CHAPTER 12

Linear Programming

Level - 1

CORE SUBJECTIVE QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQ) (1 Marks)

1. **Option (B) is correct.**

Explanation:

Corner point	Value of the objective function $Z = 4x + 3y$
1. O(0, 0)	$Z = 0$
2. R(40, 0)	$Z = 160$
3. Q(30, 20)	$Z = 120 + 60 = 180$
4. P(0, 40)	$Z = 120$

Since, the feasible region is bounded so the maximum value of the objective function $Z = 180$ is at Q(30, 20).

2. **Option (D) is correct.**

Explanation: The solution exists because the inequality $18x + 10y < 134$ does not has points in common with the feasible region.

At the corner point (3, 8), $Z = 134$

3. **Option (C) is correct.**

Explanation:

The inequality $2x + 3y < 6$ represents an open half-plane.

The boundary line is $2x + 3y = 6$, which is not included (since the inequality is strict).

Testing the origin (0, 0):

$2(0) + 3(0) = 0 < 6$

Since the origin satisfies the inequality, the solution set is the half-plane containing the origin.

4. **Option (D) is correct.**

Explanation:

Corner Points

Since $x \leq 0$ and $y \geq 0$, we check points where constraints meet:

Intersection of $4x + 3y = 12$ and $x \leq 0$

Set $x = 0$:

$4(0) + 3y = 12 \Rightarrow y = 4$

So, (0, 4) is also on this line.

Intersection of $4x + 3y = 12$ and $y = 0$

Set $y = 0$:

$4x = 12 \Rightarrow x = 3$

But since $x \leq 0$, (3, 0) is not in the feasible region.

Thus, the only relevant point is (0, 4).

At (0, 4):

$Z = 3(0) + 5(4) = 20$

Thus, the maximum value of is 20.

5. **Option (B) is correct.**

Explanation: A linear programming problem (LPP) deals with the optimization (maximization or minimization) of a linear function, subject to linear constraints.

The objective function in an LPP is always of the form:

$Z = ax + by + c$

where a, b and c are constants.

6. **Option (A) is correct.**

Explanation: : Given constraints:

$x \leq 0, y \leq 0, x + y \geq 4$

The first two constraints restrict the region to the third quadrant ($x \leq 0, y \leq 0$).

The line $x + y = 4$ does not intersect this quadrant, so no feasible region is formed.

7. **Option (D) is correct.**

Explanation: : The **common region** that satisfies all constraints in an LPP is called the **feasible region**. It contains all possible solutions.

8. **Option (B) is correct.**

Explanation::

The restrictions on the decision variables (e.g., inequalities like $x + y \leq 4$) are called constraints in an LPP.

9. **Option (C) is correct.**

Explanation: :

The two lines in the graph are:

$$x + 2y = 76$$
$$2x + y = 104$$

The shaded region is above the line $x + 2y = 76$, meaning the inequality should be $x + 2y \geq 76$.

The shaded region is below the line $2x + y = 104$, meaning the inequality should be $2x + y \leq 104$.

The region is in the first quadrant, meaning $x, y \geq 0$.

The correct group of inequalities is:

$x + 2y \geq 76, 2x + y \leq 104, x, y \geq 0$

10. **Option (C) is correct.**

Explanation: For the given feasible region with corner points:

A(0, 50), B(20, 30), C(30, 0) and D(0, 0)

Point (x, y)	$Z = 4x + y$	Value of Z
$O(0, 0)$	$4(0) + 0$	0
$A(0, 50)$	$4(0) + 50$	50
$B(20, 30)$	$4(20) + 30$	110
$C(30, 0)$	$4(30) + 0$	120

The maximum value of Z occurs at C(30, 0).
Maximum Z = 120.

11. Option (D) is correct.

Explanation: Solution Set of $2x + y \geq 5$

The boundary line is $2x + y = 5$.

Testing the origin (0, 0):

$$2(0) + 0 \not\geq 5$$

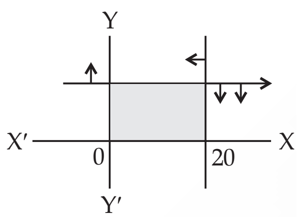
Origin is NOT in the solution set.

The solution set is the open half-plane not containing the origin, but including the boundary line.

12. Option (D) is correct.

Explanation: Given Constraints:

$x \leq 20, y \leq 10, x \geq 0, y \geq 0$



Corner Points:

(0, 10), (20, 10), (20, 0) and (0, 0)

Evaluate Z at Each Corner Point:

Point (x, y)	$Z = 3x + 8y$	Value of Z
(0, 10)	$3(0) + 8(10)$	80
(20, 10)	$3(20) + 8(10)$	140
(20, 0)	$3(20) + 8(0)$	60
(0, 0)	$3(0) + 8(0)$	0

Minimum value is 60 at (20, 0).

13. Option (C) is correct.

Explanation: : Corner Points:

(2, 72), (15, 20), (40, 15)

Evaluate Z at Each Corner Point:

Point (x, y)	$Z = 18x + 9y$	Value of Z
(2, 72)	$18(2) + 9(72)$	684
(15, 20)	$18(15) + 9(20)$	450
(40, 15)	$18(40) + 9(15)$	855

Maximum at (40, 15), Minimum at (15, 20).

14. Option (B) is correct.

Explanation: Given Constraints:

$x - y \geq 0 \Rightarrow x \geq y$

$2y \leq x + 2 \Rightarrow x - 2y \geq -2$

These lines form a bounded region in the first quadrant.

Finding intersection points gives 3 corner points.

15. Option (C) is correct.

Explanation:

Given:

$$Z = ax + by, Z(4, 6) = 42, Z(3, 2) = 19$$

Form equations:

$$4a + 6b = 42$$
$$3a + 2b = 19$$

Solve for a, b:

Multiplying the second equation by 3:

$$9a + 6b = 57$$

Subtracting from the first equation:

$$(4a + 6b) - (9a + 6b) = 42 - 57$$
$$-5a = -15 \Rightarrow a = 3$$

Substitute $a = 3$ in $3(3) + 2b = 19$:

$$9 + 2b = 19 \Rightarrow 2b = 10 \Rightarrow b = 5$$

16. Option (C) is correct.

Explanation: Solution Set of $3x + 5y < 7$

Boundary Line: $3x + 5y = 7$

Testing the Origin (0, 0):

$$3(0) + 5(0) = 0 < 7$$

$\therefore$ Origin lies in the solution set.

The solution is the open half-plane containing the origin, excluding the boundary line.

17. Option (D) is correct.

Explanation:

Point (x, y)	$2x + y \leq 10$	$x + 2y \geq 8$	Satisfies Both
(–2, 4)	$2(-2) + 4 = 0 \leq 10$	$-2 + 2(4) = 6 \not\geq 8$	No
(3, 2)	$2(3) + 2 = 8 \leq 10$	$3 + 2(2) = 7 \not\geq 8$	No
(–5, 6)	$2(-5) + 6 = -4 \leq 10$	$-5 + 2(6) = 7 \not\geq 8$	No
(4, 2)	$2(4) + 2 = 10 = 10$	$4 + 2(2) = 8 = 8$	Yes

18. Option (B) is correct.

Explanation: Given inequality:

$$2x + y \leq 4$$

Point (x, y)	LHS = $2x + y$	Check ≤ 4
(0, 8)	$2(0) + 8 = 8$	(Not in region)
(1, 1)	$2(1) + 1 = 3$	(In region)
(5, 5)	$2(5) + 5 = 15$	(Not in region)
(2, 2)	$2(2) + 2 = 6$	(Not in region

19. Option (A) is correct.

Explanation: We have the given objective function:

$$Z = 2ax + by$$

and the maximum value occurs at both points $A(250, 0)$ and $B(200, 50)$, meaning:

$$Z(A) = 2a(250) + b(0) = 500a$$
$$Z(B) = 2a(200) + b(50) = 400a + 50b$$

Since both give the same maximum value, we set them equal:

$$500a = 400a + 50b$$
$$100a = 50b$$
$$2a = b$$

ASSERTION-REASON QUESTIONS (1 Marks)

1. Option (B) is correct.

Explanation: Value of Z at corner points of feasible region

$$Z = x + 2y$$

At $P(60, 0)$, $Z = 60 + 2(0) = 60$

At $Q(120, 0)$, $Z = 120 + 2(0) = 120$ (Max.)

At $R(60, 30)$, $Z = 60 + 2(30) = 120$ (Max.)

At $S(40, 20)$, $Z = 40 + 2(20) = 80$

Since maximum value of Z occurs at two points Q and R, hence it occurs at all the points joining the line QR i.e., infinite points.

Reason (R) is true as optimal solutions in a bounded LPP occur at corner points, but this does not explain why there are infinitely many maximum points.

2. Option (D) is correct.

Explanation: Assertion is false.

Not all points in the feasible region provide the optimal solution.

Only the corner points of the feasible region need to be checked to find the optimal value of the objective function.

Interior points satisfy constraints but may not optimize the objective function.

Reason is true.

By definition, the feasible region consists of all points that satisfy the given constraints.

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. Writes that when $Z > ax_0 + by_0$ has at least one point common with the feasible region, the optimal solution of this problem does not exist.

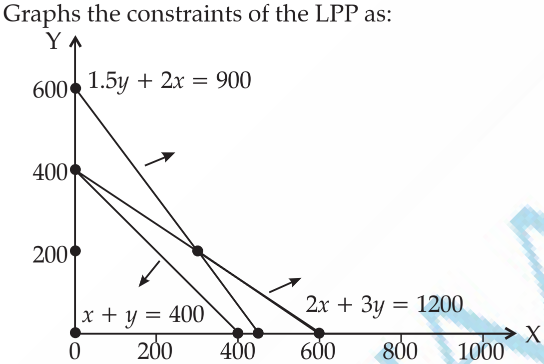
   Gives reason that when $Z > ax_0 + by_0$ has at least one point common with the feasible region, it means that there is at least one non-corner point (x_1, y_1) in the feasible region such that $ax_1 + by_1 > ax_0 + by_0$.

   Further corner that Z has no maximum value at (x_0, y_0) and thus the optimal solution of this problem does not exist as a maximum value, if exists, has to be obtained at a corner point.

2. Writes that, since the minimum of Z occurs at both (50, 30) and (20, 40), $50a + 30b = 20a + 40b$.

   Solves the above equation to find the relationship between a and b as $3a = b$.

3. Graphs the constraints of the LPP as:



   Argues that, since no overlapping region exists for the LPP, there is no optimal solution in this case.

4. Uses the given points on the graph to find the following constraints using any suitable method:

$$2x + 3y \le 120$$
$$2x + y \le 60$$

   Writes the remaining constraints by observing the graph of the feasible region as:

$$x \ge 0$$
$$y \ge 0$$

5. Writes that the given LPP has no maximum value/no optimal solution.

   Justifies the answer by evaluating the objective function $Z = 10x + 20y$ at each corner point and finding the largest value of Z as 80, which is obtained at the point (0, 4).

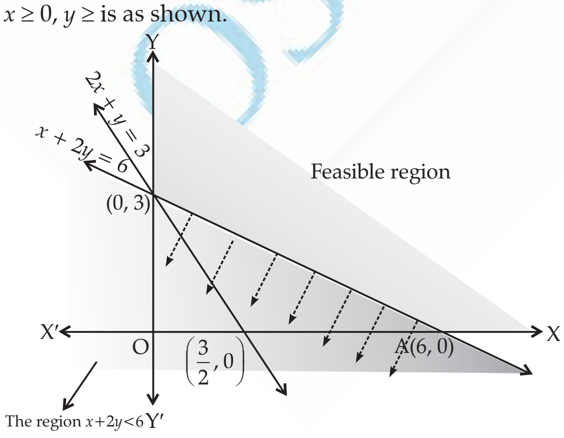
   Writes that the open plane determined by $10x + 20y > 80$ has points in common with the feasible region and hence the given LPP has no maximum value.

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. The feasible region determined by the constraints,

$$2x + y \ge 3$$
$$x + 2y \ge 6$$

   $x \ge 0, y \ge$ is as shown.



The corner points of the unbounded feasible region are A(6, 0) and B(0, 3).

The values of Z at these corner pointse are as follows:

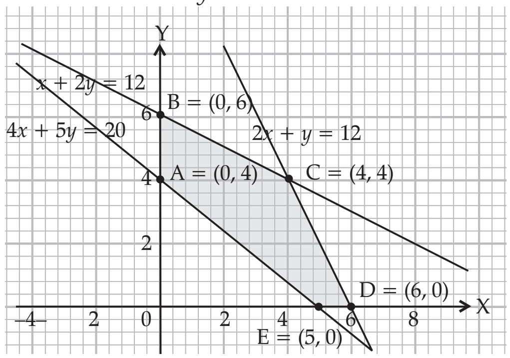
Corner points	Value of the objective function $Z = x + 2y$
A(6, 0)	6
B(0, 3)	6

We observe the region $x + 2y < 6$ have no points in common with the unbounded feasible region. Hence the minimum value of $Z = 6$.

It can be seen that the value of Z at points A and B is same. If we take any other point on the line $x + 2y = 6$ such as (2, 2) on only $x + 2y = 6$, then $Z = 6$.

Thus, the minimum value of Z occurs for more than 2 points, and is equal to 6.

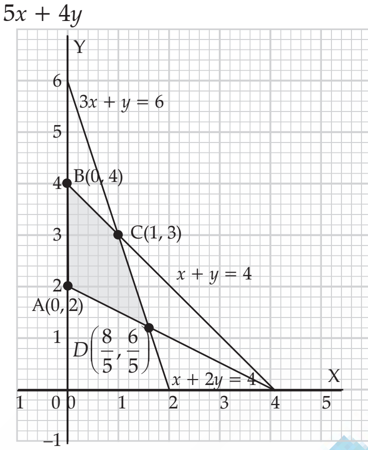
2. Max Z = $500x + 300y$



Corner point	Z = $500x + 300y$
A (0, 4)	1200
B (0, 6)	1800
C (4, 4)	3200
D (6, 0)	3000
E (5, 0)	2500

Max Z = 3200 at $x = 4, y = 4$

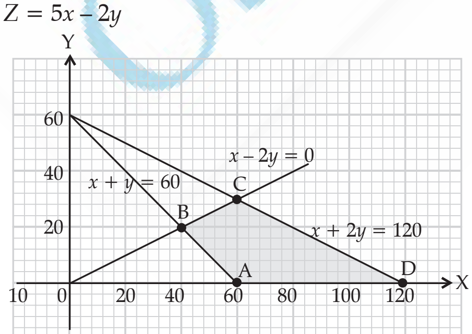
3. Max $z = 5x + 4y$



Corner Point	Z = $5x + 4y$
A(0, 2)	8
B(0, 4)	16
C(1, 3)	17
$D\left(\frac{8}{5}, \frac{6}{5}\right)$	$\frac{64}{5}$

Maximum z = 17, at $x = 1, y = 3$

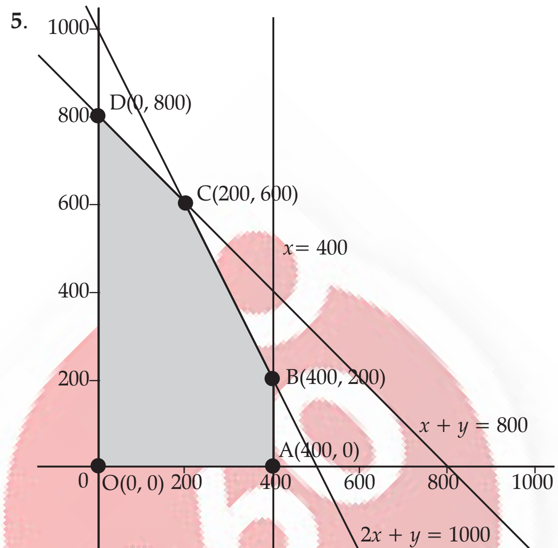
4. Min Z = $5x - 2y$



Corner Points	Z = $5x - 2y$
A(60, 0)	300
B(40, 20)	160
C(60, 30)	240
D(120, 0)	600

Min Z = 160 at $x = 40, y = 20$

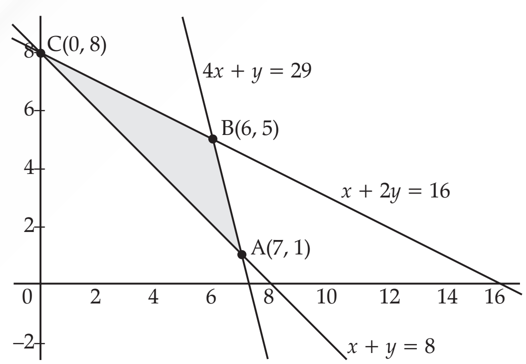
5.



Corner Point	Value of Z = $4x + 3y$
O(0, 0)	0
A(400, 0)	1600
B(400, 200)	2200
C(200, 600)	2600
D(0, 800)	2400

Z_{max} = 2600 when $x = 200, y = 600$

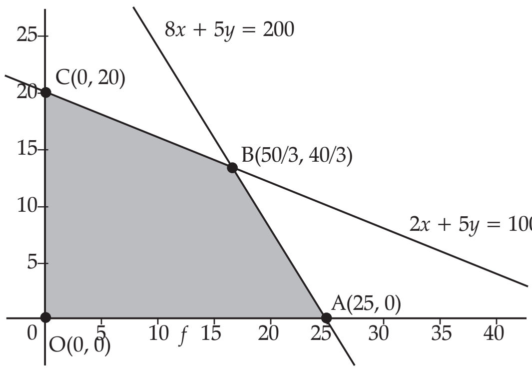
6.



Corner point	Value of $z = 600x + 400y$
A(7, 1)	4600
B(6, 5)	5600
C(0, 8)	3200

Z_{min} = 3200 when $x = 0, y = 8$

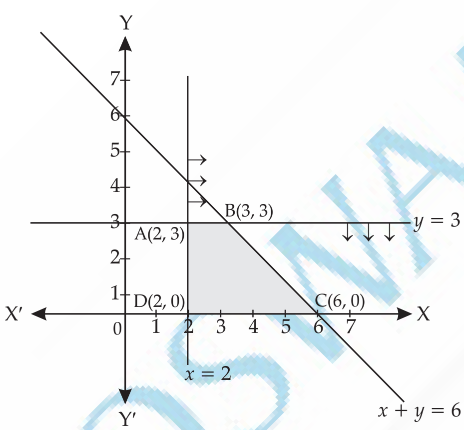
7.



Corner Point	Value of $z = x + y$
O(0, 0)	0
A(25, 0)	25
$B\left(\frac{50}{3}, \frac{40}{3}\right)$	30
C(0, 20)	20

$Z_{max} = 30$ when $x = \frac{50}{3}, y = \frac{40}{3}$

8. On plotting the graph of $x + y \leq 6, x \geq 2, y \leq 3$ & $x \geq 0$, $y \geq 0$ we get the following graph and common shaded region is the region ABCD.

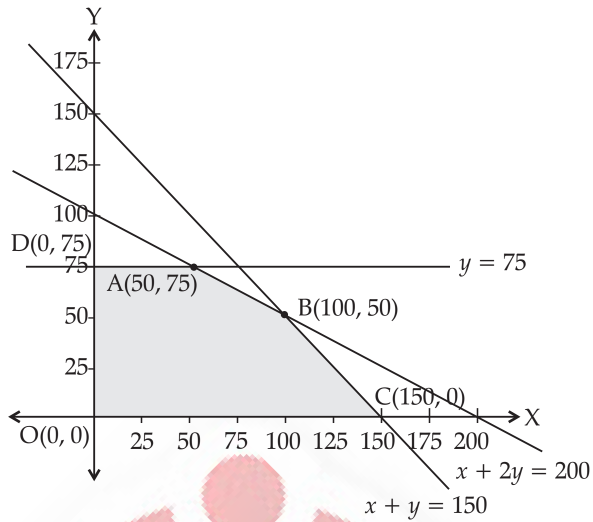


Now, Corner points of the common shaded region are A(2, 3), B(3, 3), C(6, 0) & D(2, 0). Thus,

Corner points	Value of Z = $2x + 3y$
A(2, 3)	13
B(3, 3)	15
C(6, 0)	12
D(2, 0)	4

So, maximum value of Z is 15 at $x = 3, y = 3$.

9.



Corner Point	Value of $Z = x + 3y$
O(0, 0)	Z = 0
A(50, 75)	Z = 50 + 225 = 275 (Max.)
B(100, 50)	Z = 100 + 150 = 250
C(150, 0)	Z = 150

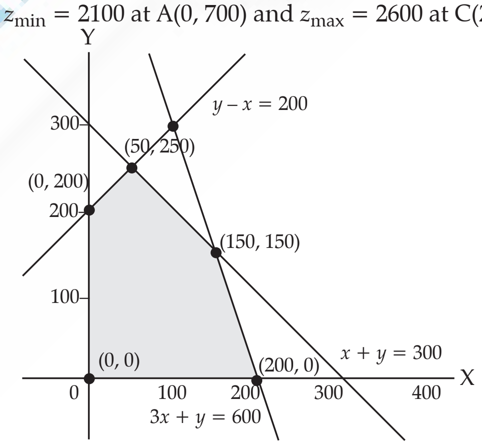
Therefore, Z is maximum at (50, 75) and maximum value of Z is 275.

10.

Corner Point	Value of $z = 4x + 3y$
A(0, 700)	2100
B(100, 700)	2500
C(200, 600)	2600
D(400, 200)	2200

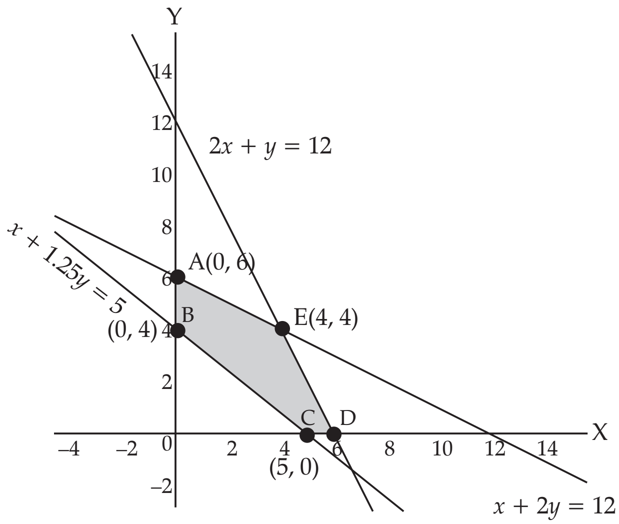
$z_{min} = 2100$ at A(0, 700) and $z_{max} = 2600$ at C(200, 600)

11.



Corner points	Value of Z = $100x + 5y$
(0, 0)	0
(200, 0)	20000 → Maximum
(150, 150)	15750
(50, 250)	6250
(0, 200)	1000

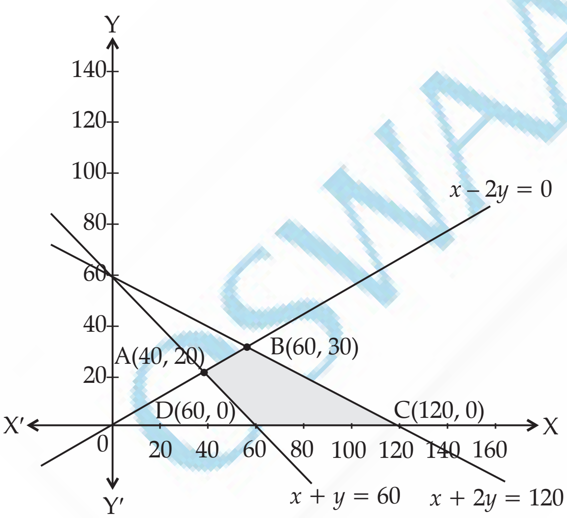
12.



Corner Points	Value of Z = 600x + 400y
(0, 4)	1600
(0, 6)	2400
(4, 4)	4000 → Maximum
(5, 0)	3000
(6, 0)	3600

Maximum value of Z is 4000 and it occurs at the point $x = 4$ and $y = 4$.

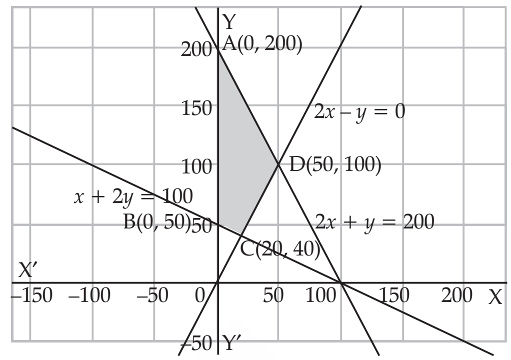
13.



Corner Points	Value of $Z = 5x + 10y$
A(40, 20)	400
B(60, 30)	600
C(120, 0)	600
D(60, 0)	300 (Min)

Min (Z) = 300 at $x = 60$; $y = 0$

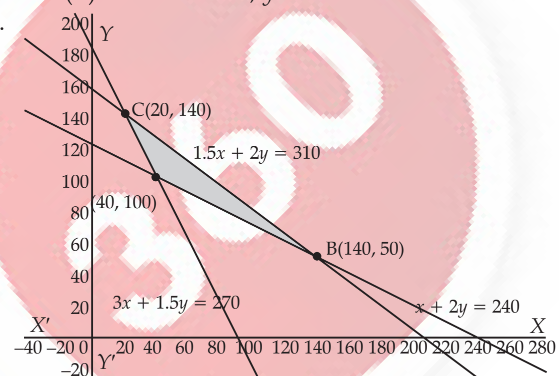
14.



Corner points	Value of $Z = x + 2y$
A(0, 200)	400 (Max)
B(0, 50)	100
C(20, 40)	100
D(50, 100)	250

Max (Z) = 400 at $x = 0$; $y = 200$

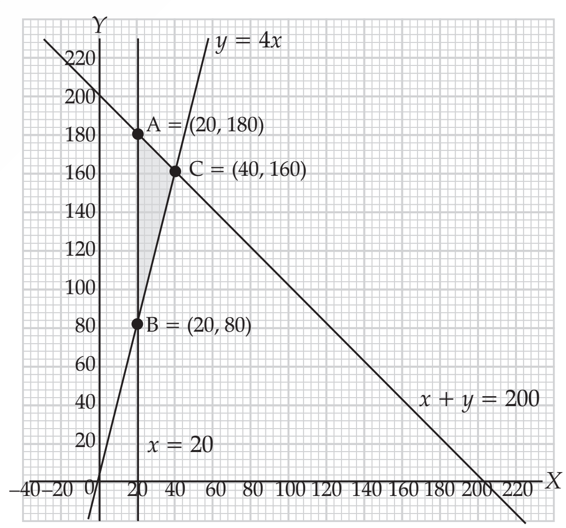
15.



Corner points	Value of $Z = 3x + 3.5y$
A(40, 100)	470
B(140, 50)	595 (Max)
C(20, 140)	550

Max(Z) = 595 at $x = 140$; $y = 50$

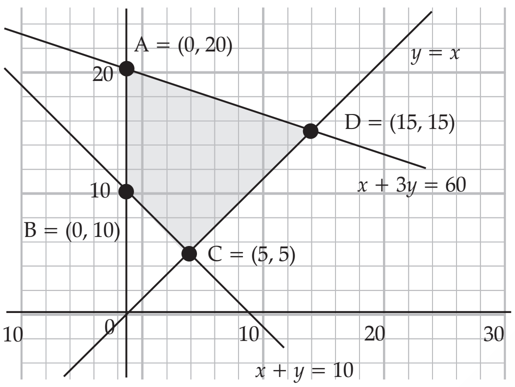
16.



Corner Points	Value of Z = 500x + 400y
A(20, 180)	82000
B(40, 160)	84000
C(20, 80)	42000 → Minimum

$Z_{min} = 4200$ at $x = 20$ and $y = 80$.

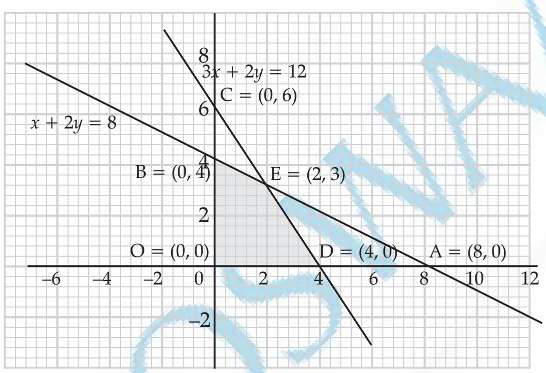
17.



Corner Points	Value of Z = 3x + 9y
A(0, 20)	180 → Maximum
B(0, 10)	90
C(5, 5)	60
D(15, 15)	180 → Maximum

Maximum lies at every point on the line segment AD.

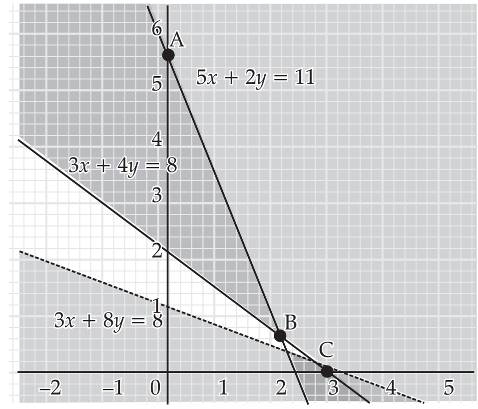
18.



Corner Points	Value of Z = –3x + 4y
O(0, 0)	0
B(0, 4)	16
D(4, 0)	–12 → Min
E(2, 3)	6

$Z_{min} = -12$ at D(4, 0).

19.

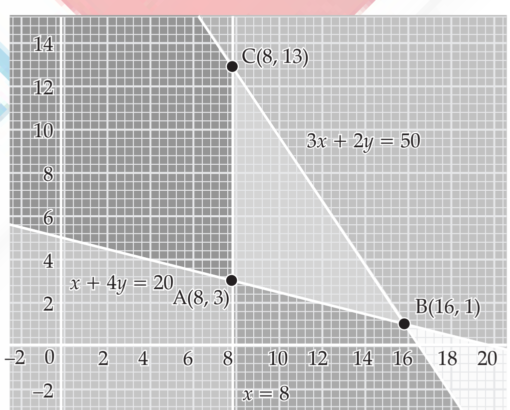


Corner Point	$Z = 3x + 8y$
$A\left(0, \frac{11}{2}\right)$	44
$B\left(2, \frac{1}{2}\right)$	10
$C\left(\frac{8}{3}, 0\right)$	8

Since $3x + 8y < 8$ do not have any point in common with the feasible region.

$Z_{min} = 8$ when $x = \frac{8}{3}, y = 0$

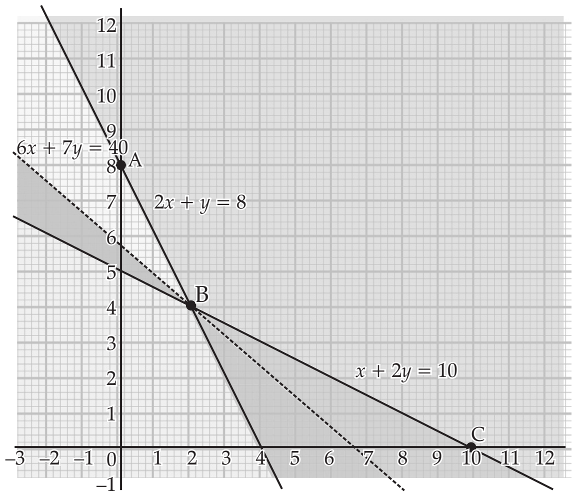
20.



Corner Point	$Z = 10x + 15y$
A(8, 3)	125
B(16, 1)	175
C(8, 13)	275

Z_{max} is 275 when $x = 8, y = 13$

21.



Corner Point	$z = 6x + 7y$
A(0, 8)	56
B(2, 4)	40
C(10, 0)	60

Since $6x + 7y < 40$ do not have any point in common with the feasible region.

$Z_{min} = 40$ when $x = 2, y = 4$

22. **(i)** Uses the graph of the feasible region and lists the consraints of the given maximisation problem as:

$$3x + 2y \leq 12$$
$$x + 2y \leq 8$$
$$x, y \geq 0$$

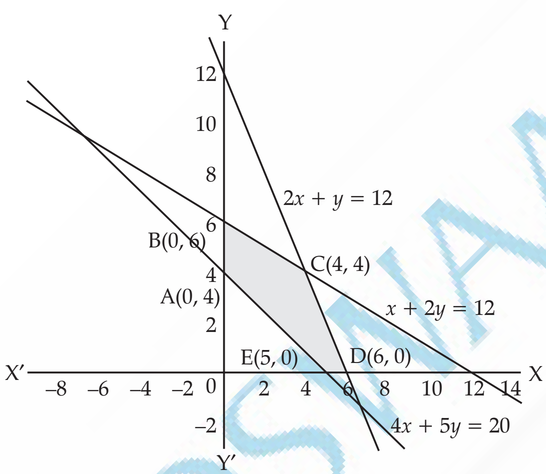
(ii) Finds the value of the objective function at corner points as:

Corner point	$Z = 5x + 3y$
(0, 0)	0
(0, 4)	12
(2, 3)	19
(4, 0)	20

Concludes that the objective function attains maximum value at (4, 0) and hence (4, 0) is the optimal solution.

LONG ANSWER TYPE QUESTIONS (5 Marks)

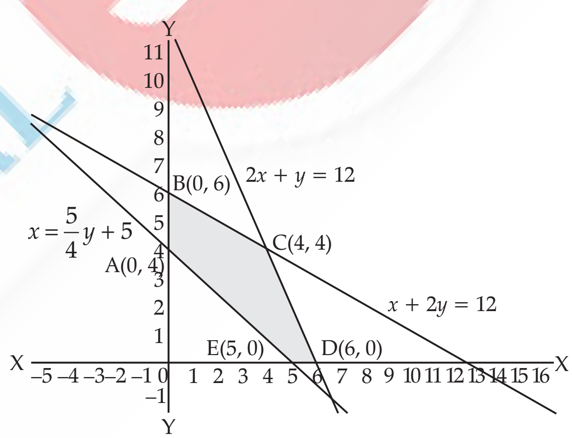
1.



Corner Points	Value of Z = $60x + 40y$
A(0, 4)	Z = 160
B(0, 6)	Z = 240
C(4, 4)	Z = 400 (Max)
D(6, 0)	Z = 360
E(5, 0)	Z = 300

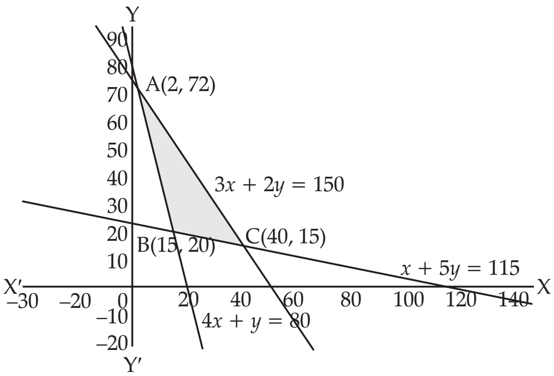
Max (Z) = 400 at $x = 4, y = 4$

2.



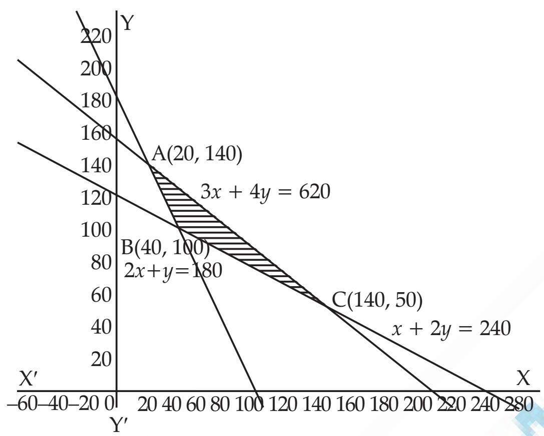
Corner Points	Value of Z = $300x + 600y$
A(0, 4)	Z = 2400
B(0, 6)	Z = 3600
C(4, 4)	Z = 3600
D(6, 0)	Z = 1800
E(5, 0)	Z = 1500

Max (Z) = 3600 at all points on the line segment BC.

3. 

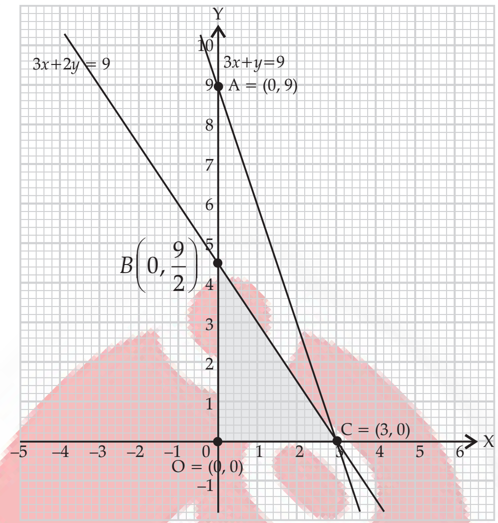
Corner Points	Value of Z = 6x + 3y
A(2, 72)	Z = 228
B(15, 20)	Z = 150
C(40, 15)	Z = 285

Max (Z) = 285 at $x = 40$, $y = 15$

4. 

Corner Points	Value of Z = 6x + 7y
A(20, 140)	Z = 1100
B(40, 100)	Z = 940
C(140, 50)	Z = 1190

Min (Z) = 940 at $x = 40$, $y = 100$

5. 

Corner Points	Value of $Z = 70x + 40y$
O(0, 0)	0
B(0, 4.5)	180
C(3, 0)	210 → Max Value

Maximum value of $Z = 210$ at $x = 3$ and $y = 0$.

6. Z(A) = 90; Z(B) = 60; Z(C) = 180; Z(D) = 180

Min(Z) = 60 at B(5, 5); Max(Z) = 180 at C(15, 15) and D(0, 20)

Level - 2 ADVANCED COMPETENCY FOCUSED QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Marks)

1. Option (B) is correct.

Explanation: Value of $Z = 10x + 3y$

At point $J(8, 10)$:

$$Z_J = 10(8) + 3(10) = 110$$

At point $K(4, 5)$:

$$Z_K = 10(4) + 3(5) = 40 + 15 = 55$$

At point $L(9.6, 0)$:

$$Z_L = 10(9.6) + 3(0) = 96$$

Thus, minimum occurs at Point K.

2. Option (A) is correct.

Explanation: Let x be servings of A and y be servings of B.

So the total protein from x servings of A and y servings of B is:

$3x + 2y$ and the minimum requirement is 12 units, so the constraint is $3x + 2y \geq 12$.

3. Option (B) is correct.

Explanation: Let x be number of tables and y be number of chairs.

So, the total carpentry time used is $4x + 3y$

Since the total time used must not exceed 240 hours, the constraint is $4x + 3y \leq 240$

4. Option (D) is correct.

Explanation: In a Linear Programming Problem, if the feasible region is bounded, the maximum or minimum value of the objective function $Z = ax + by$ will always occur at one of the corner (vertex) points of the feasible region. This is a fundamental result in linear programming, known as the Corner Point Theorem, which states, "If an optimum value (maximum or minimum) of a linear objective function exists, it will occur at a vertex (corner point) of the feasible region".

5. Option (C) is correct.

Explanation: The objective is to maximise total profit, so the objective function is:

$$Z = 50x + 70y$$

ASSERTION-REASON QUESTIONS (1 Marks)

1. **Option (A) is correct.**

   Explanation: Assertion is true. In Linear Programming, maximum or minimum of the objective function always occurs at one or more corner (vertex) points of the feasible region.

   Reason is also true: The feasible region, formed by intersecting linear inequalities, is a convex polygon. In convex sets, a linear function attains its extreme values at the boundary, typically at the corners.

2. **Option (D) is correct.**

   Explanation: Assertion is false. An unbounded feasible region does not necessarily mean that a maximum value does not exist. It is possible for a linear programming problem to have a maximum value even in an unbounded region, provided the objective function does not increase indefinitely in that unbounded direction.

   Reason is true in general, as an unbounded region may allow unlimited increase.

3. **Option (A) is correct.**

   Explanation: Assertion is true. Graphical solution methods for linear programming are only applicable when there are two variables. With more than two variables, graphical representation becomes impractical or impossible.

   Reason is also true. Graphs can only represent two dimensions on a flat surface (x and y axes). Hence, only two decision variables can be handled visually.

4. **Option (D) is correct.**

   Explanation: Assertion is false. Linear inequalities do not always produce a feasible region that is a straight line. Instead, they define a half-plane, and the feasible region is usually a polygonal region (bounded or unbounded), not just a line.

   Reason is true. Each linear inequality does represent a linear constraint and forms one of the boundaries of the feasible region, typically in the form of a straight line.

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. Writes that Sarla's claim is false.

   Give a reason. For example, every point on the line joining $\left(\frac{2}{3}, \frac{7}{3}\right)$ and (2, 1) gives the maximum value of Z which is 3. Hence, any point on the line joining $\left(\frac{2}{3}, \frac{7}{3}\right)$ and (2, 1) is an optimal solution.

2. Writes that the statement is true.

   Gives a valid reason. For example, if two adjacent corner points yield the optimal value for the objective function, then every point on the line joining them also yields the optimal value.

3. Writes that Suhas is incorrect.

   Reason that since the feasible area is unbounded, the minimum value of the objective function will exist only if the graph of $4x - 3y < (-20)$ has no point in common with the feasible region. Shows as an example that (1, 10) satisfies the inequality $4x - 3y < (-20)$ and is in common with the feasible region and hence, the minimum value of Z does not exist.

4. Writes that he is incorrect.

   Finds the value of the objective function at the corner points (3, 2) and (4, 0) as 5 and 4 respectively, and writes that $x + y > 5$ is overlapping with the feasible region and hence, no maximum value exists.

5. Writes true.

   Justifies that in certain linear programming problems, the maximum/minimum value is obtained at 2 distinct corner points of the feasible region. In such cases, every point on the line joining those two points will be an optimal solution. Hence, a linear programming problem can have infinitely many optimal solutions.

6. Writes False (F).

   Writes that, if on graphing the objective function, the open half-plane does not have any points common with the feasible region then the linear programming problem has an optimal solution.

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. Let x be number of butter cookies baked per day and y be number of chocolate cookies baked per day.

   Profit from butter cookies = ₹ 5 per cookie and Profit from chocolate cookies = ₹ 8 per cookie

   So, maximise:

   $$Z = 5x + 8y$$

   Constraints: Time required for baking: Butter cookie: 20 minutes and Chocolate cookie: 30 minutes

   Oven available: 5 hours = 300 minutes

   $$20x + 30y \leq 300$$

   Non-negativity constraints:

   $$x \geq 0, y \geq 0$$

   $$\text{Maximise } Z = 5x + 8y$$

   Subject to $20x + 30y \leq 300$

   $$x \geq 0, y \geq 0$$

2. Let x be number of indoor activities and y be number of outdoor activities

   Objective Function (Maximise total number of activities):

   Each indoor or outdoor activity counts as one activity. So, the total number of activities is:

   $$Z = x + y$$

Constraints: Time Constraint: Indoor activity takes 2 hours, Outdoor activity takes 4 hours, Total time available: 40 hours

$$2x + 4y \le 40$$

Budget Constraint: Cost per indoor activity: ₹ 200, Cost per outdoor activity: ₹ 500, Total budget: ₹ 4000

$$200x + 500y \le 4000$$

Non-negativity: $x \ge 0, y \ge 0$

Maximize $Z = x + y$

Subject to $2x + 4y \le 40$

$200x + 500y \le 4000$

$x \ge 0, y \ge 0$

3. Let x be number of units of wheat and y be number of units of rice

Objective Function (Maximise profit): Profit per unit of wheat = ₹ 40 and Profit per unit of rice = ₹ 60

So, the total profit: $Z = 40x + 60y$ (to be maximised)

Constraints: Land Constraint: Wheat requires 3 m^2/unit, Rice requires 2 m^2/unit, Total available land = 2400 m^2

$$3x + 2y \le 2400$$

Minimum Rice Requirement: At least 100 units of rice must be grown

$$y \ge 100$$

Non-negativity: $x \ge 0, y \ge 0$

Maximise: $Z = 40x + 60y$

Subject to: $3x + 2y \le 2400$

$y \ge 100$

$x \ge 0, y \ge 0$

4. Let x be number of units of Product P1 to be produced and y be number of units of Product P2 to be produced

Objective Function (Maximise profit):

Profit per unit of P1 = ₹ 50

Profit per unit of P2 = ₹ 40

So, the total profit:

$$Z = 50x + 40y \text{ (to be maximised)}$$

Constraints: Assembly Time: P1 needs 1 hour/unit and P2 needs 2 hours/unit

Maximum assembly time available = 8 hours

$$x + 2y \le 8$$

Packaging Time: P1 needs 2 hours/unit and P2 needs 1 hour/unit

Maximum packaging time available = 6 hours

$$2x + y \le 6$$

Non-negativity:

$$x \ge 0, y \ge 0$$

Maximise: $Z = 50x + 40y$

Subject to:

$x + 2y \le 8$

$2x + y \le 6$

$x \ge 0, y \ge 0$

CASE BASED QUESTIONS (4 Mark)

1. (i) Constraints are $x + 2y \ge 10$

$x + y \ge 6$

$3x + y \ge 8$

$x \ge 0$

$y \ge 0$

(ii)

Corner points	Value of $Z = 16x + 20y$
A(10, 0)	160
B(2, 4)	112
C(1, 5)	116
D(0, 8)	160

The minimum cost is ₹ 112.

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. Let x be number of apples bought and sold per day and y be number of bananas bought and sold per day

Objective Function (Maximise profit): Profit per apple = ₹ 4 and Profit per banana = ₹ 2

So, total daily profit: $Z = 4x + 2y$ (to be maximised)

Constraints: Storage Capacity: He can store at most 100 fruits (apples + bananas):

$$x + y \le 100$$

Budget Constraint: Cost of apples = ₹ 10 each, bananas = ₹ 6 each, and budget is ₹ 800:

$$10x + 6y \le 800$$

Non-negativity constraints:

$$x \ge 0, y \ge 0$$

Maximise: $Z = 4x + 2y$

Subject to:

$x + y \le 100$

$10x + 6y \le 800$

$x \ge 0, y \ge 0$

Solving it by finding the feasible region and evaluating $Z = 4x + 2y$ at the corner points of that region.

1. From $x + y = 100$:

   when $x = 0 \Rightarrow y = 100$

   when $y = 0 \Rightarrow x = 100$

2. From $10x + 6y = 800$:

   $5x + 3y = 400$

   when $x = 0 \Rightarrow y = \frac{400}{3} = 133.33$

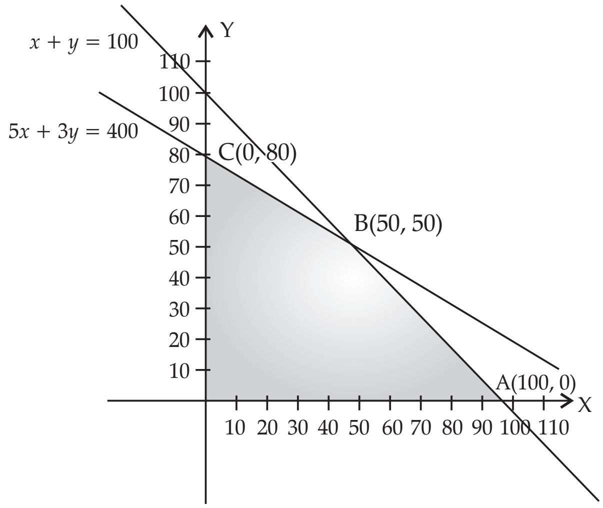
   when $y = 0 \Rightarrow x = 80$

$$x + y = 100 \Rightarrow y = 100 - x$$

Substitute into $5x + 3y = 400$:

$$5x + 3(100 - x) = 400 \Rightarrow 5x + 300 - 3x = 400$$

$$\Rightarrow \quad 2x = 100 \Rightarrow x = 50, y = 50$$



Point (x, y)	Cost $Z = 4x + 2y$
(0, 100)	$4(0) + 2(100) = 200$
(80, 0)	$4(80) + 2(0) = 320$
(50, 50)	$4(50) + 2(50) = 200 + 100 = 300$

Maximum Profit occurs at $(80, 0) \rightarrow$ 80 apples, 0 bananas

Maximum daily profit = ₹320

2. Let x be number of packets of food P and y be number of packets of food Q

Objective Function (Minimise Cost): Cost of food P = ₹ 50/packet and Cost of food Q = ₹ 40/packet

So, total cost $Z = 50x + 40y \rightarrow$ to be minimised

Constraints: Protein requirement: Food P provides 3 units/packet and Q provides 2 units/packet

Total protein needed $\geq$ 12 units

$$3x + 2y \geq 12$$

Fat requirement: Food P provides 2 units/packet and Q provides 4 units/packet

Total fat needed $\geq$ 16 units

$$2x + 4y \geq 16$$

Non-negativity: $x \geq 0, y \geq 0$

Minimise: $Z = 50x + 40y$

Subject to:

$$3x + 2y \geq 12 \text{(Protein constraint)}$$

$$2x + 4y \geq 16 \text{(Fat constraint)}$$

$$x \geq 0, y \geq 0$$

Solving the system of constraints to find the feasible region and evaluate the cost at each corner point.

From $3x + 2y = 12$

when $x = 0, y = 6$

when $y = 0, x = 4$

From $2x + 4y = 16$

when $x = 0, y = 4$

when $y = 0, x = 8$

Now finding the point of intersection of the two lines:

$$3x + 2y = 12 \quad \text{(i)}$$

$$2x + 4y = 16 \quad \text{(ii)}$$

Multiply (i) by 2:

$$6x + 4y = 24$$

Subtract (ii):

$$(6x + 4y) - (2x + 4y) = 24 - 16 \Rightarrow 4x = 8 \Rightarrow x = 2$$

Substituting $x = 2$ into (i):

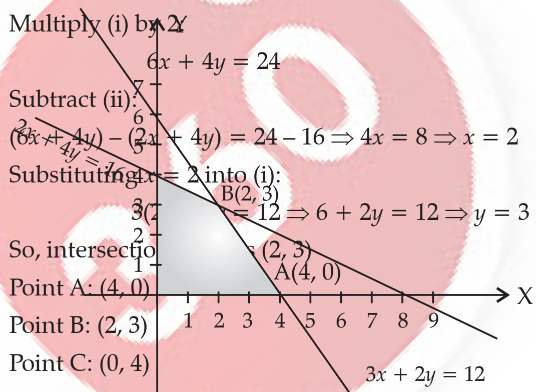
$$3(2) + 2y = 12 \Rightarrow 6 + 2y = 12 \Rightarrow y = 3$$

So, intersection point is (2, 3)

Point A: (4, 0)

Point B: (2, 3)

Point C: (0, 4)



Evaluating Cost $Z = 50x + 40y$ at each point

Point (x, y)	Cost $Z = 50x + 40y$
A (4, 0)	$50 \times 4 + 40 \times 0 = 200$
B (2, 3)	$50 \times 2 + 40 \times 3 = 100 + 120 = 220$
C (0, 4)	$50 \times 0 + 40 \times 4 = 160$

Minimum Cost = ₹ 160 at (0, 4)

The dietician should use 0 packets of food P and 4 packets of food Q and this satisfies all nutritional requirements at the minimum cost of ₹ 160.