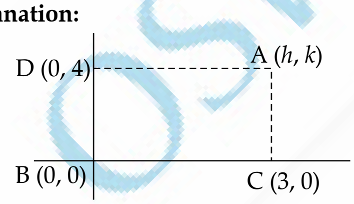
$$-3k + 6 = 0$$

$\Rightarrow$ $k = 2$

Hence, required ratio is 2 : 1.

15. Option (C) is correct

Explanation:



Let the coordinates of vertex A be (h, k).

We know that diagonals of a rectangle bisect each other

$$\left(\frac{0+h}{2},\frac{0+k}{2}\right) = \left(\frac{0+3}{2},\frac{4+0}{2}\right)$$

$\Rightarrow$ $h = 3$

and $k = 4$

Hence, coordinates of vertex A (3, 4).

16. Option (D) is correct

Explanation: To find the distance between the points $P\left(-\frac{11}{3},5\right)$ and Q $\left(-\frac{2}{3},5\right)$, we can use the distance formula:

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Since the y-coordinates are the same (both are 5), the formula simplifies to:

$$d = \sqrt{\left(\frac{-2}{3}-\frac{-11}{3}\right)^2+(5-5)^2} = \sqrt{\left(\frac{-2+11}{3}\right)^2}$$

$$= \sqrt{\left(\frac{9}{3}\right)^2} = \sqrt{3^2}$$

= 3 units

17. Option (B) is correct

Explanation: The distance of any point (x, y) from the x-axis is simply the absolute value of its y-coordinate.

For the point (–1, 7), the distance from the x-axis is:

Distance = $|y|$ = $|7|$ = 7 units

18. Option (D) is correct

Explanation: The distance of a point (x, y) from the origin is given by the formula:

$$d = \sqrt{x^2+y^2}$$

For the point (–6, 8):

$$d = \sqrt{(-6)^2+8^2} = \sqrt{36+64}$$

$$= \sqrt{100} = 10 \text{ units}$$

19. Option (A) is correct

Explanation: The distance of any point (x, y) from the x-axis is the absolute value of its y-coordinate.

For the point (4, 7), the distance from the x-axis is:

Distance = $|y|$ = $|7|$ = 7 units

20. Option (A) is correct

Explanation: The midpoint formula is:

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right) = (3, -2)$$

Substitute the values for B (7, 4) and C (3, –2):

$$\frac{x_1+7}{2} = 3 \text{ and } \frac{y_1+4}{2} = -2$$

Solve for x_1 and y_1:

$$\frac{x_1+7}{2} = 3 \Rightarrow x_1 + 7 = 6 \Rightarrow x_1 = -1$$

$$\frac{y_1+4}{2} = -2 \Rightarrow y_1 + 4 = -4 \Rightarrow y_1 = -8$$

So, the coordinates of point A are (–1, –8).

21. Option (B) is correct

Explanation: Given that AB is a chord of a circle with center O (2, 3), and the coordinates of A and B are A (4, 3) and B (x, 5), respectively.

Therefore, OA = OB (Radius)

By using distance formula

$$\sqrt{(4-2)^2+(3-3)^2} = \sqrt{(x-2)^2+(5-3)^2}$$

$$\Rightarrow \quad \sqrt{2^2+0^2} = \sqrt{(x-2)^2+2^2}$$

By squaring both sides we get

$$4 = (x-2)^2 + 4$$

$$\Rightarrow \quad (x-2)^2 = 0$$

$$\Rightarrow \quad x = 2 \text{ units}$$

22. Option (C) is correct

Explanation: As point A lies 2 units to the left of origin So its x – coordinate = –2

Now, in II[nd] Rectangle CDAE

$$\text{Breadth} = \frac{\text{Length}}{2} \text{(Given)}$$

$$\Rightarrow \quad \text{Length} = 2 \times \text{Breadth (CD)}$$

$$\Rightarrow \quad \text{Length (DA)} = 2 \times \text{Breadth (CD)}$$

$$\text{DA} = 2 \times -2$$

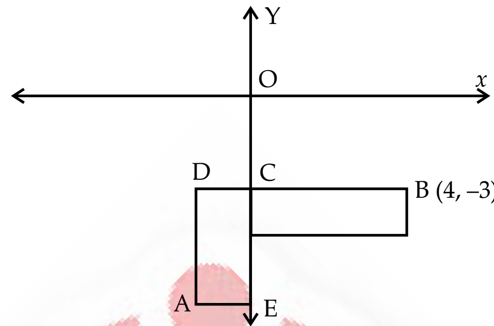
$$\text{DA} = -4$$

Thus, point A y-coordinate from Origin = [–3 + (–4)]

$$= -3 - 4$$

$$= -7$$

∴ Co-ordinates of point A = (–2, –7)



ASSERTION-REASON QUESTIONS (1 Mark)

1. Option (D) is correct

Explanation: We will use the section formula to verify Assertion (A). The section formula for the point that divides a line segment in the ratio $m_1 : m_2$ is:

$$P(x, y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

Substitute the given values for A(1, 2), B (–1, 1), and the ratio 1 : 2:

$$x = \frac{1(-1)+2(1)}{1+2} = \frac{-1+2}{3} = \frac{1}{3}$$

$$y = \frac{1(1)+2(2)}{1+2} = \frac{1+4}{3} = \frac{5}{3}$$

So, the coordinates of the point that divides the line segment are $\left(\frac{1}{3}, \frac{5}{3}\right)$.

However, Assertion (A) claims the point is $\left(-\frac{1}{3}, \frac{5}{3}\right)$ which is incorrect.

∴ Assertion (A) is false but Reason (R) is true and provides the correct formula for dividing a line segment.

2. Option (C) is correct

Explanation: Assertion (A): The midpoint of a line segment divides the line segment in the ratio 1 : 1. This is true because, by definition, the midpoint of a line segment divides it into two equal halves, meaning the ratio is 1 : 1.

Reason (R): It can be verified by using section formula:

$$P(x, y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

For A(–5, 4), B(–2, 3), and P(–3, k),the ratio $m_1 : m_2$ can be calculated using the x coordinate:

$$-3 = \frac{m_1(-2)+m_2(-5)}{m_1+m_2}$$

$$-3(m_1 + m_2) = -2m_1 - 5m_2$$

$$-3m_1 - 3m_2 = -2m_1 - 5m_2$$

$$-m_1 + 2m_2 = 0$$

$$m_1 = 2m_2$$

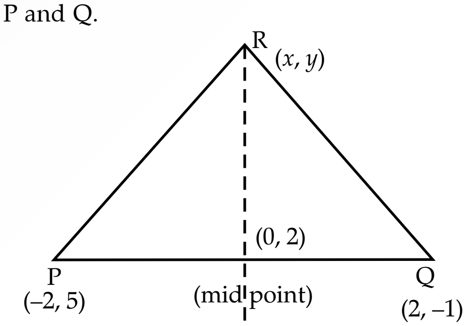
Thus, the ratio is $m_1 : m_2$ = 2 : 1, not 1 : 2.

∴ Assertion (A) is true but Reason (R) is false, as the correct ratio is 2 : 1, not 1 : 2.

3. Option (D) is correct

Explanation: Assertion (A): The midpoint (0, 2) is the only point equidistant from points P and Q.

This statement is false. As, the midpoint is indeed one point that is equidistant from P and Q, but it is not the only point. There are infinitely many points that are equidistant from P and Q lying on the perpendicular bisector of the line segment joining P and Q.



Hence, Assertion is false.

Reason (R): There are many points (x, y) where $(x + 2)^2 + (y - 5)^2 = (x - 2)^2 + (y + 1)^2$ are equidistant from P and Q.

This statement is true.

By using distance formula :

$$PR = RQ$$

$$\sqrt{(x+2)^2+(y-5)^2} = \sqrt{(x-2)^2+(y+1)^2}$$

$$(x + 2)^2 + (y - 5)^2 = (x - 2)^2 + (y + 1)^2$$

This shows that the given equation represents the set of points that are equidistant from points P and Q. Specifically, it is the equation of the perpendicular bisector of the line segment connecting P and Q.

∴ Assertion (A) is false because there are multiple points equidistant from P and Q, not just the midpoint but Reason (R) is true because the equation describes all points equidistant from P and Q.

4. Option (B) is correct

Explanation: Assertion (A): Point P(0, 2) is the point of intersection of the y–axis with the line $3x + 2y = 4$.

This statement is true. To find the intersection of the line with the y–axis, we set $x = 0$ in the equation:

$$3(0) + 2y = 4 \Rightarrow 2y = 4 \Rightarrow y = 2$$

Thus, the point of intersection is (0, 2).

Reason (R) : The distance of point P(0, 2) from the x–axis is 2 units.

This statement is also true. As, the distance of any point (x, y) from the x–axis is given by the absolute value of its y–coordinate. For point P(0, 2) :

$$\text{Distance from } x\text{-axis} = |y| = |2| = 2 \text{ units}$$

∴ Both the assertion and the reason are true statements, but the reason does not correctly explains the assertion.

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. Let the required point be $(x, 0)$

$$\sqrt{(8-x)^2+25} = \sqrt{41}$$

On squaring both the sides we get,

$\Rightarrow \quad (8 - x)^2 = 16$

$\Rightarrow \quad 8 - x = \pm 4$

$\Rightarrow \quad x = 4, 12$

Two points on the x-axis are (4, 0) and (12, 0).

2. Using distance formula we get,

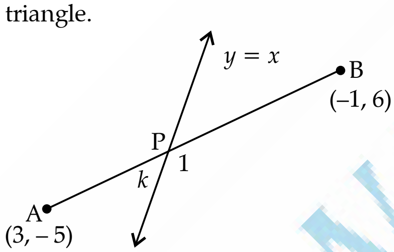
$$AB = \sqrt{(3+5)^2+(0-6)^2} = 10$$

$$BC = \sqrt{(9-3)^2+(8-0)^2} = 10$$

$$AC = \sqrt{(9+5)^2+(8-6)^2} = 10\sqrt{2}$$

Since AB = BC, therefore, ΔABC is an isosceles triangle.

3.



Let the required ratio be $k : 1$

Coordinates of point P are $\left(\dfrac{-k+3}{k+1}, \dfrac{6k-5}{k+1}\right)$

Point P lies on line $y = x$

$\Rightarrow \quad \dfrac{-k+3}{k+1} = \dfrac{6k-5}{k+1}$

$\Rightarrow \quad -k - 6k = -5 - 3$

$\Rightarrow \quad 7k = 8$

$\Rightarrow \quad k = \dfrac{8}{7}$

∴ Required ratio is 8 : 7

4. The co-ordinates of midpoint E of side AC is given by the midpoint formula:

$$E = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Substitute the coordinates of A (3, 0) and C (–1, 3):

$$E = \left(\frac{3+(-1)}{2}, \frac{0+3}{2}\right) = \left(\frac{2}{2}, \frac{3}{2}\right)$$

$$= (1, 1.5)$$

Now, use the distance formula to find the length of the median BE:

$$BE = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Substitute the coordinates of B(6, 4) and E(1, 1.5):

$$BE = \sqrt{(6-1)^2+(4-1.5)^2} = \sqrt{(5)^2+(2.5)^2}$$

$$= \sqrt{25+6.25} = \sqrt{31.25}$$

$BE \approx 5.59$ units

5. Given, the point P (x, y) is equidistant from A (7, 1) and B (3, 5).

So, $\quad AP = BP$

$\Rightarrow \quad AP^2 = BP^2$

$\Rightarrow \quad (x - 7)^2 + (y - 1)^2 = (x - 3)^2 + (y - 5)^2$

$\Rightarrow \quad -14x - 2y + 50 = -6x - 10y + 34$

$\Rightarrow \quad 8x - 8y = 16$

$\Rightarrow \quad x - y = 2$

6. The given points are A$(-1, y)$, B (5, 7) and O $(2, -3y)$.

Here, AO and BO are the radii of the circle.

So $\quad AO = BO \Rightarrow AO^2 = BO^2$

$\Rightarrow \quad (2 + 1)^2 + (-3y - y)^2 = (2 - 5)^2 + (-3y - 7)^2$

$\Rightarrow \quad 9 + (-4y)^2 = (-3)^2 + (-3y - 7)^2$

$\Rightarrow \quad 9 + 16y^2 = 9 + 9y^2 + 49 + 42y$

$\Rightarrow \quad 7y^2 - 42y - 49 = 0$

$\Rightarrow \quad y^2 - 6y - 7 = 0$

$\Rightarrow \quad y^2 - 7y + y - 7 = 0$

$\Rightarrow \quad y(y - 7) + 1(y - 7) = 0$

$\Rightarrow \quad (y - 7)(y + 1) = 0$

$\Rightarrow \quad y = -1$ or $y = 7$

Hence, $y = 7$ or $y = -1$

Now, Radius of circle OA is:

If $y = 7$

Coordinates of A (–1, 7)

and, Coordinates of O (2, –21)

∴ Radius $\quad OA = \sqrt{(2+1)^2+(-21-7)^2}$

$$= \sqrt{(3)^2 + (-28)^2}$$
$$= \sqrt{9 + 784}$$
$$= \sqrt{793}$$
$$= 28.16 \text{ units (approx)}$$

If $y = -1$

Coordinates of A (–1, –1)

Coordinates of O (2, 3)

$$\therefore \text{ Radius } \quad OA = \sqrt{(2+1)^2 + (3+1)^2}$$
$$= \sqrt{3^2 + 4^2}$$
$$= \sqrt{9 + 16}$$
$$= \sqrt{25}$$
$$= 5 \text{ units}$$

7. Point dividing a line in the ratio 1 : m formed by joining (x_1, y_1) and (x_2, y_2) is given by

$$x = \frac{mx_1 + x_2}{1+m} \; y = \frac{my_1 + y_2}{1+m}$$

We have the points A (–6, 10) and B (3, –8).

Let the point (–4, 6) divide the line joining AB in the ratio 1 : m.

$$-4 = \frac{-6m + 3}{1+m}$$
$$-4 - 4m = -6m + 3$$
$$2m = 7$$
$$m = \frac{7}{2}$$
$$1 : m = 1 : \frac{7}{2} = 2 : 7$$

So the point divides the line in the ratio 2 : 7.

8. Let A (3, 0), B (6, 4) and C (–1, 3) be the given points.

Now,

$$AB = \sqrt{(6-3)^2 + (4-0)^2} = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25}$$
$$BC = \sqrt{(-1-6)^2 + (3-4)^2} = \sqrt{(-7)^2 + (-1)^2} = \sqrt{49+1} = \sqrt{50}$$
$$AC = \sqrt{(-1-3)^2 + (3-0)^2} = \sqrt{(-4)^2 + 3^2} = \sqrt{16+9} = \sqrt{25}$$

$\therefore$ AB = AC

$$AB^2 = \left(\sqrt{25}\right)^2 = 25$$
$$BC^2 = \left(\sqrt{50}\right)^2 = 50$$
$$AC^2 = \left(\sqrt{25}\right)^2 = 25$$

$\therefore AB^2 + AC^2 = BC^2$

Thus, ΔABC is a right angled isosceles triangle.

9. Let the given points be A (–2, 3), B (8, 3) and C (6, 7)

Using distance formula

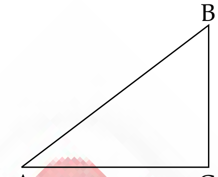
$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$AB = \sqrt{(8+2)^2 + (3-3)^2}$$
$$= 10 \text{ units}$$
$$BC = \sqrt{(6-8)^2 + (7-3)^2}$$
$$= \sqrt{20} \text{ units}$$
$$CA = \sqrt{(6+2)^2 + (7-3)^2}$$
$$= \sqrt{80} \text{ units}$$

For right angled triangle, having $\angle C = 90°$

$$BC^2 + CA^2 = AB^2$$

(By Pythagoras theorem)

$$\text{as} \quad \sqrt{20}^2 + \sqrt{80}^2 = 10^2$$
$$20 + 80 = 100$$
$$100 = 100$$



$\therefore \Delta ABC$ is a right angled triangle having right angle at vertex C.

10. We are given that R(2, 5) is the midpoint of line segment PQ. The midpoint formula is:

$$R = \left(\frac{x_1 + 0}{2}, \frac{y_1 + 0}{2}\right)$$

Substituting the coordinates of R(2, 5):

$$\left(\frac{x_1}{2}, \frac{y_1}{2}\right) = (2, 5)$$

From the x–coordinate:

$$\frac{x_1}{2} = 2 \Rightarrow x_2 = 4$$

From the y–coordinate:

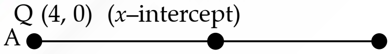
$$\frac{y_1}{2} = 5 \Rightarrow y_1 = 10$$

The coordinates of points P and Q are:

P (0, 10) (y–intercept)

Q (4, 0) (x–intercept)

11.



Let the point P (–1, k) divides the line segment AB joining (–3, 10) and (6, –8) in ratio m : 1.

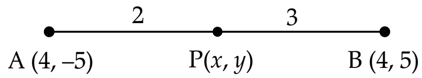
$$\text{Using} \quad x = \frac{m_1x_2 + m_1x_2}{m_1m_2}$$
$$\Rightarrow \quad -1 = \frac{m \times 6 + 1 \times -3}{m+1}$$
$$\Rightarrow \quad -m - 1 = 6m - 3$$
$$\Rightarrow \quad 2 = 7m$$
$$\Rightarrow \quad m = \frac{2}{7}$$

So the required ratio is 2 : 7

$$\text{Now using} \quad y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$
$$k = \frac{2 \times -8 + 7 \times 10}{2+7}$$
$$= \frac{54}{9}$$
$$k = 6$$

12. Given $\frac{AP}{AB} = \frac{2}{5}$

$$\frac{AP}{PB} = \frac{2}{3}$$



$$m = 2$$
$$n = 3$$

Let P (x, y)

$$x = \frac{(m \times x_2 + n \times x_1)}{(m+n)}$$

$$y = \frac{(my_2 + ny_1)}{(m+n)}$$

$$x = \frac{(2 \times 4 + 3 \times 4)}{5} = \frac{20}{5} = 4$$

$$y = \frac{(2 \times 5 - 3 \times 5)}{5} = \frac{-5}{5} = -1$$

∴ The co–ordinates of P (4, –1)

13. The distance d between two points (x_1, y_1) and (x_2, y_2) is given by the formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

The three given points are P(x, y), A(5, 1) and B(1, 5). Now let us find the distance between ' P ' and ' A '.

$$PA = \sqrt{(x-5)^2 + (y-1)^2}$$

Now, let us find the distance between 'P' and 'B'.

$$PB = \sqrt{(x-1)^2 + (y-5)^2}$$

It is given that both these distances are equal. So, let us equate both the above equations.

$$PA = PB$$

$$\sqrt{(x-5)^2 + (y-1)^2} = \sqrt{(x-1)^2 + (y-5)^2}$$

On squaring both sides of the equation we get,

$$(x-5)^2 + (y-1)^2 = (x-1)^2 + (y-5)^2$$

$$\Rightarrow x^2 + 25 - 10x + y^2 + 1 - 2y = x^2 + 1 - 2x + y^2 + 25 - 10y$$

$$\Rightarrow 26 - 10x - 2y = 26 - 10y - 2x$$

$$\Rightarrow 10y - 2y = 10x - 2x$$

$$\Rightarrow 8y = 8x$$

$$\Rightarrow y = x$$

Hence, Proved.

14. Let A (0, –1), B(6, 7), C (–2, 3) and D (8, 3) be the given points. Then,

$$AB = \sqrt{(6-0)^2 + (7+1)^2} = \sqrt{36+64} = 10 \text{ units}$$

$$CD = \sqrt{(8+2)^2 + (3-3)^2} = \sqrt{100+0} = 10 \text{ units}$$

$$AD = \sqrt{(8-0)^2 + (3+1)^2} = \sqrt{64+16} = 4\sqrt{5} \text{ units}$$

$$BC = \sqrt{(6+2)^2 + (7-3)^2} = \sqrt{64+16} = 4\sqrt{5} \text{ units}$$

∴ AB = CD and AD = BC

So, ABCD is a parallelogram

Now, $AC = \sqrt{(-2-0)^2 + (3-(-1)^2} = \sqrt{4+16} = 2\sqrt{5}$ units

and $BD = \sqrt{(8-6)^2 + (3-7)^2} = \sqrt{4+16} = 2\sqrt{5}$ units

Clearly, opposite sides are equal and diagonals are equal

Hence, ABCD is a rectangle.

15. A (6, 4), B(5, –2), C(7, –2)

$$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$= \sqrt{1^2 + 6^2} = \sqrt{37} \text{ units}$$

$$BC = \sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2}$$
$$= \sqrt{(5-7)^2 + 0^2} = 2 \text{ units}$$

$$CA = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$$
$$= \sqrt{1^2 + 6^2} = \sqrt{37} \text{ units}$$

Thus, AB = AC = $\sqrt{37}$

As, two sides of a triangle are equal in length. therefore, triangle is an isosceles triangle.

16. Distance from the center (origin) to the point (3, –5)

The formula for the distance between two points (x_1, y_1) and (x_2, y_2) is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here, the center of the circle is at (0, 0), and the point is (3, –5):

$$d = \sqrt{(3-0)^2 + (-5-0)^2} = \sqrt{3^2 + (-5)^2} = \sqrt{9+25}$$
$$= \sqrt{34}$$

Compare the distance with the radius

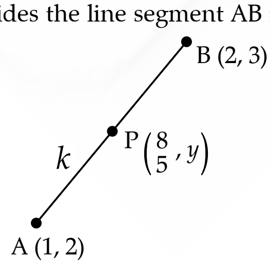
The distance from the center to the point is $\sqrt{34}$ ⊕5.83 units.

The radius of the circle is 5 units.

Since $\sqrt{34}$ (approximately 5.83) is greater than 5, the point (3, –5) lies outside the circle.

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. Let P divides the line segment AB in the ratio $k : 1$



Then by section formula

$$\frac{8}{5} = \frac{2k+1}{k+1} \quad \text{...(i)}$$

and $$y = \frac{3k+2}{k+1} \quad \text{...(ii)}$$

By solving (i) we get,

$$8k + 8 = 10k + 5$$
$$3 = 2k$$

$$\Rightarrow \qquad k = \frac{3}{2}$$

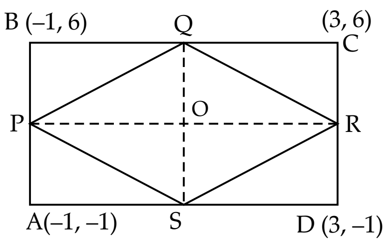
Substitute values of k is (ii)

$$y = \frac{3 \times \frac{3}{2} + 2}{\frac{3}{2} + 1}$$

$$= \frac{9+4}{5} = \frac{13}{5}$$

$\therefore$ Required ratio is 3 : 2 and value of $y = \frac{13}{5}$

2.



Coordinates of

$$P = \left(\frac{-1-1}{2}, \frac{-1+6}{2}\right)$$

$$= \left(-1, \frac{5}{2}\right)$$

$$\text{Coordinates of Q} = \left(\frac{-1+3}{2}, \frac{6+6}{2}\right)$$

$$= (1, 6)$$

$$\text{Coordinates of R} = \left(\frac{3+3}{2}, \frac{6-1}{2}\right)$$

$$= \left(3, \frac{5}{2}\right)$$

$$\text{Coordinates of S} = \left(\frac{3-1}{2}, \frac{-1-1}{2}\right)$$

$$= (1, -1)$$

Now, Midpoints of P & R which is point O.

$$x = \frac{-1+3}{2} = 1$$

$$y = \frac{\frac{5}{2} + \frac{5}{2}}{2} = \frac{5}{2}$$

$$\Rightarrow \qquad O(x, y) = \left(1, \frac{5}{2}\right)$$

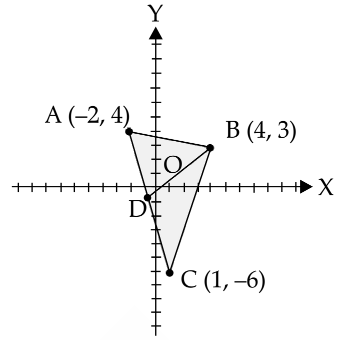
Similarly the mid-point of S and Q

$$\Rightarrow \qquad x = \frac{1+1}{2} = 1$$

$$y = \frac{6-1}{2} = \frac{5}{2}$$

Since the midpoints of PR & QS both have the same coordinate $\left(1, \frac{5}{2}\right)$ Hence, diagonals PR and SQ bisect each other.

3. Let's plot the points on the graph.



BD is the median of AC

$\therefore$ D is the midpoint of AC

We know that the coordinates of mid point of 2 coordinates (x_1, y_1) and $(x_2\ y_2)$ are given by:

$$(x, y) = \left(\frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}\right)$$

$\therefore$ Value of coordinates of midpoint D of A (–2, 4) and C (1, –6) are

$$(x, y) = \left(\frac{1+(-2)}{2}, \frac{(-6)+4}{2}\right)$$

$$= \left(\frac{-1}{2}, -1\right)$$

Now, Distance formula will be used to calculate length of BD, where B is (4, 3) and D is $\left(\frac{-1}{2}, -1\right)$

$$BD = \sqrt{\left(\frac{-1}{2} - 4\right)^2 + (-1-3)^2}$$

$$= \sqrt{\left(\frac{81}{4} + 16\right)}$$

$$= \sqrt{20.25 + 16} = \sqrt{36.25}$$

$$= 6.02$$

Therefore, the length of the median BD is 6.02 units.

4. The section formula for the point $P(x, y)$ dividing the line segment joining points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m_1 : m_2$ is:

$$P(x, y) = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$$

Here, P(0, y), A(5, 3), and B (–1, 6)

For the x–coordinate, we have:

$$0 = \frac{m_1(-1) + m_2(5)}{m_1 + m_2}$$

$$0 = \frac{-m_1 + 5m_2}{m_1 + m_2}$$

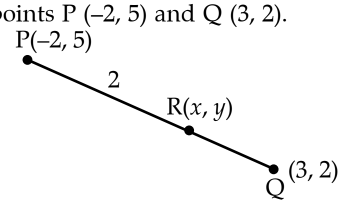
$$-m_1 + 5m_2 = 0 \Rightarrow m_1 = 5m_2$$

Hence, the ratio is 5 : 1

5. The section formula for a point dividing a line segment joining points $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the ratio $m_1 : m_2$ is:

$$R(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

Here, the ratio is 2 : 1, so $m_1 = 2$ and $m_2 = 1$, and the points P (–2, 5) and Q (3, 2).



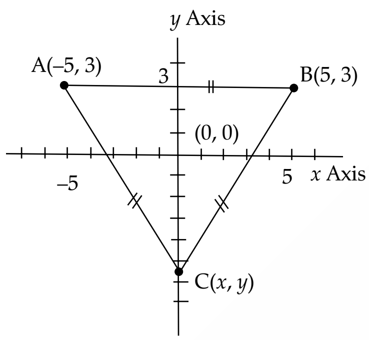
Using the section formula for the x–coordinate:

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{2(3) + 1(-2)}{2+1} = \frac{6-2}{3} = \frac{4}{3}$$

$$\text{Now, } y = \frac{2 \times 2 + 5 \times 1}{2 + 1} = \frac{4+5}{3} = \frac{9}{3} = 3$$

Therefore, the coordinates of point R are $\left(\frac{4}{3}, 3\right)$.

6.



Let the third vertex be (x, y)

Thus, 3 vertices of the triangle will be A (–5, 3) B(5, 3) C(x, y)

Since it is given that the triangle is an equilateral hence all the 3 sides will be equal

Hence, AC = BC = AB ....(i)

$$AB = \sqrt{(-5-5)^2 + (3-3)^2} = \sqrt{(-10)^2} = 10$$

$$AC = \sqrt{(-5-x)^2 + (3-y)^2}$$

$$BC = \sqrt{(5-x)^2 + (3-y)^2}$$

As AC = BC,

$$\Rightarrow \sqrt{(-5-x)^2 + (3-y)^2} = \sqrt{(5-x)^2 + (3-y)^2}$$

By squaring on both sides we get:

$$(-5-x)^2 + (3-y)^2 = (5-x)^2 + (3-y)^2$$

$$(-5-x)^2 = (5-x)^2$$

$$x^2 + 10x + 25 = x^2 - 10x + 25$$

$$20x = 0$$

$$x = 0$$

Now, BC = AB in equation (i) we get

$$\sqrt{(5-x)^2 + (3-y)^2} = 10$$

By squaring on both sides, we get

$$(5-x)^2 + (3-y)^2 = 100$$

By substituting $x = 0$ we get

$$25 + (3-y)^2 = 100$$

$$(3-y)^2 = 75$$

$$3 - y = \sqrt{75}$$

$$3 - y = \pm 5\sqrt{3}$$

$$y = 3 \mp 5\sqrt{3}$$

$$\Rightarrow \quad y = 3 - 5 \times 1.7 = 3 - 8.5 = -5.5$$

$$\text{or,} \quad y = 3 + 5\sqrt{3} = 3 + 3 \times 1.7$$

$$= 3 + 8.5 = 11.5$$

$$y = -5.5 \text{ or } 11.5$$

Hence, coordinates will be (0, – 5.5) or (0, 11.5)

Point (0, 11.5) will lie on positive side of y-axis and origin will be outside of the triangle hence $x \neq (0, 11.5)$

Point (0, –5.5) lies on negative side of y-axis and origin will be inside of the triangle, it satisfies the given condition.

Therefore, the coordinates of the third vertex are (0, –5.5)

7. Given: PQ = QR

$$\sqrt{(5-0)^2 + (-3-1)^2} = \sqrt{(0-x)^2 + (1-6)^2}$$

$$\sqrt{(5)^2 + (-4)^2} = \sqrt{(-x)^2 + (-5)^2}$$

$$\sqrt{25+16} = \sqrt{x^2 + 25}$$

$$41 = x^2 + 25$$

(on squaring both sides)

$$16 = x^2$$

$$x = \pm 4$$

8. For point P, $m_1 : m_2$ = AP : PB = 1 : 2

Now, $(x_1, y_1) = (2, 1)$

and $(x_2, y_2) = (5, -8)$

$$\text{Point P} = \left(\frac{1 \times 5 + 2 \times 2}{1+2}, \frac{1 \times -8 + 2 \times 1}{1+2}\right)$$

$$= (3, -2)$$

As, P(3, –2) lies on line $2x - y + k = 0$

$$\therefore \quad 2(3) - (-2) + k = 0$$

$$6 + 2 + k = 0$$

$$8 + k = 0$$

$$\text{Thus,} \quad k = -8$$

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. (i) Coordinates of town A (1, 7)

Coordinates of town B (4, 2)

Coordinates of station C (–4, 4)

$$\text{Distance AC} = \sqrt{(1+4)^2 + (7-4)^2}$$

$$= \sqrt{5^2 + 3^2} = \sqrt{34}$$

$$\text{Distance BC} = \sqrt{(4+4)^2 + (4-2)}$$

$$= \sqrt{64+4} = \sqrt{68}$$

Aditya will travel more distance

(ii) D is the mid-point of AB

$$= \left(\frac{1+4}{2}, \frac{7+2}{2}\right)$$

$$= (2.5, 4.5)$$

Thus, coordinates of D (2.5, 4.5)

2. Coordinates of A (–3, 2), B (1, 3), C (4, 0), D (–2, –3)

Distance travelled by Ravi

$$DC + CA = \sqrt{(-2-4)^2 + (-3-0)^2} + \sqrt{(4+3)^2 + (0-2)^2}$$

$$= \sqrt{36+9} + \sqrt{49+4}$$

$= 6.7 + 7.3$ units
$= 14.0$ units

Distance travelled by Arjun

$$DC + CB = \sqrt{(-2-4)^2+(-3-0)^2} + \sqrt{(4-1)^2+(0-3)^2}$$

$$= \sqrt{36+9}+\sqrt{9+9}$$

$$= \sqrt{45}+\sqrt{18}$$

$= 6.7 + 4.2$ units
$= 10.9$ units

So, Ravi travels more distance,

Now, $14.0 - 10.9 = 3.1$ units

Ravi travelled more by 3.1 units

Level - 2 ADVANCED COMPETENCY FOCUSED QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Mark)

1. Option (B) is correct

Explanation: Given, Coordinates of the bank: (–4, 8), Coordinates of the post office: (2, 0)

Using the distance formula:

$$\text{Distance} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(2-(-4))^2+(0-8)^2}$$

$$= \sqrt{(6)^2+(-8)^2}$$

$$= \sqrt{36+64} = \sqrt{100}$$

$= 10$ units

10 units × 50 m/unit = 500 meters

2. Option (D) is correct

Explanation: Line segment starts at origin:

O = (0, 0) and ends at point K = (20, 40)

Point G divides the line in the ratio 3 : 7, and G is closer to the origin. This means the ratio is OG : GK = 3 : 7

Using the section formula: If a point G divides the line segment between A(x_1, y_1) and B(x_2, y_2) in the ratio $m : n$, then the coordinates of G are:

$$G = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

Here:

A = O = (0, 0)
B = K = (20, 40)
$m = 3, n = 7$

So,

$$G = \left(\frac{3\times20+7\times0}{3+7}, \frac{3\times40+7\times0}{3+7}\right)$$

$$= \left(\frac{60}{10}, \frac{120}{10}\right)$$

$= (6, 12)$

3. Option (C) is correct

Explanation: Let points Q and R divide segment AB into three equal segments.

To divide a line segment in the ratio $m : n$, the formula is:

$$P = \left(\frac{nx_1+mx_2}{m+n}, \frac{ny_1+my_2}{m+n}\right)$$

Coordinates of point Q, which divides AB in the ratio 1 : 2 (since it's 1/3rd of the way from A to B):

$$Q = \left(\frac{2\times8+1\times14}{3}, \frac{2\times8+1\times11}{3}\right)$$

$$= \left(\frac{16+14}{3}, \frac{16+11}{3}\right)$$

$$= \left(\frac{30}{3}, \frac{27}{3}\right) = (10, 9)$$

Coordinates of point R, which divides AB in the ratio 2 : 1 (2/3rd of the way from A to B):

$$R = \left(\frac{1\times8+2\times14}{3}, \frac{1\times8+2\times11}{3}\right)$$

$$= \left(\frac{8+28}{3}, \frac{8+22}{3}\right)$$

$$= \left(\frac{36}{3}, \frac{30}{3}\right) = (12, 10)$$

4. Option (B) is correct

Explanation: Given: One vertex on the y-axis at –3

⇒ Point A = (0, –3)

One vertex on the x-axis at 5

⇒ Point B = (5, 0)

Third vertex = (2, 4)

⇒ Point C = (2, 4)

Using the area formula for a triangle given 3 vertices:

$$\text{Area} = \frac{1}{2}|0\,(0-4) + 5\,(4-(-3)) + 2\,(-3-0)|$$

$$= \frac{1}{2}|0 + 5\,(7) + 2(-3)|$$

$$= \frac{1}{2}|35-6| = \frac{1}{2}|29| = \frac{29}{2} = 14.5$$

Square units

5. Option (A) is correct

Explanation: Using Distance formula

$$\text{Distance} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

A(–3, 1), B (–1, 2)

$$AB = \sqrt{(-1+3)^2+(2-4)^2}$$

$$= \sqrt{(2)^2+(-2)^2}$$

$$= \sqrt{4+4} = \sqrt{8} \text{ units}$$

C(3, 4), D(1, 2) →

$$CD = \sqrt{(1-3)^2+(2-4)^2} = \sqrt{(-2)^2+(-2)^2}$$

$$= \sqrt{4+4} = \sqrt{8} \text{ units}$$

ASSERTION-REASON QUESTIONS (1 Mark)

1. Option (A) is correct

Explanation: Assertion is true as Surveyors often divide irregular plots into straight segments and use the distance formula to measure each boundary's length, then add them up.

Reason is also true because the distance between two points (x_1, y_1) and (x_2, y_2) is given by:

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

This is the standard distance formula derived from the Pythagorean Theorem.

Both assertion and reason are true and the reason correctly explains the assertion.

2. Option (D) is correct

Explanation: Assertion is false because the section formulais used to find a point dividing a line segment in a given ratio and not for finding the shortest path between two locations.

Reason is true because the section formula gives the coordinates of a point dividing a line segment in a given ratio. The midpoint is a special case when the ratio is 1 : 1.

3. Option (A) is correct

Explanation: Assertion is true because if the windows are at two known coordinates on a blueprint, the midpoint formula helps find the exact center between them.

Reason is also true because the midpoint of a line segment joining (x_1, y_1) and (x_2, y_2) is given by:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

This is the standard midpoint formula.

Both assertion and reason are true and the reason correctly explains the assertion.

4. Option (D) is correct

Explanation: Assertion is false because if a drone follows a straight path, all its position points should lie on a straight line, *i.e.*, they should be collinear. Forming a right-angled triangle implies a turn (*i.e.*, a change in direction), not a straight path.

Reason is true because this is a direct application of the Pythagorean Theorem. If $a^2 + b^2 = c^2$, where c is the longest side, the triangle is right-angled.

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. To find the distance between two points A(2, 3) and B(8, 7), we use the distance formula:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substituting the coordinates:

$$x_1 = 2, y_1 = 3$$
$$x_2 = 8, y_2 = 7$$

$$\text{Distance} = \sqrt{(8-2)^2 + (7-3)^2} = \sqrt{6^2 + 4^2}$$
$$= \sqrt{36 + 16} = \sqrt{52}$$
$$= \sqrt{4 \times 13} = 2\sqrt{13}$$
$$= 2 \times 3.61 = 7.22 \text{ units}$$

2. To find the midpoint between two points P(1, 2) and Q(7, 8), we use the midpoint formula:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Substituting the values:

$$x_1 = 1, y_1 = 2$$
$$x_2 = 7, y_2 = 8$$

$$\text{Midpoint} = \left(\frac{1+7}{2}, \frac{2+8}{2}\right) = \left(\frac{8}{2}, \frac{10}{2}\right) = (4, 5)$$

The decorative piece will be installed at midpoint (4, 5).

3. To find the coordinates of a point that divides a line segment in a given ratio, we use the section formula.

Given: Point P(0, 0), Point Q(9, 6)

Ratio = 1 : 2 (*i.e.*, point divides PQ internally in the ratio $m : n = 1 : 2$

Section Formula (for internal division):

$$(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

Substituting:

$$x_1 = 0, y_1 = 0$$
$$x_2 = 9, y_2 = 6$$
$$m = 1, n = 2$$

$$x = \frac{1 \times 9 + 2 \times 0}{1+2} = \frac{9}{3} = 3$$

$$y = \frac{1 \times 6 + 2 \times 0}{1+2} = \frac{6}{3} = 2$$

The valve should be installed at coordinates (3, 2).

4. To find the coordinates that divide the line segment AB in the ratio 1 : 2, we'll use the section formula.

Given: A(2, 4), B(8, 10), Ratio = 1 : 2 (*i.e.*, point divides AB internally in the ratio $m : n = 1 : 2$)

Using Section formula:

$$(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

Where : $x_1 = 2, y_1 = 4$

$$x_2 = 8, y_2 = 10$$
$$m = 1, n = 2$$

$$x = \frac{1 \times 8 + 2 \times 2}{1+2} = \frac{8+4}{3} = \frac{12}{3} = 4$$

$$y = \frac{1 \times 10 + 2 \times 4}{1+2} = \frac{10+8}{3} = \frac{18}{3} = 6$$

The fence should be placed at coordinates (4, 6).

5. To find the point that divides the line segment AB in the ratio 2 : 3, we use the section formula.

Given: A(5, 2), B(11, 8)

Ratio = 2:3 (*i.e.*, $m = 2, n = 3$)

Using Section Formula:

$$(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

Substituting:

$$x_1 = 5, y_1 = 2$$
$$x_2 = 11, y_2 = 8$$
$$m = 2, n = 3$$
$$x = \frac{2\times 11 + 3\times 5}{2+3} = \frac{22+15}{5} = \frac{37}{5} = 7.4$$
$$y = \frac{2\times 8 + 3\times 2}{2+3} = \frac{16+6}{5} = \frac{22}{5} = 4.4$$

The divider should be placed at coordinates (7.4, 4.4).

6. To find the coordinates of the midpoint between two points A(4, −2) and B(−2, 4), we use the midpoint formula:

Midpoint Formula:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Given:

$$x_1 = 4, y_1 = -2$$
$$x_2 = -2, y_2 = 4$$
$$x = \frac{4+(-2)}{2} = \frac{2}{2} = 1$$
$$y = \frac{-2+4}{2} = \frac{2}{2} = 1$$

The hook should be placed at coordinates (1, 1).

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. (i) Given coordinates are A(2, 3), B(6, 7) and C(4, 11)

To calculate the lengths of all three sides, we will use the distance formula for each pair of points:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

AB: Between A(2, 3) and B(6, 7)

$$AB = \sqrt{(6-2)^2 + (7-3)^2} = \sqrt{4^2 + 4^2}$$
$$= \sqrt{16+16} = \sqrt{32} \approx 5.66$$

BC: Between B(6, 7) and C(4, 11)

$$BC = \sqrt{(4-6)^2 + (11-7)^2}$$
$$= \sqrt{(-2)^2 + 4^2} = \sqrt{4+16}$$
$$= \sqrt{20} \approx 4.47$$

CA: Between C(4, 11) and A(2, 3)

$$CA = \sqrt{(2-4)^2 + (3-11)^2}$$
$$= \sqrt{(-2)^2 + (-8)^2} = \sqrt{4+64}$$
$$= \sqrt{68} \approx 8.25$$

The sides of the triangle are:

AB = 5.66, BC = 4.47 and CA = 8.25

(ii) This is a scalene triangle as all sides are different in length.

2. (i) Given, Building A: A(−3, 5) and Building B: B(7,−1)

To find the midpoint (pole location), we will use the midpoint formula:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Substituting:

$$x = \frac{-3+7}{2} = \frac{4}{2} = 2$$
$$y = \frac{5+(-1)}{2} = \frac{4}{2} = 2$$

The supporting pole should be placed at coordinates (2, 2)

(ii) To find the distance from the pole to both buildings, we will use the Distance formula between the midpoint (2, 2) and one building (A)

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_2)^2}$$
$$= \sqrt{(2-(-3))^2 + (2-5)^2}$$
$$= \sqrt{5^2 + (-3)^2} = \sqrt{25+9}$$
$$= \sqrt{34}$$
$$\text{Distance} \approx 5.83 \text{ units}$$

3. (i) Given, A(3, –2) and B(9, 4) and Ratio = 2 : 1

$\Rightarrow$ $m = 2, n = 1$

To find coordinates where the divider should be placed

Section Formula (internal division):

$$(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

Substituting

$$x_1 = 3, y_1 = -2$$
$$x_2 = 9, y_2 = 4$$
$$m = 2, n = 1$$
$$x = \frac{2\times 9 + 1\times 3}{2+1} = \frac{18+3}{3} = \frac{21}{3} = 7$$
$$y = \frac{2\times 4 + 1\times(-2)}{2+1} = \frac{8-2}{3} = \frac{6}{3} = 2$$

The divider should be placed at coordinates (7, 2).

(ii) We used the section formula already to get point (7, 2), which proves it lies on the line internally dividing AB in the ratio 2:1.

So, the verification is complete by derivation.

4. (i) Given coordinates: A(1, 2), B(1, 6), C(5, 6) and D(5, 2)

The diagonals are: Diagonal 1: AC and Diagonal 2: BD

To find midpoints of both diagonals

Midpoint Formula:

$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Diagonal AC: A(1, 2), C(5, 6)

$$\text{Midpoint of AC} = \left(\frac{1+5}{2}, \frac{2+6}{2}\right) = (3, 4)$$

Diagonal BD: B(1, 6), D(5, 2)

$$\text{Midpoint of BD} = \left(\frac{1+5}{2}, \frac{6+2}{2}\right) = (3, 4)$$

(ii) Yes, they coincide because both midpoints are (3, 4).

(iii) When both diagonals of a quadrilateral bisect each other (*i.e.*, share the same midpoint), it confirms that the shape is a parallelogram. Since all angles are right angles (from the coordinate layout) and opposite sides are equal and parallel, the shape is a rectangle. This confirms that the room is a rectangle.

5. (i) Given Points, A(1, 2), B(4, 5) and C(7, 8)

To show that the three points lie on a straight path, we will check if the slope between AB is equal to the slope of BC.

Slope Formula:

Slope between two points (x_1, y_2), (x_2, y_2)

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

Slope of AB:

$$= \frac{5-2}{4-1} = \frac{3}{3} = 1$$

Slope of BC:

$$= \frac{8-5}{7-4} = \frac{3}{3} = 1$$

Since Slope of AB = Slope of BC, the points are collinear.

(ii) Justification:

Since the points A, B, and C lie on the same straight line, a single straight security path is sufficient to patrol all three locations.

(iii) We used the slope formula from coordinate geometry to prove collinearity.

CASE BASED QUESTIONS (4 Marks)

1. (i) Since, PQRS is a square

$\therefore$ PQ = QR = RS = PS

Length of PQ = [200 – (–200)] = 400

$\therefore$ The coordinates of R = (200, 400)

and coordinates of S = (–200, 400)

(ii) (a) Area of square PQRS = $(\text{side})^2$

$= (PQ)^2$

$= (400)^2$

= 1,60,000 sq. units

OR

(b) Diagonal PR makes a right angle ΔPQR where ∠θ = 90°. By Pythagoras theorem

$(PR)^2 = (PQ)^2 + (QR)^2$

= 1,60,000 + 1,60,000

= 3,20,000

$\Rightarrow$ PR = $\sqrt{3,20,00}$

$= 400\sqrt{2}$ units

(iii) Since, point S divides CA in the ratio $k : 1$

According to figure, co-ordinates of

A = (200, 800)

C = (–600, 0)

By section formula :

$$\therefore \quad \left(\frac{kx_2+x_1}{k+1}, \frac{ky_2+y_1}{k+1}\right) = (-200, 400)$$

$$\Rightarrow \left(\frac{k(200)+(-600)}{k+1}, \frac{k(800)+0}{k+1}\right) = (-200, 400)$$

$$\Rightarrow \left(\frac{200k-600}{k+1}, \frac{800k}{k+1}\right) = (-200, 400)$$

$$\therefore \quad \frac{800k}{k+1} = 400$$

$\Rightarrow$ $800k = 400k + 400$

$\Rightarrow$ $400k = 400$

$\Rightarrow$ $k = 1$

2. (i) To find the distance between Ravi at A(2, 3) and Raghav at C(10, 5), we use the distance formula:

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

Substituting the coordinates:

$$d = \sqrt{(10-2)^2 + (5-3)^2}$$

$$d = \sqrt{(8)^2 + (2)^2} = \sqrt{64+4} = \sqrt{68} = 2\sqrt{17}$$

≈ 8.25 units

(ii) To find the distance between Vinod at B(7, 8) and Vithal at D(5, 0), we again use the distance formula:

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

Substituting the coordinates:

$$d = \sqrt{(5-7)^2 + (0-8)^2}$$

$$d = \sqrt{(-2)^2 + (-8)^2}$$

$$= \sqrt{4+64} = \sqrt{68} = 2\sqrt{17}$$

≈ 8.25 units

(iii) (a) Show that ABCD is a rectangle.

Opposite sides are equal in length.

Adjacent sides are perpendicular (i.e., their slopes multiply to –1).

Distance AB:

$$d_{AB} = \sqrt{(7-2)^2+(8-3)^2} = \sqrt{(5)^2+(5)^2}$$

$$= \sqrt{25+25} = \sqrt{50} = 7.07 \text{ units (approx)}$$

Distance CD:

$$d_{CD} = \sqrt{(5-10)^2+(0-5)^2}$$

$$= \sqrt{(-5)^2+(-5)^2} = \sqrt{25+25} = \sqrt{50}$$

$$= 7.07 \text{ units (approx)}$$

Distance AD:

$$d_{AD} = \sqrt{(5-2)^2+(0-3)^2} = \sqrt{(3)^2+(-3)^2}$$

$$= \sqrt{9+9} = \sqrt{18} = 4.24 \text{ units (approx)}$$

Distance BC:

$$d_{BC} = \sqrt{(10-7)^2+(5-8)^2} = \sqrt{(3)^2+(-3)^2}$$

$$= \sqrt{9+9} = \sqrt{18} = 4.24 \text{ units (approx)}$$

Thus, AB = CD and AD = BC, So both pairs of opposite sides are equal.

Since adjacent sides AB and BC (as well as CD and DA) are perpendicular, ABCD is a rectangle.

OR

(b) The perimeter P of a rectangle is given by:

$$P = 2 \times (\text{Length} + \text{Width})$$

Here, the length AB = 7.07 units and the width AD = 4.24 units

So, P = 2 × (7.07 + 4.24) = 2 × 11.31 = 22.62 units (approx)

3. (i) To find the distance between points A(3, 4) and B (6, 7), we use the distance formula:

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Substituting the coordinates:

$$d = \sqrt{(6-3)^2+(7-4)^2}$$

$$d = \sqrt{3^2+3^2}$$

$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$= 4.24 \text{ units}$$

(ii) To find the distance between points C (9, 4) and D(6, 1), we again use the distance formula:

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Substituting the coordinates

$$d = \sqrt{(6-9)^2+(1-4)^2}$$

$$d = \sqrt{(-3)^2-(3)^2} = \sqrt{9+9}$$

$$= \sqrt{18} = 3\sqrt{2} = 4.24 \text{ units}$$

(iii) (a) To show that ABCD forms a parallelogram, we need to prove that the midpoints of diagonals AC and BD are same. If the midpoints are equal, then ABCD is a parallelogram.

Midpoint of AC:

$$M_{AC} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{3+9}{2}, \frac{4+4}{2}\right)$$

$$= \left(\frac{12}{2}, \frac{8}{2}\right) = (6, 4)$$

Midpoint of BD:

$$M_{BD} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{6+6}{2}, \frac{7+1}{2}\right)$$

$$= \left(\frac{12}{2}, \frac{8}{2}\right) = (6, 4)$$

Since, $M_{AC} = M_{BD}$ = (6, 4), the diagonals bisect each other, confirming that ABCD forms a parallelogram.

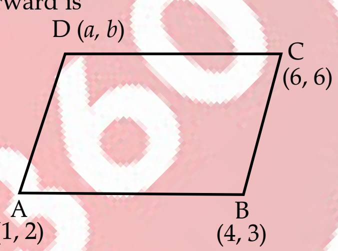
OR

(b) From the calculations above:

Midpoint of AC: (6, 4)

Midpoint of BD: (6, 4)

4. (i) Paralelogram formed by midfielders and forward is



As, diagonals of a parallelogram bisects each other.

∴ Mid point of AC = Mid point of BD

$$\left(\frac{1+6}{2}, \frac{2+6}{2}\right) = \left(\frac{4+a}{2}, \frac{3+b}{2}\right)$$

$$4 + a = 7$$

$$a = 3$$

$$3 + b = 8$$

$$b = 5$$

Central midfielder (D) is at (3, 5).

(ii) (a)

$$GH = \sqrt{(-3-3)^2+(5-1)^2}$$

$$= \sqrt{36+16}$$

$$= \sqrt{52}$$

$$= 2\sqrt{13}$$

$$GK = \sqrt{(0+3)^2+(3-5)^2}$$

$$= \sqrt{9+4}$$

$$= \sqrt{13}$$

$$HK = \sqrt{(3-0)^2+(1-3)^2}$$

$$= \sqrt{9+4}$$

$$= \sqrt{13}$$

For Point to be collinear

$$GK + HK = GH$$

⇒ G, H and K lie on a same straight line.

OR

(b) $$CJ = \sqrt{(0-5)^2+(1+3)^2} = \sqrt{25+16} = \sqrt{41}$$

$$CI = \sqrt{(0+4)^2+(1-6)^2} = \sqrt{16+25} = \sqrt{41}$$

∴ Full–back J(5, –3) and centre–back I(–4, 6) are equidistant from forward C(0, 1)

Now, Mid-point of IJ = $\left(\frac{5-4}{2}, \frac{-3+6}{2}\right) = \left(\frac{1}{2}, \frac{3}{2}\right)$

C is NOT the mid-point of IJ.

(iii) A, B and E lie on the same straight line and B is equidistant from A and E

⇒ B is the mid–point of AE

$$\left(\frac{1+a}{2}, \frac{4+b}{2}\right) = (2, -3)$$

$1 + a = 4; a = 3$

$4 + b = -6; b = -10$

Thus, position of E is (3, – 10)

5. (i) Two pairs of possible coordinates such that Rohan secured 20 and 5 points for them, will be achieved only when one arrow lie between points (1) and (2) and second arrow lie between point (3) and (4).

Now, if we take points on x-axis then coordinates of Arrow (1) are (1.5, 0) and Arrow 2 (3.5, 0).

(ii) The distance of (2, 2.5) from (0, 0) is:

$\sqrt{4+6.25} = \sqrt{10.25}$ units = 3.2 units (approx)

Since 3.2 units lies in outermost section of the concentric circular board, the player is given 5 points.

(iii) The distance of (1.2, 1.6) from the origin is:

$\sqrt{\{(1.2)^2+(1.6)^2\}}$ = 2 units

Thus, the second arrow lands on the boundary mark and the ratio in which the first arrow divides the origin and the second arrow's landing mark is the ratio of their radii =2 : 1.

Assumes the coordinates of the second arrow's landing mark as (x, y) and uses section formula to write :

$$\left(\frac{2x+0}{3}, \frac{2y+0}{3}\right) = (1.2, 1.6)$$

Solves the above equation to find the values of the coordinates of the second arrow's landing mark as (1.8, 2.4).

OR

(iii) As, per the question, the distance between the origin and the coordinate $(m, -m)$ be 2 units

Now, according to distance formula,

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$2 = \sqrt{m^2+(-m)^2}$$

⇒ $m^2 + (-m)^2 = 2^2$

⇒ $2m^2 = 4$

Thus, $m = \sqrt{2}$ and $(-\sqrt{2})$.

Hence the coordinates are $(\sqrt{2}, -\sqrt{2})$ and $(-\sqrt{2}, \sqrt{2})$

6. (i) P(3, 3), Q (8, 2), R (6, 5)

Coordinates of required point are:

$$\left(\frac{3+8+6}{3}, \frac{3+2+5}{3}\right) = \left(\frac{17}{3}, \frac{10}{3}\right)$$

(ii) According to Distance formula:

$$PR = \sqrt{(6-3)^2+(5-3)^2} = \sqrt{9+4} = \sqrt{13}$$

$$QR = \sqrt{(6-8)^2+(5-2)^2} = \sqrt{-2^2+3^2} = \sqrt{4+9} = \sqrt{13}$$

$$PQ = \sqrt{(8-3)^2+(2-3)^2} = \sqrt{25+1} = \sqrt{26}$$

Thus, PR = QR = $\sqrt{13}$

PQ = $\sqrt{26}$

and, $PQ^2 = PR^2 + QR^2$

∴Δ PQR is an isosceles right angled triangle.

(iii) (a) Area of the plot to row seeds

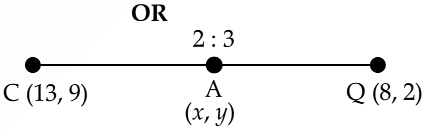
$$= 13\times 9 - \frac{1}{2}\times\sqrt{13}\times\sqrt{13}$$

$= 117 - 6.5$

$= 110.5\ m^2$

OR

(b)



Coordinates of required point (A) are

By using section formula

$$\left(\frac{2\times 8+3\times 13}{2+3}, \frac{2\times 2+3\times 9}{2+3}\right) = \left(11, \frac{31}{5}\right)$$

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. (i) Given Points, A(3, 4), B(11, 4) and C(7, 8)

To calculate the lengths of all three sides, we will use the Distance Formula:

$$\text{Distance} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Side AB:

$$AB = \sqrt{(11-3)^2+(4-4)^2} = \sqrt{8^2+0}$$
$$= \sqrt{64} = 8 \text{ units}$$

Side BC:

$$BC = \sqrt{(7-11)^2+(8-4)^2}$$
$$= \sqrt{(-4)^2+4^2} = \sqrt{16+16}$$
$$= \sqrt{32} \approx 5.66 \text{ units}$$

Side AC:

$$AC = \sqrt{(7-3)^2+(8-4)^2} = \sqrt{4^2+4^2}$$
$$= \sqrt{16+16} = \sqrt{32} \approx 5.66 \text{ units}$$

(ii) Type of triangle

$$AB = 8, AC = 5.66 \text{ and } BC \approx 5.66$$

Since AC = BC, two sides are equal, so it is an isosceles triangle.

(iii) Midpoint of side AC

Midpoint Formula:

$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

For AC : A(3, 4), C(7, 8)

$$= \left(\frac{3+7}{2}, \frac{4+8}{2}\right) = \left(\frac{10}{2}, \frac{12}{2}\right)$$
$$= (5, 6)$$

(iv) Using the Pythagorean Theorem:

Check if: $(AC)^2 + (BC)^2 = (AB)^2$

$$AC^2 = BC^2 = \left(\sqrt{32}^2\right) = 32$$
$$AB^2 = 8^2 = 64$$

So, $32 + 32 = 64$

This satisfies: $AC^2 + BC^2 = AB^2$

So, triangle ABC is right-angled at point C.

(v) Coordinate geometry provides precise location, measurement, and layout planning for better infrastructure design.

2. (i) Given vertices, P(2, 1), Q(6, 5) and R(4, 7)

To show that the points P, Q, and R do not lie on the same line, we will check the slopes of PQ and QR. If the slopes are not equal, the points are not collinear.

Slope Formula:

$$\text{Slope} = \frac{y_2-y_1}{x_2-x_1}$$

$$\text{Slope of PQ:} = \frac{5-1}{6-2} = \frac{4}{4} = 1$$

$$\text{Slope of QR:} = \frac{7-5}{4-6} = \frac{2}{-2} = -1$$

Slopes are different, so P, Q, and R do not lie on the same line.

(ii) Midpoints of sides PQ and PR

Midpoint Formula:

$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$\text{Midpoint of PQ:} = \left(\frac{2+6}{2}, \frac{1+5}{2}\right) = (4, 3)$$

$$\text{Midpoint of PR:} = \left(\frac{2+4}{2}, \frac{1+7}{2}\right) = (3, 4)$$

PQ midpoint: (4, 3) and PR midpoint: (3, 4)

(iii) To find the coordinates of Point dividing side QR in the ratio 1 : 2

Section Formula (internal division):

$$(x, y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

Divide QR in the ratio 1 : 2

Q(6, 5), R(4, 7), $m = 1, n = 2$

$$x = \frac{1\times4+2\times6}{3} = \frac{4+12}{3} = \frac{16}{3}$$

$$y = \frac{1\times7+2\times5}{3} = \frac{7+10}{3} = \frac{17}{3}$$

Tap to be installed at: $\left(\frac{16}{3}, \frac{17}{3}\right)$.

(iv) Distance between the midpoints of PQ and PR

Points: $M_1 = (4, 3)$ and $M_2 = (3, 4)$

Distance Formula:

$$\text{Distance} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(3-4)^2+(4-3)^2}$$
$$= \sqrt{(-1)^2+(1)^2} = \sqrt{1+1}$$
$$= \sqrt{2} \approx 1.41 \text{ units}$$

3. (i) Given vertices of the rectangular land: A(0, 0), B(10, 0), C(10, 6) and D(0, 6)

To find the midpoints of both diagonals AC and BD

Midpoint Formula:

$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Diagonal AC: points A (0, 0) and C(10, 6)

$$\text{Midpoint of AC} = \left(\frac{0+10}{2}, \frac{0+6}{2}\right) = (5, 3)$$

Diagonal BD: points B(10, 0) and D(0, 6)

$$\text{Midpoint of BD} = \left(\frac{10+0}{2}, \frac{0+6}{2}\right) = (5, 3)$$

(ii) Yes, both midpoints coincide as they are at (5, 3).

This confirms that the diagonals bisect each other, which is a defining property of a rectangle. Hence, the plot is indeed a rectangle.

(iii) Divide diagonal AC in the ratio 3:2

Section Formula:

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

Let A(0, 0), C(10, 6), and ratio = 3 : 2

$$x = \frac{3\times10+2\times0}{5} = \frac{30}{5} = 6$$

$$y = \frac{3\times6+2\times0}{5} = \frac{18}{5} = 3.6$$

So, the required point is (6, 3.6)

(iv) Lengths of both diagonals

Distance Formula:

$$\text{Distance} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Diagonal AC: A(0, 0), C(10, 6)

$$AC = \sqrt{(10-0)^2+(6-0)^2} = \sqrt{100+36}$$

$$= \sqrt{136} \approx 11.66 \text{ units}$$

Diagonal BD: B(10, 0), D(0, 6)

$$BD = \sqrt{(0-10)^2+(6-0)^2} = \sqrt{100+36}$$

$$= \sqrt{136} \approx 11.66 \text{ units}$$

Length of each diagonal is 11.66 units.

4. (i) Given coordinates: X(1, 2), Y(7, 6) and Z(4, 10)

To find lengths of sides XY, YZ, and XZ

Distance Formula:

$$\text{Distance} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Length of XY:

$$= \sqrt{(7-1)^2+(6-2)^2} = \sqrt{6^2+4^2}$$

$$= \sqrt{36+16} = \sqrt{52} \approx 7.21 \text{ units}$$

Length of YZ:

$$= \sqrt{(4-7)^2+(10-6)^2}$$

$$= \sqrt{(-3)^2+4^2}$$

$$= \sqrt{9+16} = \sqrt{25} = 5 \text{ units}$$

Length of XZ:

$$= \sqrt{(4-1)^2+(10-2)^2} = \sqrt{3^2+8^2}$$

$$= \sqrt{9+64} = \sqrt{73} \approx 8.54 \text{ units}$$

Side lengths: XY =7.21, YZ = 5 and XZ = 8.54

(ii) In an equilateral triangle, all sides must be equal.

Since: XY $\neq$ YZ $\neq$ XZ, the triangle is not equilateral.

Triangle XYZ is scalene (all sides of different lengths).

(iii) To find midpoints of any two sides

Midpoint Formula:

$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Midpoint of XY:

$$= \left(\frac{1+7}{2}, \frac{2+6}{2}\right) = (4, 4)$$

Midpoint of YZ:

$$= \left(\frac{7+4}{2}, \frac{6+10}{2}\right) = (5.5, 8)$$