CHAPTER 9

Introduction to Trignometry and Trignometric identities

Level - 1

CORE SUBJECTIVE QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Mark)

1. Option (A) is correct

Explanation: Since, $\frac{4\sin\theta+\cos\theta}{4\sin\theta-\cos\theta}$

On dividing by cos θ, we get $\frac{4\tan\theta+1}{4\tan\theta-1}$

Substitute $\tan\theta = \frac{5}{2}$

$$\therefore \quad \frac{4\tan\theta+1}{4\tan\theta-1} = \frac{4\times\frac{5}{2}+1}{4\times\frac{5}{2}-1} = \frac{11}{9}$$

Thus, the value of the expression is $\frac{11}{9}$.

2. Option (C) is correct

Explanation: Given:

$$\sec\theta - \tan\theta = m$$

Using the identity $\sec^2\theta - \tan^2\theta = 1$, we can express it as a difference of squares:

$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

Substitute $\sec\theta - \tan\theta = m$:

$(\sec\theta + \tan\theta).\, m = 1$

So,

$$\sec\theta + \tan\theta = \frac{1}{m}$$

3. Option (B) is correct

Explanation: We are asked to find the value of $2\sin\theta\cos\theta$ for $\theta = 30°$.

$2\sin 30°\cos 30°$

$$= 2\times\frac{1}{2}\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

4. Option (A) is correct

Explanation: If $\cos(\alpha+\beta) = 0$, then the value of $\cos\left(\frac{\alpha+\beta}{2}\right)$ is:

We know that $\cos(\alpha+\beta) = 0$ implies $\alpha+\beta = 90°$ (since $\cos 90° = 0$).

Now, $$\frac{\alpha+\beta}{2} = \frac{90°}{2} = 45°$$

Thus, $$\cos 45° = \frac{1}{\sqrt{2}}$$

5. Option (A) is correct

Explanation: We are given $\sin A = \frac{2}{3}$

We know the identity:

$$\sin^2 A + \cos^2 A = 1$$

Substitute $\sin A = \frac{2}{3}$

$$\left(\frac{2}{3}\right)^2 + \cos^2 A = 1$$

$$\frac{4}{9} + \cos^2 A = 1$$

$$\cos^2 A = 1 - \frac{4}{9} = \frac{5}{9}$$

$$\cos A = \frac{\sqrt{5}}{3}$$

Now, $\cot A = \frac{\cos A}{\sin A}$

$$\cot A = \frac{\frac{\sqrt{5}}{3}}{\frac{2}{3}} = \frac{\sqrt{5}}{2}$$

6. Option (C) is correct

Explanation: We are given $\sin\theta = \cos\theta$.

For $\sin\theta = \cos\theta$ to hold, $\theta = 45°$ because:

$$\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$$

Now, we need to find $\sec\theta\,.\,\sin\theta$:

$$\sec 45° = \sqrt{2}$$

Now, multiply sec θ and sin θ:

$$\sec 45°.\sin 45° = \sqrt{2}.\frac{1}{\sqrt{2}} = 1$$

Thus, the value of sec θ. sin θ is 1.

7. Option (C) correct

Explanation: We are given the equation $4\cot\theta - 5 = 0$

$$\therefore \quad \cot\theta = \frac{5}{4}$$

Using the identity $\cot\theta = \frac{\cos\theta}{\sin\theta'}$, we can write.

$$\frac{\cos\theta}{\sin\theta} = \frac{5}{4}$$

Thus, $\cos\theta = \frac{5}{4}\sin\theta$

Now, we need to find the value of:

$$\frac{5\sin\theta - 4\cos\theta}{5\sin\theta + 4\cos\theta}$$

Substitute $\cos\theta = \frac{5}{4}\sin\theta$ into the expression:

$$\frac{5\sin\theta - 4\left(\frac{5}{4}\sin\theta\right)}{5\sin\theta + 4\left(\frac{5}{4}\sin\theta\right)} = \frac{5\sin\theta - 5\sin\theta}{5\sin\theta + 5\sin\theta}$$

$$= \frac{0}{10\sin\theta} = 0$$

8. Option (C) is correct

Explanation: We are given that:

$$\sin\theta = \frac{a}{b}$$

Using the trigonometric identity:

$$\sin^2\theta + \cos^2\theta = 1$$

Substituting $\sin\theta$:

$$\left(\frac{a}{b}\right)^2 + \cos^2\theta = 1$$

$$\frac{a^2}{b^2} + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \frac{a^2}{b^2}$$

$$\cos^2\theta = \frac{b^2 - a^2}{b^2}$$

$$\cos\theta = \pm\sqrt{\frac{b^2 - a^2}{b^2}} = \pm\frac{\sqrt{b^2 - a^2}}{b}$$

9. Option (A) is correct

Explanation:

Given: PU = 8 cm

PQ = 17 cm

Let $\angle SPT = \theta$

$\therefore$ $\angle UPQ = 90° - \theta$

($\therefore$ $\angle SPQ = 90°$)

Now, in ΔPUQ,

$\angle UPQ + \angle U + \angle PQU = 180°$

(angle sum property)

$\Rightarrow$ $90° - \theta + 90° + \angle PQU = 180°$

$\Rightarrow$ $\angle PQU = \theta$

Thus $\angle PQU = \angle SPT = \theta$ ...(i)

Now, in Right angle ΔPUQ, $\angle U = 90°$

$$\sin\angle PQU = \frac{PU}{PQ} = \frac{8}{17}$$

Thus, $\angle SPT = \frac{8}{17}$

($\because$ $\angle PQU = \angle SPT$)

10. Option (A) is correct.

Explanation:

Given: $\dfrac{\text{cosec}\,\theta + \cot\theta - 1}{\text{cosec}\,\theta - \cot\theta + 1}$

$$= \frac{\text{cosec}\,\theta + \cot\theta - (\cot^2\theta - \text{cosec}^2\theta)}{\text{cosec}\,\theta - \cot\theta + 1} \text{ (Step 1)}$$

Here Step 1 is wrong

$\because$ According to trigonometric identities

$$1 + \cot^2\theta = \text{cosec}^2\theta$$

$$\Rightarrow \quad 1 = \text{cosec}^2\theta - \cot^2\theta$$

Not $\cot^2\theta - \text{cosec}^2\theta$ which is given in step 1

Hence, step 1 is wrong.

11. Option (D) is correct

Explanation: (sec A + tan A) (1 – sin A)

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \frac{1 + \sin A}{\cos A} \times (1 - \sin A)$$

$$= \frac{(1 + \sin A)(1 - \sin A)}{\cos A}$$

$$= \frac{1 - \sin^2 A}{\cos A}$$

$$= \frac{\cos^2 A}{\cos A}$$

$$= \cos A$$

12. Option (C) is correct

Explanation: Given 2 tan A = 3, we find:

$$\tan A = \frac{3}{2}$$

As, per trigonometric ratio $\tan A = \frac{P}{B} = \frac{3}{2}$

Now by using Pythagoras theorem.

$H^2 = P^2 + B^2 \Rightarrow H^2 = 3^2 + 2^2 \Rightarrow H^2 = 13 \Rightarrow H = \sqrt{13}$

As, $\sin A = \frac{P}{H}$ and $\cos A = \frac{B}{H}$ we get,

$$\sin A = \frac{3}{\sqrt{13}}, \cos A = \frac{2}{\sqrt{3}}$$

Substituting into the expression:

$$\frac{4\sin A + 3\cos A}{4\sin A - 3\cos A} = \frac{4 \times \frac{3}{\sqrt{13}} + 3 \times \frac{2}{\sqrt{13}}}{4 \times \frac{3}{\sqrt{13}} - 3 \times \frac{2}{\sqrt{13}}}$$

$$= \frac{\frac{12}{\sqrt{13}} + \frac{6}{\sqrt{13}}}{\frac{12}{\sqrt{13}} - \frac{6}{\sqrt{13}}} = \frac{\frac{18}{\sqrt{3}}}{\frac{6}{\sqrt{13}}}$$

$$= \frac{18}{6} = 3$$

13. Option (A) is correct

Explanation: As per trigonometric table

$$\tan 30° = \frac{1}{\sqrt{3}}, \text{ so } \tan^2 30° = \frac{1}{3}$$

$$\sec 45° = \sqrt{2}\text{, so } \sec^2 45° = 2$$

$$\sin 60° = \frac{\sqrt{3}}{2}\text{, so } \sin^2 60° = \frac{3}{4}$$

Substitute the values into the expression

$$\frac{3}{4}\tan^2 30° - \sec^2 45° + \sin^2 60°$$

$$= \frac{3}{4}\times\frac{1}{3} - 2 + \frac{3}{4}$$

$$= \frac{1}{4} - 2 + \frac{3}{4}$$

$$= \frac{1+3}{4} - 2$$

$$= 1 - 2 = -1$$

14. Option (C) is correct

Explanation: $\sec^2\theta = 1 + \tan^2\theta$, and $\tan\theta = \frac{1}{\cot\theta}$

$$\sec^2\theta = 1 + \frac{1}{\cot^2\theta}$$

Thus, sec θ in terms of cot θ becomes:

$$\sec\theta = \sqrt{1 + \frac{1}{\cot^2\theta}}$$

$$\sec\theta = \sqrt{\frac{1+\cot^2\theta}{\cot^2\theta}}$$

$$\sec\theta = \frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$$

15. Option (A) is correct

Explantion: Given that $\tan A = \frac{3}{4}$, we can use the identity

$$\tan A = \frac{\sin A}{\cos A}$$

Given, $\tan A = \frac{3}{4}$, we can represent sin A and cos A in terms of a right angled triangle where:

The opposite side (to angle A) is 3.

The adjacent side is 4.

We can find the hypotenuse (h) using the Pythagorean theorem:

$$h = \sqrt{(3^2)+(4^2)} = \sqrt{9+16}$$

$$= \sqrt{25} = 5$$

$$\cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{4}{5}$$

16. Option (C) is correct

Explanation: Using the identity $1 + \tan^2\theta = \sec^2\theta$:

$2\cos^2\theta\,(1 + \tan^2\theta) = 2\cos^2\theta\sec^2\theta$

Since $\sec^2\theta = \frac{1}{\cos^2\theta}$ we have:

$$2\cos^2\theta\sec^2\theta = 2\cos^2\theta\cdot\frac{1}{\cos^2\theta} = 2\times 1 = 2$$

So, the value of $2\cos^2\theta\,(1 + \tan^2\theta)$ is equal to 2.

17. Option (D) is correct

Explanation: We know that:

$$\cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta}$$

$$\therefore \quad \cot^2\theta - \frac{1}{\sin^2\theta} = \frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta}$$

$$= \frac{\cos^2\theta - 1}{\sin^2\theta}$$

Using the trigonometric identity $\cos^2\theta + \sin^2\theta = 1$,

$$= \frac{-\sin^2\theta}{\sin^2\theta} = -1$$

18. Option (B) is correct

Explanation: Using the identity for secant:

$$\sec^2\theta - 1 = \tan^2\theta$$

Using the identity for cosecant

$$(1 - \operatorname{cosec}^2\theta) = -\cot^2\theta$$

$$\therefore \quad (\sec^2\theta - 1)(1 - \operatorname{cosec}^2\theta) = \tan^2\theta\,.\,(-\cot^2\theta)$$

$$= -\tan^2\theta\cot^2\theta = -\left(\frac{\sin^2\theta}{\cos^2\theta}\right)\left(\frac{\cos^2\theta}{\sin^2\theta}\right)$$

$$= -\left(\frac{\sin^2\theta}{\cos^2\theta}\cdot\frac{\cos^2\theta}{\sin^2\theta}\right) = -1$$

19. Option (D) is correct

Explanation: We are given the equation $2\sin 2A = \sqrt{3}$

$$\sin 2A = \frac{\sqrt{3}}{2}$$

We know that $\sin 60° = \frac{\sqrt{3}}{2}$

So: $2A = 60°$

$$A = \frac{60°}{2} = 30°$$

Therefore, ∠A = 30°

20. Option (A) is correct

Explanation: Given: $\sqrt{3}\tan 2\theta = 3$

$$\tan 2\theta = \sqrt{3} \Rightarrow 2\theta = 60° \Rightarrow \theta = 30°$$

Now,

$$\therefore \quad \sin\theta + \sqrt{3}\cos\theta = \frac{1}{2} + \sqrt{3}\times\frac{\sqrt{3}}{2} = \frac{1}{2} + \frac{3}{2} = 2$$

21. Option (B) is correct

Explanation: Squaring both sides of the equation:

$$(\cos\theta + \sin\theta)^2 = \left(\sqrt{2}\cos\theta\right)^2$$

$\cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta = 2\cos^2\theta$

Use the identity $\cos^2\theta + \sin^2\theta = 1$

$$1 + 2\cos\theta\sin\theta = 2\cos^2\theta$$

$$2\cos\theta\sin\theta = 2\cos^2\theta - 1$$

$$2\cos\theta\sin\theta = \cos^2\theta + \cos^2\theta - 1$$

$$= \cos^2\theta + (1 - \sin^2\theta - 1)$$

$$= \cos^2\theta - \sin^2\theta$$

$$2\sin\theta\cos\theta = (\cos\theta + \sin\theta)(\cos\theta - \sin\theta)$$

$$2\sin\theta\cos\theta = (\sqrt{2}\cos\theta)(\cos\theta - \sin\theta)$$

$$\frac{2\sin\theta\cos\theta}{\sqrt{2}\cos\theta} = \cos\theta - \sin\theta$$

Thus, the value of $\cos\theta - \sin\theta$ is $\sqrt{2}\sin\theta$.

22. Option (C) is correct

Explanation: Given that, tan (A + B) = $\sqrt{3}$ and tan (A – B) = $\frac{1}{\sqrt{3}}$

Since, tan 60° = $\sqrt{3}$ and tan 30° = $\frac{1}{\sqrt{3}}$

Therefore, tan (A + B) = tan 60°

$$(A + B) = 60° \quad \text{...(i)}$$

and

$$\tan (A - B) = \tan 30°$$
$$(A - B) = 30° \quad \text{...(ii)}$$

On adding both equations (i) and (ii), we obtain.

$$A + B + A - B = 60° + 30°$$
$$2A = 90°$$
$$A = 45°$$

By substituting the value of A in equation (i) we obtani.

$$A + B = 60°$$
$$45° + B = 60°$$
$$B = 60° - 45° = 15°$$

23. Option (A) is correct

Explanation: Given: tan θ = $\frac{5}{12}$.

We know that:

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{5}{12}$$

Using the Pythagorean theorem, the hypotensue h is:

$$h = \sqrt{5^2 + 12^2} = \sqrt{25 + 144}$$
$$= \sqrt{169} = 13$$

Now, we can find sin θ and cos θ:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{5}{13}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{12}{13}$$

Now, substitute sin θ = $\frac{5}{13}$ and cos θ = $\frac{12}{13}$ into the expresssion:

$$\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{\frac{5}{13} + \frac{12}{13}}{\frac{5}{13} - \frac{12}{13}}$$

$$= \frac{\frac{17}{13}}{\frac{-7}{13}} = \frac{17}{13} \times \frac{13}{-7} = \frac{17}{-7} = -\frac{17}{7}$$

24. Option (A) is correct

Explanation: We know that,

$$\tan 30° = \frac{1}{\sqrt{3}}$$

$$\frac{2\tan 30°}{1 + \tan^2 30°} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{6}{4\sqrt{3}}$$

On Rationalising the denominator we get,

$$= \frac{3\sqrt{3}}{2 \times 3} = \frac{\sqrt{3}}{2}$$

ASSERTION-REASON QUESTIONS (1 Mark)

1. Option (A) is correct

Explanation: Assertion: From the trigonometric identity:

$\sin^2 A + \cos^2 A = 1$

Substituting sin A = $\frac{1}{3}$

$$\left(\frac{1}{3}\right)^2 + \cos^2 A = 1$$

$$\Rightarrow \frac{1}{9} + \cos^2 A = 1$$

$$\Rightarrow \cos^2 A = 1 - \frac{1}{9} = \frac{8}{9}$$

$$\Rightarrow \cos A = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

Thus, assertion is true.

The reason is also correct because $\sin^2\theta + \cos^2\theta = 1$ is a fundamental trigonometric identity that holds true for all angles.

Both assertion and reason are correct and reason is the correct explanation of assertion.

2. Option (A) is correct

Explanation: Assertion : On multiplying both the given expression we get,

(cosec θ – cot θ) (cosec θ + cot θ)

= $(\text{cosec}^2\theta - \cot^2\theta)$

As, per trigonometric idenlity

$(\text{cosec}^2\theta - \cot^2\theta) = 1$

⇒ (cosec θ – cot θ) (cosec θ + cot θ) = 1

$$\Rightarrow (\text{cosec}\,\theta - \cot\theta) = \frac{1}{(\text{cosec}\,\theta + \cot\theta)}$$

Thus both experssion are reciprocal to each other.

∴ Assertion is true.

Reason : $\text{cosec}^2\ \theta - \cot^2$ is a fundamental trigonomatric identify.

∴ Reason is also true.

And as identity has been used is assertion.

∴ Reason is correct explanation of assertion

3. Option (D) is correct

Explanation: In case of Assertion cot A is not product of cot and A. It is the cotangent of ∠A.

∴ Assertion is false.

In case of Reason: The value of sin θ increases as θ increases from 0° to 90°.
∴ Reason is correct.
Hence Assertion is false but Reason is true.

4. Option (D) is correct

Explanation: Assertion (A): The expression $\frac{1}{\sec\theta}$ = cos θ and $\frac{1}{\cos\theta}$ = sin θ, so the statement becomes cos θ + sin θ. The maximum value of cos θ + sin θ occurs when θ = 45°,

As, $\cos 45° = \sin 45° = \frac{1}{\sqrt{2}}$

and $\cos 45° + \sin 45° = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}}$

$\Rightarrow \sqrt{2} = 1.414$

Thus, $\sqrt{2}$ is greater than 1.
Hence Assertion is false.
Reason (R): The maximum value of sin θ and cos θ is indeed 1 (when sin θ = 90° and cos θ = 0°). Therefore, (R) is true.

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. Given : $\frac{2\sin^2 60° - \tan^2 30°}{\sec^2 45°}$

Substitute know values :

$$\sin 60° = \frac{\sqrt{3}}{2}$$

$$\tan 30° = \frac{1}{\sqrt{3}}$$

$$\sec 45° = \sqrt{2}$$

We get, $\frac{2\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2}{(\sqrt{2})^2}$

$$= \frac{7}{12}$$

2. Given : $2\sqrt{2}\cos 45° \sin 30° + 2\sqrt{3}\cos 30°$

Substituting known values:

$$\cos 45° = \frac{1}{\sqrt{2}}$$

$$\sin 30° = \frac{1}{2}$$

$$\cos 30° = \frac{\sqrt{3}}{2}$$

We get, $2\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) + 2\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)$

$$= 1 + 3 = 4$$

3. $2\sin^2 30° \sec 60° + \tan^2 60°$

Substituting known values:

$$\sin 30° = \frac{1}{2}$$

$$\sec 60° = \frac{2}{1} = 2$$

$$\tan 60° = \sqrt{3}$$

We get, $2\left(\frac{1}{2}\right)^2 .2 + (\sqrt{3})^2$

$$= 2 \times \frac{1}{4} \times 2 + 3$$

$$= 1 + 3 = 4$$

4.
$$\cos(A - B) = 1,$$
$$\cos(A - B) = \cos 0°$$
$\Rightarrow$ A – B = 0, or A = B

From, $2\sin(A + B) = \sqrt{3}$

$$\sin(A + B) = \frac{\sqrt{3}}{2} \Rightarrow A + B = 60°$$

Since A = B, we have:

$$2A = 60° \Rightarrow A = 30°, B = 30°$$
$$A = 30°, B = 30°$$

5. Since, sin A – cos A = 0

$\Rightarrow$ sin A = cos A, so A = 45°

$$\tan 45° = 1 \Rightarrow \tan^2 45° = 1$$
$$\text{cosec } 45° = \sqrt{2} \Rightarrow \text{cosec}^2 45° = 2$$

Now, $2\tan^2 A + \frac{1}{\text{cosec}^2 A} + 1$

$$= 2(1) + \frac{1}{2} + 1 = 2 + \frac{1}{2} + 1$$

$$= \frac{7}{2}$$

6. Given,

$2\,\text{cosec}^2 30° + x\sin^2 60° - \frac{3}{4}\tan^2 30° = 10$

Substituting known values:

$$\text{cosec } 30° = 2 \Rightarrow \text{cosec}^2 30° = 4$$

$$\sin 60° = \frac{\sqrt{3}}{2} \Rightarrow \sin^2 60° = \frac{3}{4}$$

$$\tan 30° = \frac{1}{\sqrt{3}} \Rightarrow \tan^2 30° = \frac{1}{3}$$

$$\therefore \quad 2(4) + x\left(\frac{3}{4}\right) - \frac{3}{4}\left(\frac{1}{3}\right) = 10$$

$$8 + \frac{3}{4}x - \frac{1}{4} = 10$$

$$\frac{3}{4}x = 10 - 8 + \frac{1}{4}$$

$$\frac{3}{4}x = 2 + \frac{1}{4} \Rightarrow \frac{3}{4}x = \frac{9}{4}$$

$$x = 3$$

7. Given, $\sin\theta + \cos\theta = \sqrt{3}$

Squaring both sides:

$$(\sin\theta + \cos\theta)^2 = 3$$
$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$$

Since, $\sin^2\theta + \cos^2\theta = 1$

$\therefore \quad 1 + 2\sin\theta\cos\theta = 3$

$\Rightarrow \quad 2\sin\theta\cos\theta = 2$

$\Rightarrow \quad \sin\theta\cos\theta = 1$

8. Given: $\sin\alpha = \dfrac{1}{\sqrt{2}}$ and $\cot\beta = \sqrt{3}$

Calculate cosec α:

$$\text{cosec}\,\alpha = \frac{1}{\sin\alpha} = \sqrt{2} \quad \text{...(i)}$$

Calculate cosec β:

$$\text{cosec}\,\beta = \sqrt{1+\cot^2\beta} = \sqrt{1+3} = 2 \quad \text{...(ii)}$$

Add eqn. (i) and (ii) together:

$$\text{cosec}\,\alpha + \text{cosec}\,\beta = \sqrt{2}+2$$

9. Given, $4\cot^2 45° - \sec^2 60° + \sin^2 60° + P = \dfrac{3}{4}$

Substituting known values:

$$\cot 45° = 1$$
$$\sec 60° = \frac{2}{1} = 2$$
$$\sin^2 60° = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$$

Calculate each term:

$$4\cot^2 45° = 4(1^2) = 4$$
$$\sec^2 60° = 2^2 = 4$$
$$\sin^2 60° = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$$

Substituting into the given equation:

$$4 - 4 + \frac{3}{4} + P = \frac{3}{4}$$
$$0 + \frac{3}{4} + P = \frac{3}{4}$$
$$P = \frac{3}{4} - \frac{3}{4}$$
$$P = 0$$

10. Given: $\cos A + \cos^2 A = 1$

$\Rightarrow \quad \cos A = 1 - \cos^2 A$

$\Rightarrow \quad \cos A = \sin^2 A \; (\because \sin^2 A + \cos^2 A = 1)$

Now,

$$\sin^2 A + \sin^4 A = \sin^2 A + (\sin^2 A)^2$$
$$= \cos A + (\cos A)^2 \quad (\because \sin^2 A = \cos A)$$
$$= \cos A + \cos^2 A$$
$$= 1$$

Hence, $\sin^2 A + \sin^4 A = 1$

11. Given:

$a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$

Now, $\quad m^2 + n^2 = (a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2$

Therefore,

$$m^2 + n^2 = a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta$$
$$m^2 + n^2 = (a^2 + b^2)\cos^2\theta + (a^2 + b^2)\sin^2\theta$$
$$m^2 + n^2 = (a^2 + b^2)(\cos^2\theta + \sin^2\theta)$$
$$m^2 + n^2 = a^2 + b^2 \quad (\because \cos^2\theta + \sin^2\theta = 1)$$

Hence Proved

12. $$\text{LHS} = \sqrt{\frac{(\sec A-1)(\sec A-1)}{(\sec A+1)(\sec A-1)}} + \sqrt{\frac{(\sec A+1)(\sec A+1)}{(\sec A-1)(\sec A+1)}}$$

$$= \sqrt{\frac{(\sec A-1)^2}{(\sec^2 A-1)}} + \sqrt{\frac{(\sec A+1)^2}{(\sec^2 A-1)}}$$

$$= \sqrt{\frac{(\sec A-1)^2}{\tan^2 A}} + \sqrt{\frac{(\sec A+1)^2}{\tan^2 A}}$$

$$= \frac{(\sec A-1)}{\tan A} + \frac{(\sec A+1)}{\tan A}$$

$$= \frac{(\sec A - 1 + \sec A + 1)}{\tan A}$$

$$\Rightarrow \quad 2 \times \frac{1}{\cos A} \times \frac{\cos A}{\sin A}$$

$$= \frac{2}{\sin A} = 2\,\text{cosec}\,A \quad \left(\because \frac{1}{\sin A} = \text{cosec}\,A\right)$$

= R.H.S. Hence Proved.

13. Since $\tan A = 1$, $A = 45°$

and $\tan B = \sqrt{3}$, $B = 60°$

$\therefore \cos 45° \cos 60° + \sin 45° \sin 60°$

$$= \frac{1}{\sqrt{2}} \times \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1+\sqrt{3}}{2\sqrt{2}}$$

14. We know that

$$\sin^2\theta + \cos^2\theta = 1$$

Given, $\quad \dfrac{1-\cos\theta}{1+\cos\theta} = (\text{cosec}\,\theta - \cot\theta)^2$

Now multiplying the numerator and denominator of LHS by $(1 - \cos\theta)$:

$$\therefore \text{LHS} = \frac{1-\cos\theta}{1+\cos\theta} \times \frac{1-\cos\theta}{1-\cos\theta} = \frac{(1-\cos\theta)^2}{(1-\cos^2\theta)}$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta} = \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 = (\text{cosec}\,\theta - \cot\theta)^2 = \text{RHS}$$

Hence Proved.

15. $$\text{LHS} = \frac{\dfrac{\sin A}{\cos A}}{1 - \dfrac{\cos A}{\sin A}} + \frac{\dfrac{\cos A}{\sin A}}{1 - \dfrac{\sin A}{\cos A}}$$

$$= \frac{\sin A}{\cos A} \times \frac{\sin A}{\sin A - \cos A} + \frac{\cos A}{\sin A} \times \frac{\cos A}{\cos A - \sin A}$$

$$= \frac{\sin^2 A}{\cos A(\sin A - \cos A)} + \frac{\cos^2 A}{\sin A(\cos A - \sin A)}$$

$$= \frac{\sin^2 A}{\cos A(\sin A - \cos A)} - \frac{\cos^2 A}{\sin A(\sin A - \cos A)}$$

$$= \frac{\sin^3 A - \cos^3 A}{\text{sim } A \cos A (\sin A - \cos A)}$$

$$= \frac{(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A \cdot \cos A)}{\sin A \cdot \cos A(\sin A - \cos A)}$$

Apply identity : $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$$= \frac{1 + \sin A \cdot \cos A}{\sin A \cdot \cos A} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{1}{\sin A \cdot \cos A} + \frac{\sin A \cdot \cos A}{\sin A \cdot \cos A}$$

$$= \frac{1}{\sin A} \cdot \frac{1}{\cos A} + 1$$

= sec A. cosec A + 1

= RHS Hence Proved

16. $\dfrac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \sin^2 60^\circ}$

Use known values of trigonometric functions.

$\cos 60^\circ = \dfrac{1}{2}$, so $\cos^2 60^\circ = \left(\dfrac{1}{2}\right)^2 = \dfrac{1}{4}$

$\sec 30^\circ = \dfrac{2}{\sqrt{3}}$, $\sec^2 30^\circ = \left(\dfrac{2}{\sqrt{3}}\right)^2 = \dfrac{4}{3}$

$\tan 45^\circ = 1$, so $\tan^2 45^\circ = 1^2 = 1$

$\sin 30^\circ = \dfrac{1}{2}$, so $\sin^2 30^\circ = \left(\dfrac{1}{2}\right)^2 = \dfrac{1}{4}$

$\sin 60^\circ = \dfrac{\sqrt{3}}{2}$, so $\sin^2 60^\circ = \left(\dfrac{\sqrt{3}}{2}\right)^2 = \dfrac{3}{4}$

On substituting in given expression, we get

$$\frac{5 \times \frac{1}{4} + 4 \times \frac{4}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{1+3}{4}}$$

$$= \frac{\frac{5}{4} + \frac{16}{3} - 1}{1}$$

$$= \frac{5}{4} + \frac{16}{3} - 1 = \frac{5}{4} + \frac{16}{3} - \frac{4}{4}$$

$$= \frac{5-4}{4} + \frac{16}{3} = \frac{1}{4} + \frac{16}{3}$$

$$= \frac{3}{12} + \frac{64}{12} = \frac{67}{12}$$

17. Given:

$$\left(\frac{\cos A}{1 + \sin A}\right) + \left(\frac{1 + \sin A}{\cos A}\right) = 2 \sec A$$

L.H.S. =

$$\left(\frac{\cos A}{1 + \sin A}\right) + \left(\frac{1 + \sin A}{\cos A}\right) = \frac{\cos^2 A + (1 + \sin A)^2}{\cos A(1 + \sin A)}$$

$[\because (a + b)^2 = a^2 + b^2 + 2.a.b]$

$$= \frac{\cos^2 A + (1)^2 + \sin^2 A + 2.\sin A.1}{\cos A(1 + \sin A)}$$

$$= \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{\cos A(1 + \sin A)}$$

$(\because \cos^2 A + \sin^2 A = 1)$

$$= \frac{1 + 1 + 2\sin A}{\cos A(1 + \sin A)}$$

$$= \frac{2 + 2\sin A}{\cos A(1 + \sin A)}$$

$$= \frac{2(1 + \sin A)}{\cos A(1 + \sin A)}$$

$$= \frac{2}{\cos A}$$

$= 2 \sec A \qquad \left(\because \dfrac{1}{\cos A} = \sec A\right)$

= R.H.S. Hence Proved.

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. $$\text{LHS} = \frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} \qquad \left(\because \cot\theta = \frac{1}{\tan\theta}\right)$$

$$= \frac{1 + \cot^2\theta - 1 - \tan^2\theta}{1 + \cot^2\theta + 1 + \tan^2\theta}$$

$$= \frac{\cot^2\theta - \tan^2\theta}{2 + \cot^2\theta + \tan^2\theta}$$

On substituting $\tan\theta = \dfrac{1}{\sqrt{7}}$ we get,

$$= \frac{7 - \frac{1}{7}}{2 + 7 + \frac{1}{7}}$$

$$= \frac{\frac{49-1}{7}}{\frac{63+1}{7}}$$

$$= \frac{48}{64}$$

$$= \frac{3}{4}$$

= RHS Hence Proved.

2. LHS $= \dfrac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}$

$= \dfrac{(\sec^2\theta-\tan^2\theta)+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta}$

$= \dfrac{(\sec\theta-\tan\theta)(\sec\theta+\tan\theta+1)}{(1+\sec\theta+\tan\theta)}$

$= \sec\theta-\tan\theta$

$= \dfrac{1}{\cos\theta}-\dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{1-\sin\theta}{\cos\theta}$

= RHS Hence Proved

3. LHS $= (\sin\theta+\text{cosec}\,\theta)^2+(\cos\theta+\sec\theta)^2$

$= (\sin^2\theta+\text{cosec}^2\theta+2\sin\theta\,\text{cosec}\,\theta)$
$+ (\cos^2\theta+\sec^2\theta+2\cos\theta\sec\theta)$

$= (\sin^2\theta+\cos^2\theta)+(\text{cosec}^2\theta+\sec^2\theta)$
$+ 2\sin\theta\left(\dfrac{1}{\sin\theta}\right)+2\cos\theta\left(\dfrac{1}{\cos\theta}\right)$

$= (1)+(1+\cot^2\theta+1+\tan^2\theta)$
$+ (2)+(2)$

$= 7+\tan^2\theta+\cot^2\theta$ = RHS

Hence Proved

4. LHS $= \dfrac{(\tan\theta+\sec\theta)-(\sec^2\theta-\tan^2\theta)}{\tan\theta-\sec\theta+1}$

$= \dfrac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}$

$= \dfrac{(\tan\theta+\sec\theta)(1-\sec\theta+\tan\theta)}{\tan\theta-\sec\theta+1}$

$= \tan\theta+\sec\theta$

$= \dfrac{1+\sin\theta}{\cos\theta}$ = RHS Hence Proved

5. LHS $= \left(\dfrac{1}{\cos\theta}-\cos\theta\right)\left(\dfrac{1}{\sin\theta}-\sin\theta\right)$

$= \left(\dfrac{1-\cos^2\theta}{\cos\theta}\right)\left(\dfrac{1-\sin^2\theta}{\sin\theta}\right)$

$= \dfrac{\sin^2\theta}{\cos\theta}\times\dfrac{\cos^2\theta}{\sin\theta}$

$= \sin\theta\cos\theta$

Now,

RHS $= \dfrac{1}{\tan\theta+\cot\theta} = \dfrac{1}{\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}}$

$= \dfrac{\cos\theta\sin\theta}{\sin^2+\cos^2\theta}$

$= \sin\theta\cos\theta$

∴ LHS = RHS Hence Proved.

6. LHS =

$\dfrac{1+\sin\theta}{1-\sin\theta}-\dfrac{1-\sin\theta}{1+\sin\theta}$

$\Rightarrow \dfrac{(1+\sin\theta)^2-(1-\sin\theta)^2}{(1-\sin^2\theta)}$

$\Rightarrow \dfrac{(1+2\sin\theta+\sin^2\theta)-(1-2\sin\theta+\sin^2\theta)}{\cos^2\theta}$

$\Rightarrow \dfrac{4\sin\theta}{\cos^2\theta} = 4\times\dfrac{\sin\theta}{\cos\theta}\times\dfrac{1}{\cos\theta}$

$= 4\tan\theta\sec\theta$ = RHS

Hence Proved

7. Given, $1+\sin^2\theta = 3\sin\theta\cos\theta$

Dividing both sides by $\cos^2\theta$

$\dfrac{1}{\cos^2\theta}+\dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{3\sin\theta\cos\theta}{\cos^2\theta}$

$\sec^2\theta+\tan^2\theta = 3\tan\theta$

$1+\tan^2\theta+\tan^2\theta = 3\tan\theta$

$1+2\tan^2\theta = 3\tan\theta$

$2\tan^2\theta-3\tan\theta+1 = 0$

If $\tan\theta = x$, then the equation becomes

$2x^2-3x+1 = 0$

$\Rightarrow (x-1)(2x-1) = 0$

$\Rightarrow \quad x = 1$ or $\dfrac{1}{2}$

Thus, $\tan\theta = 1$ or $\dfrac{1}{2}$

Hence Proved.

8. LHS $= (\sin^4\theta-\cos^4\theta+1)\,\text{cosec}^2\theta$

$= [(\sin^2\theta-\cos^2\theta)(\sin^2\theta+\cos^2\theta)+1]\,\text{cosec}^2\theta$

$= (\sin^2\theta-\cos^2\theta+1)\,\text{cosec}^2\theta$

$= 2\sin^2\theta\,\text{cosec}^2\theta$

$= 2\sin^2\theta\times\dfrac{1}{\sin^2\theta} = 2$ Hence Proved.

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. LHS: $m^2-n^2 = (m+n)(m-n)$

$= (\tan\theta+\sin\theta+\tan\theta-\sin\theta)(\tan\theta+\sin\theta-\tan\theta+\sin\theta)$

$= (2\tan\theta)(2\sin\theta)$

$= 4\tan\theta\sin\theta$ ...(i)

RHS: $4\sqrt{mn} = 4\sqrt{(\tan\theta+\sin\theta)(\tan\theta-\sin\theta)}$

$= 4\sqrt{\tan^2\theta-\sin^2\theta}$

$= 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta}$

$= 4\sqrt{\frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}}$

$= 4\sqrt{\frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta}}$

$= 4\sqrt{\tan^2\theta\sin^2\theta}$

$= 4\tan\theta\sin\theta$ ...(ii)

From (i) and (ii)

LHS = RHS

Hence, $m^2 - n^2 = 4\sqrt{mn}$ Hence Proved

2. We are given the following conditions in an acute angled triangle ΔABC:

$\sec(B + C - A) = 1$

$\tan(C + A - B) = \frac{1}{\sqrt{3}}$

Understanding the trigonometric conditions.

$\sec(B + C - A) = 1$ implies:

$\therefore$ $\sec(x) = 1 \Rightarrow x = 0°$

Therefore:

$B + C - A = 0° \Rightarrow B + C = A$

This gives us the relation: A = B + C ...(i)

Now, $\tan(C + A - B) = \frac{1}{\sqrt{3}}$ implies:

As, $\tan(30°) = \frac{1}{\sqrt{3}}$

Therefore:

$C + A - B = 30°$

This gives us the equation: C + A – B = 30°

From (i) A = B + C

Substitute Value of A into the second conditon:

$C + (B + C) - B = 30°$

On Simplifying:

$2C = 30° \Rightarrow C = 15°$

Now substitute C = 15° into A = B + C

$A = B + 15°$

Since the sum of the angles in any triangle is 180°, we have:

$A + B + C = 180°$

On substituting A = B + 15° and C = 15°:

$(B + 15°) + B + 15° = 180°$

$\Rightarrow$ $2B + 30° = 180° \Rightarrow 2B = 150° \Rightarrow B = 75°$

Now substitute B = 75° into A = B + 15°

$A = 75° + 15° = 90°$

Thus, the three angles of the triangle are:

$A = 90°, B = 75°, C = 15°$

3. $\frac{\tan^2 60° + 4\sin^2 45° + 3\sec^2 60° + 5\cos^2 90°}{\text{cosec } 30° + \sec 60° - \cot^2 30°}$

On evaluating each trigonometric term

$\tan 60° = \sqrt{3}$, so $\tan^2 60° = (\sqrt{3})^2 = 3$

$\sin 45° = \frac{1}{\sqrt{2}}$, so $\sin^2 45° = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$

Thus,

$4\sin^2 45° = 4 \times \frac{1}{2} = 2$

$\sec 60° = 2$, so $\sec^2 60° = 2^2 = 4$

Thus, $3\sec^2 60° = 3 \times 4 = 12$

$\cos 90° = 0$

so $5\cos^2 90° = 5 \times 0^2 = 0$

On substituting into the numerator we get,

$\tan^2 60° + 4\sin^2 45° + 3\sec^2 60° + 5\cos^2 90°$

$= 3 + 2 + 12 + 0 = 17$ ..(i)

Now evaluate the denominator

$\text{cosec } 30° = 2$

$\sec 60° = 2$

$\cot 30° = \sqrt{3}$, so

$\cot^2 30° = (\sqrt{3})^2 = 3$

Thus, the denominator becomes

$\text{cosec } 30° + \sec 60° - \cot^2 30° = 2 + 2 - 3 = 1$ ...(ii)

Thus, on substituting value

From (i) and (ii) in the given expression we get,

$\frac{17}{1} = 17$

Thus, the value of the expression is 17

4. $\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A$

On solving LHS we get,

$= \frac{\sin A(1 - 2\sin^2 A)}{\cos A(2\cos^2 A - 1)}$

$= \frac{\sin A(\sin^2 A + \cos^2 A - 2\sin^2 A)}{\cos A[2\cos^2 A - (\sin^2 A + \cos^2 A)]}$

$= \frac{\sin A(\cos^2 A - \sin^2 A)}{\cos A(2\cos^2 A - \sin^2 - \cos^2 A)}$

$\Rightarrow \frac{\sin A(\cos^2 A - \sin^2 A)}{\cos A(\cos^2 A - \sin^2 A)}$

$\Rightarrow \frac{\sin A}{\cos A} = \tan A$ Hence Proved

6. $(1 + \tan x + \sec x)^2 = 2(1 + \sec x)(\sec x + \tan x)$

LHS:

$(1 + \tan x + \sec x)^2$

Using the identity $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$ expand:

$(1 + \tan x + \sec x)^2 = 1 + \tan^2 x + \sec^2 x + 2\tan x + 2\tan x\sec x + 2\sec x$

Using the identity $\sec^2 x = 1 + \tan^2 x$, this becomes:

$1 + \tan^2 x + (1 + \tan^2 x) + 2\tan x + 2\tan x\sec x + 2\sec x$

$= 2 + 2\tan^2 x + 2\tan x + 2\tan x\sec x + 2\sec x$

$= 2(1 + \tan^2 x + \tan x + \tan x\sec x + \sec x)$ ...(i)

Now, RHS:

$2(1 + \sec x)(\sec x + \tan x)$

$= 2(1.\sec x + 1.\tan x + \sec x.\sec x + \sec x.\tan x)$

$= 2(\sec x + \tan x + \sec^2 x + \sec.x\tan x)$

Using $\sec^2 x = 1 + \tan^2 x$, this becomes:

$2(\sec x + \tan x + 1 + \tan^2 x + \sec x\tan x)$

$= 2(1 + \sec x + \tan x + \tan^2 x + \sec x\tan x)$

On comparing (i) and (ii) we get,

The LHS and RHS are identical:
$2(1 + \tan^2 x + \tan x + \tan x \sec x + \sec x)$
Hence, the identity is proven:

$(1 + \tan x + \sec x)^2 = 2(1 + \sec x)(\sec x + \tan x)$
Hence Proved.

Level - 2 ADVANCED COMPETENCY FOCUSED QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Mark)

1. Option (A) is correct

Explanation: In a right triangle, when we refer to the adjacent and opposite sides with respect to an angle $\angle x$, the adjacent side is the side next to $\angle x$ (but not the hypotenuse).

The opposite side is the side opposite to $\angle x$.

Now, for the complementary angle $(90° - x)$: The side that was adjacent to $\angle x$ becomes opposite to $(90° - x)$. The side that was opposite to $\angle x$ becomes adjacent to $(90° - x)$.

So, the adjacent and opposite sides are reversed for complementary angles.

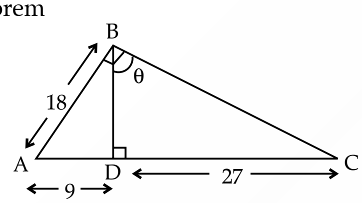
2. Option (D) is correct

Explanation: In triangle ADC, $\angle y$ is at vertex C. The two sides forming $\angle y$ in ΔADC are: CD and AC. So, sides AC and CD both are adjacent to $\angle y$ in ΔADC.

In triangle ABC, again, $\angle y$ is at vertex C. The two sides forming $\angle y$ in ΔABC are: AC and BC. So, side AC and BC both are adjacent to $\angle y$ in ΔABC.

3. Option (A) is correct

Explanation: In right ΔADB, using Pythagorean Theorem



$$BD^2 = 18^2 - 9^2$$
$$BD^2 = 324 - 81$$
$$BD^2 = 243$$
$$\Rightarrow \quad BD = \sqrt{243} = 9\sqrt{3}$$

Now, in right ΔBDC, using Pythagoras theorem,

$$BC^2 = BD^2 + DC^2$$
$$= (9\sqrt{3})^2 + (27)^2$$
$$= 243 + 729$$
$$= 972$$
$$\therefore \quad BC = \sqrt{972} = 18\sqrt{3}$$

Now, in ΔBDC,

$$\cos\theta = \frac{BD}{BC} = \frac{9\sqrt{3}}{18\sqrt{3}} = \frac{1}{2}$$
$$\therefore \quad \cos\theta = \frac{1}{2}$$

4. Option (B) is correct

Explanation: Given, tan A = 1 and angle A is acute.

We know that:

$$\tan(30°) = \frac{1}{\sqrt{3}}$$
$$\tan(45°) = 1$$
$$\tan(60°) = \sqrt{3}$$

tan (90°) is undefined

Since tan A = 1, and that only happens when A = 45°, and 45° is acute.

5. Option (C) is correct

Explanation: The sine of any angle is never greater than 1. For acute angles $(0° < \theta < 90°)$, $\sin\theta$ is between 0 and 1.

The cosine of any angle is also never greater than 1.

For acute angles, $\cos\theta$ is between 0 and 1.

$\tan\theta = \cos\theta$ is not always true. It only holds at specific angles, like $\theta = 0°$, where both are 0.

But in general, $\tan\theta \neq \cos\theta$ for all acute angles.

6. Option (C) is correct

Explanation: In a right triangle, if one of the angles in 30°, the ratio of the side opposite to 30° angle to hyptenuse is

$$\sin 30° = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{2} \text{ or } 1:2$$

ASSERTION-REASON QUESTIONS (1 Mark)

1. Option (A) is correct

Explanation: Assertion is true as this is a fundamental trigonometric identity valid for all acute angles in a right-angled triangle.

Reason is also true because these are the definitions of sine and cosine in a right triangle.

From the definitions of sine and cosine:

$$\sin\theta = \frac{P}{H}, \cos\theta = \frac{B}{H}$$

Then:

$$\sin^2\theta + \cos^2\theta = \left(\frac{P}{H}\right)^2 + \left(\frac{B}{H}\right)^2 = \frac{P^2 + B^2}{H^2}$$

Using the Pythagoras theorem:

$$P^2 + B^2 = H^2 \Rightarrow \frac{H^2}{H^2} = 1$$

Hence, the assertion is provedusing the definitions in the reason.

2. Option (A) is correct

Explanation: Assertion is true because this is a standard trigonometric value:

$$\text{Tan } 45° = \frac{\text{Perpendicular}}{\text{Base}} = \frac{a}{a} = 1$$

Reason is also true because in a right triangle, if two angles are equal and one is 90°, the other two must each be 45°.

The triangle described in the reason leads to equal sides (perpendicular = base), which makes the corresponding angles 45°. This setup gives tan 45° =1, directly explaining the assertion.

3. Option (A) is correct

Explanation: Assestion is true as value of sin 30° = $\frac{1}{2}$ (according to trigonometric table. Reason is also true as in right triangle with 30° angle, the side opposite to this angle is always half of the hypotenuse. Reason corretly explains assestion based on triangle geometry

4. Option (A) is correct

Explanation: The reciprocal relationship between tangent and cotangent is directly derived from their definitions. Since:

$$\tan\theta = \frac{P}{B}, \cot\theta = \frac{B}{P} \Rightarrow \tan\theta = \frac{1}{\cot\theta}$$

Hence, both assertion and reason are true, and the reason explains the assertion correctly.

5. Option (A) is correct

Explanation: Since cos 90° = 0, calculating sec 90° $= \frac{1}{\cos 90^\circ} = \frac{1}{0}$ is mathematically undefined. Both assertion and reason are true and the reason directly explains why the assertion is true.

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. We know the cosine of an angle in a right triangle is defined as:

$$\cos\theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$$

Substituting the values:

$$\cos 60^\circ = \frac{5}{10} = \frac{1}{2}$$

Standard value of cos 60° = $\frac{1}{2}$

The triangle satisfies the identity:

$$\cos 60^\circ = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{5}{10} = \frac{1}{2}$$

Thus, the value is verified correctly using the triangle.

2. To find the base, we use the cosine ratio:

$$\cos\theta = \frac{\text{Adjacent side (base)}}{\text{Hypotenuse}}$$

Substituting values:

$$\cos 60^\circ = \frac{\text{Base}}{2}$$

We know:

$$\cos 60^\circ = \frac{1}{2}$$

So, $\frac{1}{2} = \frac{\text{Base}}{2} \Rightarrow \text{Base} = 1\,\text{m}$

Verification

We used:

$$\cos 60^\circ = \frac{1}{2}$$

Using the calculated base:

$$\cos 60^\circ = \frac{1}{2} \text{ (matches standard value)}$$

The length of the base is 1 metre, and the trigonometric value cos 60° = $\frac{1}{2}$ is verified using this triangle.

3. $\tan\theta = \frac{\text{Adjacent side}}{\text{Opposite side}} = \frac{x}{x} = 1$

So, $\tan\theta = 1$

From standard trigonometric values:

$$\tan 45^\circ = 1$$

Therefore, $\theta = 45^\circ$

4. Let the adjacent side be x.

By Pythagoras theorem:

$$x^2 + 5^2 = 13^2 \Rightarrow x^2 + 25 = 169$$

$$\Rightarrow x^2 = 169 - 25 = 144$$

$$\Rightarrow x = 12\,\text{cm}$$

Now,

Opposite side = 5 cm

Adjacent side = 12 cm

Hypotenuse = 13 cm

$$\sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{5}{13}$$

$$\cos\theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{12}{13}$$

Verifying the identity $\sin^2\theta + \cos^2\theta = 1$

$$\sin^2\theta = \left(\frac{5}{13}\right)^2 = \frac{25}{169}$$

$$\cos^2\theta = \left(\frac{12}{13}\right)^2 = \frac{144}{169}$$

$$\sin^2\theta + \cos^2\theta = \frac{25}{169} + \frac{144}{169} = \frac{169}{169} = 1.$$

Verified

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. (i) To find the hypotenuse using cos 60°:

$$\cos 60^\circ = \frac{\text{Base}}{\text{Hypotenuse}}$$

Using, $\cos 60^\circ = \frac{1}{2}$

$$\Rightarrow \quad \frac{1}{2} = \frac{6}{\text{Hypotenuse}}$$

$$\text{Hypotenuse} = 6 \times 2 = 12 \text{ cm}$$

(ii) To find vertical height using sin 60°:

$$\sin 60° = \frac{\text{Height}}{\text{Hypotenuse}}$$

Using

$$\sin 60° = \frac{\sqrt{3}}{2},$$

$$\sin 60° = \frac{\text{Height}}{\text{Hypotenuse}}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{\text{Height}}{12}$$

$$\text{Height} = 12 \times \frac{\sqrt{3}}{2} = 6\sqrt{3} \text{ cm}$$

$$\text{Height} = 6\sqrt{3} \text{ cm}$$

2. (i)

$$\sin 45° = \frac{\text{Opposite (height)}}{\text{Hypotenuse}}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{h}{10}$$

$$\Rightarrow \quad h = \frac{10}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2} \text{ m}$$

Vertical height gained = $5\sqrt{2}$ m

(ii) For $\theta = 45°$

$$\sin^2 45° = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} \text{ and}$$

$$\cos^2 45° = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$$

$$\sin^2\theta + \cos^2\theta = \frac{1}{2} + \frac{1}{2} = 1$$

Verified: The identity $\sin^2\theta + \cos^2\theta = 1$ holds true for $\theta = 45°$

3. (i) Given, $\cos\theta = \frac{4}{5}$

Using the identity:

$$\sin^2\theta + \cos^2\theta = 1$$

Substituting $\cos\theta = \frac{4}{5}$

$$\sin^2\theta + \left(\frac{4}{5}\right)^2 = 1$$

$$\sin^2\theta + \frac{16}{25} = 1$$

$$\sin^2\theta = 1 - \frac{16}{25} = \frac{9}{25}$$

$$\sin^2\theta = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$\sin\theta = \frac{3}{5}$$

(ii)

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\tan\theta = \frac{3/5}{4/5} = \frac{3}{4}$$

$$\tan\theta = \frac{3}{4}$$

4. (i) We know:

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$= \frac{\text{height}}{10}$$

Substituting the values:

$$\sin 30° = \frac{1}{2}$$

$$\frac{1}{2} = \frac{h}{10} \Rightarrow h = 10 \times \frac{1}{2} = 5 \text{ m}$$

(ii) Verifying using $\sin 30° = \frac{1}{2}$:

Already used in part (i):

$$\sin 30° = \frac{h}{10} = \frac{5}{10} = \frac{1}{2}$$

Verified: Height = 5 m and matches with $\sin 30° = \frac{1}{2}$

CASE BASED QUESTIONS (4 Marks)

1. (i) No, the value of sin A is not greatest for triangle PQA. All triangles have the same value of sin A, because the value of sin A does not depend on side lengths but is a ratio of side lengths.

(ii) We have,

Measure of angle A = 45° (for right isosceles triangle)

Now, $2 \sin A \cos A = 2 \times \sin 45° \times \cos 45°$

$$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 1$$

(iii) (a) The value of sin A depends on the measure of the angle or the ratio of the corresponding sides. So, the value of sin A is not different in triangles ABC and ADE.

OR

(b) We have, $\angle A = 45°$

$$\therefore \sin A = \sin 45° = \frac{1}{\sqrt{2}}$$

$$\cos A = \cos 45° = \frac{1}{\sqrt{2}}$$

and, $\tan A = \tan 45° = 1$

2. (i) The value of sin A increase as the measure of angle A increases. It reaches to 1 when the person comes directly under the camera.

(ii) In instant 5,
The person comes directly under camera and points A and B coincides so the measure of angle A becomes 90°.

(iii) (a) We have,
In an isosceles triangle ABC,
We have, $\angle C = \angle A = 30°$

So, $\cos 30° = \frac{\sqrt{3}}{2}$

OR

(b) We know that $\sin 0° = 0$

$\sin 90° = 1$

$\therefore \sin 0° + \sin 90° = 0 + 1 = 1$

3. Given, Angle of inclination θ = 30°, Base (horizontal distance) = 4 m

$$\sin 30° = \frac{1}{2}, \cos 30° = \frac{\sqrt{3}}{2}$$

(i) Trigonometric ratio for height and hypotenuse:

$$\sin \theta = \frac{\text{height}}{\text{hypotenuse}}$$

(ii)

$$\sin 30° = \frac{\text{height}}{\text{hypotenuse}} = \frac{1}{2}$$

$$\Rightarrow \text{height} = \frac{\text{hypotenuse}}{2}$$

(iii) (a) Using,

$$\cos \theta = \frac{\text{base}}{\text{hypotenuse}}$$

$$\cos 30° = \frac{\sqrt{3}}{2} = \frac{4}{\text{hypotenuse}}$$

$$\Rightarrow \text{hypotenuse} = \frac{4\times 2}{\sqrt{2}} = \frac{8}{\sqrt{3}} = \frac{8\sqrt{3}}{3} \text{ m}$$

OR

(b) Verify the identity $\sin^2\theta + \cos^2\theta = 1$:

$$\sin 30° = \frac{1}{2}$$

$$\Rightarrow \sin^2 30° = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\cos 30° = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \cos^2 30° = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$$

$$\sin^2 30° + \cos^2 30° = \frac{1}{4} + \frac{3}{4} = 1$$

4. **(i)** Trigonometric ratio relating height to hypotenuse:

$$\sin \theta = \frac{\text{side}}{\text{hypotenuse}}$$

$$= \frac{\text{height}}{\text{panel length}}$$

(ii)

$$\sin 45° = \frac{2}{\text{hypotenuse}}$$

$$\frac{1}{\sqrt{2}} = \frac{2}{\text{hypotenuse}}$$

$$\Rightarrow \text{hypotenuse} = 2\sqrt{2}$$

hypotenuse ≈ 2 × 1.414 = 2.8 m (approx)

(iii) (a)

$$\cos 45° = \frac{\text{base}}{\text{hypotenuse}} = \frac{\text{base}}{2.8}$$

$$\frac{1}{\sqrt{2}} = \frac{\text{base}}{2.8}$$

$$\Rightarrow \text{base} = \frac{2.8}{\sqrt{2}} = 2.8 \times \frac{1}{\sqrt{2}} = 2 \text{ m}$$

OR

(b) Verify identity $\sin^2\theta + \cos^2\theta = 1$

$$\sin2\ 45° + \cos2\ 45° = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{1}{2} + \frac{1}{2} = 1 \qquad \text{Verified}$$

5. **(i)** To find height (opposite side) when base (adjacent side) is given:
we apply,

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{\text{height}}{\text{base}}$$

(ii)

$$\tan 60° = \frac{\text{height}}{4}$$

$$\Rightarrow \sqrt{3} = \frac{\text{height}}{4} \Rightarrow \text{height} = 4\sqrt{3} \text{ m}$$

Approximating:
height ≈ 4 × 1.732 = 6.928 m

(iii) (a)

$$\cos 60° = \frac{\text{hypotenuse}}{\text{base}} = \frac{\text{hypotenuse}}{4} = \frac{1}{2}$$

⇒ hypotenuse length of diogonal support = 8 m

OR

(b) Using standard values:

$$\sin 60° = \frac{\sqrt{3}}{2}$$

$$\cos 60° = \frac{1}{2}$$

$$\therefore \sin^2 60° + \cos^2 60° = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{4} + \frac{1}{4} = 1$$

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. (i) Using the cosine ratios:

$$\cos\theta = \frac{\text{base}}{\text{hypotenuse}}$$

$$\Rightarrow \quad \cos 60^\circ = \frac{2.5}{\text{hypotenuse}}$$

$$\Rightarrow \quad = \frac{1}{2} = \frac{2.5}{\text{hypotenuse}}$$

$$\Rightarrow \quad \text{hypotenuse} = \frac{2.5}{1/2} = 5\text{ m}$$

Length of ladder $= 5$ m

(ii) Using the sine ratios

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\text{height}}{5}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{\text{height}}{5}$$

$$\Rightarrow \quad \text{height} = 5\times\frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{2}$$

Approximating:

$$\text{height} \approx \frac{5\times 1.732}{2} \approx \frac{8.66}{2} = 4.33\text{ m}$$

Height of the wall $= 4.33$ m

(iii)

$$\sin^2 60^\circ = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$$

$$\cos^2 60^\circ = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\Rightarrow \quad \sin^2\theta + \cos^2\theta = \frac{3}{4}+\frac{1}{4} = 1$$

Verified: $\sin^2\theta + \cos^2\theta = 1$

2. (i) Using the sine ratios

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\Rightarrow \quad \sin 30^\circ = \frac{1.2}{\text{hypotenuse}}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{1.2}{\text{hypotenuse}}$$

$$\Rightarrow \quad \text{hypotenuse} = \frac{1.2}{1/2} = 2.4\text{ m}$$

Length of the ramp $= 2.4$ m

(ii) Using the cosine ratios

$$\cos\theta = \frac{\text{base}}{\text{hypotenuse}}$$

$$\Rightarrow \quad \cos 30^\circ = \frac{\text{base}}{2.4}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{\text{base}}{2.4}$$

$$\Rightarrow \quad \text{base} = 2.4\times\frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \text{base} \approx 2.4\times 0.866 = 2.0784\text{ m}$$

$$\Rightarrow \quad \text{Base} \approx 2.08\text{ m}$$

(iii)

$$\sin^2 30^\circ = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\cos^2 30^\circ = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$$

$$\Rightarrow \quad \sin^2\theta + \cos^2\theta = \frac{1}{4}+\frac{3}{4} = 1$$

Verified: $\sin^2\theta + \cos^2\theta = 1$