CHAPTER 8

Circles

Level - 1 CORE SUBJECTIVE QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Mark)

1. Option (D) is correct

Explanation: In the given statements:

(A) A number of secants can be drawn from any point on a circle. This is true, as a secant line intersects the circle at two distinct points.

(B) Only one tangent can be drawn at any point on the circle. This is true, as a tangent touches the circle at exactly one point.

(C) A chord is a line segment joining two points on the circle. This is true, as the definition of a chord is the line segment between any two points on the circle.

(D) From a point inside a circle, only two tangents can be drawn. This is false because tangents can only be drawn from a point outside the circle or on the circle but not from inside the circle.

2. Option (C) is correct

Explanation: A tangent is drawn at point P on the circle, then $\angle OPT = 90°$ and we are given that $\angle TPQ = 110°$.

Then, $\angle OPQ = \angle TPQ - \angle TPO$
$= 110° - 90°$
$= 20°$

In ΔOPQ, $OP = OQ$ (radii)

Then, $\angle OPQ = \angle OQP = 20°$

$\therefore$ $\angle POQ = 180° - 2 \times \angle OPQ$
$= 180° - 40°$
$= 140°$

3. Option (B) is correct

Explanation: $OT \perp PT$ (Radius is $\perp$ to the tangent)

$\therefore$ $\angle OTP = 90°$

Now, in right angled ΔOPT

According to the Pythagoras theorem:

$$OP^2 = OT^2 + PT^2$$
$$OP = 11 \text{ cm}$$
$$OT = 7 \text{ cm (radius)}$$
$$\Rightarrow \quad 11^2 = 7^2 + PT^2$$
$$121 = 49 + PT^2$$
$$PT = \sqrt{72} = 8.5 \text{ cm}$$

Therefore, the length of the tangent is approximately 8.5 cm.

4. Option (C) is correct

Explanation: Given:

OT (radius) = 4 cm

PT (tangent) = 15 cm

$OT \perp PT$ (Radius is $\perp$ to tangent)

$\therefore$ $\angle OTP = 90°$

In Right angled ΔOTP

Using the Pythagoras theorem:

Substitute the values:

$$OP^2 = OT^2 + PT^2$$
$$OP^2 = 4^2 + 15^2$$
$$OP^2 = 16 + 225 = 241$$
$$OP = \sqrt{241} \approx 15.52 \text{ cm}$$

Thus, the length of OP is approximately 15.52 cm.

5. Option (B) is correct

Explanation: $OP \perp PR$ [As, tangent and radius are $\perp$ to each other at the point of contact]

$\Rightarrow$ $\angle OPQ = 90° - 50° = 40°$

$OP = OQ$ [Radii]

$\therefore$ $\angle OPQ = \angle OQP = 40°$

In ΔOPQ

$\Rightarrow$ $\angle POQ + \angle OPQ + \angle OQP = 180°$

$\angle POQ = 180° - 80° = 100°$

6. Option (D) is correct

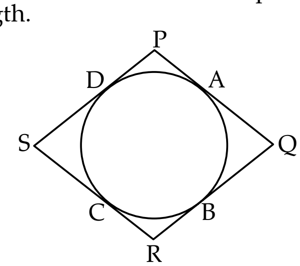
Explanation: A quadrilateral PQRS is drawn to circumscribe a circle.

Hence,

$$PQ + RS = QR + SP$$
$$12 + 14 = 15 + SP$$
$$SP = 26 - 15$$
$$SP = 11 \text{ cm}$$

7. Option (C) is correct

Explanation: We know that tangents drawn to a circle from the same external point will be equal in length.



Therefore,

$PD = PA$ ...(i)

$SD = SC$ ...(ii)

Adding equations (i) and (ii), we get,

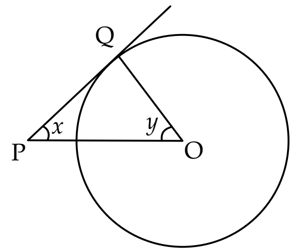
$$PD + SD = PA + SC$$

By looking at the figure we can say,

$$PS = PA + CS$$

8. Option (B) is correct

Explantion: In the given figure, PQ is a tangent to the circle with centre O.



Given, $\angle OPQ = x, \angle POQ = y$

Now, $\angle OQP = 90°$

($\because$ Radius is perpendicular to the tangent at the point of contact)

In ΔOPQ

$$\angle OPQ + \angle POQ + \angle OQP = 180°$$

(angle sum property)

$\Rightarrow \quad x + y + 90° = 180°$

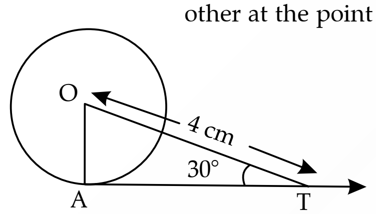
$\Rightarrow \quad x + y = 180° - 90° = 90°$

9. Option (A) is correct

Explanation: $\angle OTA = 30°$ (Given)

$\angle OAT = 90°$

[$\because$ Tangent and radius are $\perp$ to each other at the point of contact]



In right-angled ΔOAT,

$$\frac{AT}{OT} = \cos 30°$$

$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{AT}{4}$

$\Rightarrow \quad AT = \frac{\sqrt{3} \times 4}{2}$

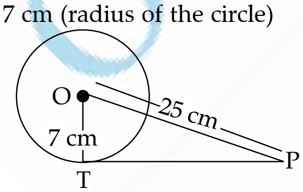
$\Rightarrow \quad AT = 2\sqrt{3}$ cm

10. Option (B) is correct

Explanation: Given:

OP = 25 cm (distance from point P to the center O)

OT = 7 cm (radius of the circle)



$OT \perp TP$ ($\because$ Radius is $\perp$ to the tangent at the point of contact)

In right angled ΔOTP,

Using the Pythagoras theorem:

$$OP^2 = OT^2 + PT^2$$

Substitute the values:

$$25^2 = 7^2 + PT^2$$

$$625 = 49 + PT^2$$

$$PT^2 = 625 - 49 = 576$$

$$PT = \sqrt{576} = 24 \text{ cm}$$

11. Option (A) is correct

Explanation: Given: A circle with center O and PA and PB are tangents to circle from a common external point P to point A and B respectively and $\angle APB = 50°$

$OA \perp AP$ and $OB \perp PB$ [As tangent to at any point on the circle is perpendicular to the radius through point of contact]

$$\angle OBP = \angle OAP = 90° \quad ...(i)$$

In Quadrilateral AOBP

$$\angle OBP + \angle OAP + \angle AOB + \angle APB = 360°$$

$$90° + 90° + \angle AOB + 50° = 360°$$

[By angle sum property of quadrilateral]

$$\angle AOB = 130° \quad ...(ii)$$

Now in ΔOAB

$$OA = OB$$

[Radii of same circle]

$\therefore \quad \angle OBA = \angle OAB \quad ...(iii)$

(angles opposite to equal sides)

Also, by angle sum property of triangle

$$\angle OBA + \angle OAB + \angle AOB = 180°$$

$$\angle OAB + \angle OAB + 130° = 180°$$

[Using (ii) and (iii)]

$$2\angle OAB = 50°$$

$$\angle OAB = 25°$$

12. Option (D) is correct

Explanation: Given:

$$BC = 7 \text{ cm}$$

$$CD = 4 \text{ cm}$$

$$AD = 3 \text{ cm}$$

Using the property:

$$AB + CD = BC + AD$$

Substitute the values:

$$AB + 4 = 7 + 3$$

$$AB + 4 = 10$$

$$AB = 10 - 4 = 6 \text{ cm}$$

13. Option (C) is correct

Explanation: Let

$$AP = x = CQ$$

$$AP = AR = x$$

$$CQ = CR = x$$

$$AC = 7$$

$$AR + CR = 7$$

$$x + x = 7$$

$$x = 3.5$$

$\therefore \quad BP = 10 - AP = 10 - x$

$= 10 - 3.5$

$= 6.5$ cm

14. Option (A) is correct

Explanation: OB = radius

$\angle OBA = 90°$ (Radii & tangent are $\perp$ to each other)

In right angled ΔOAB

$$\sin 30° = \frac{OB}{OA}$$

$$\frac{1}{2} = \frac{OB}{6}$$

$$OB = 3 \text{ cm}$$

15. Option (A) is correct

Explanation: In ΔAOB,

$$OA = OB \text{ (radii of the same circle)}$$

$$\Rightarrow \quad \angle OBA = \angle OAB = x$$

Using angle sum property, $x + x + 95° = 180°$

$$\Rightarrow \quad 2x = 85°$$

$$x = 42.5°$$

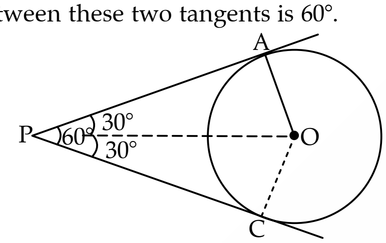
Now, $\angle OBQ = 90°$ ($OB \perp PQ$)

$$\Rightarrow \quad \angle ABQ = \angle OBQ - \angle OBA$$

$$= 90° - 42.5° = 47.5°$$

16. Option (D) is correct

Explanation: Let P be an external point and a pair of tangents is drawn from point P and angle between these two tangents is 60°.



Radius of the circle = 5 cm

Join OA and OP

Also, OP is a bisector line of $\angle APC$

$$\therefore \quad \angle APO = \angle CPO = 30°$$

$$OA \perp AP$$

As, tangent at any point of a circle is perpendicular to the radius through the point of contact.

In right angled ΔOAP, we have

$$\tan 30° = \frac{OA}{AP} = \frac{5}{AP}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{5}{AP}$$

$$\Rightarrow \quad AP = 5\sqrt{3} \text{ cm}$$

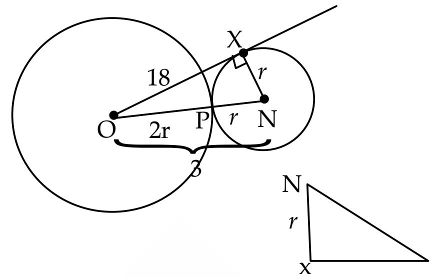
$AP = CP = 5\sqrt{3}$ cm [Tangents drawn from an external point are equal]

Hence, the length of each tangent is $5\sqrt{3}$ cm

17. Option (C) is correct

Explanation: In right-angled ΔONX

$$\angle OXN = 90° \quad \text{(Point of contact)}$$



By using Pythagoras theorem

$$ON^2 = NX^2 + OX^2$$

$$(3r)^2 = r^2 + 18^2$$

$$9r^2 = r^2 + 18^2$$

$$9r^2 - r^2 = 324$$

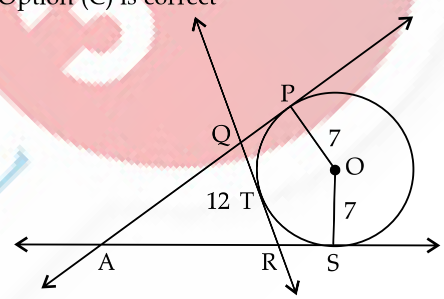
$$8r^2 = 324$$

$$r^2 = \frac{324}{8}$$

$$= \frac{81}{2}$$

$$r = \frac{9}{\sqrt{2}} \text{ cm}$$

18. Option (C) is correct



As, per tangents theorem:

The lengths of tangents drawn from an external point to a circle are equal.

So,

$$AP = AS$$

Let $QT = QP = x$

$$RS = RT = 12 - x$$

Now, Perimeter of PQTRSO

$= QP + PO + OS + RS + RT + QT$

$= x + 7 + 7 + 12 - x + 12 - x + x$

$= 7 + 7 + 12 + 12 = 38$ cm

ASSERTION-REASON QUESTIONS (1 Mark)

1. Option (A) is correct

Explanation: Two parallel tangents always lie at the end points of the diameter of the circle. So, assertion is true. Also, both tangents are perpendicular to the same diameter, and the dines perpendicular to the same line are parallel. Reason is also true, and it's the fundamental property that explains why the tangents are parallel.

2. Option (B) is correct

Explanation: Assertion and Reason both are true. However, (R) does not explain (A). The reason talks about a different property of tangents, unrelated to the perpendicularity mentioned in the assertion.

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. Given:

$\angle BAC = 65°$

Now, $OB \perp AB$

$\therefore \angle OBA = 90°$

$OC \perp AC$

$\therefore \angle OCA = 90°$

The angle sum property in quadrilateral OABC:

$\angle OBA + \angle OCA + \angle BAC + \angle BOC = 360°$

Substitute the known values:

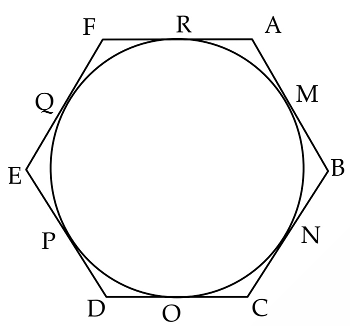
$90° + 90° + 65° + \angle BOC = 360°$

$\Rightarrow 245° + \angle BOC = 360°$

$\angle BOC = 360° - 245° = 115°$

Thus, the measure of $\angle BOC$ is 115°.

2.



As, length of the tangents drawn from an external external point to the circle are equal.

$AM = AR$ ...(i)

$BM = BN$ ...(ii)

$CO = CN$ ....(iii)

$DO = DP$ ...(iv)

$EQ = EP$ ...(v)

$FQ = FR$ ...(vi)

Adding (i) and (ii) we get

$AM + BM = AR + BN$

$\Rightarrow AB = AR + BN$

Adding (iii) and (iv) we get

$CO + DO = CN + DP$

$\Rightarrow CD = CN + DP$

Adding (v) and (vi) we get

$EQ + FQ = EP + FR$

$\Rightarrow EF = EP + FR$

Adding all these we obtain

$AB + CD + EF = AR + (BN + CN) + (DP + EP) + FR = BC + DE + FA$

$\therefore AB + CD + EF = BC + DE + FA$

Hence Proved.

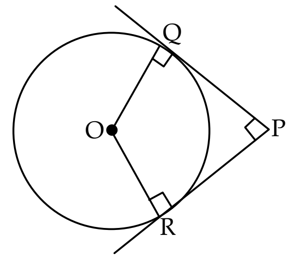
3. Given, $\angle OPR = 45°$

$\therefore \angle OPQ = \angle OPR = 45°$

($\because$ tangents from an external point are equally inclined to the line segment joining the centre to that point)

Thus, $\angle QPR = \angle OPQ + \angle OPR$

$= 90°$



We know that the lengths of tangents drawn from an external point to a circle are equal.

$\therefore PQ = PR$ ...(i)

We know that the radius is perpendicular to the tangent at the point of contact.

$\therefore \angle PQO = 90°$ and $\angle ORP = 90°$

In quadrilateral OQPR:

$\angle QPR + \angle PQO + \angle QOR + \angle ORP = 360°$

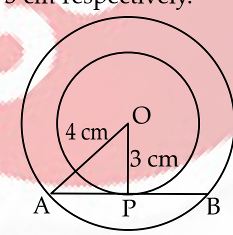
$\Rightarrow 90° + 90° + \angle QOR + 90° = 360°$

$\Rightarrow \angle QOR = 360° - 270° = 90°$

$\therefore \angle QPR = \angle PQO = \angle QOR = \angle ORP = 90°$ ...(ii)

From (i) and (ii), it can be concluded that PQOR is a square.

4. Let O be the centre of the concentric circle of radii 4 cm and 3 cm respectively.



Let AB be a chord of the larger circle touching the smaller circle at P.

Then

$AP = PB$ and $OP \perp AB$

Applying Pythagoras theorem in right angled ΔOPA, we have

$OA^2 = OP^2 + AP^2$

$\Rightarrow 16 = 9 + AP^2$

$\Rightarrow AP^2 = 7$

$\Rightarrow AP = \sqrt{7}$cm

As, $\perp$ drawn from the centre of the circle to the chord bisects the chord

$\therefore AB = 2AP = 2\sqrt{7}$ cm

5. Let the radius of the circle $OP = r$

OP is perpendicular to the tangent l.

$RS \parallel l$ (given)

$\therefore \angle OQS = \angle OPl = 90°$ (Corresponding $\angle$S)

$\Rightarrow OQ \perp RS$

Therefore, $QS = \frac{1}{2} RS = 6$ cm

(Perpendicular drawn from centre to the chord bisects the chord)

In, right angled ΔOQS,

$$\angle Q = 90°$$
$$OS^2 = OQ^2 + QS^2$$
(By Pythagoras theorem)
$$r^2 = \left(\frac{r}{2}\right)^2 + 6^2$$
$$r^2 - \frac{r^2}{4} = 36$$
$$\frac{3r^2}{4} = 36$$
$$r^2 = 48$$
$$r = 4\sqrt{3} \text{ cm}$$

6. OA is perpendicular to AP (Radius is ⊥ to tangent)

Now, in right angled ΔOAP

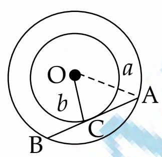
By Applying Pythagoras Theorem:

$$OP^2 = OA^2 + AP^2$$
$$\Rightarrow \quad OA = \sqrt{100-64}$$
$$\Rightarrow \quad OA = \sqrt{36}$$
$$\Rightarrow \quad OA = 6 \text{ cm} = \text{radius}$$

As, Diameter = $2r$

$$\Rightarrow \quad D = 12 \text{ cm}$$

7. Let O be the common centre of the two circles and AB be the chord of the larger circle which touches the smaller circle at C.



Join OA and OC.

Then OC ⊥ AB

Let OA = a and OC = b.

Since OC ⊥ AB, OC bisects AB

[∵ perpendicular from the centre to a chord bisects the chord].

In right angled ΔACO, we have

$$OA^2 = OC^2 + AC^2$$
[by Pythagoras theorem]
$$\Rightarrow \quad AC = \sqrt{OA^2 - OC^2} = \sqrt{a^2 - b^2}$$
$$\therefore \quad AB = 2AC = 2\sqrt{a^2 - b^2}$$
[∵ C is the midpoint of AB]

∴ Length of the chord AB = $2\sqrt{a^2 - b^2}$

8. Given:

CD, CB and FE are tangents to the circle with center A.

$$\angle DCF = 60°$$
$$\angle EFC = 75°$$

A tangent to a circle is perpendicular to the radius at the point of tangency.

Since CD, CB and FE are tangents, and all are perpendicular to the radii at points D, B and E respectively:

$$\therefore \quad \angle ADC = 90°$$
$$\angle AEF = 90°$$
and $$\angle ABC = 90°$$

Now $\angle EFC + \angle EFB = 180°$ (straight line ∠s)

$$\Rightarrow \quad \angle EFB = 180° - 75° = 105°$$

Now, in quadrilateral EFBA: $\angle EAB + \angle EFB = 180°$

$$\Rightarrow \quad \angle EAB = 180° - 105° = 75° \quad ...(i)$$

Again, In quadrilateral ADCB: $\angle DCB + \angle DAB = 180°$

$$\angle DAB = 180° - 60° = 120°$$
$$\angle DAB = \angle DAE + \angle EAB$$
$$\Rightarrow \quad \angle DAE = 120° - 75° = 45°$$

9. Given : Three tangents at points A, E and C.

AE is a diameter

To Prove : At least one pair of opposite sides of AEDB is parallel

Proof : If we extend AE and BD they will touch each other thus AE and BD are not parallel.

Now, OA ⊥ AB and OE ⊥ ED

(∵ radius are ⊥ to the tangent through the point of contact)

$$\Rightarrow \quad \angle OAB = \angle OED = 90° \quad ...(i)$$

We can see that,

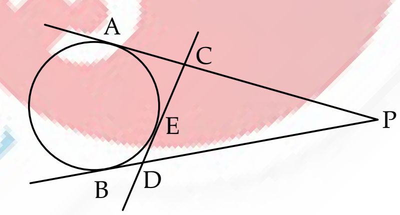
$$\angle OAB + \angle OED = 90° + 90° = 180°$$

As, adjacent interior angles are supplementary.

$$\therefore \quad AB \parallel ED$$

Hence, atleast one pair of opposite sides of AEDB is parallel.

10.



PA = PB; CA = CE; DE = DB [Tangents form external point to a circle]

Perimeter of ΔPCD

$$= PC + CD + PD$$
$$= PC + CE + ED + PD$$
$$= PC + CA + BD + PD$$
$$= PA + PB$$

Perimeter of ΔPCD = PA + PA = 2PA = 2(10) = 20 cm

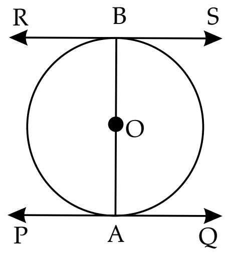
11. We know that the radius and tangent are perpendicular at their point of contact

As, $\angle PRO + \angle ROQ + \angle OQP + \angle QPR = 360°$

$$\Rightarrow \quad \angle QPR + \angle ROQ + 90° + 90° = 360°$$
$$\Rightarrow \angle QPR + \angle ROQ = 180°$$

Since, the sum of the opposite angles of the quadrilateral is 180°. Hence, PROQ is a cyclic quadrilateral.

12. Given: Let AB be a diameter of a given circle, and let PQ and RS be the tangent lines drawn to the circle at points A and B respectively.



To prove: PQ || RS

Proof: BA ⊥ PQ and AB ⊥ RS

(∵ Radius and tangent are ⊥ to each other at their point of contact.)

$\Rightarrow \quad \angle PAB = 90°$

and $\quad \angle ABS = 90°$

$\Rightarrow \quad \angle PAB = \angle ABS$

$\Rightarrow \quad PQ \,||\, RS$

[∵ ∠PAB and ∠ABS are alternate angles]

13. Given, PT is a tangent to a circle from an external point P. A line segment PAB is drawn to a circle with centre O.

OC is perpendicular on the chord AB.

From the figure,

$$PA = PC - AC$$
$$PB = PC + BC$$

So, PA. PB = (PC – AC) (PC + BC)

Since, OC is perpendicular on the chord AB.

$\therefore \quad AC = BC$

(∵ ⊥ drawn from the centre of a circle to the chord bisects the chord)

Now, PA. PB = (PC – AC) (PC + AC)

By using algebraic identity,

$(a^2 - b^2) = (a + b)(a - b)$ we get,

$$(PC - AC)(PC + AC) = PC^2 - AC^2$$
$$= RHS$$

14. Given:

OP = 6 cm (radius of the first circle).

O′P = 8 cm (radius of the second circle).

∠OPO′ = 90° because the tangents OP and O′P are perpendicular to the radii.

Using Pythagoras Theorem: In right angled ΔOPO′:

$$(OO')^2 = (OP)^2 + (O'P)^2$$

Substituting the given values:

$$(OO')^2 = (6)^2 + (8)^2 = 36 + 64 = 100$$
$$OO' = \sqrt{100} = 10 \text{ cm}$$

Using Pythagoras in right angled ΔOPA

∠OAP = 90° (where OA = x and PA is the perpendicular from P):

$$(OP)^2 = (OA)^2 + (PA)^2$$

Substituting OP = 6cm:

$$6^2 = x^2 + (PA)^2$$
$$36 = x^2 + (PA)^2 \quad \text{(i)}$$

Similarly, in right angled ΔO′PA

∠O′AP = 90° (where O′A = 10 – x):

$$(O'P)^2 = (O'A)^2 + (PA)^2$$

Substituting O′P = 8 cm

$$8^2 = (10 - x)^2 + (PA)^2$$
$$64 = (100 - 20x + x^2) + (PA)^2 \quad \text{(ii)}$$

Substitute value of $(PA)^2$ into Equation (ii):

$$64 = (36 - x^2) + (100 - 20x + x^2)$$
$$\Rightarrow \quad 64 = 136 - 20x$$
$$\Rightarrow \quad 20x = 72$$
$$x = \frac{72}{20} = 3.6 \text{ cm}$$

Substitute x = 3.6 into Equation (i):

$$(PA)^2 = 36 - (3.6)^2 = 36 - 12.96$$
$$= 23.04$$
$$PA = \sqrt{23.04} \oplus 4.8 \text{ cm}$$

As ⊥ drawn from the centre of the circle to the chord bisects the chord.

∴ The length of the common chord PQ = 2 × PA:

$$PQ = 2 \times 4.8 = 9.6 \text{ cm}$$

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. We know that tangents drawn from an external point to a circle are equal

$\therefore \quad TP = TQ$

In ΔTPQ, $\quad TP = TQ$

$\angle TQP = \angle TPQ \quad$ ...(i)

[As opposite equal sides have equal angles]

Now, using angle sum property

$$\angle PTQ + \angle TQP + \angle TPQ = 180°$$
$$2\angle TPQ + \angle PTQ = 180° \quad \text{[using eq. (i)]}$$
$$\Rightarrow \quad \angle PTQ = 180° - 2\angle TPQ \quad \text{...(ii)}$$

Now, $\quad OP \perp PT$

$$\Rightarrow \quad \angle OPT = 90°$$
$$\Rightarrow \quad \angle OPQ + \angle QPT = 90°$$
$$\angle OPQ = 90° - \angle QPT$$

Multiply both sides by 2

$$2\angle OPQ = 2(90° - \angle QPT)$$
$$2\angle OPQ = 180° - 2\angle QPT \quad \text{...(iii)}$$

From eqns. (ii) and (iii)

$\angle PTQ = 2\angle OPQ \quad$ Proved.

2. Given:

$$\angle B = 90°$$
$$AD = 17 \text{ cm},$$
$$AB = 20 \text{ cm},$$
$$DS = 3 \text{ cm}$$

From the property of tangents drawn from an external point to a circle:

Tangents from A are equal: RA = AQ,
Tangents from D are equal: DR = DS,
Tangents from B are equal : PB = BQ
Since DS = 3 cm, we also have DR = 3 cm

AD = 17 cm, so :

DR + RA = AD ⇒ RA = 17 – 3

⇒ RA = 14 cm

Since, RA = AQ (from the propety of tangents), we also have :

AQ = 14 cm

AB = 20 cm, and AQ + QB = AB. Therefore :

14 + QB = 20 ⇒ QB = 6 cm

Now, considering quadrilateral OPBQ, where O is the center of the circle, and OP = OQ = r we observe that it is a square because:

OP = OQ (radii of the circle)
PB = BQ
(from the tangent property)

Thus, OP = QB = 6 cm, meaning the radius r of the circle is 6 cm.

∴ The radius of the circle is 6 cm.

3. In the given figure OA = OP [radii of the same circle]

In ΔOAP, ∠OPA = ∠OAP (angles opposite to equal sides) ...(i)

Since tangent is perpendicular to radius

∠OPR = 90°
∠OPA + ∠APR = 90°
∠OAP + ∠APR = 90° (from (i))

Since ∠OAP = ∠QAP
[On extending line AO to point Q]

Therefore, ∠QAP + ∠APR = 90°

Hence Proved.

4. Given : Three tangents AQ, AR and BC

To Prove : AQ = $\frac{1}{2}$ (perimeter of ΔABC)

Proof :

Perimeter of the triangle ΔABC,

= AB + AC + BC

∴ = AB + AC + BP + CP
(since P; lies on the line BC)

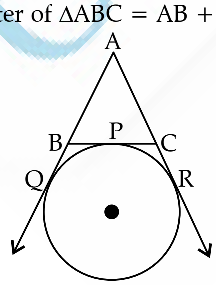
Now BP and BQ are tangents on the circle from point B,

∴ BP = BQ

Similarly, CP and CR are tangents from point C on the circle

∴ CP = CR

As, Perimeter of ΔABC = AB + AC + BP + CP



∴ = AB + AC + (BQ) + (CR)
∴ = (AB + BQ) + (AC + CR)
∴ = AQ + AR

From the diagram, we can see that AQ and AR are tangents on the circle from point A,

∴ AQ = AR

Substituting this relation into equation of P, we get:

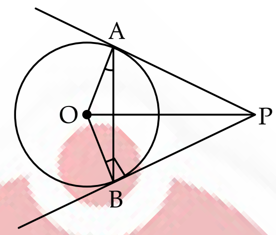
∴ Peumeter of ΔABC= AQ + AR = AQ + (AQ)

= 2AQ

∴ AQ = $\frac{1}{2}$ (Perimeter of ΔABC)

Hence Proved.

5.



Given : ∠OAB = 30°
AB = 6 cm
PA & PB are two tangents

As, ∠OAB = 30°
∠OAP = 90°
[Angle between the tangent and the radius at the point of contact]

So, ∠PAB = ∠OAP – ∠OAB
= 90° – 30°
= 60°

AP = BP
[Tangents to a circle from an external point]

∠PAB = ∠PBA
[Angles opposite to equal sides of a triangle]

In ΔABP, ∠PAB + ∠PBA + ∠APB = 180°
[Angle Sum Property]

60° + 60° + ∠APB = 180°
∠APB = 60° ...(i)

∴ ΔABP is an equilateral triangle, where AP = BP = AB.

Thus, length of PA = 6 cm

In Right angled ΔOAP, ∠OPA = 30°

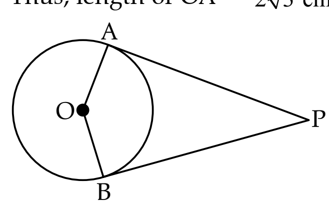
$$\tan 30° = \frac{OA}{PA}$$

$$\frac{1}{\sqrt{3}} = \frac{OA}{6}$$

$$OA = \frac{6}{\sqrt{3}} = 2\sqrt{3} \text{ cm}$$

Thus, length of OA = $2\sqrt{3}$ cm

6.



Given: PA and PB are the tangents drawn from a point P to a circle with center O. Also the line segment OA and OB are drawn.

To Prove:

$\angle APB + \angle AOB = 180°$

Proof : We know that the tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore$ $PA \perp OA$

$\Rightarrow$ $\angle OAP = 90°$

$PB \perp OB$

$\Rightarrow$ $\angle OBP = 90°$

$\therefore$ $\angle OAP + \angle OBP = (90° + 90°)$

$= 180°$ ...(i)

But we know that the sum of all the angles of a quadrilateral is 360°

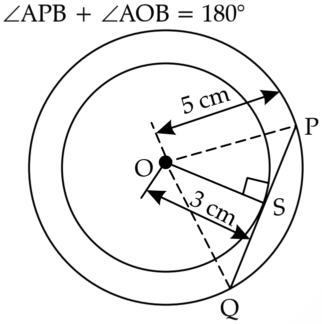
$\angle OAP + \angle OBP + \angle APB + \angle AOB$

$= 360°$ ...(ii)

From (i) and (ii), we get

$\angle APB + \angle AOB = 180°$ Hence Proved

7.



Given : OP = 5 cm, OS = 3 cm

PQ is a chord of a larger circle and a tangent of a smaller circle.

Tangent PQ is perpendicular to the radius at the point of contact S.

Therefore, $\angle OSP = 90°$

In ΔOSP (Right-angled triangle)

By the Pythagoras Thorem,

$OP^2 = OS^2 + SP^2$

$5^2 = 3^2 + SP^2$

$SP^2 = 25 - 9$

$SP^2 = 16$

$SP = \pm 4$

SP is the length of the tangent and cannot be negative

Hence, SP = 4 cm

QS = SP (Perpendicular from center bisects the chord considering QP to be the larger circle's chord)

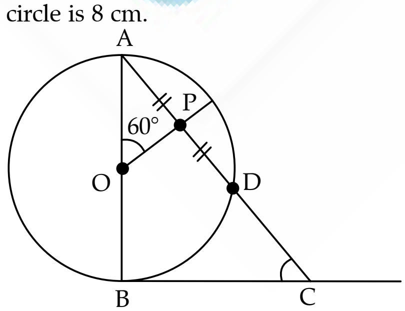
Therefore, QS = SP = 4 cm

Length of the chord PQ = QS + SP = 4 + 4

PQ = 8 cm

Therefore, the length of the chord of the larger circle is 8 cm.

8.



Given : AB is diameter

$\angle AOP = 60°$

$AP = PD$

BC is tangent

Since, OP bisects the chord AD, therefore, $\angle OPA = 90°$ [$\because$ The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord]

Now, in right angled ΔAOP

$\angle APO = 90°$

$\angle A = 180° - (60° + 90°)$

$= 180° - 150°$

$= 30°$

Also, we know that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore$ $\angle ABC = 90°$

Now, in rightangled ΔABC $\angle B = 90°$

$\angle C = 180° - \angle A - \angle B$

$= 180° - 30° - 90°$

$= 150° - 90°$

$= 60°$

9. Given, $\angle ABO = 40°$

Here, $\angle YAO = 90°$

(Angle between radius and tangent)

$OA = OB$...(Radii of same circle)

$\Rightarrow$ $\angle OAB = \angle OBA$

(Angles opposite to equal sides)

$\therefore$ $\angle OAB = 40°$

Now,

$\angle OAB + \angle YAB = \angle YAO$

$40° + \angle YAB = 90°$

$\angle BAY = 90° - 40°$

$\therefore$ $\angle BAY = 50°$

Now, In ΔAOB,

$\angle AOB + \angle OAB + \angle OBA = 180°$

or, $\angle AOB + 40° + 40° = 180°$

or, $\angle AOB = 180° - 80° = 100°$

10. Given : $\angle QPT = 60°$

PQ is a chord

Here, $\angle OPT = 90°$

(tangent at any point of a circle is perpendicular to the radius)

$\angle QPT = 60°$ (given)

$\angle OPQ = \angle DPT - \angle QPT$

$\angle OPQ = 90° - 60°$

$\Rightarrow$ $\angle OPQ = 30° = \angle OQP$

(Angles opposite to equal sides)

$\Rightarrow$ $\angle POQ = 120°$

{Angle sum property of a triangle}

Now, reflex $\angle POQ = 360° - 120° = 240°$

$\Rightarrow$ $\angle PRQ = \frac{1}{2}$ reflex $\angle POQ$

(Angle subtended at centre is double)

$= \frac{1}{2} \times 240° = 120°$

Thus, $\angle PRQ = 120°$

11. PQ = PR

Given : $\angle RPQ = 30°$, RQ is chord

$PQ = PR$

(Tangents drawn from an external Point)

$\Rightarrow \quad \angle PQR = \angle PRQ$

(Angles opposite to equal sides)

In ΔPQR, $\angle RQP + \angle PRQ + \angle QPR = 180°$

$\Rightarrow \quad \angle RQP = 75° = \angle PRQ$

Now, $\angle OQR + \angle PQR = \angle OQP$

$\Rightarrow \quad \angle OQR = 90° - 75° = 15°$

In ΔORQ

$OQ = OR$ (Radius)

$\angle OQR = \angle ORQ = 15°$

(Angles opposite to equal sides)

$\angle OQR + \angle ORQ + \angle QOR = 180°$

(Angle sum property of a triangle)

$\Rightarrow \quad \angle QOR = 150°$

$\Rightarrow \quad \angle RSQ = \frac{1}{2} \times 150°$

(Angle at the centre is double)

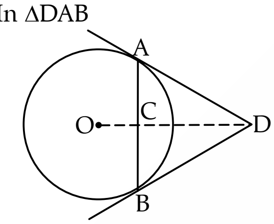
$= 75°$

Thus, $\angle RQP = 75°$ and $\angle RSQ = 75°$

12. Given : DA and DB are tangents to a circle

To Prove : $\angle DAB = \angle DBA$

Proof : In ΔDAB



As, the length of two tangents drawn from an external point to a circle are equal

$\therefore \quad DA = DB$

Thus, $\angle DAB = \angle DBA$

($\because$ angles opposite to equal sides are equal)

Hence Proved.

13. Given : PQ is the diameter

$PR \parallel OS$

To show : SQ is a tangent

$OP = OR$

(Radii of Circle)

$\therefore \quad \angle OPR = \angle ORP$ ...(i)

(angles opposite to equal sides)

$PR \parallel OS$ (Given)

$\therefore \quad \angle ORP = \angle ROS$ (alternate $\angle$s) ...(ii)

Also, $\angle OPR + \angle ORP = \angle QOR$

(External angle Property)

$\Rightarrow \quad \angle OPR + \angle ORP = \angle QOS + \angle SOR$

($\because \angle QOR = \angle QOS + \angle SOR$)

From equation (i) and (ii) we get

$\angle ROS + \angle ROS = \angle QOS + \angle ROS$

$\Rightarrow \quad \angle ROS = \angle QOS$

Now, In ΔROS and ΔQOS

$OS = OS$ (common)

$OR = OQ$

(Radii of same circle)

$\angle ROS = \angle QOS$ (Proved above)

$\therefore \quad \Delta ROS \cong \Delta QOS$ (SAS case)

$\therefore \quad \angle SRO = \angle SQO$ (By CPCT)

Thus, $\angle SQO = 90°$ (As $\angle SRO = 90°$)

As tangent is $\perp$ to the radius.

Therefore, SQ is a tangent to the circle.

14. Given : Tangent at A and C interest at B.

$OA \perp OC$

To Prove : OABC is a square

Proof : AB = BC, as they are tangents from an external point to a circle.

OA = OC as they are radius.

$\angle BAO = \angle BCO = 90°$ as AB and BC are tangents.

$OA \parallel BC$ as $\angle AOC + \angle OCB = 180°$ (adjacent interior angles)

$OC \parallel AB$ as $\angle AOC + \angle OAB = 180°$ (adjacent interior angles)

Thus, OABC is a parallelogram.

As opposite sides in a parallelogram are equal, OA = BC and OC = AB. Also, as opposite angles in a parallelogram are equal, $\angle AOC = \angle ABC = 90°$.

Therefore, OABC is a square as all of its angles are 90°, and OA = AB = BC= OC.

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. Given, radius of circle = 5 cm

PA and BC are two tangent at point A and B

$OP = 13$ cm

OA is perpendicular on tangent AP (OA is radius of the circle.

In right angle triangle ΔOAP

$(OP)^2 = (OA)^2 + (AP)^2$

$\Rightarrow \quad (AP)^2 = (OP)^2 - (OA)^2$

$\Rightarrow \quad (AP)^2 = (13)^2 - (5)^2 = 169 - 25 = 144$

$AP = \sqrt{144} = 12$

$AP = 12$ cm

Let length of BC be x

As, AC = BC = x (tangnet from an external point)

So length of PC = $12 - x$

and PB = OP – OB = 13 – 5 = 8 cm (OB is the radius and length of OP is given)

OB is perpendicular on tangent CB, so $\angle OBC = \angle CBP = 90°$

In right arigle triangle ΔCBP

$(CP)^2 = (BP)^2 + (BC)^2$

$\Rightarrow \quad (12 - x)^2 = (8)^2 + (x)^2$

$\Rightarrow \quad 144 - 24x + x^2 = 64 + x^2$

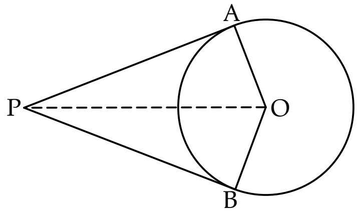
$\Rightarrow \quad 144 - 24x - 64 = 0$

$\Rightarrow \quad 80 - 24x = 0 \Rightarrow x = \frac{80}{24} = 3.33$ cm

$\therefore$ Length of tangents PA = 12 cm end BC = 3.33 cm

2. (i) O is be the center of the circle, P be an external point, and PA and PB are two tangents drawn from P to the circle at points A and B, respectively.

To prove, : PA = PB.



Proof: Join O to P, O to A and O to B. Since PA and PB are tangents to the circle from the external point P, PA ⊥ OA (the tangent is perpendicular to the radius at the point of contact)

Now, in ΔOPA and ΔOPB,

OP is a common side.

OA = OB (radii of the circle)

$\angle OAP = \angle OBP = 90°$

ΔOPA ≅ ΔOPB (By RHS)

PA = PB (CPCT)

This proves that the lengths of the tangents drawn from an external point to a circle are equal.

(ii) Given : $\angle B = 90°$, AB = 8 cm and BC = 6 cm

In right nagled ΔABC

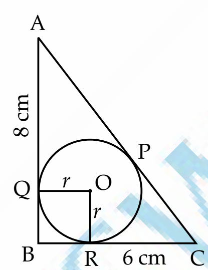
$\angle B = 90°$

By using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$AC^2 = 8^2 + 6^2$

$AC = \sqrt{64+36} = \sqrt{100} = 10 \text{ cm}$



Now, OR = OQ = radius (r)

$\angle OQB = \angle ORB = 90°$

(Radius is ⊥ to the tangent at the point of contact)

$\angle B = 90°$ (Given)

⇒ $\angle QOR = 90°$

(Sum of all angles of quadrilateral is 360°)

∴ OQBR is a square (As, adjacent sides are equal and all angles are right angle).

⇒ OR = OQ = OB = BR = 'r'

As, tangents drawn from an external point to a circle are equal.

∴ BR = BQ = r

CR = CP = $(6 - r)$

AQ = AP = $(8 - r)$

Now, AC = AP + PC

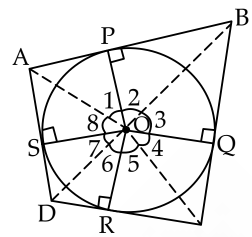
$10 = 8 - r + 6 - r$

⇒ $10 = 14 - 2r$

$2r = 14 - 10 \Rightarrow 2r = 4$

⇒ $r = 2$ cm ∴ OR = 2 cm

3.



Given : Quadrilateral ABCD circumscribes a circle with centre O.

To Prove : $\angle AOB + \angle COD = 180°$

and $\angle BOC + \angle DOA = 180°$

Construction : Join OP, OQ, OR and OS.

Proof : In ΔOAP and ΔOAS.

AP = AS

(Tangents from the same point)

OP = OS (Radii of the same circle)

OA = OA (Common side)

ΔOAP ≅ ΔOAS

(SSS congruence criterion)

And thus, $\angle POA = \angle AOS$ (CPCT)

⇒ $\angle 1 = \angle 8$

Similarly,

$\angle 2 = \angle 3$

$\angle 4 = \angle 5$

$\angle 6 = \angle 7$

Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

$(\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360°$

⇒ $2\angle 1 + 2\angle 2 + 2\angle 5 + 2\sqrt{6} = 360°$

⇒ $(\angle 1 + \angle 2) + (\angle 5 + \angle 6) = 180°$

$\angle AOB + \angle COD = 180°$

Similarly, we can prove that $\angle BOC + \angle DOA = 180°$

Hence, opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

4. **Given :** Two circles with centres at O and O′ of radii $2r$ and r respectively.

Chord AB of a bigger circle.

To Prove : AC = CB

Proof : In smaller circle

OA is the diameter

∴ $\angle OCA = 90°$ (Angle in a semicricle)

⇒ $\angle OCB = 90°$ (Linear pair)

Now, In right ΔOCA and ΔOCB

OA = OB (equal radii)

$\angle OCA = \angle OCB = 90°$

(Proved above)

$$OC = OC \quad \text{(common)}$$

$\therefore \quad \Delta OCA \cong \Delta OCB$ (RHS case)

Thus, AC = CB (CPCT)

Hence, C bisects AB **Proved.**

5. Given : Chord PQ = 8 cm

Radius OP = 5 cm

PT and PQ are tangent

Join OP and OT

TR bisects PQ, as perpendicular drawn from the center of the circle to a chord bisect the chord.

$\therefore \quad PR = RQ = 4$ cm

In right angled ΔORP, $\angle R = 90°$

$$OR = \sqrt{OP^2 - PR^2}$$
$$= \sqrt{5^2 - 4^2} \text{ cm}$$
$$= \sqrt{25-16} \text{ cm} = \sqrt{9} \text{ cm}$$
$$= 3 \text{ cm}$$

Let TP = x cm

and TR = y cm

In right ΔTRP, $\angle R = 90°$ we get

$$TP^2 = TR^2 + PR^2$$
$$\Rightarrow \quad x^2 = y^2 + 16$$
$$\Rightarrow \quad x^2 - y^2 = 16 \quad \text{...(i)}$$

In right ΔOPT, $\angle P = 90°$ we get

$$TP^2 + OP^2 = OT^2$$
$$\Rightarrow \quad x^2 + 5^2 = (y + 3)^2$$

$[OT^2 = (OR + RT)^2]$

$$\Rightarrow \quad x^2 - y^2 = 6y - 16 \quad \text{...(ii)}$$

From (i) and (ii), we get

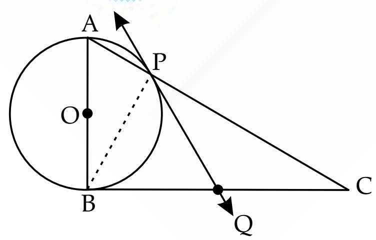
$$6y - 16 = 16 \Rightarrow 6y = 32 \Rightarrow y = \frac{16}{3}$$

Putting $y = \frac{16}{3}$ in (i) we get

$$x^2 = 16 + \left(\frac{16}{3}\right)^2 = \frac{16+256}{9}$$
$$= \frac{400}{9}$$
$$\Rightarrow \quad x = \sqrt{\frac{400}{9}} = \frac{20}{3}$$

Hence, length TP = x cm = 6.67 cm

6.



Given : AB is diameter, $\angle B = 90°$

To Prove : BQ = QC

Proof : $\angle APB = 90°$ (Angle in semicricle)

$\angle APB + \angle BPC = 180°$ (Linear pair)

$\Rightarrow \quad \angle BPC = 180° - 90° = 90°$

$\Rightarrow \quad \angle BPQ + \angle QPC = 90°$ ...(i)

Now, In ΔBPC

$\angle BPC + \angle PBC + \angle PCB = 180°$

(Angle sum Property)

$\Rightarrow \quad \angle PBC + \angle PCB = 180° - 90°$

$= 90°$ ...(ii)

Now, PQ = BQ

(tangents drawn from an external point)

$\Rightarrow \quad \angle QPB = \angle QBP$

(angles oppsite to equal sides) ...(iii)

From (i) and (ii)

$\angle BPQ + \angle QPC = \angle PBC + \angle PCB$

$\Rightarrow \quad \angle QPC = \angle PCB$

$(\because \angle QPB = \angle PBC$ from (iii))

$\Rightarrow$ PQ = QC (sides opposite to equals)

and PQ = BQ (Proved above)

$\therefore$ BQ = QC

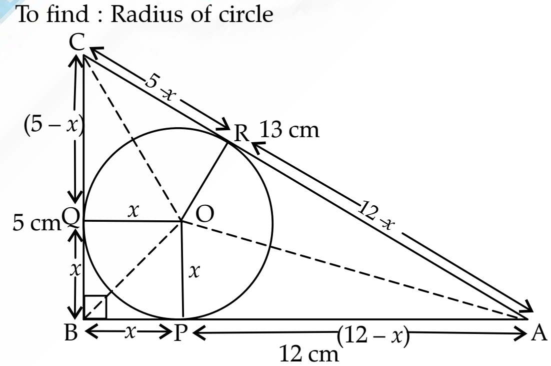
Hence, PQ bisects BC Proved.

7. Given : $\angle B = 90°$

AB = 12 cm

BC = 5 cm

To find : Radius of circle



Sol. Since, ΔABC is a right angle, therefore, by Pythagoras Theorem,

$$\Rightarrow \quad AC = \sqrt{AB^2 + BC^2} = \sqrt{12^2 + 5^2}$$
$$= \sqrt{144 + 25}$$
$$= \sqrt{13^2} = 13 \text{ cm}$$

Now, OQ, OP and OR are radius of the circle therefore, OQ = OP = OR = x (say) and are also perpendicular to respective sides BC, AB and AC. [Radius $\perp$ Tangent]

Also, join OA, OB and OC.

$\therefore$ arΔABC = arΔAOB + arΔBOC + arΔAOC

$\Rightarrow \frac{1}{2} \times AB \times BC = \frac{1}{2} \times AB \times PO + \frac{1}{2} \times QO \times BC + \frac{1}{2} \times AC \times RO$

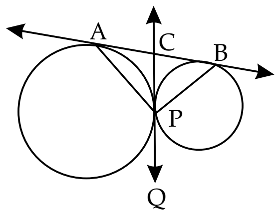
$\Rightarrow 5 \times 12 = 12 \times x + 5 \times x + 13 \times x$

$\Rightarrow 60 = (5 + 12 + 13)x$

$\therefore x = \frac{60}{30} = 2$ cm

Hence, radius of incircle is 2 cm.

8.



Given : AB is a common tangent

P is also a common tangent meeting AB at C.

To Prove : $\angle APB = 90°$

Proof : CA = CP (Lengths of tangents drawn from an external point to a circle are equal)

$\therefore$ CB = CP (Lengths of tangents drawn from an external point to a circle are equal)

In ΔACP,

$$CA = CP$$

$\therefore \quad \angle APC = \angle PAC$ ... (i)

(In a triangle, equal sides have equal angles opposite to them)

In ΔBCP

$$CB = CP$$

$\therefore \quad \angle BPC = \angle PBC$ ...(ii)

(In a triangle, equal sides have equal angles opposite to them)

Now, in ΔAPB,

$\angle APB + \angle PAB + \angle PBA = 180°$ (Angle sum property)

$\Rightarrow \angle APB + \angle APC + \angle BPC = 180°$ [From (i) and (ii)]

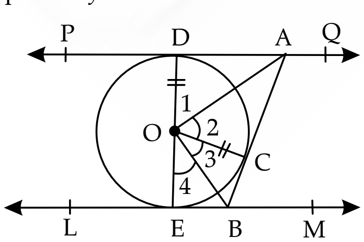
$\Rightarrow \angle APB + \angle APB = 180°$

$\Rightarrow 2\angle APB = 180°$

$\Rightarrow \angle APB = 90°$

Thus, the value of $\angle APB$ is 90°. Hence Proved.

9. Given : PQ and LM are two parallel tangents to a circle with centre O and AB is a tangent to the circle at a point C intersecting PQ and RS at A and B respectively.



To prove: $\angle AOB = 90°$

Proof: Since PQ and LM are tangents to the circle at D and E respectively and DOE is a diameter of the circle, we have

$$\angle ODA = 90° \text{ and } \angle OEB = 90°$$

$\Rightarrow \angle ODA + \angle OEB = 180°$

$\Rightarrow$ PA || LM

We know that the tangnets to a circle from a external point are equal in length.

$\therefore$ AD = AC

Now, In ΔODA and ΔOCA

OA = OA (common)

OD = OC (equal radii)

AD = AC (Proved above)

$\therefore \Delta ODA \cong \Delta OCA$ (SSS case)

$\Rightarrow \angle 1 = \angle 2$ (CPCT)...(i)

Similarly, $\Delta OCB \cong \Delta OBE$

$\therefore \angle 3 = \angle 4$ (CPCT) ...(ii)

Now, DE is a diameter

$\therefore \angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$ (straight line angles)

$\Rightarrow \angle 2 + \angle 2 + \angle 3 + \angle 3 = 180°$ (From i and ii)

$\Rightarrow 2(\angle 2 + \angle 3) = 180°$

$\Rightarrow \angle 2 + \angle 3 = 90°$

Thus, $\angle AOB = 90°$ Hence Proved.

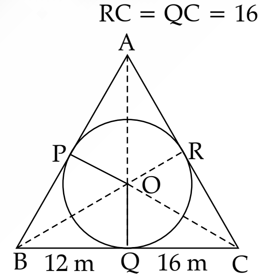
10. Given : Radius = 8 m

Cost of fencing = ₹55/m

Since the length of tangents from an external point to a circle are equal

$\therefore$ In ΔABC PB = QB = 12 m,

RC = QC = 16 m



and AP = AR = x (say)

Also, the radius is perpendicular to the tangent at the point of contact.

$\therefore ar(\Delta ABC) = ar(\Delta BOC) + ar(\Delta AOB) + ar(\Delta AOC)$

$= \frac{1}{2} \times 8 \times (28 + 12 + x + 16 + x) = 8(28 + x)$

square meters. ...(i)

Also, semi perimeter 's' of ΔABC = $(28 + x)$ m

And by Heron's formula,

$$ar(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{(28+x)\times x \times 12 \times 16}$$

$$= 4\sqrt{(28+x)\times x \times 3 \times 4}$$

$= 4\sqrt{(28+x)\times x\times 12}$

square meters ...(ii)

Now, from equations (i) and (ii), we get

$$8(28 + x) = 4\sqrt{(28+x)\times x\times 12}$$

Taking square on both the sides, we get

$$4(28 + x)^2 = 12x\,(28 + x)$$

$\Rightarrow$ $(28 + x)(28 + x - 3x) = 0$

$\Rightarrow$ $-2x = -28$

or $x = 14$

Hence, the perimeter of the triangular park

$= 2\,(28 + x) = 2(28 + 14)$

$= 2 \times 42$

$= 84$ meters

Cost of fencing = ₹ 55 × 84

= ₹ 4620

Level - 2 ADVANCED COMPETENCY FOCUSED QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Mark)

1. Option (A) is correct

Explanation: Line AG intersects the circle at exactly one point (K). This makes AG a tangent.

Line CL intersects the circle at exactly one point (D). This makes CL a tangent.

Line EF passes through the circle, intersecting it at two distinct points (D and another point). This makes EF a secant.

Lines AG and CL are the tangents and line EF is a secant to the circle.

2. Option (C) is correct

Explanation: From a point on the circle, exactly one tangent can be drawn, it touches the circle only at that point. Therefore, number of tangents: 1

A secant intersects the circle at two points. From point P (on the circle), infinitely many lines can be drawn that pass through another point on the circle, each of these is a secant. Therefore, number of secants are infinite

3. Option (A) is correct

Explanation: Statement I is correct. Line OK is the radius. A line making 90° with the radius at the point of contact is called a tangent. Through any point on a circle, exactly one tangent can be drawn, which is perpendicular to the radius at that point.

Statement II is correct. The shortest distance from the center O to any tangent is the perpendicular distance. For a tangent at point L, the perpendicular from O to the tangent is along the radius OL. So, the shortest distance is indeed equal to the radius OL.

Statement III is incorrect. A tangent to a circle touches it at only one point. If a line passes through two points on the circle, it is a chord or secant, not a tangent.

4. Option (B) is correct

Explanation: OT is vertical $\Rightarrow$ the perpendicular would be a horizontal line. Through point T, only one horizontal line can pass, i.e. line GH (the tangent). So, option (B) is correct.

5. Option (D) is correct

Explanation: In this case, triangle JRL is isosceles, with RJ = RL (tangent from extend point R to the circle) and angles at J and L are equal.

Given: $\angle LJR = 42°$

Then, $\angle JRL = 180° - 2 \times 42°$

$\Rightarrow$ $\angle JRL = 180° - 84°$

$\Rightarrow$ $\angle JRL = 96°$

6. Option (C) is correct

Explanation: Let the length of the tangent be x.

Using the Pythagoras theorem:

$x^2 + 6^2 = 10^2$

$\Rightarrow$ $x^2 + 36 = 100$

$\Rightarrow$ $x^2 = 64$

$\Rightarrow$ $x = 8$

ASSERTION-REASON QUESTIONS (1 Mark)

1. Option (B) is correct

Explanation: Assertion is true because the length of tangents drawn from an external point to a circle are equal.

Reason is also true. Since the two tangents are of equal length and form two congruent right-angled triangles (with radii as one side and tangents as another), they subtend equal angles at the center.

Both the assertion and the reason are true bur the reason does not correctly explain the assertion. The tangents are equal in length due to congruent triangles formed (by SSS or RHS criteria), not because they subtend equal angles at the center.

2. Option (A) is correct

Explanation: Assertion is true because a line (or rope) describes a tangent, touching a circle at exactly one point and not crossing into the circle.

Reason is also true because this is the definition of a tangent. It touches the circle at only one point and does not enter the circle's interior.

The reason correctly explains why the rope is acting as a tangent because it touches the fountain (circle) at one point and does not enter the circle.

3. Option (C) is correct

Explanation: Assertion is true. This describes two tangents drawn from an external point (the top of the post) to the circle (the playground). Each wire touches the circle at exactly one point.

Reason is false because tangents drawn from the same external point to a circle are always equal in length.

4. Option (A) is correct

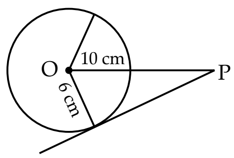
Explanation: Assertion is true. If the cutter just touches the pizza at one point on the edge and does not cut through, then it acts as a tangent to the circle.

Reason is also true because a tangent to a circle is always perpendicular to the radius drawn to the point of contact.

The definition and property in the reason helps explain why the cutter's position (touching only one point and forming a right angle with the radius) makes it a tangent. Thus, both Assertion and Reason are true, and the Reason is the correct explanation of the Assertion.

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. Let tangent be PT. It is perpendicular to the radius at the point of contact. Therefore, right triangle OPT is formed.



Using Pythagoras Theorem:

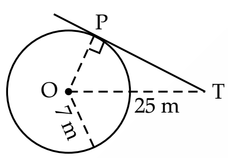
$$PT^2 = OP^2 - OT^2$$

$$PT^2 = 10^2 - 6^2 = 100 - 36 = 64$$

$$PT = 8 \text{ cm}$$

2. To determine the length of the tangent from a point 25 meters away from the center of a circular park with a radius of 7 meters, we can use the tangent length formula derived from the Pythagorean theorem:

$$\text{Tangent Length} = \sqrt{d^2 - r^2}$$



Where:

$d = 25$ (distance from the external point to the center)

$r = 7$ (radius of the circle)

Substituting the values:

$$\text{Tangent Length, PT} = \sqrt{25^2 - 7^2} = \sqrt{625 - 49}$$

$$= \sqrt{576} = 24 \text{ m}$$

The straight path from the external point to the point of contact on the park's boundary will be 24 meters long.

3. Given: Distance from the center of the tank to the robot's starting point = 15 m

Radius of the tank = 9 m

The robot's path is a tangent to the circular tank, and the radius is perpendicular to the tangent at the point of contact. So we form a right-angled triangle with hypotenuse = 15 m

(from robot to center) and one side = 9 m (radius)

Using Pythagoras Theorem:

$$\text{Tangent length}^2 = 15^2 - 9^2 = 225 - 81 = 144$$

$$\text{Tangent length} = \sqrt{144} = 12 \text{ m}$$

The robot travels 12 meters along the tangent.

4. Given: Radius of the circular stage $r = 10$ m

Distance from the center to the external point where the barrier is placed $d = 26$ m

Using the Pythagorean theorem:

Tangent Length Formula:

$$\text{Tangent length} = \sqrt{d^2 - r^2}$$

$$\text{Substitute the values: } = \sqrt{26^2 - 10^2} = \sqrt{676 - 100}$$

$$= \sqrt{576} = 24 \text{ m}$$

The length of the barrier that touches the circular stage is 24 m.

5. Given: Radius of the circular garden $r = 5$ m

Distance from the person to the center of the garden $d = 13$ m

$$\text{Tangent length} = \sqrt{d^2 - r^2} = \sqrt{13^2 - 5^2}$$

$$= \sqrt{169 - 25} = \sqrt{144} = 12 \text{ m}$$

So, the person walks 12 meters along the tangent path.

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. Given: Radius of the circular park: $r = 21$ m

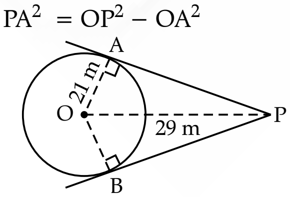
Distance from point P (external point) to the center O:

$$OP = 29 \text{ m}$$

Tangents from point P touch the circle at points A and B, so $PA = PB$

Using the Pythagoras theorem in right-angled triangle OAP

$$PA^2 = OP^2 - OA^2$$



$$= 29^2 - 21^2$$

$$= 841 - 441 = 400$$

$$PA = \sqrt{400} = 20 \text{ m}$$

Triangle OAP has sides:

OA = 21 m (radius),

AP = 20 m (tangent),

and OP = 29 m (given)

Perimeter = OA + AP + OP

= 21 + 20 + 29 = 70 m

Length of each tangent = 20 m

Perimeter of triangle OAP = 70 m

2. To Prove:

$$\Delta OAP \cong \Delta OBP$$

Line OP bisects $\angle APB$

In ΔOAP and ΔOBP:

OA = OB (Radii of the same circle)

OP = OP (Common side)

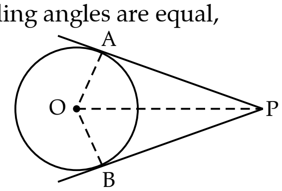
PA = PB (Tangents from the same external point are equal in length)

So, by SSS congruence,

$\Delta OAP \cong \Delta OBP$

Since the triangles are congruent:

Corresponding angles are equal,



So, $\angle APO = \angle BPO$

Thus, line OP bisects angle $\angle APB$

3. (i) A tangent to a circle is perpendicular to the radius at the point of contact.

So, $\angle OPT = 90°$

(Since radius OP $\perp$ tangent TP)

Hence proved.

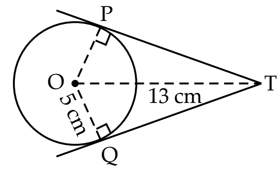
(ii) In right-angled triangle ΔOPT:

$OT^2 = OP^2 + TP^2$

Substituting known values:

$13^2 = 5^2 + TP^2$

$169 = 25 + TP^2$



$TP^2 = 169 - 25 = 144$

$TP = 12$ cm

Length of each tangent = 12 cm

4. (i) The lengths of tangents drawn from an external point to a circle are equal.

So, PA = PB

Hence, the two support cables are equal in length.

(ii) Using the Pythagoras theorem in right-angled triangle ΔOAP (since radius is perpendicular to the tangent at point of contact):

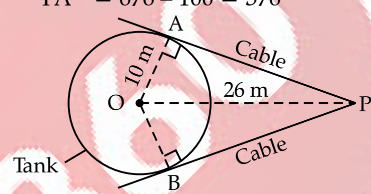
$OP^2 = OA^2 + PA^2$

Substituting values:

$26^2 = 10^2 + PA^2$

$676 = 100 + PA^2$

$PA^2 = 676 - 100 = 576$



PA = 24m

Each cable is 24 meters long.

CASE BASED QUESTIONS (4 Marks)

1. (i) $\angle OSA = 90°$

($\because$ A tangent is $\perp$ to the radius at the point of contact)

(ii) Given : AB = AD (adjacent sides)

$\therefore$ AP + PB = AS + SD

$\therefore$ PB = SD

[$\because$ AP = AS Tangent from external point A]

Also, QB = RD

[$\because$ PB = QB and SO = RD]

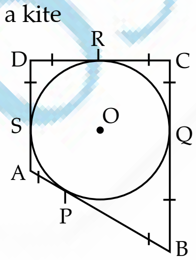
$\therefore$ QB + RC = RD + RC

or QB + QC = RD + RC [$\because$ RC = QC]

$\therefore$ BC = CD [adjacent sides]

$\Rightarrow$ DC = BC

So, ABCD is a kite



(iii) (a) DS = DR = 7 cm and AS = AP = x

(Length of tangents drawn from an external Point to a circle are equal.)

As, AD = AS + SD $\Rightarrow$ 11 = AP + 7

$\Rightarrow$ AP = 11 – 7

= 4 cm

OR

(b) $\angle QCR = 60°$,

OQCR is a quadrilateral

$\therefore \angle CQO + \angle RCQ + \angle ORC + \angle QOR = 360°$

$\Rightarrow 90° + 60° + 90° + \angle QOR = 360°$

[Radius $\perp$ on tangents]

$\Rightarrow \angle QOR = 360° - 240° = 120°$

2 (i) The lengths of tangents drawn from an external point to a circle are equal.

Justification: According to the tangent-segment theorem,

Tangents drawn from the same external point to a circle are equal in length.

So, PA = PB

(ii) The angle between the radius and the tangent at the point of contact is 90°.

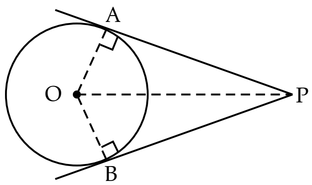
Justification: A tangent to a circle is always perpendicular to the radius at the point of contact.

$\angle OAP = 90°$

(iii) (a) Using the Pythagoras theorem in triangle ΔOAP, which is right-angled at A:

$OP^2 = OA^2 + PA^2$

$\Rightarrow PA^2 = OP^2 - OA^2$



Substituting values:

$$PA^2 = 25^2 - 7^2 = 625 - 49 = 576$$

$$\Rightarrow \quad PA = 24\,m$$

Each fencing rod is 24 meters long.

OR

(b) The angle subtended by two tangents at an external point (∠APB) and the angle subtended at the center by the radii to the tangent points (∠AOB) are supplementary.

So: $\angle AOB = 180° - \angle APB$

$= 180° - 60° = 120°$

$\angle AOB = 120°$

3. (i) The two wires PA and PB are equal in length because they are tangents drawn from the same external point P to a circle.

PA and PB satisfies the geometric property that tangents drawn from an external point to a circle are equal in length.

So, $PA = PB$

(ii) Yes, triangle ΔOAP is a right-angled triangle.

Justification: A tangent to a circle is perpendicular to the radius at the point of contact.

So, $\angle OAP = 90°$

Hence, triangle OAP is a right triangle with right angle at A.

(iii) (a) Using the Pythagoras theorem in triangle ΔOAP:

$$OP^2 = OA^2 + PA^2$$

$$\Rightarrow \quad PA^2 = OP^2 - OA^2$$

Substituting the values:

$$PA^2 = 172 - 82 = 289 - 64 = 225$$

$$\Rightarrow \quad PA = 15\,m$$

Each wire is 15 meters long.

OR

(b) When two tangents are drawn from an external point, the angle between them (∠APB) and the angle at the center (∠AOB) made by the radii to the points of contact are supplementary.

So, $\angle AOB = 180° - \angle APB$

$= 180° - 80° = 100°$

$\angle AOB = 100°$

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. (i) Tangents drawn from an external point to a circle are equal in length.

So, $PA = PB$ **Proved**

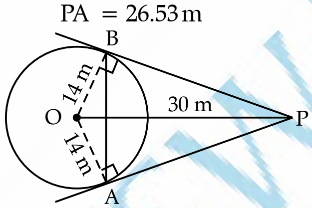
(ii) In right triangle ΔOAP, applying the Pythagoras Theorem:

$$PA^2 = OP^2 - OA^2$$

Substituting the values:

$$PA^2 = 30^2 - 14^2 = 900 - 196 = 704$$

$$\Rightarrow \quad PA = 26.53\,m$$



(iii) Perimeter $= PA + PB + AB$

We already have $PA = PB \approx 26.53\,m$

ΔPAB is an isosceles triangle with PA = PB. Since, ∠APB = 60°, the other two angles are $\dfrac{180° - 60°}{2}$ = 60° each.

∴ Δ PAS in an equilateral triangle

Thus, AB = PA = PB = 26.53 m

So, Perimeter $= 26.53 + 26.53 + 26.53$

$= 3 \times 26.53 = 79.59\,m$

(iv) The angle between the two radii ∠AOB and the angle between the tangents ∠APB are supplementary.

So, $\angle AOB = 180° - \angle APB$

$= 180° - 60° = 120°$