CHAPTER 1
Relations and Functions

Level - 1 CORE SUBJECTIVE QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQ) (1 Mark)

1. Option (A) is correct.

Explanation: Given, function $f : \mathbb{R}_+ \to \mathbb{R}$

$$f(x) = 4x + 3$$

where $\mathbb{R}_+$ is the set of all non-negative real numbers $(x \geq 0)$.

For One-One :

A function is one-one if:

$$f(x_1) = f(x_2)$$

$$\Rightarrow \quad x_1 = x_2$$

for all $x_1, x_2 \in \mathbb{R}_+$.

$$f(x_1) = f(x_2)$$

$$\Rightarrow \quad 4x_1 + 3 = 4x_2 + 3$$

$$4x_1 = 4x_2$$

$$x_1 = x_2$$

The function is one-one

For Onto :

A function is onto if for every $y \in \mathbb{R}$, there exists $x \in \mathbb{R}_+$ such that:

$$f(x) = y$$

$$y = 4x + 3$$

$$x = \frac{y-3}{4}$$

For $f(x)$ to be onto, x should always exist in $\mathbb{R}_+$.

However, since x must be non - negative $(x \geq 0)$, we require:

$$\frac{y-3}{4} \geq 0$$

$$y - 3 \geq 0$$

$$y \geq 3$$

Thus, not all real numbers are covered by $f(x)$; only $y \geq 3$ are possible. Since $f(x)$ does not cover all of $\mathbb{R}$, it is not onto.

2. Option (D) is correct.

Explanation: The given relation $R = \{(x, y) : x = y\}$ on the set $A = \{x \mid x \in \mathbb{Z}, 0 \leq x \leq 10\}$ means that each element is only related to itself.

The set A contains the integers from 0 to 10, thus

$$A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

This gives us a total of 11 elements in the set A.

Each element forms its own equivalence class because the only relation it holds is with itself.

The equivalence classes are:

$$\{0\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}, \{7\}, \{8\}, \{9\}, \{10\}$$

Thus, the total number of equivalence classes is 11.

3. Option (D) is correct

Explanation: The relation $R = \{(x, y) : x$ is 5 cm shorter than $y\}$

Not reflexive – x can't be 5 cm shorter than itself.

Not symmetric – if x is 5 cm shorter than y, y is not 5 cm shorter than x.

Not transitive – if x is 5 cm shorter than y and y is 5 cm shorter than z, then x is 10 cm shorter than z, not 5 cm.

4. Option (C) is correct.

Explanation:

One-one:

Let $x_1, x_2 \in \mathbb{R}_+$ and $f(x_1) = f(x_2)$

$$9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$$

$$9x_1^2 + 6x_1 = 9x_2^2 + 6x_2$$

$$3x_1^2 + 2x_1 = 3x_2^2 + 2x_2$$

$$\Rightarrow \quad 3x_1^2 - 3x_2^2 + 2x_1 - 2x_2 = 0$$

$$\Rightarrow \quad 3(x_1^2 - x_2^2) + 2(x_1 - x_2) = 0$$

$$3(x_1 - x_2)(x_1 + x_2) + 2(x_1 - x_2) = 0$$

$$(x_1 - x_2)(3x_1 + 3x_2 + 2) = 0$$

Now, $3x_1 + 3x_2 + 2 = 0 \Rightarrow 3x_1 = -3x_2 - 2$

As $x_1, x_2 \in \mathbb{R}_+$ (non-negative real numbers), then $(3x_1 + 3x_2 + 2 = 0)$ cannot be possible. So, $x_1 - x_2 = 0 \Rightarrow x_1 = x_2$

Hence it is one-one.

Onto:

Let $y = f(x)$ and $y \in [-5, \infty)$

So $y = 9x^2 + 6x - 5$

$9x^2 + 6x - 5 - y = 0$

$$x = \frac{-6 \pm \sqrt{6^2 - 4 \times 9 \times (-y-5)}}{2 \times 9}$$ (by using formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for quadratic equation $ax^2 + bx + c = 0$)

$$x = \frac{-6 \pm \sqrt{36 + 36(y+5)}}{18}$$

$$\Rightarrow \quad x = \frac{-6 \pm 6\sqrt{(y+6)}}{18}$$

$x = \dfrac{-1-\sqrt{y+6}}{3}$ and $x = \dfrac{-1+\sqrt{y+6}}{3}$.

We need to prove either one solution (x) exists for every $y \in [-5, \infty)$.

The solution $x = \dfrac{-1-\sqrt{y+6}}{3}$ is negative, which is invalid since $x \geq 0$.

The other solution $x = \dfrac{-1+\sqrt{y+6}}{3}$ is non-negative for all $y \geq -5$.

Thus, for every $y \geq -5$, there exists an $x \geq 0$ such that $f(x) = y$.

So, f is onto

Hence, the function is bijective (one - one and onto).

5. Option (A) is correct.

Explanation: The function $f : \mathbb{R}_+ \to \mathbb{R}_+$ defined by

$$f(x) = x^2 + 1$$

One-One:

A function is one-one if

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

for all $x_1, x_2 \in \mathbb{R}_+$.

Let $\quad f(x_1) = f(x_2),$

$$x_1^2 + 1 = x_2^2 + 1$$

$$(x_1 - x_2)(x_1 + x_2) = 0$$

$\therefore \quad x_1 = x_2 \quad (x_1 + x_2 \neq 0)$

Hence, $f(x)$ is one-one.

Onto:

A function is onto if for every $y \in \mathbb{R}_+$, there exists $x \in \mathbb{R}_+$ such that

$$f(x) = y$$

$$y = x^2 + 1$$

$$x^2 = y - 1$$

For x to be real and non - negative,

$$y - 1 \geq 0 \Rightarrow y \geq 1$$

Since $\mathbb{R}_+ = [0, \infty)$, but the range of $f(x)$ is only $[1, \infty)$, values in $(0, 1)$ are missing.

Hence, $f(x)$ is not onto.

6. Option (C) is correct.

Explanation: The function $f : \mathbb{R} \to A$ defined as

$$f(x) = x^2 + 1$$

to determine for which set A, $f(x)$ is onto.

The function $f(x) = x^2 + 1$ is a parabola opening upwards.

The minimum value occurs at $x = 0$, where

$$f(0) = 0^2 + 1 = 1$$

Since $x^2 \geq 0$ for all $x \in \mathbb{R}$, we conclude that

$$f(x) \geq 1, \text{ for all } x \in \mathbb{R}$$

Hence, the range of $f(x)$ is $[1, \infty)$.

For $f(x)$ to be onto, its codomain must match this range.

7. Option (B) is correct.

Explanation: The given function is $f : \mathbb{Z} \to \mathbb{Z}$ defined as

$$f(x) = x^3 - 1$$

One-One:

For $f(x)$ to be one-one, we check if $f(x_1) = f(x_2)$ implies $x_1 = x_2$.

$$x_1^3 - 1 = x_2^3 - 1 \Rightarrow x_1^3 = x_2^3$$

Since the cube function is strictly increasing for all integers, this implies:

$$x_1 = x_2$$

Thus, $f(x)$ is one – one.

Onto:

For $f(x)$ to be onto, every integer $y \in \mathbb{Z}$ must have some $x \in \mathbb{Z}$ such that:

$$y = x^3 - 1$$

$$x^3 = y + 1$$

For y to be an integer, $y + 1$ must be a perfect cube, but not every integer is of this form (e.g., $y = 2$ gives $x^3 = 3$, which has no integer solution).

Thus, $f(x)$ is not onto.

8. Option (D) is correct.

Explanation: Given function:

$$f(x) = x^2 - 4x + 5$$

One – One (Injectivity):

Let $x_1, x_2 \in \mathbb{Z}$ and assume $f(x_1) = f(x_2)$, then:

$$x_1^2 - 4x_1 + 5 = x_2^2 - 4x_2 + 5$$

$$x_1^2 - 4x_1 = x_2^2 - 4x_2$$

$$x_1^2 - x_2^2 = 4x_1 - 4x_2$$

$$(x_1 - x_2)(x_1 + x_2 - 4) = 0$$

$$x_1 + x_2 = 4$$

$\therefore \quad x_1 \neq x_2$

This means different values of x can map to the same $f(x)$, proving that $f(x)$ is not one – one.

Onto (Surjectivity):

Let $y = f(x)$, so:

$$y = x^2 - 4x + 5$$

$$x^2 - 4x + 5 - y = 0$$

This is a quadratic equation in x:

$$x = \frac{4 \pm \sqrt{16 - 4(5-y)}}{2}$$

$$= \frac{4 \pm \sqrt{16 - 20 + 4y}}{2}$$

$$= \frac{4 \pm \sqrt{4y - 4}}{2}$$

For a real solution, the discriminant must be non – negative:

$$4y - 4 \geq 0$$

$$4y \geq 4$$

$$y \geq 1$$

Thus, $f(x)$ only takes values $y \geq 1$ and does not cover all of $\mathbb{R}$, meaning it is not onto.

ASSERTION-REASON QUESTIONS (1 Mark)

1. Option (A) is correct.

***Explanation:* Assertion:** We are given the function:

$$f(x) = \sec x$$

$$D = \mathbb{R} - \left\{\frac{(2n+1)\pi}{2}, n \in \mathbb{Z}\right\}$$

and the codomain:

$$(-\infty, -1] \cup [1, \infty)$$

This means the function is defined for all real numbers except the odd multiples of $\frac{\pi}{2}$, where $\sec x$ is undefined.

A function is one – one (injective) if:

$$f(a) = f(b) \Rightarrow a = b$$

The secant function $\sec x$ is periodic with period 2π:

$$\sec(x) = \sec(x + 2\pi k), \forall k \in \mathbb{Z}$$

This means that multiple values of x map to the same output, making it not one - one.

Assertion (A) is true because the function is not one – one.

Reason (R): "The line $y = 2$ meets the graph of $f(x)$ at more than one point."

The graph of $y = \sec x$ has multiple points where $\sec x = 2$, confirming that $f(x)$ is not one – one.

This directly shows that different values of x give the same function output.

Thus, Both (A) and (R) are true, and (R) is the correct explanation for (A).

2. Option (C) is correct.

Explanation:

Given Relation:

$R = \{(x, y) \mid (x + y)$ is a prime number, $x, y \in \mathbb{N}\}$

Assertion:

A relation is reflexive if:

$$(x, x) \in R \ \forall\ x \in \mathbb{N}$$

This means $x + x = 2x$ should be prime for all $x \in \mathbb{N}$.

If $x = 1$, then $2(1) = 2$ (which is prime).

If $x = 2$, then $2(2) = 4$ (which is not prime).

If $x = 3$, then $2(3) = 6$ (which is not prime).

Since $x + x$ (i.e., $2x$) is not always prime, R is not reflexive.

Assertion (A) is true.

Reason: "The number $2n$ is composite for all natural numbers n."

If $n = 1$, then $2(1) = 2$, which is prime, not composite.

Reason (R) is false.

3. Option (C) is correct.

Explanation: Assertion (A) is true for a finite set X, as a one – one function is always onto and vice-versa.

Reason (R) is false because this statement is not true for infinite sets.

So, (A) is true, but (R) is false.

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. Since $f(x) = 2x$ is one-one onto function. So, every element in A will have a unique corresponding element in B, and every element will have corresponding element in B, and every element in B will have corresponding element is A. Thus, $f(1) = 2, f(2) = 4, f(3) = 6, f(4) = 8$

   $\therefore B = \{2, 4, 6, 8\}$

2. For Injectivity

   Let $x_1, x_2 \in \mathbb{N}$; Let $f(x_1) = f(x_2)$

   $\Rightarrow x_1^3 = x_2^3 \Rightarrow x_1 = x_2 \Rightarrow f$ is injective function.

   For Surjectivity

   Let $y \in \mathbb{N}$, then $f(x) = y \Rightarrow x^3 = y \Rightarrow x = y^{1/3} \notin \mathbb{N}$ for some y

   $\Rightarrow f$ is not surjective.

   Note: Student can justify f as non-surjective by taking some counter examples also.

3. For not one-one:

   $1.1, 1.2 \in \mathbb{R}$ (domain)

   Now, $1.1 \neq 1.2$ but $f(1.1) = f(1.2) = 1 \Rightarrow f$ is not one-one.

   For not onto:

   Let $\frac{1}{2} \in \mathbb{R}$ (co-domain), but $[x] = \frac{1}{2}$ is not possible for x in domain.

   So, f is not onto.

4. Given relation R can be written in roaster form as:

   $R = \{(3, 3), (6, 2), (9, 1)\}$

   $\therefore$ Domain (R) = {3, 6, 9}

   Range (R) = {1, 2, 3}

5. $f(-1) = f(1) = 1$, but $-1 \neq 1$, $\therefore$ 'f' is not a one-one function.

   $0 \le x^4 < \infty \Rightarrow R_f = [0, \infty) \Rightarrow$ Range $\neq$ Co-domain.

   $\therefore$ 'f' is not an onto function.

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. **Reflexive:**

   $\because |x^2 - x^2| < 8\ \forall\ x \in A \Rightarrow (x, x) \in R$

   $\therefore$ R is reflexive.

   Symmetric:

   Let $(x, y) \in R$ for some $x, y \in R$

   If $|x^2 - y^2| < 8$, then $|y^2 - x^2| < 8$.

   Hence R is symmetric.

   Transitive:

   $(1, 2), (2, 3) \in R$ as $|1^2 - 2^2| < 8$, $|2^2 - 3^2| < 8$ respectively.

   But $|1^2 - 3^2| \not< 8 \Rightarrow (1, 3) \notin R$

Hence R is not transitive.

2. Given, $f(x) = ax + b, f(1) = 1$ and $f(2) = 3$

Solving $a + b = 1$ and $2a + b = 3$ to get $a = 2, b = -1$

$\therefore \quad f(x) = 2x - 1$

Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in \mathbb{R}$

$2x_1 - 1 = 2x_2 - 1$

$\Rightarrow \quad x_1 = x_2$

Hence f is one-one.

Let $y = 2x - 1, y \in \mathbb{R}$ (Codomain)

$\Rightarrow x = \dfrac{y+1}{2} \in \mathbb{R}$ (domain)

Also, $f(x) = f\left(\dfrac{y+1}{2}\right) = y$

$\therefore f$ is onto.

3. (i) Set X as $\{1 \times 2, 2 \times 3, 3 \times 4, 4 \times 5, ...n(n+1), ...\}$.

Defines a funciton $f : X \to Y$ as:

$f\{n(n+1)\} = n + 1$

(ii) Shows that the above function is one-one as:

$f(n(n+1)) = f(m(m+1))$

$\Rightarrow \quad n + 1 = m + 1$

$\Rightarrow \quad n = m$

4. Reflexive: Every country shares its boundary with itself.

That is, $(x, x) \in R$, for each element $x \in G$. Hence, R is reflexive.

Symmetric: Whenever x shares a boundary with y, y also shares a boundary with x.

That is, $(x, y) \in R \Rightarrow (y, x) \in R$. Hence, R is symmetric.

Transitive: If x shares a boundary with y and y shares a boundary with z, then x need not share is boundary with z.

That is, $(x, y) \in R$, $(y, z) \in R$ need not imply $(x, z) \in R$.

Hence, R is not transitive.

From the above steps, concludes that R is not an equivalence relation.

5. (i) The function must be of the form $f(x) = k$, where 'k' is a real number.

f is not one-one because.

$f(1) = k = f(2)$, but $1 \neq 2$.

(ii) f is not onto because.

Consider $\beta\ (\neq k) \in \mathbb{R}$ (codomain), there is no element $x \in \mathbb{R}$ (domain) such that $f(x) = \beta$.

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in \mathbb{R}$

Then $\dfrac{2x_1}{1+x_1^2} = \dfrac{2x_2}{1+x_2^2}$

$\Rightarrow \quad x_1 + x_1x_2^2 = x_2 + x_1^2x_2$

$\Rightarrow (x_1 - x_2) - x_1x_2(x_1 - x_2) = 0$

$\Rightarrow \quad (x_1 - x_2)(1 - x_1x_2) = 0$

$\Rightarrow \quad x_1 - x_2 = 0$ or $1 - x_1x_2 = 0$

$\Rightarrow x_1 = x_2$ or $x_1x_2 = 1$, so if $x_1x_2 = 1, x_1 \neq x_2$

Hence f is not one-one.

Let $y = f(x)$ where $x \in \mathbb{R}$

$\Rightarrow \quad yx^2 - 2x + y = 0$

$\Rightarrow \quad x = \dfrac{2 \pm \sqrt{4 - 4y^2}}{2y}$

For x to be real, $4 - 4y^2 \geq 0$

$\Rightarrow y^2 \leq 1$

$\Rightarrow -1 \leq y \leq 1$

Hence, range $= [-1, 1] \neq$ codomain.

Hence, f is not onto.

For the given function is become onto, $A = [-1, 1]$

2. Let $(a, b) \in \mathbb{N} \times \mathbb{N}$

We have

$a - a = b - b$

This implies that $(a, b)\ R\ (a, b)\ \forall\ (a, b) \in \mathbb{N} \times \mathbb{N}$

Hence R is reflexive.

Let $(a, b)\ R\ (c, d)$ for some $(a, b), (c, d) \in \mathbb{N} \times \mathbb{N}$

Then $\quad a - c = b - d$

$\Rightarrow \quad c - a = d - b$

$\Rightarrow (c, d)\ R\ (a, b)$

Hence, R is symmetric.

Let $(a, b)\ R\ (c, d)$, $(c, d)\ R\ (e, f)$ for some $(a, b), (c, d), (e, f) \in \mathbb{N} \times \mathbb{N}$

Then $\quad a - c = b - d,$

$c - e = d - f$

$\Rightarrow \quad a - c + c - e = b - d + d - f \quad$ (On adding)

$\Rightarrow \quad a - e = b - f$

$\Rightarrow (a, b)\ R\ (e, f)$

Hence, R is transitive.

Thus, R is an equivalence relation.

3. Let $f(x_1) = f(x_2)$ for some, $x_1, x_2 \in R$

Then $\quad x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$

$\Rightarrow \quad (x_1 - x_2)(1 + x_1 + x_2) = 0$

$\Rightarrow \quad x_1 - x_2 = 0$ or $x_1 + x_2 = -1$

$\Rightarrow \quad x_1 = x_2$ or $x_1 + x_2 = -1$ so if $x_1 + x_2 = -1, x_1 \neq x_2$

Hence f is not one-one.

Let $y = f(x)$ where $x \in R$

Then $\quad y = x^2 + x + 1$

$\Rightarrow \quad x^2 + x + 1 - y = 0$

$\Rightarrow \quad x = \dfrac{-1 \pm \sqrt{4y - 3}}{2}$

For x to be real, $4y - 3 \geq 0$

$\Rightarrow y \geq \dfrac{3}{4}$

Hence, range $= \left[\dfrac{3}{4}, \infty\right] \neq$ codomain

Hence, f is not onto.

$f(x) = 3 \Rightarrow x^2 + x + 1 = 3 \Rightarrow x^2 + x - 2 = 0$

$\Rightarrow \quad x = \dfrac{-1 \pm \sqrt{9}}{2} = -2, 1$

4. Let $(a, b) \in \mathbb{N} \times \mathbb{N}$

We have, $\quad \dfrac{a}{a} = \dfrac{b}{b}$

This implies that (a, b) R (a, b) $\forall$ $(a, b) \in \mathbb{N} \times \mathbb{N}$

Hence R is reflexive.

Let (a, b) R (c, d) for some $(a, b), (c, d) \in \mathbb{N} \times \mathbb{N}$

Then $$\frac{a}{c} = \frac{b}{d}$$

$$\Rightarrow \quad \frac{c}{a} = \frac{d}{b}$$

$\Rightarrow (c, d)$ R (a, b)

Hence, R is symmetric.

Let (a, b) R (c, d), (c, d) R (e, f) for some $(a, b), (c, d), (e, f) \in \mathbb{N} \times \mathbb{N}$

Then $$\frac{a}{c} = \frac{b}{d}, \frac{c}{e} = \frac{d}{f}$$

$$\Rightarrow \quad \frac{a}{c} \times \frac{c}{e} = \frac{b}{d} \times \frac{d}{f} \quad \text{(On multiplying)}$$

$$\Rightarrow \quad \frac{a}{e} = \frac{b}{f}$$

$\Rightarrow (a, b)$ R (e, f).

Hence, R is transitive.

Thus, R is an equivalence relation.

5. For reflexive: Clearly $x + x$ i.e. $2x$ is integer divisible by 2.

$\Rightarrow (x, x) \in R \Rightarrow R$ is reflexive.

For symmetric: $(x, y) \in R \Rightarrow x + y$ is integer divisible by 2.

$\Rightarrow y + x$ is integer divisible by 2 $\Rightarrow (y, x) \in R$

For transitive: $(x, y) \in R \Rightarrow x + y$ is integer divisible by 2.

$\Rightarrow \quad x + y = 2m$ ...(i)

and $(y, z) \in R \Rightarrow y + z$ is integer divisible by 2.

$\Rightarrow \quad y + z = 2n$ ...(ii)

On adding eq. (i) & (ii), we get

$$x + z + 2y = 2m + 2n$$

$$x + z = 2(m + n - y)$$

$\Rightarrow x + z$ is integer divisible by 2 $\Rightarrow (x, z) \in R$

Equivalence class $[2] = \{x : x \in A, (x, a) \in R\}$
$= \{-4, -2, 0, 2, 4\}$

6. For reflexive relation

To prove $(x, x) \in R$, $x - x = 0$ which is divisible by 5

$\therefore (x, x) \in R \Rightarrow R$ is reflexive.

For symmetric relation

Let $(x, y) \in R \Rightarrow x - y$ is divisible by 5

$\Rightarrow \; x - y = 5m \Rightarrow y - x = 5(-m)$

$\Rightarrow \; y - x$ is divisible by 5

$\therefore \; (y, x) \in R \Rightarrow R$ is symmetric.

For transitive relation

Let $(x, y) \in R$ and $(y, z) \in R$

$x - y$ is divisible by 5 $\quad \Rightarrow x - y = 5m$ ...(i)

$y - z$ is divisible by 5 $\quad \Rightarrow y - z = 5n$ ...(ii)

On adding eqs. (i) & (ii)

$x - y + y - z = 5(m - n) \quad \Rightarrow x - z = 5(m - n)$

$\therefore x - z$ is divisible by 5.

$\Rightarrow (x, z) \in R \therefore R$ is transitive.

Thus, R is an equivalence relation.

Equivalence class of $[5] = \{-10, -5, 0, 5, 10\}$

7. For onto, let $f(x) = y$

$$\frac{x-2}{x-3} = y$$

$$\Rightarrow \quad x - 2 = xy - 3y$$

$$\Rightarrow \quad x(1 - y) = 2 - 3y$$

$$\Rightarrow \quad x = \frac{2-3y}{1-y}$$

For $y = 1$, $x \in A$ $\therefore$ Range $= R - \{1\}$ $\therefore a = 1$

For one-one

Let $f(x_1) = f(x_2)$ where $x_1, x_2 \in A$

$$\Rightarrow \quad \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$$

$$\Rightarrow \; x_1x_2 - 3x_1 - 2x_2 + 6 = x_2x_1 - 2x_1 - 3x_2 + 6$$

$$\Rightarrow \quad x_1 = x_2$$

$\therefore f$ is one-one.

8. **Reflexive:** For $a \in S$

$\Rightarrow \; a - a + \sqrt{2}$ is irrational nmber

$\Rightarrow \; \sqrt{2}$ is irrational number

$\Rightarrow \; (a, a) \in S$

Thus, S is Reflexive Relation.

Symmetric: Let $(a,b) \in S \Rightarrow a - b + \sqrt{2}$ is an irrational number but $b - a + \sqrt{2}$ may not be irrational number.

For example, $(\sqrt{2}, 1) \in S \Rightarrow \sqrt{2} - 1 + \sqrt{2} = 2\sqrt{2} - 1$ is an irrational number but $(1, \sqrt{2}) \notin S$ as $1 - \sqrt{2} + \sqrt{2} = 1$ is an not irrational number.

$\therefore \; (b, a) \notin S$. So S is not symmetric relation.

Transitive: Let $(a, b) \in S \Rightarrow a - b + \sqrt{2}$ is an irrational number and $(b, c) \in S \Rightarrow b - c + \sqrt{2}$ is an irrational number, then $(a, c) \in S \Rightarrow a - c + \sqrt{2}$ is not an irrational number.

For example, $(1, \sqrt{3}) \in S \Rightarrow 1 - \sqrt{3} + \sqrt{2}$ an is irrational number, $(\sqrt{3}, \sqrt{2}) \in S \Rightarrow \sqrt{3} - \sqrt{2} + \sqrt{2} = \sqrt{3}$ an is irrational number.

But $(1, \sqrt{2}) \notin S$ as $1 - \sqrt{2} + \sqrt{2} = 1$ is not irrational number.

$\therefore \; (a, c) \notin S$, So, S is not transitive relation.

Thus, S is reflexive but neither symmetric nor transitive relation.

9. Reflexive: Let $\frac{1}{2} \le \left(\frac{1}{2}\right)^3$,

$\therefore \; S$ is not reflexive.

Symmetric: $1 \le 2^3 \Rightarrow (1, 2) \in S$ but $2 \not\le 1^3 \Rightarrow (2, 1) \notin S$.

$\therefore \; S$ is not symmetric.

Transitive: $10 \le 6^3 \Rightarrow (10, 6) \in S$ & $6 \le 2^3 \Rightarrow (6, 2) \in S$ but $10 \not\le 2^3 \Rightarrow (10, 2) \notin S$

$\therefore \; S$ is not transitive.

Level - 2 ADVANCED COMPETENCY FOCUSED QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Mark)

1. Option (D) is correct.

Explanation:

Given set: $X = \{x, y, z\}$

Given relation:

$$R = \{(x, x), (x, y), (y, x), (y, z), (x, z)\}$$

Reflexive:

A relation R is reflexive if $(a, a) \in R$ for all $a \in X$.

Here, we have $(x, x) \in R$, but $(y, y) \notin R$ and $(z, z) \notin R$, so R is not reflexive.

Symmetric:

A relation R is symmetric if $(a, b) \in R \Rightarrow (b, a) \in R$.

- $(x, y) \in R$, and $(y, x) \in R$
- $(y, z) \in R$, but $(z, y) \notin R$

Since $(y, z) \in R$ but $(z, y) \notin R$, R is not symmetric.

Transitive:

A relation R is transitive if $(a, b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$.

- $(x, y) \in R$ and $(y, z) \in R, \Rightarrow (x, z) \in R$
- (y, x) and $(x, y) \in R$, but $(y, y) \notin R$,

Thus, R is not transitive.

2. Option (A) is correct.

Explanation: For a function $f : A \to B$ to be onto, every element in B must have at least one pre-image in A. However, since $m < n$, there are fewer elements in A than in B. This means that at least one element in B will not have a pre-image, making it impossible for any function from A to B to be onto. Thus, the number of onto functions in this case is 0.

3. Option (A) is correct.

Explanation: A function from set X to set Y is a relation in which each element of X has exactly one image in Y (i.e., no element of X should map to more than one element of Y).

Aabha's defined:

$$f = \{(1, 2), (1, 4), (1, 6), (1, 8)\}$$

Here, 1 is mapped to multiple elements (2, 4, 6, 8), which is not a function.

Bhakti's defined:

$$f = \{(1, 2), (3, 4), (5, 6), (7, 8)\}$$

Each element of $X = \{1, 3, 5, 7, 9\}$ should be mapped uniquely. However, 9 is missing.

A function must be defined for all elements of X.

So, not a function.

Chirag's defined:

$$f = \{(1, 4), (3, 4), (5, 4), (7, 4), (9, 4)\}$$

Each element of X has exactly one corresponding element in Y.

This satisfies the function rule.

Chirag's definition is correct.

4. Option (A) is correct.

Explanation: The correct answer is (A) R_1 is an equivalence relation because it satisfies:

Reflexive: (1, 1), (4, 4), (9, 9) are present

Symmetry: Only self - pairs exist, so it's symmetric

Transitivity: No cross - relations to check, so it's trivially transitive

Other options fail reflexivity, symmetry, or transitivity, so they are not equivalence relations.

5. Option (D) is correct.

Explanation: Given $P(A)=\{\phi, \{a\}, \{b\}, \{a, b\}\}$

Find Ordered Pairs (r, s) Where $r \subseteq s$

We list all subset relations:

- $\phi \subseteq \{a\}$
- $\phi \subseteq \{b\}$
- $\phi \subseteq \{a, b\}$
- $\{a\} \subseteq \{a, b\}$
- $\{b\} \subseteq \{a, b\}$
- $\phi \subseteq \phi$ (trivially holds but not included in the relation form)
- $\{a\} \subseteq \{a\}$ (trivial self – pair, usually not listed)
- $\{b\} \subseteq \{b\}$ (trivial self – pair, usually not listed)
- $\{a, b\} \subseteq \{a, b\}$

So,

$R = \{(\phi,\{a\}), (\phi,\{b\}), (\phi,\{a, b\}), (\{a\}, \{a, b\}), (\{b\}, \{a, b\}), (\{a, b\},\{a, b\})\}$

6. Option (B) is correct.

Explanation: The function $f : \mathbb{Z}^* \to \mathbb{Z}^*$ defined as:

$$f(n) = \begin{cases} \dfrac{n+1}{2}, & \text{if } n \text{ is odd} \\ 0, & \text{if } n \text{ is even} \end{cases}$$

where $\mathbb{Z}^*$ is the set of non – negative integers:

$$\mathbb{Z}^* = \{0, 1, 2, 3, 4, \ldots\}$$

One – One (Injective):

A function is one – one (injective) if $f(a) = f(b)$ implies $a = b$.

If n is even, $f(n) = 0$. This means all even numbers map to 0 , causing multiple inputs to have the same output.

If n is odd, $f(n) = \dfrac{n+1}{2}$, which is unique for every odd n.

Since multiple even numbers map to the same output (0), f is not injective.

Onto (Surjective):

A function is onto (surjective) if every element in the codomain $\mathbb{Z}^*$ has a pre-image in $\mathbb{Z}^*$.

- The output values of $f(n)$ are $0, 1, 2, 3, \ldots$, which cover all non – negative integers.
- Any $k \in \mathbb{Z}^*$ can be achieved by choosing $n = 2k - 1$ for odd n, or choosing any even n for 0.

Thus, f is onto.

7. Option (D) is correct.

Explanation: **Reflexive:**

A relation is reflexive if $(x, x) \in R$ for all $x \in$ G.

Every student has the same teacher as themselves, so $(x, x) \in R$.

R is reflexive.

Symmetry:

A relation is symmetric if $(x, y) \in R \Rightarrow (y, x) \in R$.

If student x and student y have the same teacher, then y and x also have the same teacher.

R is symmetric.

Transitivity:
A relation is transitive if $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$.

If x and y have the same teacher, and y and z also have the same teacher, then x and z must have the same teacher.
R is transitive.

ASSERTION-REASON QUESTIONS (1 Marks)

1. **Option (A) is correct.**
Explanation: Assertion is true. A function $f : A \rightarrow B$ is one-one (injective) if no two different elements in A have the same image in B. That is, for all $x_1, x_2 \in A$, if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.
Reason is also true as this condition is the mathematical definition of a one-one function.
Both assertion and reason are true and reason is the correct explanation of assertion.

2. **Option (A) is correct.**
Explanation: Let the relation $R = \{(a, b) \in Z \times Z \mid a - b$ is even$\}$
Checking for the three properties of an equivalence relation:
Reflexive: For any $a \in Z$, $a - a = 0$, which is even. So, $(a, a) \in R$
Symmetric: If $(a, b) \in R$, then $a - b$ is even
$\Rightarrow b - a$ is also even
$\Rightarrow (b, a) \in R$
Transitive: If $(a, b) \in R$ and $(b, c) \in R$, then both $a - b$ and $b - c$ are even
$\Rightarrow (a-b) + (b-c) = a - c$ is also even $\Rightarrow (a, c) \in R$
Since R satisfies reflexivity, symmetry, and transitivity, it is indeed an equivalence relation.
Both assertion and reason are true, and reason is the correct explanation of assertion.

3. **Option (A) is correct.**
Explanation: A function is invertible if there exists another function f^{-1} such that:
$f^{-1}(f(x)) = x$ and $f(f^{-1}(y)) = y$ for all $x \in A$, $y \in B$.
This is only possible when the function is both:
One-one (injective): Every element of the domain maps to a distinct element of the codomain.
Onto (surjective): Every element of the codomain is the image of some element of the domain.
When both conditions are satisfied, the function is bijective, and every bijective function is invertible.
Both the assertion and the reason are correct, and the reason correctly explains the assertion.

4. **Option (D) is correct.**
Explanation: Assertion is false. The function $f(x) = x^2$, with domain $x \in \mathbb{R}$, is not one-one because:
$f(-2) = (-2)^2 = 4 = (2)^2 = f(2)$
So, two different inputs give the same output $\Rightarrow$ not injective $\Rightarrow$ not one-one.
Reason is true because it correctly shows that $f(x_1) = f(x_2)$ even when $x_1 \neq x_2$.

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. Shreyas is not correct.
Shreyas' relation is not reflexive as (–2, –2) is not a part of it.
Since the relation is not reflexive, it cannot be an equivalence relation.

2. (i) Here, $f\left(\frac{8}{5}\right) = \frac{\frac{24}{5}}{0} \notin R$. Thus, f is not a function as it is not well defined.
(ii) f can be made a function by removing the element $\frac{8}{5}$ from the domain $\mathbb{R}$.

3. False (F).
Because the sine function is onto but not one-one (for example such as $\sin\left(\frac{\pi}{2}\right) = \sin\left(\frac{5\pi}{2}\right) = 1$, therefore it is not bijective in nature.

4. A function from A to B that is neither one-one nor onto is given as:
$f : A \rightarrow B$ defined by $f(x) = 4$ for all $x \in A$.

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. **Reflexive:** A relation R is reflexive if every element of the set A is related to itself, i.e.,
for all $a \in A$, the pair $(a, a) \in R$.
Set A = {1, 2, 3}:
$(1, 1) \in R$
$(2, 2) \in R$
$(3, 3) \in R$
All elements are related to themselves $\Rightarrow R$ is reflexive.
Symmetric: A relation R is symmetric if for every $(a, b) \in R$, the pair $(b, a) \in R$ also exists.
Non-reflexive pairs in R:
$(1, 2) \in R \Rightarrow (2, 1) \in R$
$(2, 1) \in R \Rightarrow (1, 2) \in R$
All symmetric pairs are present. R is symmetric.
Yes, the relation R is both reflexive and symmetric.
Justification:
All elements $1, 2, 3 \in A$ satisfy $(a, a) \in R$, so R is reflexive.
For each pair $(a, b) \in R$, the reverse pair $(b, a) \in R$ also exists, so R is symmetric.

2. **One-one (Injective):**

Assume: $f(x_1) = f(x_2)$

$\Rightarrow 3x_1 + 5 = 3x_2 + 5 \Rightarrow 3x_1 = 3x_2 \Rightarrow x_1 = x_2$

So, f is one-one.

Onto (Surjective):

For every $y \in \mathbb{R}$, there exists $x \in \mathbb{R}$ such that:

$f(x) = y \Rightarrow 3x + 5 = y \Rightarrow x = \frac{y-5}{3}$

Since $x \in \mathbb{R}$ for every $y \in \mathbb{R}$, f is onto.

3. **Given:** $f: \mathbb{R} \to \mathbb{R}$ $f(x) = x^2$

A function is one-one if:

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

But in this case:

$f(2) = 4$ and $f(-2) = 4 \Rightarrow f(2) = f(-2)$, but $2 \neq -2$

So, the function is not one-one.

A function is onto if every real number $y \in \mathbb{R}$ has a pre-image in $\mathbb{R}$.

But:

$f(x) = x^2 \geq 0$ for all $x \in \mathbb{R}$

So, no real number exists such that $f(x) = -1$, or any negative number.

Hence not all $y \in \mathbb{R}$ have pre-image $\Rightarrow f$ is not onto.

If we restrict the domain to only non-negative real numbers:

$f : [0, \infty) \to \mathbb{R}, f(x) = x^2$

Then, for $x_1, x_2 \geq 0$,

$f(x_1) = f(x_2) \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = x_2$

If the domain is restricted to $[0, \infty)$, then f becomes one-one.

4. Given two functions:

$f(x) = x^2 + 1$

$g(x) = \sqrt{x-1}$

To find the domain of the composite function:

$(g \circ f)(x) = g(f(x)) = \sqrt{f(x)-1}$

$(g \circ f)(x) = \sqrt{f(x)-1} = \sqrt{(x^2+1)-1} = \sqrt{x^2}$

$\Rightarrow (g \circ f)(x) = |x|$

Since $|x|$ is defined for all real values of x, the domain of $(g \circ f)(x)$ is $\mathbb{R}$ (All real numbers)

5. A function $f : A \to B$ is invertible if and only if it is bijective, i.e.,

One-one (Injective): No two elements in A map to the same element in B.

Onto (Surjective): Every element in B has a pre-image in A.

Example:

Let $f : \mathbb{R} \to \mathbb{R}$, defined by

$f(x) = 2x + 3$

This function is:

One-one: $f(x_1)$, then $2x_1 + 3 = 2x_2 + 3 \Rightarrow x_1 = x_2$

Onto: For every $y \in \mathbb{R}$ take $x = \frac{y-3}{2} \in \mathbb{R}$ such that $f(x) = y$

CASE BASED QUESTIONS (4 Marks)

1. **(i)** Number of relations is equal to the number of subsets of the set $B \times G = 2^{n(B \times G)} = 2^{n(B) \times n(G)}$ $= 2^{3 \times 2} = 2^6 = 64$

(Where $n(B)$ and $n(G)$ denote the number of the elements in the finite sets B and G, respectively)

(ii) Smallest equivalence relation on G is $\{(g_1, g_1), (g_2, g_2)\}$

(iii) (a) (A) reflexive but not symmetric

$\{(b_1, b_2), (b_2, b_1), (b_1, b_1), (b_2, b_2) (b_3, b_3), (b_2, b_3)\}$.

So the minimum number of elements to be added are

$(b_1, b_1), (b_2, b_2), (b_3, b_3), (b_2, b_3)$

(**Note:** It can be a any one of the pair from, (b_3, b_2), (b_1, b_3), (b_3, b_1) in place of (b_2, b_3) also}

(B) reflexive and symmetric but not transitive:

$\{(b_1, b_2), (b_2, b_1), (b_1, b_1), (b_2, b_2), (b_3, b_3), (b_2, b_3), (b_3, b_2)\}$

So the minimum number of elements to be added are

$(b_1, b_1), (b_2, b_2), (b_3, b_3), (b_2, b_3), (b_3, b_2)$

OR

(b) One-one and onto function

$x^2 = 4y$

Let $y = f(x) = \frac{x^2}{4}$

Let $x_1, x_2 \in [0, 20\sqrt{2}]$ such that

$f(x_1) = f(x_2) \Rightarrow \frac{x_1^2}{4} = \frac{x_2^2}{4}$

$\Rightarrow {x_1}^2 = {x_2}^2 \Rightarrow (x_1 - x_2)(x_1 + x_2) = 0 \Rightarrow x_1 = x_2$ as $x_1, x_2 \in [0, 20\sqrt{2}]$

$\therefore f$ is one-one function.

Now, $0 \leq y \leq 200$ hence the value of y is non-negative and $f(2\sqrt{y}) = y$

$\therefore$ For any arbitrary $y \in [0, 200]$, the pre-image of y exists in $[0, 20\sqrt{2}]$ hence f is onto function.

2. **(i)** Let $(l_1, l_2) \in R \Rightarrow l_1 \parallel l_2 \Rightarrow l_2 \parallel l_1 \Rightarrow (l_2, l_1) \in R$,

$\therefore R$ is a symmetric relation.

(ii) Let $(l_1, l_2), (l_2, l_3) \in R \Rightarrow l_1 \parallel l_2, l_2 \parallel l_3 \Rightarrow l_1 \parallel l_3$ $\Rightarrow (l_1, l_3) \in R$,

$\therefore R$ is a transitive relation.

(iii) (a) The given rail line equation $y = 3x + 2$ the slope of this line is 3. Any rail line parallel to this line must have same slope.

The set is $\{l : l$ is a line of type $y = 3x + c, c \in R\}$

OR

(b) Let $(l_1, l_2) \in R \Rightarrow l_1 \perp l_2 \Rightarrow l_2 \perp l_1 \Rightarrow (l_2, l_1) \in R$

$\therefore$ R is a symmetric relation.

Let $(l_1, l_2), (l_2, l_3) \in R \Rightarrow l_1 \perp l_2, l_2 \perp l_3 \Rightarrow l_1 \parallel l_3$
$\Rightarrow (l_1, l_3) \in R$
$\therefore$ R is not a transitive relation.

3. (i) Number of relations $= 2^{n(B) \times n(G)} = 2^{3 \times 2} = 2^6 = 64$.

(ii) Number of possible functions $= 2^3 = 8$.

(iii) (a) Given, $R = \{(x, y) : x$ and y are students of same sex$\}$

Reflexive:

Since x and x of the same sex.

So, $(x, x) \in R \; \forall \; x \in R$

$\therefore$ R is reflexive.

Symmetric:

If x and y are of same sex, then y and x are of same sex.

Thus, $(x, y) \in R \Rightarrow (y, x) \in R \; \forall \; (x, y \in R)$

$\therefore$ R is symmetric.

Transitive:

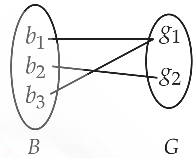
If x and y are of same sex and y and z are of same sex, the x and z one also of same sex.

Thus, if $(x, y) \in R$ and $(y, x) \in R \Rightarrow (x, z) \in R \; \forall \; (x, y, z \in R)$

Since, R is reflexive, symmetric and transitive, hence R is on equivalence relation.

OR

Given, $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$
and $R = \{(b_1, g_1)(b_2, g_2), (b_3, g_1)\}$



From the above diagram, it can be easily seen that R is onto but not one-one as b_1 and b_3 have same image g_1.
Hence, R is not bijective (one-one and onto).

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. (i) **Domain:** Set of all vehicle serial numbers = A = {1, 2, 3,..., 500}

Co-domain = Set of possible registration codes

= B = {1001, 1002,..., 1500}

(ii) Using $f(x) = x + 1000$:

$f(1) = 1 + 1000 = 1001$

$f(250) = 250 + 1000 = 1250$

$f(500) = 500 + 1000 = 1500$

(iii) Yes, the function is one-one (injective).

Reason: Assume $f(x_1) = f(x_2)$

$\Rightarrow x_1 + 1000 = x_2 + 1000 \Rightarrow x_1 = x_2$

Since equal outputs imply equal inputs, the function is one-one.

(iv) Yes, the function is onto (surjective) in this case.

Reason: The function maps each $x \in A$ to exactly one value in B.

The smallest value of $f(x) = 1 + 1000 = 1001$

The largest value of $f(x) = 500 + 1000 = 1500$

So, range of f = {1001, 1002,..., 1500} = Co-domain

Since the range = co-domain, the function is onto.

(v) This relation is a function, not a general relation.

A function maps each input to exactly one output.

This mapping $f(x) = x + 1000$ satisfies that it is not reflexive, symmetric, or transitive, as those properties are defined for relations on a set where pairs are like (a, a), (a, b), (b, a), etc.

2. (i) Domain = Set of all valid project counts

= {0, 1, 2,..., 10}

Range: Compute values from $f(0)$ to $f(10)$:

$f(0) = 25, f(10) = 5(10) + 25 = 75$

Since it's a linear function increasing by 5, the values will be:

{25, 30, 35,..., 75}

(ii) $f(0) = 5(0) + 25 = 25$

$f(5) = 5(5) + 25 = 50$

$f(10) = 5(10) + 25 = 75$

(iii) Yes, the function is linear.

Reason: The equation $f(x) = 5x + 25$ is in the standard form of a linear function:

$f(x) = mx + c$

where $m = 5, c = 25$

(iv) Yes, the function is one-one (injective).

Reason: If $f(x_1) = f(x_2)$, then:

$5x_1 + 25 = 5x_2 + 25 \Rightarrow x_1 = x_2$

No two different project counts give the same marks
$\Rightarrow$ Function is one-one.

(v) Yes, every function is also a relation.

Explanation: A relation is a set of ordered pairs.

A function is a special kind of relation where each input has exactly one output.

Since $f(x) = 5x + 25$ assigns one unique value for each x, it satisfies the definition of a function — and thus also qualifies as a relation.

3. (i) No, R is not a function.

Reason: In a function, each input (from domain) must map to exactly one output. In this relation, the input "Ravi" is mapped to two different outputs: 202 and 204. Hence, the same input has more than one image, which violates the definition of a function.

(ii) To make it a function, we must ensure that each input has only one output.

Choose only one ordered pair for "Ravi".

One possible corrected relation:

R = {(Anita, 201), (Ravi, 202), (Meena, 203)}

(iii) Yes, a function can map different inputs to the same output.

Explanation: The rule for a function is that one input → one output.

But two (or more) inputs can have the same output.

Example: $f(x) = x^2$

$f(2) = 4$ and $f(-2) = 4$ are different inputs, same output → still a function.

(iv) From the given relation:

R = {(Anita, 201), (Ravi, 202), (Meena, 203), (Ravi, 204)}

Domain = set of all first elements (names)

= {Anita, Ravi, Meena}

Range = set of all second elements (codes)

= {201, 202, 203, 204}

(v) The property violated is uniqueness of image (well-definedness) — every input should have only one output.

"Ravi" has two outputs (202 and 204), so the relation is not well-defined as a function.