CHAPTER 11

Three Dimensional Geometry

Level - 1 CORE SUBJECTIVE QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQ) (1 Marks)

1. **Option (C) is correct.**

Explanation: The distance of a point $P(a, b, c)$ from the y-axis is given by:

$$\text{Distance} = \sqrt{(a-0)^2 + (c-0)^2}$$
$$= \sqrt{a^2 + c^2}$$

2. **Option (B) is correct.**

Explanation: The given lines are:

$$\frac{x+1}{2} = \frac{2-y}{-5} = \frac{z}{4} \text{ or } \frac{x+1}{2} = \frac{y-2}{5} = \frac{z-0}{4}$$
$$\frac{x-3}{1} = \frac{y-7}{2} = \frac{5-z}{3} \text{ or } \frac{x-3}{1} = \frac{y-7}{2} = \frac{z-5}{3}$$

The direction ratios of these lines are:

From the first equation: (2, 5, 4)

From the second equation: (1, 2, –3)

The angle θ between the two lines is given by the formula:

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\cdot\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\cos\theta = \frac{(2)(1) + (5)(2) + (4)(-3)}{\sqrt{2^2 + (5)^2 + 4^2}\cdot\sqrt{1^2 + 2^2 + (-3)^2}}$$

$$= \frac{2 + 10 - 12}{\sqrt{4 + 25 + 16}\cdot\sqrt{1 + 4 + 9}}$$

$$= \frac{0}{\sqrt{45}\cdot\sqrt{14}}$$

$$= 0$$

$$\theta = \frac{\pi}{2}$$

3. **Option (C) is correct.**

Explanation: Given Cartesian equations:

$$6x - 2 = 3y + 1 = 2z - 2$$

$$6\left(x - \frac{1}{3}\right) = 3\left(y + \frac{1}{3}\right) = 2(z-1)$$

$$\Rightarrow \frac{x - \frac{1}{3}}{1} = \frac{y + \frac{1}{3}}{2} = \frac{z-1}{3} \quad \text{[On dividing by 6]}$$

Thus, Drs are 1, 2, 3.

4. **Option (D) is correct.**

Explanation: Vector equation for x-axis is $\vec{r} = \lambda\hat{i}$ where $x = \lambda$. Let a point P(λ, 0, 0) is on the x-axis perpendicular to the given point A(0, 1, 2) then $\overrightarrow{AP}$ is perpendicular to $\lambda\hat{i}$.

$\overrightarrow{AP}\cdot\lambda\hat{i} = 0$ where $\overrightarrow{AP} = \lambda\hat{i} - \hat{j} - 2\hat{k}$

$$(\lambda\hat{i} - \hat{j} - 2\hat{k})\cdot(\lambda\hat{i} + 0\hat{j} + 0\hat{k}) = 0$$
$$\lambda^2 = 0$$

λ = 0, so the foot of the perpendicular drawn from the point (0, 1, 2) is (0, 0, 0).

5. **Option (C) is correct.**

Explanation: The given symmetric form of the line is:

$$\frac{x-1}{2} = y = \frac{2z+1}{6}$$

or
$$\frac{x-1}{2} = \frac{y-0}{1} = \frac{z - \left(-\frac{1}{2}\right)}{3}$$

We compare this with the general symmetric form:

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Thus, the direction ratios are (2, 1, 3).

6. **Option (D) is correct.**

Explanation: The line makes angles α, β, γ with the positive x, y, z axes.

The known fundamental identity is:

$$\cos^2\theta + \cos^2\theta + \cos^2\theta = 1$$

(A) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ (True, fundamental identity)

(B) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

Since $\sin^2\theta = 1 - \cos^2\theta$, we get:

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$

$= 3 - 1 = 2$ (True)

(C) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

$= 2\cos^2\alpha - 1 + 2\cos^2\beta - 1 + 2\cos^2\gamma - 1$

$= 2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) - 3$

$= 2 - 3$

$= -1$ (True)

(D) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = 1$ (Not True)

7. **Option (B) is correct.**

Explanation: We know that vector equation of a line is

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

Given $\vec{a} = (1,-1,0)$

Here, $\vec{b} = (0,1,0)$ [$\because$ parallel to y-axis]

$$\vec{r} = \hat{i} - \hat{j} + \lambda\hat{j}$$

8. **Option (C) is correct.**

Explanation:

L$_1$: $$\frac{1-x}{2} = \frac{y-1}{3} = \frac{z}{1}$$

or $$\frac{x-1}{-2} = \frac{y-1}{3} = \frac{z}{1}$$

L$_2$: $$\frac{2x-3}{2p} = \frac{y}{-1} = \frac{z-4}{7}$$

or $$\frac{x-\frac{3}{2}}{p} = \frac{y-0}{-1} = \frac{z-4}{7}$$

On Comparing with

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

Direction ratio of line (i) are

$$a_1 = -2, b_1 = 3, c_1 = 1$$

Direction ratio of line (ii) are

$$a_2 = p, b_2 = -1, c_2 = 7$$

when $L_1 \perp L_2$ then $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\therefore -2 \times p + 3 \times (-1) + 1 \times 7 = 0$

$\therefore -2p - 3 + 7 = 0 \Rightarrow p = 2$

9. **Option (B) is correct.**

Explanation: We are given:

A point on the line with position vector $a = \hat{i} - \hat{j}$, which corresponds to the Cartesian coordinates (1, –1, 0).

A line in vector form:

$$r = \hat{i} + \hat{k} + \mu(2\hat{i} - \hat{j})$$

Here, the direction vector of the given line is $\vec{d} = 2\hat{i} - \hat{j} + 0\hat{k}$, or in coordinate form: (2, –1, 0).

Equation of the Required Line

A line passing through point (1, –1, 0) and parallel to direction vector (2, –1, 0) has the Cartesian equation:

$$\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z}{0}$$

10. **Option (B) is correct.**

Explanation: We are given the equation of a line in symmetrical form:

$$\frac{x}{1} = \frac{y}{-1} = \frac{z}{0}$$

From this equation, the direction ratios (d.r.s) of the line are:

$$(1, -1, 0)$$

The formula for the angle θ between a line with direction ratios (a, b, c) and the positive Y-axis (0, 1, 0) is:

$$\cos\theta = \frac{(1\cdot 0 + (-1)\cdot 1 + 0\cdot 0)}{\sqrt{1+1+0} \times \sqrt{1}}$$

$$= \frac{0-1+0}{\sqrt{2}\times 1} = \frac{-1}{\sqrt{2}}$$

$$= -\frac{1}{\sqrt{2}}$$

$$\cos(135^\circ) = \cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$

Thus,

$$\theta = \frac{3\pi}{4}$$

11. **Option (D) is correct.**

Explanation: Given line is $\vec{r} = (2+\lambda)\hat{i} + \lambda\hat{j} + (2\lambda - 1)\hat{k}$

$$= 2\hat{i} - \hat{k} + \lambda(\hat{i} + \hat{j} + 2\hat{k})$$

which is of the form $\vec{r} = a + \lambda\vec{b}$

$\therefore$ Required line is $\dfrac{x-1}{1} = \dfrac{y+3}{1} = \dfrac{z-2}{2}$

[As lines passes through (1, –3, 2) and having Direction cosines (1, 1, 2)]

12. **Option (C) is correct.**

Explanation: Given points:

A(0, 0, 2) and B(3, –2, 5)

Position vector of $\vec{a}$:

$$\vec{a} = 0\hat{i} + 0\hat{j} + 2\hat{k}$$

Direction vector $\vec{b}$:

$$\vec{b} = (3-0, -2-0, 5-2) = (3, -2, 3)$$

or in vector form:

$$\vec{r} = 2\hat{k} + \lambda(3\hat{i} - 2\hat{j} + 3\hat{k})$$

13. **Option (B) is correct.**

Explanation: The given lines can be written as:

$$\frac{x-5}{7} = \frac{y-2}{-5} = \frac{z-0}{2} \text{ and } \frac{x-0}{1} = \frac{y-\frac{1}{2}}{\frac{\lambda}{2}} = \frac{z-0}{3}$$

Dr's of line 1 = (7, –5, 1)

Dr's of line 2 = $\left(1, \frac{\lambda}{2}, 3\right)$

Two lines are perpendicular if the dot product of their direction ratios is zero:

$$(7 \times 1) + \left(-5 \times \frac{\lambda}{2}\right) + (1 \times 3) = 0$$

$$7 - \frac{5\lambda}{2} + 3 = 0$$

$$10 - \frac{5\lambda}{2} = 0$$

$$\frac{5\lambda}{2} = 10$$

$$\lambda = 4$$

14. Option (D) is correct.

Explanation:

$$\frac{x-1}{2} = \frac{y-1}{-3} = \frac{z-\frac{1}{2}}{6}$$

Comparing with $\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$

we get direction ratios (d.r.s) = (2, –3, 6)

Direction Cosines (d.c.s)

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}},\quad m = \frac{b}{\sqrt{a^2+b^2+c^2}},\quad n = \frac{c}{\sqrt{a^2+b^2+c^2}}$$

$$\sqrt{a^2+b^2+c^2} = \sqrt{2^2+(-3)^2+6^2}$$

$$= \sqrt{4+9+36}$$

$$= \sqrt{49} = 7$$

Now, the direction cosines:

$$l = \frac{2}{7},\quad m = \frac{-3}{7},\quad n = \frac{6}{7}$$

15. Option (D) is correct.

Explanation: A line parallel to the z-axis, x and y are zero, so the direction ratios must be:

(0, 0, 1)

16. Option (D) is correct.

Explanation: A line parallel to the z-axis has direction ratios (0, 0, 1), passing though (1, 1, 1)

$$\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{1}$$

17. Option (B) is correct.

Explanation: The direction cosines (l, m, n) are given by:

$$l = \cos 30^\circ = \frac{\sqrt{3}}{2},\quad m = \cos 60^\circ = \frac{1}{2},\quad n = \cos 90^\circ = 0$$

Direction ratios (d.r.s):

$$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}, 0\right)$$

Equation of line passing through origin

$$\frac{x-0}{\frac{\sqrt{3}}{2}} = \frac{y-0}{\frac{1}{2}} = \frac{z-0}{0}$$

$$\Rightarrow \quad \frac{2x}{\sqrt{3}} = \frac{2y}{1} = \frac{z}{0}$$

ASSERTION-REASON QUESTIONS (1 Marks)

1. Option (A) is correct.

Explanation: If a line is perpendicular to all three coordinate axes, its direction cosines should be zero for each axis.

However, the sum of squares of the direction cosines must always be 1:

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

This equation cannot hold if all cosines are zero, which means such a line does not exist.

(A) True, (R) True, and (R) explains (A).

2. Option (D) is correct.

Explanation: The direction vector $\vec{b}$ of the line through A(–1, 0, 2) and B(3, 4, 6) is:

$$\vec{b} = (3-(-1))\hat{i} + (4-0)\hat{j} + (6-2)\hat{k} = 4\hat{i} + 4\hat{j} + 4\hat{k}$$

The correct equation should be:

$$\vec{r} = (-\hat{i} + 2\hat{k}) + \lambda(4\hat{i} + 4\hat{j} + 4\hat{k})$$

The given equation is incorrect as the direction vector is wrongly written as $\hat{i} + \hat{j} + \hat{k}$.

(A) False, (R) True.

3. Option (D) is correct.

Explanation: The correct direction ratios are:

(3 – 1, –1 –2, 3 – 3) = (2, –3, 0)

The correct equation of the line is:

$$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z-3}{0}$$

The given equation incorrectly swaps (x_1, y_1, z_1) with (x_2, y_2, z_2).

(A) False, (R) True.

4. Option (A) is correct.

Explanation: The given assertion states:

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$$

Using the identity:

$$\sin^2\theta = 1 - \cos^2\theta$$

$$(1-\cos^2\alpha) + (1-\cos^2\beta) + (1-\cos^2\gamma) = 2$$

$$3 - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = 2$$

Since, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$3 - 1 = 2$$

$$2 = 2$$

(A) True, (R) True, and (R) explains (A).

5. Option (C) is correct.

Explanation: Direction vector of first line: (2 – 4, 3 – 7, 4 – 8) = (–2, –4, –4).

Direction vector of second line: (1 – (–1), 2 – (–2), 5 – 1) =(2, 4, 4).

Since both vectors are scalar multiples of each other, the lines are parallel.

However, the reason is incorrect because parallel vectors should be proportional, not necessarily perpendicular (which happens if $\vec{b}_1.\vec{b}_2 = 0$).

(A) True, (R) False.

6. **Option (A) is correct.**

Explanation: Assertion (A): The lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$ are perpendicular when

$$\vec{b}_1.\vec{b}_2 = 0$$

Two lines are perpendicular if their direction vectors satisfy $\vec{b}_1.\vec{b}_2 = 0$.

Reason (R): The angle θ between two lines is given by:

$$\cos\theta = \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}$$

The formula for cos θ correctly explains the angle between two lines.

7. **Option (A) is correct.**

Explanation: Assertion (A): The vector equation of a line passing through (1, 2, 3) and (3, 0, 2) is:

$$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(2\hat{i} - 2\hat{j} - \hat{k})$$

Where direction vector is calculated as:

$$(3 - 1, 0 - 2, 2 - 3) = (2, -2, -1)$$

Reason (R): Equation of a line passing through two points is

$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$$

Given equation is correct.

(A) True, (R) True, and (R) explains (A).

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1.
$$\overrightarrow{OA} = 4\hat{i} + 3\hat{k}$$
$$\overrightarrow{OB} = \hat{k}$$
$$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB} = 4\hat{i} + 2\hat{k}$$
$$|\overrightarrow{BA}| = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$
$$\hat{BA} = \frac{4}{2\sqrt{5}}\hat{i} + \frac{2}{2\sqrt{5}}\hat{k}$$

or
$$\hat{BA} = \frac{2}{\sqrt{5}}\hat{i} + \frac{1}{\sqrt{5}}\hat{k}$$

So, the angles made by vector $\overrightarrow{BA}$ with *x*, *y* and *z* axes are respectively

$$\cos^{-1}\left(\frac{2}{\sqrt{5}}\right), \frac{\pi}{2}, \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

2. cos α = cos β = cos γ = *l*

$\Rightarrow \quad l^2 + l^2 + l^2 = 1$

$\Rightarrow \quad 3l^2 = 1$

$\therefore \quad l = \frac{1}{\sqrt{3}}$

Direction cosines of the line are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

⇒ The direction ratios are 1, 1, 1

∴ Vector equation of the line is:

$$r = 2\hat{i} + 3\hat{j} - 5\hat{k} + \lambda(\hat{i} + \hat{j} + \hat{k})$$

3. The direction ratios of the two lines are: 7, – 5, 1 and 1, 2, 3 respectively.

∴ The angle between the two lines is given by:

$$\theta = \cos^{-1}\left(\frac{7(1) - 5(2) + 1(3)}{\sqrt{49+25+1}\sqrt{1+4+9}}\right)$$

$\Rightarrow \quad \theta = \cos^{-1} 0 = \frac{\pi}{2}$

4.
$$\cos\theta = \frac{(\hat{i} - 2\hat{j} - 2\hat{k}).(3\hat{i} + 2\hat{j} - 6\hat{k})}{|\hat{i} - 2\hat{j} - 2\hat{k}|.|3\hat{i} + 2\hat{j} - 6\hat{k}|}$$
$$= \frac{(1)(3) + (-2)(2) + (-2)(-6)}{\sqrt{(1)^2 + (-2)^2 + (-2)^2}\sqrt{(3)^2 + (2)^2 + (-6)^2}}$$
$$= \frac{11}{21}$$
$$\Rightarrow \theta = \cos^{-1}\left(\frac{11}{21}\right)$$

5. D.r.s. of lines are < 2, 7, – 3 > and <–1, 2, 4>

Now 2 × – 1 + 7 × 2 + – 3 × 4 = 0

∴ Given lines are perpendicular.

6. General point on the curve is *P*(*k*, 2*k* + 1, 2*k* – 1), $k \in \mathbb{R}$

$$OP = \sqrt{11} \Rightarrow OP^2 = 11$$

$\therefore k^2 + (2k+1)^2 + (2k-1)^2 = 11 \Rightarrow k = \pm 1$

∴ Coordinates of points are (1, 3, 1) & (–1, –1, –3)

7. The equation of the line can be written as:

$$\frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c}$$

∴ The direction ratios are *a*, 1, *c*

A point on the line is (*b*, 0, *d*)

8. d.r.'s of lines are <–2, 3*p*, 4> and <4*p*, 2, –7>

As lines are perpendicular

$$-8p + 6p - 28 = 0$$

$\Rightarrow \quad p = -14$

9.
$$\cos\frac{\pi}{4} = \frac{|\alpha \times 1 + -5 \times 0 + \beta \times 1|}{\sqrt{\alpha^2 + \beta^2 + 25}\sqrt{2}}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{|\alpha + \beta|}{\sqrt{\alpha^2 + \beta^2 + 25}\sqrt{2}}$$

$$\Rightarrow \quad |\alpha + \beta| = \sqrt{\alpha^2 + \beta^2 + 25}$$

Squaring both sides, we get

$$\alpha^2 + \beta^2 + 2\alpha\beta = \alpha^2 + \beta^2 + 25$$

$$\Rightarrow \qquad \alpha\beta = \frac{25}{2}$$

10. Vector equation of the line passing through (2, 1, 3) is

$$\vec{r} = (2\hat{i} + \hat{j} + 3\hat{k}) + \lambda(a\hat{i} + b\hat{j} + c\hat{k})$$

Line $\vec{r}$ is perpendicular to the given lines then

$$a + 2b + 3c = 0;$$

$$-3a + 2b + 5c = 0$$

$$\Rightarrow \frac{a}{4} = \frac{b}{-14} = \frac{c}{8} = k \text{ (say)}$$

$\Rightarrow a = 2k, b = -7k$ and $c = 4k$.

Thus, the required vector equation is

$$\vec{r} = (2\hat{i} + \hat{j} + 3\hat{k}) + \lambda(2\hat{i} - 7\hat{j} + 4\hat{k})$$

11. The equation of the given line is

$$\frac{x - \frac{3}{5}}{\frac{1}{5}} = \frac{y + \frac{7}{15}}{\frac{1}{15}} = \frac{z - \frac{3}{10}}{-\frac{1}{10}}$$

Its direction ratios are.

$$\left(\frac{1}{5}, \frac{1}{15}, -\frac{1}{10}\right) \text{ or } (6, 2, -3)$$

Direction cosines are $\left(\pm\frac{6}{7}, \pm\frac{2}{7}, \mp\frac{3}{7}\right)$

Point through which line is passing $= \left(\frac{3}{5}, \frac{-7}{15}, \frac{3}{10}\right)$

12. Let θ be the angle between the given lines. Then

$$\cos\theta = \left|\frac{(3\hat{i} + 2\hat{j} + 6\hat{k}).(\hat{i} + 2\hat{j} + 2\hat{k})}{\sqrt{9 + 4 + 36}\sqrt{1 + 4 + 4}}\right| = \frac{19}{21}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{19}{21}\right)$$

13. The given line is

$$\frac{x - 5}{\frac{1}{5}} = \frac{y - 2}{-\frac{1}{7}} = \frac{z}{\frac{1}{35}} \text{ or } \frac{x - 5}{7} = \frac{y - 2}{-5} = \frac{z}{1}$$

So, the required vector equation of the line passing through (1, 2, –1) is

$$\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(7\hat{i} - 5\hat{j} + \hat{k})$$

Cartesian equation of the line is

$$\frac{x - 1}{7} = \frac{y - 2}{-5} = \frac{z + 1}{1}$$

14. D-ratios of the two lines are 1, –2, 2 and –3, 2, 6

$$\cos\theta = \frac{-3 - 4 + 12}{3 \times 7} = \frac{5}{21} \Rightarrow \theta = \cos^{-1}\left(\frac{5}{21}\right)$$

15. The equation of the lines in standard form are.

$$\frac{x - 1}{-3} = \frac{y - 1}{2k} = \frac{z - 3}{2}$$

and $$\frac{x - 1}{3k} = \frac{y - \frac{1}{3}}{2} = \frac{z - 6}{-5}$$

Lines are perpendicular

$\therefore -9k + 4k - 10 = 0 \Rightarrow k = -2$

16. D.c's are cos α, cos β, cos γ

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\Rightarrow (1 - \sin^2\alpha) + (1 - \sin^2\beta) + (1 - \sin^2\gamma) = 1$$

$$\Rightarrow \qquad \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$$

17. Required equation of line is given by

$$\frac{x - 2}{3} = \frac{y - 2}{-1} = \frac{z - 1}{-3}$$

Putting $z = -2$, we get $\frac{y - 2}{-1} = \frac{-3}{-3} = 1$

$$y - 2 = -1 \Rightarrow y = 1$$

18. $l = \cos\alpha, m = \cos\beta, n = \cos\gamma$

$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos 2\alpha + \cos 2\beta + \cos 2\gamma$

$$= 2\cos^2\alpha - 1 + 2\cos^2\beta - 1 + 2\cos^2\gamma - 1$$

$$= 2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) - 3$$

$$= 2 - 3$$

$$= -1$$

19. Equation of line

$$\frac{x - \frac{1}{2}}{\frac{12}{2}} = \frac{y + 2}{2} = \frac{z - 3}{3}$$

Direction ratios of line are 6, 2, 3

Direction cosines of line are $\left\langle \pm\frac{6}{7}, \pm\frac{2}{7}, \pm\frac{3}{7} \right\rangle$

20. Vector parallel to the required line is $2i - 2\hat{j} + \hat{k}$

Required vector equation of the line

$$\vec{r} = (2\hat{i} - \hat{j} + \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$$

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. (i)

$$\theta = \cos^{-1}\left(\frac{\vec{l_1}.\vec{l_2}}{|\vec{l_1}||\vec{l_2}|}\right)$$

$$= \cos^{-1}\left(\frac{(\hat{i} - 2\hat{j} + 3\hat{k}).(3\hat{i} - 2\hat{j} + \hat{k})}{|(\hat{i} - 2\hat{j} + 3\hat{k})||(3\hat{i} - 2\hat{j} + \hat{k})|}\right)$$

$$= \cos^{-1}\left(\frac{3 + 4 + 3}{\sqrt{1 + 4 + 9}\sqrt{9 + 4 + 1}}\right)$$

$$= \cos^{-1}\left(\frac{10}{14}\right) = \cos^{-1}\left(\frac{5}{7}\right)$$

(ii) Scalar projection of

$$\vec{l}_1 \text{ on } \vec{l}_2 = \frac{\vec{l}_1.\vec{l}_2}{|\vec{l}_2|} = \frac{(\hat{i}-2\hat{j}+3\hat{k}).(3\hat{i}-2\hat{j}+\hat{k})}{|(3\hat{i}-2\hat{j}+\hat{k})|}$$

$$= \frac{3+4+3}{\sqrt{9+4+1}} = \frac{10}{\sqrt{14}}$$

2. Line perpendicular to the lines

$$\vec{r} = 2\hat{i}+\hat{j}-3\hat{k}+\lambda(\hat{i}+2\hat{j}+5\hat{k})$$

and $$\vec{r} = 3\hat{i}+3\hat{j}-7\hat{k}+\mu(3\hat{i}-2\hat{j}+5\hat{k})$$

has a vector parallel it is given by

$$\vec{b} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 5 \\ 3 & -2 & 5 \end{vmatrix}$$

$$= 20\hat{i}+10\hat{j}-8\hat{k}$$

∴ Equation of line in vector form is

$$\vec{r} = -\hat{i}+2\hat{j}+7\hat{k}+\gamma(10\hat{i}+5\hat{j}-4\hat{k})$$

And equation of line in cartesian form is

$$\frac{x+1}{10} = \frac{y-2}{5} = \frac{z-7}{-4}$$

3. General point on the given is M(5λ – 3, 2λ + 1, 3λ – 4)

Direction ratios of *PM* are 5λ – 3, 2λ – 1, 3λ – 7

If this point is the foot of the perpendicular from the point *P*(0, 2, 3), then *PM* is perpendicular to the line. Thus,

$$(5\lambda-3).5 + (2\lambda-1).2 + (3\lambda-7).3 = 0$$

$\Rightarrow \lambda = 1$

Hence co-ordinates of *M* are (2, 3, –1)

4. Here

$\vec{a}_1 = \hat{i}+2\hat{j}-4\hat{k}, \vec{b}_1 = 2\hat{i}+3\hat{j}+6\hat{k}$

$\vec{a}_2 = 3\hat{i}+3\hat{j}-5\hat{k}, \vec{b}_2 = 4\hat{i}+6\hat{j}+12\hat{k}$

Here, $\vec{b}_1$ and $\vec{b}_2$ are parallel vectors.

$$\vec{a}_2 - \vec{a}_1 = 2\hat{i}+\hat{j}-\hat{k}$$

$$\vec{b} = 2\hat{i}+3\hat{j}+6\hat{k}$$

Thus, $$(\vec{a}_2-\vec{a}_1)\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$$

$$= 9\hat{i}-14\hat{j}+4\hat{k}$$

Distance between the line $$= \left|\frac{(\vec{a}_2-\vec{a}_1)\times\vec{b}}{|\vec{b}|}\right|$$

$$= \frac{\sqrt{81+196+16}}{\sqrt{4+9+36}}$$

$$= \frac{\sqrt{293}}{7} \text{ units.}$$

5. A general point on the given line is M(3λ + 15, 8λ + 29, –5λ + 5).

DRs of *PM* are (3λ + 10, 8λ + 22, –5λ + 2)

This general point for some specific value of λ will be the foot of the perpendicular drawn from (5, 7, 3) on the given line is PM ⊥ line.

i.e. if (3λ + 10)(3) + (8λ + 22)(8) + (–5λ + 2)(–5) = 0

$\Rightarrow \lambda = -2$

Hence, M is (9, 13, 15) is the required foot of the perpendicular.

6. Let the required point on given line be (3λ – 2, 2λ – 1, 2λ + 3) for some λ.

According to question

$$\sqrt{(3\lambda-3)^2+(2\lambda-4)^2+(2\lambda)^2} = 5$$

$$17\lambda^2 - 34\lambda + 25 = 25$$

17λ(λ – 2) = 0 gives λ = 0, λ = 2

∴ Coordinates of required points are (–2, –1, 3) and (4, 3, 7)

7. $\vec{a}_1 = 3\hat{i}+5\hat{j}+7\hat{k}$ $\quad \vec{b}_1 = (\hat{i}-2\hat{j}+\hat{k})$

$\vec{a}_2 = -\hat{i}-\hat{j}-\hat{k}$ $\quad \vec{b}_2 = 7\hat{i}-6\hat{j}+\hat{k}$

$$(\vec{a}_2-\vec{a}_1) = -4i-6j-8k$$

$$\vec{b}_1\times\vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 7 & -6 & 1 \end{vmatrix}$$

$$= 4\hat{i}+6\hat{j}+8\hat{k}$$

$$(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2) = -16-36-64 = -116$$

$$|\vec{b}_1\times\vec{b}_2| = \sqrt{16+36+64} = \sqrt{116}$$

$$\text{S.D.} = \frac{|-116|}{\sqrt{116}} = \sqrt{116} \text{ units}$$

8. $l = \cos 60° = \frac{1}{2}, n = \cos 45° = \frac{1}{\sqrt{2}}$

Now, $l^2 + m^2 + n^2 = 1$

$\Rightarrow \frac{1}{4}+m^2+\frac{1}{2} = 1$

$\Rightarrow m^2 = \frac{1}{4} \Rightarrow m = \pm\frac{1}{2}$

θ = 60°

Required direction cosines are $\left\langle \frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}} \right\rangle$

9. Let, $\vec{a}_1 = \hat{i}+2\hat{j}+3\hat{k}, \vec{a}_2 = 4\hat{i}+\hat{j}$

$\vec{b}_1 = 2\hat{i}+3\hat{j}+4\hat{k}, \vec{b}_2 = 5\hat{i}+2\hat{j}+\hat{k}$

$\vec{a}_2-\vec{a}_1 = 3\hat{i}-\hat{j}-3\hat{k}$

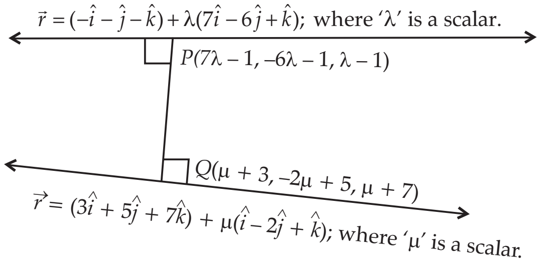
$\vec{b}_1\times\vec{b}_2 = -5\hat{i}+18\hat{j}-11\hat{k}$

Here, $(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2) = -15-18+33 = 0$

Hence given lines are not skew lines.

LONG ANSWER TYPE QUESTIONS (5 Marks)

1.



Given that equation of lines are

$$\vec{r} = (-\hat{i} - \hat{j} - \hat{k}) + \lambda(7\hat{i} - 6\hat{j} + \hat{k}) \quad \text{...(i)}$$

and $$\vec{r} = (3\hat{i} + 5\hat{j} + 7\hat{k}) + \mu(\hat{i} - 2\hat{j} + \hat{k}) \quad \text{...(ii)}$$

The given lines are non-parallel lines as vectors $\hat{i} - 2\hat{j} + \hat{k}$ and $7\hat{i} - 6\hat{j} + \hat{k}$ are not parallel. There is a unique line segment PQ(P lying on line (i) and Q on the other line (ii)), which is at right angles to both the lines PQ is the shortest distance between the lines.

Hence, the shortest possible distance between the lines $= PQ$

Let the position vector of the point P lying on the inner $\vec{r} = (-\hat{i} - \hat{j} - \hat{k}) + \lambda(7\vec{i} - 6\vec{j} + \hat{k})$ where 'λ' is a scalar, is $(7\lambda - 1)\hat{i} - (6\lambda + 1)\hat{j} + (\lambda - 1)\hat{k}$, for some λ and the position vector of the point Q lying on the line $\vec{r} = (3\hat{i} + 5\hat{j} + 7\hat{k})$ $+\mu(\hat{i} - 2\hat{j} + \hat{k})$ where 'μ' is a scalar, is $(\mu + 3)\hat{i} + (-2\mu + 5)\hat{j} + (\mu + 7)\hat{k}$, for some μ. Now, the vector

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = (\mu + 3 - 7\lambda + 1)\hat{i} + (-2\mu + 5 + 6\lambda + 1)\hat{j} + (\mu + 7 - \lambda + 1)\hat{k}$$

i.e. $\overrightarrow{PQ} = (\mu - 7\lambda + 4)\hat{i} + (-2\mu + 6\lambda + 6)\hat{j} + (\mu - \lambda + 8)\hat{k}$;

(where 'O' is the origin), is perpendicular to both the lines, so the vector $\overrightarrow{PQ}$ is perpendicular to both the vectors $7\hat{i} - 6\hat{j} + \hat{k}$ and $\hat{i} - 2\hat{j} + \hat{k}$.

$\Rightarrow (\mu - 7\lambda + 4).7 + (-2\mu + 6\lambda + 6).(-6) + (\mu - \lambda + 8).1 = 0$

and $(\mu - 7\lambda + 4).1 + (-2\mu + 6\lambda + 6)(-2) + (\mu - \lambda + 8).1 = 0$

$$20\mu - 86\lambda = 0$$
$$6\mu - 20\lambda = 0$$

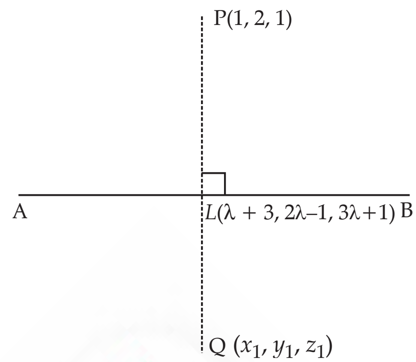
On solving the above equations, we get $\mu = \lambda = 0$

So, the position vector of the points P and Q are $-\hat{i} - \hat{j} - \hat{k}$ and $3\hat{i} + 5\hat{j} + 7\hat{k}$ respectively.

$$\overrightarrow{PQ} = 4\hat{i} + 6\hat{j} + 8\hat{k}$$

and $|\overrightarrow{PQ}| = \sqrt{4^2 + 6^2 + 8^2} = \sqrt{116} = 2\sqrt{29}$ units.

2.



Let $P(1, 2, 1)$ be the given point and L be the foot of the perpendicular from P to the given line AB (as shown in the figure above).

Let's put $\dfrac{x-3}{1} = \dfrac{y+1}{2} = \dfrac{z-1}{3} = \lambda$,

Then, $x = \lambda + 3$, $y = 2\lambda - 1$, $z = 3\lambda + 1$

Let the coordinates of the point L be $(\lambda + 3, 2\lambda - 1, 3\lambda + 1)$.

So, direction ratios of PL are $(\lambda + 3 - 1, 2\lambda - 1 - 2, 3\lambda + 1 - 1)$ i.e. $(\lambda + 2, 2\lambda - 3, 3\lambda)$

Direction ratios of the given line are 1, 2 and 3, which is perpendicular to PL. Therefore, we have

$$(\lambda + 2).1 + (2\lambda - 3).2 + 3\lambda.3 = 0 \Rightarrow 14\lambda = 4 \Rightarrow \lambda = \frac{2}{7}$$

Then, $\lambda + 3 = \dfrac{2}{7} + 3 = \dfrac{23}{7}$; $2\lambda - 1 = 2\left(\dfrac{2}{7}\right) - 1 = -\dfrac{3}{7}$;

$$3\lambda + 1 = 3\left(\frac{2}{7}\right) + 1 = \frac{13}{7}$$

Therefore, coordinates of the point L are $\left(\dfrac{23}{7}, -\dfrac{3}{7}, \dfrac{13}{7}\right)$

Let $Q(x_1, y_1, z_1)$ be the image of P(1, 2, 1) with respect to the given line. Then, L is the mid-point of PQ.

Therefore, $\dfrac{1 + x_1}{2} = \dfrac{23}{7}, \dfrac{2 + y_1}{2} = -\dfrac{3}{7}, \dfrac{1 + z_1}{2} = \dfrac{13}{7}$

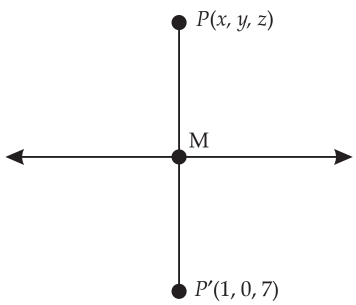
$$\Rightarrow x_1 = \frac{39}{7}, y_1 = -\frac{20}{7}, z_1 = \frac{19}{7}$$

Hence, the image of the point P(1, 2, 1) with respect to the given line $Q\left(\dfrac{39}{7}, -\dfrac{20}{7}, \dfrac{19}{7}\right)$.

The equation of the line joining P(1, 2, 1) and $Q\left(\dfrac{39}{7}, -\dfrac{20}{7}, \dfrac{19}{7}\right)$ is

$$\frac{x-1}{32/7} = \frac{y-2}{-34/7} = \frac{z-1}{12/7} \Rightarrow \frac{x-1}{16} = \frac{y-2}{-17} = \frac{z-1}{6}$$

3. Let foot of the perpendicular on the given line from point P be $M(\lambda, 2\lambda + 1, 3\lambda + 2)$



DR's of PP' are $\lambda - 1, 2\lambda + 1, 3\lambda - 5$

$1(\lambda - 1) + 2(2\lambda + 1) + 3(3\lambda - 5) = 0$

$\Rightarrow \lambda = 1$

Coordinates of M(1, 3, 5)

$$\frac{x+1}{2} = 1, \frac{y+0}{2} = 3, \frac{z+7}{2} = 5$$

$\Rightarrow x = 1, y = 6, z = 3 \Rightarrow$ P(1, 6, 3)

4. Let direction ratios of the required line be a, b, c

$\therefore$ The required line is perpendicular to both the given lines

$\therefore \quad 3a - 16b + 7c = 0$

and $\quad 3a + 8b - 5c = 0$

$$\Rightarrow \frac{a}{24} = \frac{b}{36} = \frac{c}{72}$$

$$\Rightarrow \frac{a}{2} = \frac{b}{3} = \frac{c}{6}$$

The mid-point of the line-segment AB is (3, 4, 6).

Hence, the required equation of the line is

$$\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-6}{6}$$

5. The mid-point of the BC is $\left(\frac{-1}{2}, 2, 0\right)$

The equation of the median through A is

$$\frac{x-1}{\frac{-1}{2}-1} = \frac{y-1}{2-1} = \frac{z}{0}$$

$$\Rightarrow \quad \frac{x-1}{-3} = \frac{y-1}{2} = \frac{z}{0} \quad ...(1)$$

The mid-point of the AC is $\left(\frac{-1}{2}, \frac{3}{2}, \frac{-1}{2}\right)$

The equation of the median through B is

$$\frac{x-1}{\frac{-1}{2}-1} = \frac{y-2}{\frac{3}{2}-2} = \frac{z-1}{\frac{-1}{2}-1}$$

$$\Rightarrow \quad \frac{x-1}{-3} = \frac{y-2}{-1} = \frac{z-1}{-3} \quad ...(2)$$

Any point on the line (1) is $(-3\lambda + 1, 2\lambda + 1, 0)$

Any point on the line (2) is $(-3\mu + 1, -\mu + 2, -3\mu + 1)$

For the point of intersection,

$$-3\lambda + 1 = -3\mu + 1,$$

$$2\lambda + 1 = -\mu + 2,$$

$$0 = -3\mu + 1$$

$$\Rightarrow \lambda = \mu = \frac{1}{3}$$

The coordinates of the centroid are $\left(0, \frac{5}{3}, 0\right)$.

6. $l_1 : \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda; l_2 : \frac{x-1}{0} = \frac{y}{-3} = \frac{z-7}{2} = \mu$

any point on l_1 is $(\lambda, 2\lambda + 1, 3\lambda + 2)$ & any point on l_2 is $(1, -3\mu, 2\mu + 7)$

If l_1 and l_2 intersect,

$\lambda = 1, 2\lambda + 1 = -3\mu$ and $3\lambda + 2 = 2\mu + 7 \Rightarrow \lambda = 1$ and $\mu = -1$

Point of intersection of l_1 and l_2 is (1, 3, 5).

Let d.r's of required line be $<a, b, c>$. Then,

$a + 2b + 3c = 0$ and $-3b + 2c = 0 \Rightarrow \frac{a}{13} = \frac{b}{-2} = \frac{c}{-3}$

Required equation of line is $\frac{x-1}{13} = \frac{y-3}{-2} = \frac{z-5}{-3}$

7. 

d.r's of CD are <1, –2, 2>

$\therefore$ d.r's of AB are <1, –2, 2>

$\therefore$ Equation of AB is $\frac{x+1}{1} = \frac{y-2}{-2} = \frac{z-1}{2}$

$\therefore$ Equation of CD is $\frac{x-4}{1} = \frac{y+7}{-2} = \frac{z-8}{2}$

Let $\vec{a}_1 = -\hat{i} + 2\hat{j} + \hat{k}, \vec{a}_2 = 4\hat{i} - 7\hat{j} + 8\hat{k}$ & $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$

Now, $\vec{a}_2 - \vec{a}_1 = 5\hat{i} - 9\hat{j} + 7\hat{k}$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -9 & 7 \\ 1 & -2 & 2 \end{vmatrix} = -4\hat{i} - 3\hat{j} - \hat{k}$$

Distance between AB and CD is given by

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$$

$$d = \frac{\sqrt{16+9+1}}{\sqrt{1+4+4}} = \frac{\sqrt{26}}{3} \text{ units}$$

$$AB = \sqrt{2^2 + (-4)^2 + (4)^2} = 6 \text{ units}$$

Area of parallelogram ABCD

$$= AB \times d = 6 \times \frac{\sqrt{26}}{3} = 2\sqrt{26} \text{ sq. units}$$

8. Equation of the given in standard form is

$L_1 : \dfrac{x}{2} = \dfrac{y-3}{2} = \dfrac{z-1}{1}$

Equation of the line parallel to L_1 & passing through (4, 0, –5) is

$L_2 : \dfrac{x-4}{2} = \dfrac{y}{2} = \dfrac{z+5}{1}$

Vector Equation of Lines are

$L_1 : \vec{r} = (0\hat{i} + 3\hat{j} + \hat{k}) + \lambda(2\hat{i} + 2\hat{j} + \hat{k})$

$L_2 : \vec{r} = (4\hat{i} + 0\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 2\hat{j} + \hat{k})$

Now,

$\vec{a}_2 - \vec{a}_1 = (4\hat{i} + 0\hat{j} - 5\hat{k}) - (0\hat{i} + 3\hat{j} + \hat{k}) = (4\hat{i} - 3\hat{j} - 6\hat{k})$

$\vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -3 & -6 \\ 2 & 2 & 1 \end{vmatrix} = 9\hat{i} - 16\hat{j} + 14\hat{k}$$

$|\vec{b}| = \sqrt{4+4+1} = 3$

Thus, distance between the lines is

$$\text{S.D.} = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|} = \frac{\sqrt{81+256+196}}{3} = \frac{\sqrt{533}}{3} \text{ units}$$

9. $L_1 : \dfrac{x-1}{-3} = \dfrac{y-2}{2k} = \dfrac{z-3}{2}$

$\Rightarrow$ direction ratio's of L_1 = <–3, 2*k*, 2>

$L_2 : \dfrac{x-1}{3k} = \dfrac{y-1}{1} = \dfrac{z-6}{-7}$

$\Rightarrow$ direction ratio's of L_2 = <3*k*, 1, –7>

Since $L_1 \perp L_2$,

$-9k + 2k - 14 = 0 \Rightarrow k = -2$

Thus, d.r's of L_1 = <–3, –4, 2>, d.r's of L_2 = <–6, 1, –7>

Now the vector perpendicular to both L_1 & L_2 is given by

$$\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -4 & 2 \\ -6 & 1 & -7 \end{vmatrix} = 26\hat{i} - 33\hat{j} - 27\hat{k}$$

Thus, equation of the required line is

$\vec{r} = (3\hat{i} - 4\hat{j} + 7\hat{k}) + \lambda(26\hat{i} - 33\hat{j} - 27\hat{k})$

10. The standard form of the equation of the line is

$$\frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3}$$

Let foot of the perpendicular from the point A(2, 3, –8) to the given line be $B(-2\lambda + 4, 6\lambda, -3\lambda + 1)$

D-ratios of AB is: $-2\lambda + 2, 6\lambda - 3, -3\lambda + 9$

As AB is perpendicular to the given line:

$-2(-2\lambda + 2) + 6(6\lambda - 3) - 3(-3\lambda + 9) = 0 \Rightarrow \lambda = 1$

$\therefore$ Foot of the perpendicular is: B(2, 6, –2)

Perpendicular distance = $AB = 3\sqrt{5}$ units

11. Equation of L_1 : $\vec{r} = 2\hat{i} - \hat{j} + \hat{k} + \lambda(\hat{i} + \hat{j} + 3\hat{k})$

Equation of L_2 : $\vec{r} = (\hat{i} + \hat{j} - 2\hat{k}) + \mu(2\hat{j} - \hat{k})$

Taking

$$\vec{a}_1 = 2\hat{i} - \hat{j} + \hat{k}, \vec{b}_1 = \hat{i} + \hat{j} + 3\hat{k}$$

$$\vec{a}_2 = \hat{i} + \hat{j} - 2\hat{k}, \vec{b}_2 = 2\hat{j} - \hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = -\hat{i} + 2\hat{j} - 3\hat{k}, \vec{b}_1 \times \vec{b}_2 = -7\hat{i} + \hat{j} + 2\hat{k}$$

$$\text{Shortest distance} = \frac{|(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} = \frac{1}{\sqrt{6}} \text{ unit}$$

12. $\left.\begin{array}{l} \vec{r} = (8\hat{i} - 9\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k}) \\ \vec{r} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k}) \end{array}\right\}$

We have, $\vec{a}_1 = 8\hat{i} - 9\hat{j} + 10\hat{k}, \vec{b}_1 = 3\hat{i} - 16\hat{j} + 7\hat{k}$,

$\vec{a}_2 = 15\hat{i} + 29\hat{j} + 5\hat{k}, \vec{b}_2 = 3\hat{i} + 8\hat{j} - 5\hat{k}$

$$\vec{a}_2 - \vec{a}_1 = 7\hat{i} + 38\hat{j} - 5\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$$

$$= 24\hat{i} + 36\hat{j} + 72\hat{k}$$

$$\text{Shortest distance} = \left|\frac{(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right| = \frac{98}{7} = 14$$

13. If the given lines intersect then,

$$\hat{i} - \hat{j} + 6\hat{k} + \lambda(3\hat{i} - \hat{k}) = -3\hat{j} + 3k + \mu(\hat{i} + 2\hat{j} - \hat{k})$$

Solving the equations,

$1 + 3\lambda = \mu, -1 = 2\mu - 3,$ we get $\lambda = 0, \mu = 1$

which do not satisfy the equation, $6 - \lambda = 3 - \mu$

$\therefore$ The lines do not intersect, hence no point of intersection. They are skew lines.

Now the line perpendicular to both the lines will be:

- Perpendicular to both direction vctors $\vec{b}_1$ and $\vec{b}_2$
- Passes throgh the shortest line segment between skew lines.

$$\therefore \vec{d} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & -1 \\ 1 & 2 & -1 \end{vmatrix} = 2\hat{i} + 2\hat{j} + 6\hat{k}$$

Point *P* on line 1 is $(1 + 3\lambda, -1, 6 - \lambda)$

Point *Q* on line 2 is $(\mu, -3 + 2\mu, 3 - \mu)$

Now, $\overrightarrow{PQ}.\vec{b}_1 = 0$ and $\overrightarrow{PQ}.\vec{b}_2 = 0$

On solving, we get $\lambda = \dfrac{2}{11}$ and $\mu = \dfrac{10}{11}$

$\therefore$ Point $P = \left(\dfrac{17}{11}, -1, \dfrac{64}{11}\right)$ and $Q = \left(\dfrac{10}{11}, \dfrac{-13}{11}, \dfrac{23}{11}\right)$

Now, mid-point of $PQ = \left(\frac{27}{22}, \frac{-24}{22}, \frac{87}{22}\right)$ or

$\left(\frac{27}{22}, \frac{-12}{11}, \frac{87}{22}\right)$

Thus, equation of required line is $\vec{r} = \left(\frac{27}{22}\hat{i} - \frac{12}{11}\hat{j} + \frac{87}{22}\hat{k}\right)$

$+ t(2\hat{i} + 2\hat{j} + 6\hat{k})$

14. $\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}, \vec{a}_2 = 3\hat{i} - 3\hat{j} - 5\hat{k}$

$\vec{b}_1 = 2\hat{i} + 3\hat{j} + 6\hat{k}, \vec{b}_2 = -2\hat{i} + 3\hat{j} + 8\hat{k}$

here, $\vec{a}_2 - \vec{a}_1 = 2\hat{i} - 5\hat{j} - \hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ -2 & 3 & 8 \end{vmatrix} = 6\hat{i} - 28\hat{j} + 12\hat{k}$$

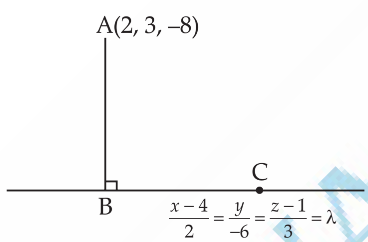
$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{964}$$

$$\text{S.D.} = \left|\frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$$

$$= \left|\frac{(2)(6) + (-5)(-28) + (-1)(12)}{\sqrt{964}}\right|$$

$$= \frac{140}{\sqrt{964}} \text{ or } \frac{70}{\sqrt{241}}$$

15.



Any point on line is $(2\lambda + 4, -6\lambda, 3\lambda + 1)$ for some λ.
Let $B(2\lambda + 4, -6\lambda, 3\lambda + 1)$

d.r. of $AB = <2\lambda + 2, -6\lambda - 3, 3\lambda + 9>$

d.r. of $BC = <2, -6, 3>$

$AB \perp BC \Rightarrow 2(2\lambda + 2) - 6(-6\lambda - 3) + 3(3\lambda + 9) = 0$

$\Rightarrow \lambda = -1$

$\therefore$ Foot of the perpendicular, $B(2, 6, -2)$

Now,

$AB = \sqrt{(2-2)^2 + (6-3)^2 + (-2+8)^2} = \sqrt{45}$ or $3\sqrt{5}$ units

16. Vector equation of required line through (1, 2, –4) is

$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

and cartesian equation $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$

Equation of line through A(3, 3, –5) and B(1, 0, –11) is

$\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$

Distance between parallel lines is given by

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$$

Here $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}, \vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}, \vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}$

$(\vec{a}_2 - \vec{a}_1) = 2\hat{i} + \hat{j} - \hat{k}$

$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = 9\hat{i} - 14\hat{j} + 4\hat{k}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix} = 9\hat{i} - 14\hat{j} + 4\hat{k}$$

$\therefore d = \frac{\sqrt{293}}{7}$ units

17. Equation of line AB is $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{6}$

Let coordinates of required point on AB be $(2\lambda + 1, 3\lambda + 2, 6\lambda + 3)$ for some $\lambda \in \mathbb{R}$

According to Question,

$(2\lambda - 2)^2 + (3\lambda - 3)^2 + (6\lambda - 6)^2 = 14^2$ gives $\lambda^2 - 2\lambda - 3 = 0$

Solving we get $\lambda = 3$ and -1

$\therefore$ Required points are (7, 11, 21) and (–1, –1, –3)

18. Equation of diagonal $PR : \frac{x-4}{8} = \frac{y-2}{2} = \frac{z+6}{11}$

Equation of diagonal $QS : \frac{x-5}{6} = \frac{y+3}{12} = \frac{z-1}{-3}$

General points on PR & QS are $(8k + 4, 2k + 2, 11k - 6)$ and $(6t + 5, 12t - 3, -3t + 1)$ for real numbers 'k' and 't' respectively.

For point of intersection of PR and QS:

$$8k + 4 = 6t + 5, 2k + 2 = 12t - 3$$

Solving, we get $k = \frac{1}{2}, t = \frac{1}{2}$

$\therefore$ The point of intersection is $\left(8, 3, -\frac{1}{2}\right)$

19. Let direction ratios of the required line be a, b, c.

Since it is perpendicular to the two given lines,

$a + 2b + 3c = 0; -3a + 2b + 5c = 0$

Solving together, $a = 4k, b = -14k, c = 8k$

$\therefore$ Equation of line is: $\frac{x+1}{4k} = \frac{y-3}{-14k} = \frac{z+2}{8k}$

$\Rightarrow \frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}$

Vector equation: $\vec{r} = -\hat{i} + 3\hat{j} - 2\hat{k} + \lambda(2\hat{i} - 7\hat{j} + 4\hat{k})$

Distance from origin $= \frac{|(-\hat{i} + 3\hat{j} - 2\hat{k}) \times (2\hat{i} - 7\hat{j} + 4\hat{k})|}{|2\hat{i} - 7\hat{j} + 4\hat{k}|}$

$$= \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & -2 \\ 2 & -7 & 4 \end{vmatrix}}{|2i - 7j + 4k|}$$

$$= \frac{|-2\hat{i} + \hat{k}|}{|2\hat{i} - 7\hat{j} + 4\hat{k}|} = \frac{\sqrt{5}}{\sqrt{69}} \text{ or } \sqrt{\frac{5}{69}} \text{ units}$$

20. As lines are intersecting, $(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2) = 0$

$$\Rightarrow \begin{vmatrix} 4-1 & 1-b & -3 \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix} = 0$$

$-15 + 18 - 18b + 33 = 0$

$\Rightarrow b = 2$

Any point on the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is

$(2\lambda + 1, 3\lambda + 2, 4\lambda + 3), \lambda \in \mathbb{R}$

For the point of intersection, this point must lie on the line

$$\frac{x-4}{5} = \frac{y-1}{2} = z$$

$$\Rightarrow \frac{2\lambda+1-4}{5} = \frac{3\lambda+2-1}{2} = 4\lambda+3$$

$\Rightarrow \lambda = -1$

$\therefore$ point of intersection is $(-1, -1, -1)$

21. Equation of the line $AB: \frac{x-4}{2} = \frac{y-7}{4} = \frac{z-8}{4}$

Equation of line $BC: \frac{x-2}{3} = \frac{y-3}{5} = \frac{z-4}{3}$

Equation of line $CD: \frac{x+1}{1} = \frac{y+2}{2} = \frac{z-1}{2}$

Equation of line $DA: \frac{x-1}{3} = \frac{y-2}{5} = \frac{z-5}{3}$

Let P be foot of perpendicular from A to CD.

$\therefore$ Coordinates of P are $(\lambda - 1, 2\lambda - 2, 2\lambda + 1)$ for some $\lambda \in \mathbb{R}$

d.r's of AP are $(\lambda - 5, 2\lambda - 9, 2\lambda - 7)$

since $AP \perp CD$

$\Rightarrow 1(\lambda - 5) + 2(2\lambda - 9) + 2(2\lambda - 7) = 0$

$\Rightarrow 9\lambda = 37 \quad \Rightarrow \lambda = \frac{37}{9}$

$\therefore$ Coordinates of P are $\left(\frac{28}{9}, \frac{56}{9}, \frac{83}{9}\right)$

22. Shortest distance

$$= \frac{\begin{vmatrix} -3 & 2 & -2 \\ 3 & 2 & 5 \\ 4 & 3 & -2 \end{vmatrix}}{\sqrt{(2\times-2-5\times3)^2 + (3\times-2-4\times5)^2 + (3\times3-2\times4)^2}}$$

$$= \frac{-3(-19) - 2(-26) - 2(1)}{\sqrt{361+676+1}}$$

$$= \frac{57+52-2}{\sqrt{1038}}$$

$$= \frac{107}{\sqrt{1038}} \neq 0$$

So, the line will not intersect each other.

23. The given lines are

$$\frac{x-0}{1/2} = \frac{y-0}{1/3} = \frac{z-0}{-1} \text{ or } \frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$$

and $$\frac{x-0}{1/6} = \frac{y-0}{-1} = \frac{z-0}{-1/4} \text{ or } \frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$$

Let θ be the angle between the two lines, then

$$\cos\theta = \left|\frac{(3\times2)+(2\times-12)+(-6)(-3)}{\sqrt{9+4+36}\sqrt{4+144+9}}\right|$$

$$= \left|\frac{6-24+18}{7\times\sqrt{157}}\right|$$

$= 0$

$\Rightarrow \theta = 90°$

24. $(\vec{a}_2 - \vec{a}_1) = -\hat{j}$ and $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & -1 & 2 \end{vmatrix} = -\hat{i} - \hat{j}$

$(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = 1 \neq 0 \Rightarrow$ Lines are not intersecting.

25. General point on the line, say, $P(2\lambda, 3\lambda + 2, 4\lambda + 3)$

Direction ratios of the perpendicular from the point $(3, -1, 11)$ to the line are

$2\lambda - 3, 3\lambda + 3, 4\lambda - 8$

And direction ratios of the line are 2, 3, 4

$\therefore 2(2\lambda - 3) + 3(3\lambda + 3) + 4(4\lambda - 8) = 0 \Rightarrow \lambda = 1$

$\therefore$ (2, 5, 7) is the foot of the perpendicular.

Equation of the perpendicular

Cartesian form: $\frac{x-3}{1} = \frac{y+1}{-6} = \frac{z-11}{4}$

Or, Vector form: $\vec{r} = (3\hat{i} - \hat{j} + 11\hat{k}) + \lambda(\hat{i} - 6\hat{j} + 4\hat{k})$

26. line 1 : $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = \lambda$ ...(1)

line 2 : $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = \mu$ ...(2)

General points on (1) and (2) are

$(3\lambda - 1, 5\lambda - 3, 7\lambda - 5)$ and $(\mu + 2, 3\mu + 4, 5\mu + 6)$

for the lines to intersect,

$3\lambda - 1 = \mu + 2$ ...(3)

$5\lambda - 3 = 3\mu + 4$ ...(4)

$7\lambda - 5 = 5\mu + 6$ ...(5)

Solving (3) and (4) gives $\lambda = \frac{1}{2}$ and $\mu = -\frac{3}{2}$

Clearly these values of λ and μ satisfies (5)

⇒ given lines intersect.

Point of intersection is $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$

27. Given lines are $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-0}{1}$

and $\frac{x+1}{5} = \frac{y-2}{1} = \frac{z-2}{0}$

In vector form, lines are

$$\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + 3\hat{j} + \hat{k}) = \vec{a}_1 + \lambda\vec{b}_1$$

and $\vec{r} = (-\hat{i} + 2\hat{j} + 2\hat{k}) + \mu(5\hat{i} + \hat{j}) = \vec{a}_2 + \lambda\vec{b}_2$

Now, $\vec{a}_2 - \vec{a}_1 = -2\hat{i} + 3\hat{j} + 2\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 5 & 1 & 0 \end{vmatrix} = -\hat{i} + 5\hat{j} - 13\hat{k}$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{195}$$

$$\text{S.D.} = \left|\frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}\right|$$

$$= \left|\frac{2 + 15 - 26}{\sqrt{195}}\right| = \frac{9}{\sqrt{195}} \text{ units}$$

28. Assumes that $O(x, y, z)$ is the point of intersection of PQ and line m.

Writes that $\frac{x+8}{2} = \frac{5-y}{3} = \frac{z-4}{5} = \lambda$, where λ is a constant.

Using the equations from step 1, finds x, y and z as follows:

$$x = 2\lambda - 8$$
$$y = 5 - 3\lambda$$
$$z = 5\lambda + 4$$

Notes that $O(x, y, z)$ and P both line on line PQ.

Finds the direction ratios of PQ as:

$(2\lambda - 8 - (-2), 5 - 3\lambda - (-7), 5\lambda + 4 - 2)$

$= (2\lambda - 6, 12 - 3\lambda, 5\lambda + 2)$

Since PQ is perpendicular to line m, equates the dot product of their direction ratio to 0 to find λ as 1.

$\Rightarrow 2(2\lambda - 6) - 3(12 - 3\lambda) + 5(5\lambda + 2) = 0$

$\Rightarrow 4\lambda - 12 - 36 + 9\lambda + 25\lambda + 10 = 0$

$\Rightarrow 38\lambda = 38$

$\Rightarrow \lambda = 1$

Finds the $O(x, y, z)$, the coordinates of the foot of the perpendicular from P to line m as (–6, 2, 9).

Assumes the coordinates of Q as (a, b, c). Finds Q as (–10, 11, 16) using the midpoint theorem as:

$$-2 + a = 2(-6)$$
$$\Rightarrow \quad a = -10$$
$$-7 + b = 2(2)$$
$$\Rightarrow \quad b = 11$$
$$2 + c = 2(9)$$
$$\Rightarrow \quad c = 16$$

Level - 2 ADVANCED COMPETENCY FOCUSED QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Marks)

1. Option (A) is correct.

Explanation: To find the direction ratios of the line joining points A(2, 3, 4) and B(6, 7, 8), we subtract the coordinates of A from B:

Direction ratios = B – A = (6 – 2, 7 – 3, 8 – 4) = (4, 4, 4)

2. Option (A) is correct.

Explanation: $\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

$$\vec{d}_1.\vec{d}_2 = (3)(1) + (2)(-2) + (1)(2)$$
$$= 3 - 4 + 2 = 1$$
$$|\vec{d}_1| = 3^2 + 2^2 + 1^2$$
$$= 9 + 4 + 1 = \sqrt{14}$$
$$|\vec{d}_2| = \sqrt{1^2 + (-2)^2 + 2^2}$$
$$= \sqrt{1+4+4} = 3$$
$$\cos\theta = \frac{1}{3\sqrt{14}}$$

so, $\theta = \cos^{-1}\left(\frac{1}{3\sqrt{14}}\right)$

3. Option (B) is correct.

Explanation: Let, Direction vector of Line 1,

- Direction vector of Line 1,

$$\vec{d}_1 = 2\hat{i} - \hat{j} + \hat{k}$$

- Direction vector of Line 2,

$$\vec{d}_2 = \hat{i} + 2\hat{j} + \hat{k}$$

- A point on Line 1: $P_1 = (1, 2, 3)$
- A oint on Line 2: $P_2 = (3, 1, 1)$

Joining the two points:

$$\vec{P_1P_2} =$$

$$(3-1)\hat{i} + (1-2)\hat{j} + (1-3)\hat{k} = 2\hat{i} - \hat{j} - 2\hat{k}$$

The shortest distance is the projection of $\vec{P}_1.\vec{P}_2$ and $\vec{d}_1 \times \vec{d}_2$ (normal vector to both lines)

4. Option (C) is correct.

Explanation: If two lines in space intersect, they must lie in the same plane — i.e., they are coplanar.

ASSERTION-REASON QUESTIONS (1 Mark)

1. **Option (A) is correct.**

   Explanation: Assertion is true. Skew lines are lines that do not intersect and are not parallel, often seen in 3D space.The shortest distance between such lines is the length of the perpendicular segment connecting them. This represents the minimum separation between these two lines.

   Reason is also true and aligns with the standard method in vector geometry.

2. **Option (A) is correct.**

   Explanation: Assertion is true. If two roads intersect and the angle between them is 90°, then they meet at a point and are perpendicular at that point. The direction vectors of the roads will also be perpendicular. When two vectors are perpendicular, their dot product is zero.

   Reason is also true because this is a basic identity in vector algebra.

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. Writes that the sum of the squares of the direction cosines is equal to 1 and finds α as

   $$\sqrt{\left\{1-\left(\frac{9}{98}+\frac{25}{98}\right)\right\}}=\frac{-8}{\sqrt{98}} \text{ or } \frac{8}{\sqrt{98}}.$$

   (Award 1 mark if only the value of α is found correctly.)

   Writes that the angle between the line and the z-axis is:

   $$\cos^{-1}\frac{8}{\sqrt{98}} \text{ or } \cos^{-1}\frac{-8}{\sqrt{98}}$$

2. Assumes the angle between the two lines as θ and writes:

   $$\cos\theta=\left|\frac{\vec{b}_1.\vec{b}_2}{|\vec{b}_1||\vec{b}_2|a}\right|$$

   where, $\vec{b}_1=(-\hat{i}+\hat{j}+\hat{k})$ and $\vec{b}_1=(2\hat{i}+G\hat{j}-\hat{k})$.

   Substitutes θ as 90° and finds G as:

   $$\cos 90°=0=\left|\frac{(-1)(2)+(1)(G)+(1)(-1)}{\sqrt{3}\sqrt{5+4^2}}\right|$$

   $\Rightarrow G = 3$

3. Writes that te lines are parallel and so $\frac{2}{k}=\frac{3}{-2}=\frac{-j}{-2}$.

   Finds k as $\frac{-4}{3}$ and j as (–3).

4. Since the two lines are perpendicular, writes the equation:

   $$3(2) + 2(1) + 2(-k) = 0$$

   Solves the above equation to find the value of k as 4.

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. Writes that the direction ratios of the given line are $(a, b, c) = (-3, 1, 1)$.

   Writes that any point on the given line is:

   $(-1 + \lambda a, 2 + \lambda b, 0 + \lambda c) = (-3\lambda - 1, \lambda + 2, \lambda)$, where λ is a parameter.

   Uses the distance formula between (–1, 2, 0) and $(-3\lambda - 1, \lambda + 2, \lambda)$ to find λ as:

   $$(-3\lambda)^2+\lambda^2+\lambda^2=(6\sqrt{11})^2$$

   $\Rightarrow \lambda = \pm 6$

   Substitutes λ = 6 and (–6) in $(-3\lambda - 1, \lambda + 2, \lambda)$ to get (–19, 8, 6) and (17, –4, –6) respectively.

   Concludes that the coordinates of P are (–19, 8, 6) or (17, –4, –6).

2. Writes that the helicopters are flying parallel to one another and the shortest distance, d, between them is:

   $$d=\left|\frac{\vec{b}\times(\vec{a}_2-\vec{a}_1)}{|\vec{b}|}\right|$$

   Where, $\vec{a}_1=2\hat{i}+3\hat{j}+2\hat{k}, \vec{a}_2=\hat{i}+2\hat{j}+3\hat{k}, \vec{b}=3\hat{i}+\hat{j}+2\hat{k}$

   Simplifies the above expression as:

   $$\left|\frac{\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & 2\\ -1 & -1 & 1\end{vmatrix}}{\sqrt{9+1+4}}\right|=\left|\frac{3\hat{i}-5\hat{j}-2\hat{k}}{\sqrt{14}}\right|=\left|\frac{\sqrt{9+25+4}}{\sqrt{14}}\right|$$

   Evaluates the above expression as $\frac{\sqrt{19}}{\sqrt{7}}$ units.

CASE BASED QUESTIONS (4 Mark)

1. **(i)** $\vec{a}_1=0\hat{i}+0\hat{j}+0\hat{k}, \vec{a}_2=3\hat{i}+3\hat{j}$

   $$\vec{a}_2-\vec{a}_1=3\hat{i}+3\hat{j}$$

   $$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -1\\ 2 & 1 & 1\end{vmatrix}$$

   $$=3\hat{i}-3\hat{j}-3\hat{k}$$

   $$SD=\frac{|(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)|}{|\vec{b}_1\times\vec{b}_2|}$$

   Now, $(\vec{a}_2-\vec{a}_1).(\vec{b}_1\times\vec{b}_2)=(3\hat{i}+3\hat{j})(3\hat{i}-3\hat{j}-3\hat{k})$

$= 9 - 9 = 0$

Shortest distance between two lines = 0

(ii) Any point on the line $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ is $\lambda\hat{i} + 2\lambda\hat{j} - \lambda\hat{k}$

Any point on the line $\vec{r} = 3\hat{i} + 3\hat{j} + \mu(2\hat{i} + \hat{j} + \hat{k})$ is $(2\mu + 3)\hat{i} + (\mu + 3)\hat{j} + \mu\hat{k}$

As the lines are intersecting,

$$\lambda = 2\mu + 3,\ 2\lambda = \mu + 3$$

On solving $\mu = -1, \lambda = 1$

Point of intersection is $i + 2j - k$ or $(1, 2, -1)$

1. (i) To find the direction ratios of vector $\overrightarrow{AB}$, we subtract the coordinates of point A from point B.

$\overrightarrow{AB}$, $= (5 - 3, 3 - 4, 3 - 0) = (2, -1, 3)$

(ii) $\overrightarrow{CD} = (13\hat{i} - 5\hat{j} - 4\hat{k}) - (6\hat{i} - 4\hat{j} + \hat{k}) = 7\hat{i} - \hat{j} - 5\hat{k}$

$$\text{Unit vector} = \frac{7\hat{i} - \hat{j} - \hat{k}}{\sqrt{(7)^2 + (-1)^2 + (-5)^2}}$$

$$\frac{7\hat{i} - \hat{j} - 5\hat{k}}{\sqrt{(7)^2 + (-1)^2 + (-5)^2}} = \frac{1}{\sqrt{75}}(7\hat{i} - \hat{j} - 5\hat{k})$$

$$= \frac{1}{5\sqrt{3}}(7\hat{i} - \hat{j} - 5\hat{k})$$

(iii) (a) $\overrightarrow{AB} = (5-3)\hat{i} + (3-4)\hat{j} + (3-0)\hat{k}$

or $\overrightarrow{AB} = 2\hat{i} - \hat{j} + 3\hat{k}$

Also, $\overrightarrow{CD} = 7\hat{i} - \hat{j} - 5\hat{k}$

Let angle between $\overrightarrow{AB}$ and $\overrightarrow{CD}$ is θ.

$$\therefore \quad \cos\theta = \frac{(\overrightarrow{AB}).(\overrightarrow{CD})}{|\overrightarrow{AB}|.|\overrightarrow{CD}|}$$

$$= \frac{(2i - j + 3k).(7i - j - 5k)}{\sqrt{(2^2 + (-1)^2 + 3^2}\sqrt{(7^2 + (-1)^2 + (-5)^2}}$$

$$= \frac{14 + 1 - 15}{\sqrt{14}\sqrt{75}} = 0$$

$$\therefore \cos\theta = 0 \Rightarrow \cos\theta = \cos\frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{2}$$

So, $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are perpendicular.

OR

(b) Vector perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{CD}$ is $(\overrightarrow{AB} \times \overrightarrow{CD})$

$$\text{So,} \quad \overrightarrow{AB} \times \overrightarrow{CD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 7 & -1 & -5 \end{vmatrix}$$

$$\overrightarrow{AB} \times \overrightarrow{CD} = \hat{i}(5+3) - \hat{j}(-10-21) + \hat{k}(-2+7)$$

$$\overrightarrow{AB} \times \overrightarrow{CD} = 8\hat{i} + 31\hat{j} + 5\hat{k}$$

2. (i) $\overrightarrow{AV}$ = Position Vector of V – Position Vector of A

$= -3\hat{i} + 7\hat{j} + 11\hat{k} - 7\hat{i} - 5\hat{j} - 8\hat{k}$

$= -10\hat{i} + 2\hat{j} + 3\hat{k}$

Thus, $|\overrightarrow{AV}| = \sqrt{100 + 4 + 9} = \sqrt{113}$ units

(ii) $\overrightarrow{DA}$ = Position Vector of A – Position Vector of D

$= 7\hat{i} + 5\hat{j} + 8\hat{k} - 2\hat{i} - 3\hat{j} - 4\hat{k}$

$= 5\hat{i} + 2\hat{j} + 4\hat{k}$

Unit vector in the direction of $\overrightarrow{DA} = \dfrac{5\hat{i} + 2\hat{j} + 4\hat{k}}{3\sqrt{5}}$

(iii) (a) $\overrightarrow{DV} = -5\hat{i} + 4\hat{j} + 7\hat{k}$

$$\angle VDA = \cos^{-1}\left(\frac{\overrightarrow{DV}.\overrightarrow{DA}}{|\overrightarrow{DV}||\overrightarrow{DA}|}\right) = \cos^{-1}\left(\frac{11\sqrt{2}}{90}\right)$$

OR

(b) $\overrightarrow{DV} = -5\hat{i} + 4\hat{j} + 7\hat{k}$

$$\text{Projection of } \overrightarrow{DV} \text{ on } \overrightarrow{DA} = \left(\frac{\overrightarrow{DV}.\overrightarrow{DA}}{|\overrightarrow{DA}|}\right) = \frac{11\sqrt{5}}{15}$$

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. (i) The general vector form of a straight line in space is:

$$\vec{r}(t) = \vec{a} + t\vec{b}$$

where, $\vec{a}$ is a point on the line and $\vec{b}$ is the direction vector.

Satellite's position vector can be rewritten as:

$$\vec{r}(t) = 1\hat{i} + 1\hat{j} - 1\hat{k} + t(2\hat{i} + 1\hat{j} + 1\hat{k})$$

So, it is in the form $\vec{r} = \vec{a} + t\vec{b};$; where:

- $\vec{a} = (1, 1, -1)$,
- $\vec{b} = (2, 1, 1)$

Hence, the satellite moves along a straight line.

(ii) We check if there exists a value of t such that:

$(2t + 1, t + 1, t - 1) = (1, 2, 3)$

Equating components:

$$2t + 1 = 1 \Rightarrow t = 0$$
$$t + 1 = 2 \Rightarrow t = 1$$
$$t - 1 = 3 \Rightarrow t = 4$$

We get three different values of t — which is not possible.

Hence, the satellite does not pass through point A.

(iii) Let the line pass through point $P_0(1, 1, -1)$ and have direction vector $\vec{d} = (2, 1, 1)$.

Let point B(4, 5, 6) be the external point.

Let $\overrightarrow{PB} = \vec{B} - \vec{P_0} = (4-1, 5-1, 6-(-1))$

$= (3, 4, 7)$

The shortest distance D is given by:

$$D = \frac{|\overrightarrow{PB} \times \vec{d}|}{|\vec{d}|}$$

$\overrightarrow{PB} = (3, 4, 7)$, $\vec{d} = (2, 1, 1)$

$$\overrightarrow{PB} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 7 \\ 2 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}(4 \times 1 - 7 \times 1) - \hat{j}(3 \times 1 - 7 \times 2) + \hat{k}(3 \times 1 - 4 \times 2)$$

$$= \hat{i}(-3) - \hat{j}(-11) + \hat{k}(-5) = (-3, 11, -5)$$

Magnitude of cross product:

$$|\overrightarrow{PB} \times \vec{d}| = \sqrt{(-3)^2 + 11^2 + (-5)^2}$$

$$= \sqrt{9 + 121 + 25} = \sqrt{155}$$

Magnitude of direction vector $\vec{d} = (2, 1, 1)$

$$|\vec{d}| = \sqrt{2^2 + 1^2 + 1^2} = \sqrt{4 + 1 + 1} = \sqrt{6}$$

$$D = \frac{\sqrt{155}}{\sqrt{6}} = \sqrt{\frac{155}{6}} \text{ units}$$