CHAPTER 7

Triangles

Level - 1 CORE SUBJECTIVE QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Mark)

1. Option (C) is correct

Explanation:

Here DE || BC

In Δ ADE and ΔABC

$$\angle A = \angle A \quad \text{(Common)}$$

$$\angle ADE = \angle ABC \quad \text{(Corresponding angles)}$$

By AA similarity criterion

$$\Delta ADE \sim \Delta ABC$$

$$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$$

$$\frac{5}{7.5} = \frac{DE}{12}$$

$$7.5\ DE = 60$$

$$DE = 8 \text{ cm}$$

2. Option (B) is correct

Explanation:

According to Basic Proportionality Theorem, (Thales' Theorem)

When a line is drawn parallel to one side of a triangle, it divides the other two sides in the same ratio. This means:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Given, AD = 4 cm and AB = 9 cm

Calculate DB:

$$DB = AB - AD = 9 - 4 = 5 \text{ cm}$$

We know from the theorem that:

$$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{4}{5} = \frac{AE}{EC}$$

Let's set EC = x:

Now substituting, we have:

$$\frac{4}{5} = \frac{AE}{x}$$

$$\Rightarrow AE = \frac{4x}{5}$$

We also know AC = 13.5 cm and so:

Substituting into AC = 13.5 = AE + EC:

$$13.5 = \frac{4}{5}x + x \Rightarrow 13.5 = \frac{9}{5}x$$

Solving for x:

$$x = 13.5 \times \frac{5}{9} = 7.5 \text{ cm}$$

3. Option (C) is correct

Explanation: Given:

$$AD = 2.4 \text{ cm}$$

$$DB = 4 \text{ cm}$$

$$AE = 2 \text{ cm}$$

Let AC = x,

According to the Basic Proportionality Theorem, we have:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

First, we need to find EC. Since AC = AE + EC, we can express EC as:

$$EC = AC - AE = x - 2$$

Now substituting the known values into the proportion:

$$\frac{2.4}{4} = \frac{2}{x-2}$$

$$2.4(x - 2) = 4 \times 2$$

$$2.4x - 4.8 = 8$$

$$2.4x = 12.8$$

$$x = \frac{12.8}{2.4} = \frac{128}{24} = \frac{16}{3} \text{ cm}$$

Thus, the length of AC is $\frac{16}{3}$ cm

4. Option (C) is correct

Explanation: $\frac{OM}{MP} = \frac{ON}{NS}$ (by BPT)

$$\frac{OM}{8.5} = \frac{4.8}{6}$$

$$OM = \frac{40.8}{6} = 6.8 \text{ cm}$$

$$OP = OM + MP = 6.8 + 8.5 = 15.3 \text{ cm}$$

5. Option (B) is correct

Explanation: Let smaller part of line QR, QT be $2x$

Hence, RT $= 3x$ (Ratio 2 : 3)

In Δ PQR and ΔSTQ

Given $\angle PRQ = \angle STQ$

$\angle RQP = \angle TQS$ (common)

By AA similarity criterion

$\Delta PQR \sim \Delta SQT$

$\because$ length of QT $= 2x$

$\therefore$ length of QR $= 5x$

$$\frac{ST}{PR} = \frac{QT}{QR}$$

$$\frac{ST}{20} = \frac{2x}{5x} = ST \Rightarrow \frac{2}{5} \times 20$$

$= 8$ cm.

6. Option (A) is correct

Explanation: In ΔPQR and ΔABC

$\angle P = \angle C$ (corresponding angles)

$\angle R = \angle A$ (corresponding angles)

By AA similarity

Δ PQR is similar to ΔCBA

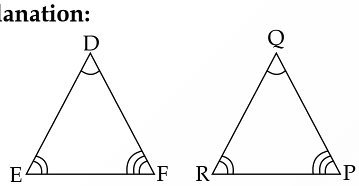
∴ Anas is right

∴ ΔPQR and ΔCBA are not congruent

∴ Rishi is not correct

7. Option (B) is correct

Explanation:



By AAA similarity

ΔDEF is similar to ΔPQR

If the two triangles are similar by AAA criterion, then their corresponding sides are in the same proportion

$$\therefore \quad \frac{DE}{QR} = \frac{DF}{PQ} = \frac{EF}{PR}$$

Thus, Option (B) is incorrect as $\frac{EF}{RP} \uparrow \frac{DE}{PQ}$

8. Option (B) is correct

Explanation: Since DE || AB

In ΔCDE & ΔCAB

$\angle C = \angle C$ (Common)

$\angle CED = \angle CBA$

(Corresponding ∠s are equal as DE || AB)

∴ ΔCDE ~ ΔCAB (By AA criterion)

$$\Rightarrow \quad \frac{DE}{AB} = \frac{EC}{BC} \quad \text{(sides of similar Δs)}$$

$$\Rightarrow \quad \frac{x}{a} = \frac{c}{BE+EC}$$

$$\Rightarrow \quad x = \frac{ac}{b+c}$$

Thus, x is expressed as $\frac{ac}{b+c}$

9. Option (C) is correct

Explanation: Given, ΔABC ~ ΔPQR

$\Rightarrow \quad \angle A = \angle P,$

$\angle B = \angle Q$

and $\angle C = \angle R$

$\therefore \quad \angle C = 65°$ $(\because \angle R = 65°)$

Now, In ΔABC, using Angle sum property

$32° + \angle B + 65° = 180°$

$\Rightarrow \quad \angle B = 83°$

10. Option (B) is correct

Explanation: ΔABC ~ ΔQPR

$$\frac{AC}{QR} = \frac{BC}{PR}$$

$$\Rightarrow \quad \frac{6}{3} = \frac{5}{x}$$

$$x = \frac{5}{2}$$

$= 2.5$ cm

11. Option (B) is correct

Explanation: Given, the ratio $\frac{AB}{DE} = \frac{BE}{FD}$, we have two sides of triangles proprotional

For triangles to be similar, we need included pair of angles to be equal

∴ The correct option is (B) i.e., ∠B = ∠D, which ensures similarity by SAS criterion.

12. Option (B) is correct

Explanation: Since DE || BC, by the Basic Proportionality Theorem (Thales' Theorem), we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

We are given AD = 3 cm, AB = 7 cm, and EC = 3 cm. Therefore, DB = AB – AD = 7 – 3 = 4 cm

Substitute the values into the equation:

$$\frac{3}{4} = \frac{AE}{3}$$

$$\Rightarrow \quad AE = \frac{(3 \times 3)}{4} = \frac{9}{4} = 2.25 \text{ cm}$$

13. Option (D) is correct

Explantion: Using the Basic Proportionality Theorem:

$$\frac{AB}{PQ} = \frac{OB}{OP}$$

Substitute the given values:

$$\frac{6}{2} = \frac{3}{OP}$$

Cross multiply to solve for OP:

$6 \times OP = 3 \times 2$

$$OP = \frac{6}{6} = 1 \text{ cm}$$

Thus, the length of OP is 1 cm.

14. Option (A) is correct

Explanation: In two triangles PQR and LMN, if:

$$\frac{PQ}{MN} = \frac{QR}{LN} = \frac{PR}{LM}$$

Then:

By the SSS similarity criterion, when the sides of two triangles are proprotional, the triangles are similar.

Thus, the correct similarity statement is:

(A) ΔLMN ~ ΔRPQ.

15. Option (D) is correct

Explanation: In triangle ABC

Since, DE || BC, by the Basic Proportionality Theorem (Thales' Theorem), we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Substitute the given values:

$$\frac{2}{3} = \frac{3}{x}$$

$$x = \frac{(3\times3)}{2} = \frac{9}{2}$$

16. Option (A) is correct

Explanation: To determine which may NOT be similar:

(i) Any two circles: Always similar (same shape, different sizes).

(ii) Any two rhombuses: Not always similar (angles can differ).

(iii) Any two regular hexagons: Always similar (same shape, equal sides and angles).

The answer is (A) only (ii) as not all rhombuses are similar.

17. Option (A) is correct

Explanation: Given that ΔMNO is similar to ΔPQR.

We can set up the proportion as:

$$\frac{MN}{PQ} = \frac{NO}{QR}$$

$$\Rightarrow \quad \frac{10}{6} = \frac{15}{QR}$$

$$\Rightarrow \quad QR = 9 \text{ cm}$$

18. Option (D) is correct

19. Option (B) is correct

Explanation: Using the Basic Proportionality Theorem (BPT):

$$\frac{XV}{XY} = \frac{XW}{XZ}$$

$$XV = XY - VY = 14 - 6 = 8 \text{ cm}$$

Substitute the known values:

$$\frac{8}{14} = \frac{12}{XZ}$$

Cross multiply to solve for XZ :

$$8 \times XZ = 14 \times 12$$

$$XZ = \frac{168}{6} = 21 \text{ cm}$$

Thus, the length of XZ is 21 cm

ASSERTION-REASON QUESTIONS (1 Mark)

1. Option (C) is correct

Explanation:

Assertion : In ΔDEF and ΔXYZ

$$\angle E = \angle Y = 90° \quad \text{(Given)}$$

and, $$\frac{DE}{XY} = \frac{EF}{YZ} \quad \text{(Given)}$$

∴ ΔDEF ~ ΔXYZ (By SAS Criterion)

Hence, Assertion is true.

Reasion : All right angled triangles are not similar to each other, because to be similar the other two angles, and the ratios of the corresponding side should be equal, which is not necessary.

∴ Reason is False

Hence, Assertion is true but Reason is false

2. Option (B) is correct

Explanation: In ΔADC and ΔABC

$\angle C = \angle C$ (Common)

$\angle ADC = \angle BAC$ (Given)

∴ ΔADC ~ ΔABC (AA Criterion)

$$\Rightarrow \quad \frac{AC}{CB} = \frac{CD}{CA}$$

$$\Rightarrow \quad CA^2 = CB \times CD$$

∴ Assertion is True.

Reason : In ΔADE and ΔABC

$\angle A = \angle A$ (Common)

$\angle ADE = \angle ACB$ (given)

∴ ΔADE ~ ΔABC (AA Creterion)

Thus, Reason is true.

Here, both Assertion and Reason are ture but Reason is not correct explanation.

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. Given that,

$$\frac{EA}{EC} = \frac{EB}{ED}$$

To Prove :

ΔEAB ~ ΔECD

Proof : Now, In ΔEAB and ΔECD,

$$\frac{EA}{EC} = \frac{EB}{ED} \quad \text{(given)}$$

and $\angle AEB = \angle DEC$ [Vertically opposite angles]

∴ ΔEAB ~ ΔECD [By SAS similarity criteria]

Hence, Proved.

2. (i) Given: Diagonal BD bisects ∠B and ∠D

To Prove: ΔABD ~ ΔCBD

Proof:

In ΔABD and ΔCBD

$\angle ABD = \angle CBD$ (BD Bisects ∠B)

$\angle ADB = \angle CDB$ (BD bisects ∠D)

∴ ΔABD ~ ΔCBD (AA criterion)

Proved.

(ii) Since $\Delta ABD \sim \Delta CBD$

$$\therefore \quad \frac{AB}{BC} = \frac{BD}{BD}$$

$$\Rightarrow \quad \frac{AB}{BC} = 1$$

$$\therefore \quad AB = BC$$

3. Given : BC || DE

By Basic Proportionality theorem

$$\frac{AB}{AD} = \frac{BC}{DE}$$

$$\Rightarrow \quad \frac{x}{x+5} = \frac{3}{8}$$

$$\Rightarrow \quad 8x = 3x + 15$$

$$\Rightarrow \quad 5x = 15$$

$$\Rightarrow \quad x = 3 \text{ cm}$$

4. Given $AB = BC$

$\therefore \quad \angle B = \angle C$ ...(i)

In ΔABD and ΔECF

$\angle B = \angle C$ (Proved above)

$\angle ADB = \angle EFC = 90°$

(As $AD \perp BC$ and $EF \perp AC$)

$\therefore$ By AA criterion of similarity

$\Delta ABD \sim \Delta ECF$ Hence Proved.

5. In ΔABC and ΔAED

$\angle ABC = \angle AED = 90°$

$\angle BAC = \angle DAE$ (Common angle)

$\therefore \quad \Delta ABC \sim \Delta AED$

(By using AA criterion)

Hence Proved.

6. In ΔABC, DE | | AC

$\Rightarrow \quad \frac{BD}{DA} = \frac{BE}{EC}$ [From BPT] ....(i)

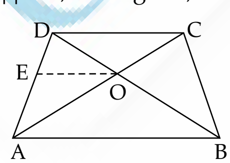
Now, in ΔBPA, DC | | AP

$\Rightarrow \quad \frac{BC}{CP} = \frac{BD}{DA}$ [From BPT] ...(ii)

From eqns.(i) and (ii)

$\frac{BE}{EC} = \frac{BC}{CP}$ Hence Proved.

7. Draw OE | | AB, through O, meeting AD at E



In ΔADC

EO| | DC (EO | | AB | | DC)

$\therefore \quad \frac{AE}{ED} = \frac{OA}{OC}$

(By Thales Theorem) ...(i)

In ΔDAB EO || AB

$\frac{AE}{ED} = \frac{OB}{OD}$

(By Thales Theorem) ...(ii)

From eqns. (i) and (ii)

$\frac{OA}{OC} = \frac{OB}{OD}$ Hence Proved.

8. No the above ratio of the sides is not correct.

Reason: As, corresponding sides of similar ΔPQR and ΔSRT are QR and RT respectively.

Hence, the ratio of the corresponding sides $\frac{QR}{RT} = 1$

9. In ΔBED and ΔACB, we have

$\angle BED = \angle ACB = 90°$

$\because \quad \angle B + \angle C = 180°$

$\therefore \quad BD \,||\, AC$

$\angle EBD = \angle CAB$ (Alternate angles)

Therefore, by AA similarity theorem, we get.

$\Delta BED \sim \Delta ACB$

$\Rightarrow \quad \frac{BE}{AC} = \frac{DE}{BC}$

$\Rightarrow \quad \frac{BE}{DE} = \frac{AC}{BC}$

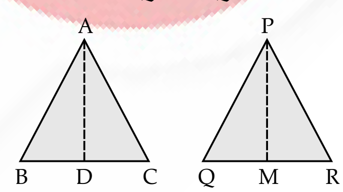
Hence Proved.

10. Given, $\Delta ABC \sim \Delta PQR$

$\Rightarrow \quad \angle ABC = \angle PQR$ ...(i)

(corresponding angles)

$\Rightarrow \quad \frac{AB}{PQ} = \frac{BC}{QR}$ (corresponding sides)



$$\Rightarrow \quad \frac{AB}{PQ} = \frac{\left(\frac{BC}{2}\right)}{\left(\frac{QR}{2}\right)}$$

$\Rightarrow \quad \frac{AB}{PQ} = \frac{BD}{QM}$ ...(ii)

(D and M are mid points of BC and QR)

Now, In ΔABD and ΔPQM,

$\angle ABD = \angle PQM$ (From (i))

$\frac{AB}{PQ} = \frac{BD}{QM}$ (From (ii))

$\Rightarrow \quad \Delta ABD \sim \Delta PQM$ (SAS criterion)

$\Rightarrow \quad \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

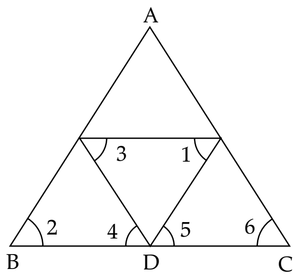
(corresponding sides)

$\Rightarrow \quad \frac{AB}{PQ} = \frac{AD}{PM}$

Hence Proved.

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1.



Since D, E, F are the mid points of BC, CA, AB respectively

$\therefore$ EF || BC, DF || AC, DE || AB

So, BDEF is a parallelogram

$\therefore$ $\angle 1 = \angle 2$ (opposite $\angle$s of a ||gm)

$\angle 3 = \angle 4$ (alternate interior $\angle$s)

$\therefore$ $\Delta FBD \sim \Delta DEF$ (AA criterion)

Also, DCEF is a parallelogram

Now, in ΔDEF and ΔABC

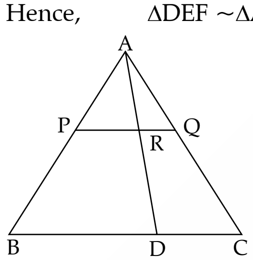
$\angle 3 = \angle 6$ (opposite $\angle$s of a || gm)

$\angle 1 = \angle 2$

(Proved above)

Hence, $\Delta DEF \sim \Delta ABC$ (AA Criterion)

2.



Given : PQ // BC

BD = DC

To Prove : PR = RQ

Proof : As, PQ || BC

$\angle PAR = \angle BAD$ (Common)

$\angle APR = \angle ABD$

(corresponding $\angle$s)

$\therefore$ $\Delta APR \sim \Delta ABD$ (By AA Criterion)

$\Rightarrow \frac{AP}{AB} = \frac{PR}{BD}$ ...(i)

Similarly, $\Delta AQR \sim \Delta ACD$

$\Rightarrow \frac{AQ}{AC} = \frac{RQ}{DC}$ ...(ii)

As, PQ || BC

$\therefore \frac{AP}{AB} = \frac{AQ}{AC}$ (By BPT) ...(iii)

Using (i), (ii) and (iii),

$$\frac{PR}{BD} = \frac{RQ}{DC}$$

But BD = DC

$\Rightarrow$ PR = RQ or AD bisects PQ

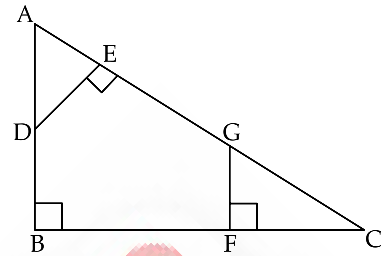
3. In the given figure, we have

AB $\perp$ BC

FG $\perp$ BC, and

DE $\perp$ AC

To Prove : $\Delta ADE \sim \Delta GCF$



In ΔADE, we have

$\angle A + \angle D = 90°$ ...(i)

In ΔABC, we have

$\angle A + \angle C = 90°$ ....(ii)

From equation (i) and equation (ii), we have

$\angle A + \angle C = \angle A + \angle D$

$\Rightarrow \angle C = \angle D$

Similarly, we have

$\angle A = \angle G$

Since ΔADE and ΔGCF are equiangular, therefore $\Delta ADE \sim \Delta GCF$

4. Given, XZ || BC

$\Rightarrow \frac{AX}{BX} = \frac{AZ}{ZC}$ (By BPT)

$\frac{AX}{BX} = \frac{3}{2}$ ...(i)

In ΔAXY and ΔABM

$\angle AXY = \angle ABM$ (corresponding $\angle$s)

$\angle XAY = \angle BAM$. (common angle)

$\therefore$ $\Delta XAY \sim \Delta ABM$ (By AA criterion)

$\Rightarrow \frac{AX}{AB} = \frac{XY}{BM} \Rightarrow \frac{3}{5} = \frac{XY}{3}$

$XY = \frac{9}{5} = 1.8$ cm

5. Given, PR || AC

So, from ΔOAC

$$\frac{OP}{PA} = \frac{OR}{RC}$$

(By Basic Proportionality Theorem) ...(i)

Gives, PQ || AB, so from ΔOAB

$\frac{OQ}{QB} = \frac{OP}{PA}$ ...(ii)

From eqns. (i) and (ii)

$$\frac{OQ}{QB} = \frac{OR}{RC}$$

So, by the converse of Basic Proportionality Theorem

QR || BC Hence Proved.

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. In ΔCAP and ΔCBQ

$$\angle CAP = \angle CBQ = 90°$$

$$\angle PCA = \angle QCB \quad \text{(common angle)}$$

So, $\Delta CAP \sim \Delta CBQ$ (By AA similarity criterion)

Hence,

$$\frac{BQ}{AP} = \frac{BC}{AC} \Rightarrow \frac{y}{x} = \frac{BC}{AC} \quad \text{...(i)}$$

Now, in Δ ACR and ΔABQ

$$\angle ACR = \angle ABQ = 90°$$

$$\angle CAR = \angle BAQ \quad \text{(Common angle)}$$

So, $\Delta ACR \sim \Delta ABQ$

(By AA similarity criterion)

Hence,

$$\frac{BQ}{CR} = \frac{AB}{AC} \Rightarrow \frac{y}{z} = \frac{AB}{AC} \quad \text{...(ii)}$$

On adding eq. (i) and (ii), we get

$$\frac{y}{x} + \frac{y}{z} = \frac{BC}{AC} + \frac{AB}{AC}$$

$$y\left(\frac{1}{x} + \frac{1}{z}\right) = \frac{BC+AB}{AC}$$

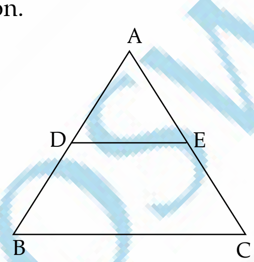
$$y\left(\frac{1}{x} + \frac{1}{z}\right) = \frac{AC}{AC}$$

$$\Rightarrow \quad y\left(\frac{1}{x} + \frac{1}{z}\right) = 1$$

$$\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$$

Hence Proved.

2. Basic Proportionality Theorem states that if a line is drawn parallel to a side of a triangle which intersects the other sides into two distinct points, then the line divides those sides of the triangle in proportion.



In ΔABC, DE || BC

$$\Rightarrow \quad \frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \quad \frac{x}{x-2} = \frac{x+2}{x-1}$$

$$x^2 - x = (x-2)(x+2)$$

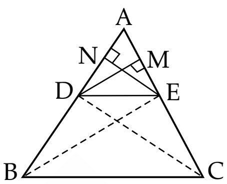
$$x^2 - x = x^2 - 2^2$$

$$x^2 - x = x^2 - 4$$

$$x = 4$$

3. Given, ΔABC where DE || BC

To Prove: $\dfrac{AD}{DB} = \dfrac{AE}{EC}$



Construction: Join BE and CD

Draw DM ⊥ AC and EN ⊥ AB

$$\text{area } \Delta \text{ ADE} = \frac{1}{2} \times \text{Base} \times \text{height}$$

$$= \frac{1}{2} \times AD \times EN \quad \text{...(i)}$$

$$\text{area } \Delta BDE = \frac{1}{2} \times DB \times EN \quad \text{...(ii)}$$

Divide (i) and (ii)

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN}$$

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{AD}{DB} \quad \text{...(1)}$$

Now, $\quad \text{area } \Delta AED = \frac{1}{2} \times AE \times DM \quad \text{...(iii)}$

$$\text{area } \Delta DEC = \frac{1}{2} \times EC \times DM \quad \text{...(iv)}$$

Divide (iii) and (iv)

$$\frac{ar(\Delta ADE)}{ar(\Delta DEC)} = \frac{AE}{AC} \quad \text{...(2)}$$

Now, ΔBDE and ΔDEC are on the same base DE and between same parallel lines BC and DE

$\therefore$ area ΔBDE = area ΔDEC ...(3)

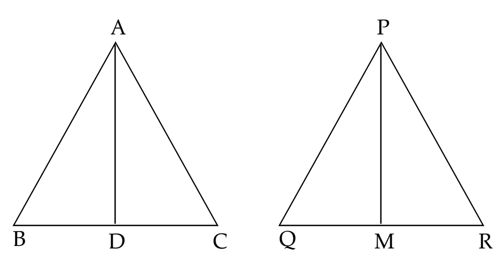
From (1), (2) and (3) we get

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence Proved.

4. Given, ΔABC and ΔPQR, AB, BC and median AD of ΔABC are proportional to sides PQ, QR and median PM of ΔPQR

i.e. $\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AD}{PM}$



To Prove : $\Delta ABC \sim \Delta PQR$

Proof :

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM} \text{ (given)}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM} \quad ...(i)$$

(D is the mid-point of BC. M is the mid-point of QR)

⇒ ΔABD ~ ΔPQM [SSS similarity criterion]

∴ ∠ABD = ∠PQM

[Corresponding angles of two similar triangles are equal]

⇒ ∠ABC = ∠PQR

Now, in ΔABC and ΔPQR

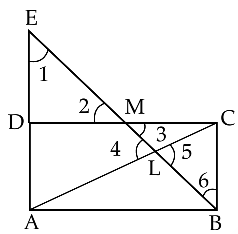
$$\frac{AB}{PQ} = \frac{BC}{QR}$$

∠ABC = ∠PQR (Proved above)

∴ ΔABC ~ ΔPQR [By SAS similarity criterion]

Hence Proved.

5.



In ΔDEM and ΔCBM

∠1 = ∠6 (Alternate interior angles)

∠2 = ∠3 (Vertically opposite angles)

DM = MC (M is the mid point of CD)

∴ ΔDEM ≅ ΔCBM (AAS congruence criterion)

So, DE = BC (CPCT)

Also, AD = BC

AE = AD + DE = 2 BC

In ΔELA and ΔBLC

∠1 = ∠6 and ∠4 = ∠5

∴ ΔELA ~ ΔBLC (AA similarity)

$$\Rightarrow \frac{EL}{BL} = \frac{EA}{BC}$$

$$\Rightarrow \frac{EL}{BL} = \frac{2BC}{BC} = 2$$

⇒ EL = 2BL Hence Proved.

6. (i) In ΔEFG and ΔCDG,

∠GFE = ∠GDC (alternate interior angles, EF || DC)

∠EGF = ∠CGD (Vertically opposite angles)

∴ ΔEFG ~ ΔCDG (By AA similarity criterion)

$$\frac{EF}{EG} = \frac{CD}{CG} \Rightarrow \frac{EF}{5} = \frac{18}{10}$$

EF = 9 cm

(ii) In ΔCAB and ΔCEF,

∠CAB = ∠CEF (AB || EF)

∠C = ∠C (Common angle)

∴ ΔCAB ~ ΔCEF (By AA similarity criterion)

$$\Rightarrow \frac{AC}{CE} = \frac{AB}{EF} \Rightarrow \frac{AC}{15} = \frac{15}{9}$$

AC = 25 cm

7. Given, ∠PQR = 90°, QS ⊥ PR

To Prove: ΔPSQ ~ ΔQSR

Proof: In ΔPQR & ΔQSR

∠PRQ = ∠SRQ (common)

∠PQR = ∠QSR (90°)

∴ ΔPQR ~ ΔQSR (AA Criterion)...(i)

Similarly, ΔPQR ~ ΔPQS ...(ii)

From (i) and (ii), we get

ΔQSR~ ΔPQS

Since, ΔPSQ ~ ΔQSR

$$\frac{PS}{QS} = \frac{QS}{SR}$$

$QS^2 = PS \times SR$

$x^2 = 10 \times 8$

$x^2 = 80$

$x = 4\sqrt{5}$

In ΔPQS

$z^2 = 10^2 + x^2$

$= 100 + 80 = 180$

$z = 6\sqrt{5}$

Similarly, in ΔQSR

$y^2 = 8^2 + x^2$

$y^2 = 64 + 80$

⇒ $y = 12$

Thus, $x = 4\sqrt{5}$

$y = 12$

and, $z = 6\sqrt{5}$

Level - 2 ADVANCED COMPETENCY FOCUSED QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Mark)

1. Option (A) is correct

Explanation: Azhar's claim is true because congruent figures have the same shape and size, they also meet the conditions of similar figures (same shape, sides in proportion with ratio 1:1). Ayub's claim is false because similar figures can be of different sizes. So, they are not necessarily congruent.

2. Option (C) is correct

Explanation: Statement 1 is correct because all circles have the same shape regardless of size — they all have the same constant ratio of circumference to diameter. This satisfies the condition for similarity (same shape, sides in proportion).

Statement 2 is correct because all squares have equal angles and the same shape. Their sides are in proportion, so they are always similar, even if their sizes differ.

Statement 3 is incorrect because right triangles may all have a 90° angle, but unless all sides are equal, they are not necessarily congruent. Congruence requires all sides and angles to match exactly.

Statement 4 is incorrect because all equilateral triangles have equal angles (each 60°) and all sides equal within the same triangle, but not all equilateral triangles are the same size, so they are not necessarily congruent — only similar.

3. Option (C) is correct

Explanation: Given, in ΔPQR and ΔDEF: ∠P = ∠E (both labeled as y)

In ΔPQR: PQ = 3.2, PR = 4

In ΔDEF: DE = 6, EF = 4.8

Check the ratio of corresponding sides:

$$\frac{PQ}{DE} = \frac{3.2}{6} = \frac{8}{15} \text{ and } \frac{PR}{EF} = \frac{4}{4.8} = \frac{5}{6}$$

Clearly,

$$\frac{8}{15} \neq \frac{5}{6}$$

So, the sides are not in proportion.

4. Option (C) is correct

Explanation: Triangle ABC is isosceles with two equal sides of 5 cm. Triangle PQR is isosceles with two equal sides of 10 cm.

So, the sides are in the ratio 1:2, and both triangles are isosceles.

But similarity of triangles requires: Corresponding angles equal and/or Sides in proportion with included angles equal (SAS)

In triangle PQR, the angle between the two equal sides (∠P) is given as 40°. For triangle ABC to be similar by SAS, the angle between its equal sides (∠A) must also be 40°.

5. Option (C) is correct

Explanation: Since both Paramjeet and the pole cast shadows at the same time, the triangles formed are similar (same angle of elevation of the sun).

So, we can set up a proportion:

$$\frac{\text{Height of paramjeet}}{\text{Shadow of Paramjeet}} = \frac{\text{Height of Pole}}{\text{Shadow of Pole}}$$

$$\frac{6}{5} = \frac{h}{30}$$

Now, h (height of the pole)

$$h = \frac{6}{5} \times 30 = \frac{180}{5} = 36$$

6. Option (A) is correct

Explanation: When Purohit sees the top of the building via the mirror, the triangles formed by Purohit and the building are similar. So, we set up the proportion:

$$\frac{\text{Height of Purohit}}{\text{Distance of Purohit From mirror}} = \frac{\text{Height of Building}}{\text{Distance of Building from mirror}}$$

$$\frac{5}{2} = \frac{h}{48}$$

$$h = \frac{5}{2} \times 48 = \frac{280}{2} = 120 \text{ feet}$$

7. Option (C) is correct

Explanation: For similar triangles, the ratio of areas is the square of the ratio of corresponding sides:

$$\frac{\text{Area}_{small}}{\text{Area}_{large}} = \left(\frac{\text{side}_{small}}{\text{side}_{large}}\right)^2$$

$$\frac{9}{16} = \left(\frac{15}{x}\right)^2$$

Take square roots on both sides:

$$\frac{3}{4} = \frac{15}{x}$$

$3x = 60 \Rightarrow x = 20$

Difference = 20 – 15 = 5 cm

ASSERTION-REASON QUESTIONS (1 Mark)

1. Option (A) is correct

Explanation: Assertion is true because if the flyover is constructed parallel to the base, and intersects the other two sides, it will divide those two sides in the same ratio

Reason is also true as this is a direct statement of the Basic Proportionality Theorem (Thales' Theorem). The flyover acts as the line parallel to one side of the triangle, and according to the Basic Proportionality Theorem, such a line divides the other two sides in the same ratio. So the Reason (R) correctly explains Assertion (A).

2. Option (A) is correct

Explanation: Assertion is true because this is a common application of similar triangles. If the tree and the pole both cast shadows at the same time, the triangles formed by the tree and its shadow and by the pole and its shadow are similar. Hence, the ratio of height to shadow length is the same, and we can use this to find the unknown height.

Reason is also true because this is the definition of similar triangles. Similarity is established by either AA (Angle-Angle), SAS (Side-Angle-Side with proportion), or SSS (Side-Side-Side with proportion). So this statement is a valid explanation of triangle similarity.

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. Since the sun's rays fall at the same angle, the triangles formed by the pole and its shadow and the tree and its shadow are similar. So, we can set up a proportion using the corresponding sides:

$$\frac{\text{Height of pole}}{\text{Shadow of pole}} = \frac{\text{Height of tree}}{\text{Shadow of tree}}$$

Substituting the known values:

$$\frac{2}{4} = \frac{h}{10}$$

$$\Rightarrow \frac{1}{2} = \frac{h}{10}$$

$$\Rightarrow h = \frac{1}{2} \times 10 = 5$$

The height of the tree is 5 metres.

2. Since DE is parallel to BC, by the Basic Proportionality Theorem (Thales' Theorem):

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Substituting the known values:

$$\frac{3}{2} = \frac{4.5}{EC}$$

$$\Rightarrow EC = \frac{4.5 \times 2}{3} = \frac{9}{3} = 3$$

The length of EC is 3 m.

3. Since ST is parallel to PQ, by the Basic Proportionality Theorem, the segments on the other two sides are divided in the same ratio:

$$\frac{PS}{SQ} = \frac{PT}{TQ}$$

Substitute the known values:

$$\frac{3}{6} = \frac{2.5}{TQ} \Rightarrow \frac{1}{2} = \frac{2.5}{TQ}$$

TQ = 2.5 x 2 = 5 m

The length of TQ is 5 meters.

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. Since the sun's rays fall at the same angle, the triangles formed are similar. Therefore, their corresponding sides are in proportion:

$$\frac{\text{Height of person}}{\text{Shadow of Person}} = \frac{\text{Height of building}}{\text{Shadow of building}}$$

Substitute the known values:

$$\frac{1.8}{2.7} = \frac{h}{12}$$

2.7 h = 12 x 1.8

h = 8

The height of the building is 8 meters.

2. Using the Basic Proportionality Theorem (BPT)

Since DE is parallel to BC,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Substitute the known values:

$$\frac{4.5}{3} = \frac{6}{EC}$$

$$\Rightarrow \frac{3}{2} = \frac{6}{EC}$$

$$EC = 6 \times \frac{2}{3} = 4$$

The length of EC is 4 meters.

To verify whether DE divides the triangle in the correct proportion, we check if the ratios match:

$$\frac{AD}{DB} = \frac{4.5}{3} = \frac{3}{2} \text{ and } \frac{AF}{EC} = \frac{6}{4} = \frac{3}{2}$$

Both ratios are equal, so DE divides the triangle in the correct proportion.

Hence **Verified**

3. Let's assume the 3 m side corresponds to the 9 m side of the larger triangle. Then the scale factor is:

$$\frac{3}{9} = \frac{1}{3}$$

Now multiply the other sides of the larger triangle by 1/3:

Side corresponding to 12 m: $12 \times \frac{1}{3} = 4$ m

Side corresponding to 15 m: $15 \times \frac{1}{3} = 5$m

The lengths of the other two sides of the smaller triangle are 4 meters and 5 meters.

CASE BASED QUESTIONS (4 Mark)

1. (i) In ΔDPQ and ΔDEF

$\angle DPQ = \angle DEF$ ($\because$ PQ || EF, corresponding $\angle$s)

$\angle PDQ = \angle EDF$ (common)

$\therefore$ ΔDPQ ~ ΔDEF (By AA critertion)

(ii) DE = 50 + 70 = 120 cm

As, ΔDPQ ~ Δ DEF

$\therefore \quad \frac{DP}{DE} = \frac{PQ}{EF} = \frac{50}{120}$

$\therefore \quad \frac{PQ}{EF} = \frac{50}{120}$ or $\frac{5}{12}$

(iii) (a) As, $\Delta ABC \sim \Delta DEF$

$\therefore \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Given, $2AB = 5DE$

$\Rightarrow \quad AB = \frac{5}{2} DE$

Similarly, $\quad BC = \frac{5}{2} EF$

and, $\quad AC = \frac{5}{2} DF$

Now,

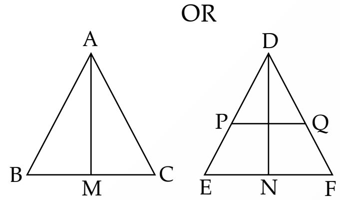
$$\frac{\text{Perimeter of ØABC}}{\text{Perimeter of ØDEF}} = \frac{AB+BC+CA}{DE+EF+DF}$$

$$\Rightarrow \frac{\text{Perimeter of ØABC}}{\text{Perimeter of ØDEF}} = \frac{\frac{5}{2}(DE+EF+FD)}{DE+EF+FD}$$

$$= \frac{5}{2} \text{ (constant)}$$

OR

(b)



$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{BC/2}{EF/2} = \frac{BM}{EN}$$

Also, $\quad \angle B = \angle E$

$\therefore \quad \Delta ABM \sim \Delta DEN$ (By SAS criterion)

2. (i) Given: AD = 3 m and DB = 2 m

So, the ratio in which D divides AB is:

$$\frac{AD}{DB} = \frac{3}{2}$$

D divides AB in the ratio 3 : 2

(ii) Using Basic Proportionality Theorem, since DE is parallel to BC,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Substitute known values:

$$\frac{3}{2} = \frac{4.5}{EC}$$

$$\Rightarrow EC = \frac{4.5 \times 2}{3} = \frac{9}{3} = 3\text{m}$$

The length of EC is 3 meters.

(iii) (a) We already have:

$$\frac{AD}{DB} = \frac{3}{2}$$

$$\frac{AE}{EC} = \frac{4.5}{3} = \frac{3}{2}$$

Since both ratios are equal, line DE divides triangle ABC in the same ratio on both sides.

The theorem used is the Basic Proportionality Theorem.

OR

(b) Since DE is parallel to BC, and intersects sides AB and AC, by the AAA (Angle-Angle-Angle) similarity criterion:

$\angle ADE = \angle ABC$ (corresponding angles)

$\angle AED = \angle ACB$ (corresponding angles)

$\angle A$ is common to both triangles

So:

$\Delta ADE \sim \Delta ABC$

Triangle ADE is similar to triangle ABC by AAA similarity criterion.

3. (i) Using the Pythagorean Theorem:

$AB^2+AC^2=3^2+4^2=9+16=25$

$BC = 5$

Since the sum of the squares of the two shorter sides equals the square of the longest side, triangle ABC is right-angled at A.

Yes, triangle ABC is a right-angled triangle.

(ii) The Converse of the Pythagorean Theorem: justifies the conclusion in (i).

The theorem states that if the square of one side of a triangle is equal to the sum of the squares of the other two sides, the triangle is right-angled.

(iii) (a) Wire from pole to B: Horizontal distance = AB = 3 m and Vertical height = 2 m

Wire length $= \sqrt{3^2+2^2} = \sqrt{9+4} = \sqrt{13} \sim 3.61$ m

Wire from pole to C: Horizontal distance = AC = 4 m and Vertical height = 2 m

Wire length $= \sqrt{4^2+2^2} = \sqrt{16+4} = \sqrt{20} \sim 4.47$ m

Total wire length = 3.61 + 4.47 = 8.08 meters.

OR

(b) If triangle scaled up by a factor of 2

Multiply each side by 2:

$AB = 3 \times 2 = 6$ m

$AC = 4 \times 2 = 8$ m

$BC = 5 \times 2 = 10$ m

New side lengths = 6 m, 8 m, 10 m

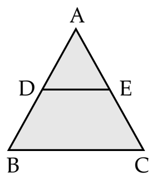
LONG ANSWER TYPE QUESTIONS (5 Marks)

1. (i) Given: AB = 6 m, AD = 4

$\Rightarrow DB = AB - AD = 6 - 4 = 2$ m

AC = 7.5 m, AE = 5

$\Rightarrow EC = AC - AE = 2.5$ m



Checking the ratios:

$$\frac{AD}{DB} = \frac{4}{2} = 2, \frac{AE}{EC} = \frac{5}{2.5} = 2$$

Since both ratios are equal, by Basic Proportionality Theorem, it can be concluded:

DE || BC.

(ii) DB = 2 m

EC = 2.5 m (Calculated above)

(iii) We know:

$\Delta ADE \sim \Delta ABC$ (from DE || BC)

Area of similar triangles is in the square of the ratio of corresponding sides

Since,

$$\frac{AD}{AB} = \frac{4}{6} = \frac{2}{3}$$

So, ratio of areas:

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta ABC} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Let area of triangle ABC = A. Then:

$$\frac{24}{A} = \frac{4}{9} \Rightarrow A = \frac{24 \times 9}{4} = 54\text{ m}^2$$

Area of triangle ABC = 54 m^2

(iv) In similar triangles, the ratio of perimeters = ratio of corresponding sides.

Since,

$$\frac{\text{Perimeter of } \Delta ADE}{\text{Perimeter of } \Delta ABC} = \frac{AD}{AB} = \frac{4}{6} = \frac{2}{3} = 2:3$$

2 (i) Let the base point of the tower be point A, and the top be point B.

The two ground support points be C and D, such that the total distance CD = 15 m, and the tower stands vertically at midpoint A.

So, AC = AD = 7.5 m, and AB = 20 m

To show congruence:

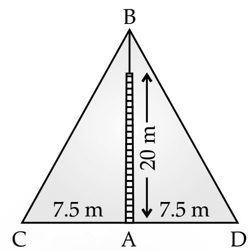
AB is common (hypotenuse)

AC = AD = 7.5 m

$\angle ACB = \angle ADB = 90°$

So, by RHS Congruence Criterion (Right angle, Hypotenuse, Side):

$\Delta ABC \cong \Delta ABD$



(ii) Each wire acts as the hypotenuse of a right-angled triangle with Height = 20 m and Base = 7.5 m

Using Pythagoras Theorem:

Wire length $= \sqrt{20^2 + 7.5^2} = \sqrt{400 + 56.25} = \sqrt{456.25}$

Wire length ≈ 21.37 m

(iii) The area of land used is the area of the triangle formed by the wires and the ground, i.e., a right triangle on each side.

Area of each triangle $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 7.5 \times \text{tower height}$

For 20 m tower:

Area on one side $= \frac{1}{2} \times 7.5 \times 20 = 75\text{ m}^2 \Rightarrow$ Total area = 2 × 75 = 150 m^2

For 12 m tower:

Area on one side $= \frac{1}{2} \times 7.5 \times 12 = 45\text{ m}^2 \Rightarrow$ Total area = 2 × 45 = 90 m^2

Area of land used: 150 m^2 (20 m tower) vs. 90 m^2(12 m tower)