CHAPTER 3

Matrices

Level - 1 CORE SUBJECTIVE QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQ) (1 Mark)

1. **Option (A) is correct.**

Explanation:

$$\begin{array}{cccc} P & Y & W & Y \\ \downarrow \text{Order} & \downarrow \text{Order} & \downarrow \text{Order} & \downarrow \text{Order} \\ p \times k & 3 \times k & n \times 3 & 3 \times k \end{array}$$

For PY to exist $\quad$ Order of WY

$k = 3 \quad = n \times k$

Order of $PY = p \times k$

For $PY + WY$ to exist, order (PY) = order (WY)

$$p \times k = n \times k$$

$$\therefore p = n$$

2. **Option (A) is correct.**

Explanation: $A = \begin{bmatrix} 0 & 1 & c \\ -1 & a & -b \\ 2 & 3 & 0 \end{bmatrix}$

It matrix A is skew symmetric then $A^T = -A \Rightarrow a_{ij} = -a_{ji}$;
$\Rightarrow c = -2$; $a = 0$ and $b = 3$

So, $a + b + c = 0 + 3 - 2 = 1$.

3. **Option (D) is correct.**

Explanation: A matrix with 36 elements has an order of $m \times n$, where m is the number of rows and n is the number of columns, and their product must be 36:

$$m \times n = 36$$

All possible pairs (m, n) where their product is 36 are (1, 36), (2, 18), (3, 12), (4, 9), (6, 6), (9, 4), (12, 3), (18, 2), (36, 1)

These are 9 possible orders.

4. **Option (D) is correct.**

Explanation: Given the matrix equation:

$$\begin{bmatrix} x+y & 2 \\ 5 & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

Equate Elements

$$x + y = 6$$
$$xy = 8$$

Now, $\dfrac{24}{x} + \dfrac{24}{y}$

$$\frac{24}{x} + \frac{24}{y} = 24\left(\frac{x+y}{xy}\right) = 24 \times \frac{6}{8} = 18$$

5. **Option (B) is correct.**

Explanation: Given:

$$(A + B)^2 = A^2 + B^2$$
$$\Rightarrow (A + B)(A + B) = A^2 + B^2$$
$$\Rightarrow A^2 + AB + BA + B^2 = A^2 + B^2$$
$$\Rightarrow AB + BA = 0$$
$$\Rightarrow AB = -BA$$

6. **Option (D) is correct.**

Explanation: We are given:

Matrix A is of order 1×3

Matrix B is of order 3×1

The transpose of A, denoted A', will have order 3×1

The transpose of B, denoted B', will have order 1×3

Matrix multiplication is defined when the number of columns in the first matrix matches the number of rows in the second matrix.

A' is 3×1

B' is 1×3

Multiplying $(3 \times 1) \times (1 \times 3)$ results in a 3×3 matrix.

7. **Option (A) is correct.**

Explanation: A 3×3 scalar matrix has the form:

$$A = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$$

Since non-diagonal elements are 0, their product remains 0. Thus, the product of all elements is 0.

8. **Option (C) is correct.**

Explanation: We are given a 3×3 matrix $A = [a_{ij}]$, where:

$$a_{ij} = i - 3j$$

Required Elements

(A) a_{11}:

$$a_{11} = 1 - 3(1) = 1 - 3 = -2$$

Since $a_{11} = -2 < 0$, statement $a_{11} < 0$ (option A) is true.

(B) $a_{12} + a_{21}$:

$$a_{12} = 1 - 3(2) = 1 - 6 = -5$$
$$a_{21} = 2 - 3(1) = 2 - 3 = -1$$
$$a_{12} + a_{21} = -5 + (-1) = -6$$

Since $a_{12} + a_{21} = -6$, option (B) is true.

(C) $a_{13} > a_{31}$:

$$a_{13} = 1 - 3(3) = 1 - 9 = -8$$

$$a_{31} = 3 - 3(1) = 3 - 3 = 0$$
$$-8 > 0 \text{ (false)}$$

Since $a_{13} \not> a_{31}$, option (C) is false.

(D) $a_{31} = 0$:

From above, $a_{31} = 0$ is true, so option (D) is true.

9. Option (B) is correct.

Explanation: Given:

$$F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Squaring $F(x)$:

$$[F(x)]^2 = \begin{bmatrix} \cos 2x & -\sin 2x & 0 \\ \sin 2x & \cos 2x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and
$$F(kx) = \begin{bmatrix} \cos kx & -\sin kx & 0 \\ \sin kx & \cos kx & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Given $F(x)^2 = F(kx)$, we compare:

Thus, $kx = 2x$ gives $k = 2$.

10. Option (B) is correct.

Explanation: There are 3 possible scalar matrices of order 3 with entries $-1, 0$ or 1.

11. Option (C) is correct.

Explanation: Since,

$$A = \begin{bmatrix} \tan x & 1 \\ -1 & \tan x \end{bmatrix}$$

$$A' = \begin{bmatrix} \tan x & -1 \\ 1 & \tan x \end{bmatrix}$$

Given

$$A + A' = 2\sqrt{3}I$$

$$A + A' = \begin{bmatrix} 2\tan x & 0 \\ 0 & 2\tan x \end{bmatrix} = 2\sqrt{3}I$$

$$= \begin{bmatrix} 2\sqrt{3} & 0 \\ 0 & 2\sqrt{3} \end{bmatrix}$$

$$2 \tan x = 2\sqrt{3} \Rightarrow \tan x = \sqrt{3}$$

Thus, $x = \dfrac{\pi}{3}$.

12. Option (D) is correct.

Explanation: An identity matrix I_n of order $n \times n$ is a square matrix where:

All diagonal elements (where row index i equals column index j) are 1.

All non - diagonal elements (where $i \neq j$) are 0.

For example, a 3×3 identity matrix looks like:

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

This means that for an identity matrix:

$$a_{ij} = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases}$$

13. Option (A) is correct.

Explanation: Given:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \ \text{adj}(A) = A$$

Now,

$$\text{adj}(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Equating with A :

$$d = a$$
$$-b = b \Rightarrow b = 0$$
$$-c = c \Rightarrow c = 0$$

Thus,

$$a + b + c + d = a + 0 + 0 + a = 2a$$

14. Option (B) is correct.

Explanation: Given A and B are skew – symmetric, i.e.,

$$A^T = -A, B^T = -B$$

Transpose of $AB + BA$:

$$(AB + BA)^T = B^TA^T + A^TB^T$$
$$= (-B)(-A) + (-A)(-B)$$
$$= BA + AB$$

Since $(AB + BA)^T = AB + BA$, it is symmetric.

15. Option (C) is correct.

Explanation:

Let $B = A - A'$. To determine if B is symmetric or skew-symmetric, we need to find the transpose of B, denoted by B'.

$B' = (A - A')' = A' - (A')' = A' - A = -(A - A') = -B$

Since $B' = -B$, the matrix $B = A - A'$ is a skew-symmetric matrix.

16. Option (D) is correct.

Explanation: A scalar matrix is a diagonal matrix where all diagonal elements are equal and all non – diagonal elements are zero.

Since,

All non-diagonal elements must be 0 $\rightarrow$ So, $c = 0$ and $b = 0$.

All diagonal elements must be equal $\rightarrow$ So, $a = d = 5$.

Now, $a + 2b + 3c + 4d$

$$a + 2b + 3c + 4d = 5 + 2(0) + 3(0) + 4(5)$$
$$= 5 + 0 + 0 + 20 = 25$$

17. Option (A) is correct.

Explanation: Given:

$$A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$$

Compute A^2 :

$$A^2 = A \times A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} (2\times 2 + 1\times -4) & (2\times 1 + 1\times -2) \\ (-4\times 2 + -2\times -4) & (-4\times 1 + -2\times -2) \end{bmatrix}$$

$$= \begin{bmatrix} 4-4 & 2-2 \\ -8+8 & -4+4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Since $A^2 = 0$, all higher powers vanish.

$$I - A + A^2 - A^3 + \cdots = I - A$$

$$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$$

18. Option (C) is correct.

Explanation: Given:

$$[1 \quad x] \times \begin{bmatrix} 4 & 0 \\ -2 & 0 \end{bmatrix} = O$$

Multiplication:

$$[(4 - 2x)0] = [0 \quad 0]$$

On comparing $4 - 2x = 0$

$\Rightarrow x = 2$

19. Option (B) is correct.

Explanation: Since,

$$A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} \text{ and } A^2 - kA - 5I = O.$$

$$A^2 = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix}$$

Now, $A^2 - kA - 5I = O$

$$\begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - k\begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 5-k & 15-3k \\ 15-3k & 20-4k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

From $5 - k = 0$, we get $k = 5$.

From $15 - 3k = 0$, we get $k = 5$.

From $20 - 4k = 0$, we get $k = 5$.

So, $k = 5$.

20. Option (C) is correct.

Explanation: Given:

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}, \ A^2 + 7I = kA$$

$$A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$\therefore \quad A^2 + 7I = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$$

$$\text{and} \quad kA = \begin{bmatrix} 3k & k \\ -k & 2k \end{bmatrix}$$

Since $A^2 + 7I = kA$, comparing elements:

$$15 = 3k \quad \Rightarrow k = 5$$
$$-k = -5 \quad \Rightarrow k = 5$$
$$2k = 10 \quad \Rightarrow k = 5$$

21. Option (C) is correct.

Explanation: Given:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}, \ B = \frac{1}{3}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & \lambda \end{bmatrix}$$

Since $AB = I$, we compute the (3, 3) entry of AB:

$(3 \times 1) + (-2 \times -3) + (4 \times \lambda) = 3 + 6 + 4\lambda = 9 + 4\lambda$

Dividing by 3:

$$\frac{9 + 4\lambda}{3} = 1$$

Solving:

$$9 + 4\lambda = 3 \Rightarrow 4\lambda = -6 \Rightarrow \lambda = -\frac{3}{2}$$

22. Option (A) is correct.

Explanation: Given:

$$A = \begin{bmatrix} a & c & -1 \\ b & 0 & 5 \\ 1 & -5 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} a & b & 1 \\ c & 0 & -5 \\ -1 & 5 & 0 \end{bmatrix}$$

Since $A^T = -A$, on comparing

$a = -a \Rightarrow 2a = 0 \Rightarrow a = 0$

$b = -c \Rightarrow b + c = 0$

Now,

$$2a - (b + c)$$
$$2(0) - (b + c) = 0 - 0 = 0$$

23. Option (A) is correct.

Explanation: We are given the matrix equation:

$$[x \quad 2 \quad 0] \times \begin{bmatrix} 5 \\ -1 \\ x \end{bmatrix} = [3 \quad 1] \times \begin{bmatrix} -2 \\ x \end{bmatrix}$$

$$x(5) + 2(-1) + 0(x) = 3(-2) + 1(x)$$
$$5x - 2 = -6 + x$$
$$5x - x = -6 + 2$$
$$4x = -4$$
$$x = -1$$

24. Option (C) is correct.

Explanation: The given rule for matrix elements is:

$$a_{ij} = \max(i, j) - \min(i, j)$$

Now,

$a_{11} = \max(1, 1) - \min(1, 1) = 1 - 1 = 0$

$a_{12} = \max(1, 2) - \min(1, 2) = 2 - 1 = 1$

$a_{21} = \max(2, 1) - \min(2, 1) = 2 - 1 = 1$

$a_{22} = \max(2, 2) - \min(2, 2) = 2 - 2 = 0$

Thus, the matrix A is:

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 = A \times A$$

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (0\times 0 + 1\times 1) & (0\times 1 + 1\times 0) \\ (1\times 0 + 0\times 1) & (1\times 1 + 0\times 0) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

25. Option (A) is correct.

Explanation: Given:

$$A = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}, \ B = \begin{bmatrix} 4 & 0 \\ -1 & 1 \end{bmatrix}$$

Compute A^2 :

$$A^2 = \begin{bmatrix} x^2 & 0 \\ x+1 & 1 \end{bmatrix}$$

Equating with B:

$x^2 = 4 \Rightarrow x = \pm 2$

$x + 1 = -1 \Rightarrow x = -2$

Thus, $x = -2$.

26. Option (C) is correct.

Explanation: Let the order of P be $m \times 3$.

Given that $P \times Q$ results in a diagonal matrix, it implies:

$$(m \times 3) \times (3 \times 2) = (m \times 2)$$

Since the resulting matrix is square matrix, we equate $m = 2$.

Thus, the order of P is 2×3.

27. Option (C) is correct.

Explanation: For A to be the identity matrix I,

$$A = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\cos x = 1, \sin x = 0$$

The smallest positive solution is $x = 0$.

28. Option (D) is correct.

Explanation: Given the skew - symmetric matrix:

$$A = \begin{bmatrix} 0 & 5 & -7 \\ a & 0 & 3 \\ b & -3 & 0 \end{bmatrix}$$

For skew – symmetry: $A^T = -A, \Rightarrow A_{ij} = -A_{ji}$.

Now,

$a = -A_{12} = -5$

$b = -A_{13} = 7$

29. Option (C) is correct.

Explanation: We are given the matrices X, Y, and their product XY with the following orders:

X is of order 2×3

Y is of order $m \times n$

XY is of order 2×5

The number of columns of X must be equal to the number of rows of Y, i.e.,

$$3 = m$$

Also, the product XY results in a matrix of order 2×5, which means $n = 5$.

Number of element in matrix $Y = 3 \times 5 = 15$.

30. Option (B) is correct.

Explanation:

$$2A + B = O$$

$$B = -2A$$

$$2A = 2 \times \begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 4 \end{bmatrix}$$

$$B = -\begin{bmatrix} 6 & 8 \\ 10 & 4 \end{bmatrix} = \begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$$

31. Option (C) is correct.

Explanation:

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \; I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$(3I + 4A)(3I - 4A) = x^2 I$$

Now, $(3I + 4A)(3I - 4A) = 9I - 16A^2$

$$A^2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I$$

$\therefore \quad 9I - 16(-I) = 9I + 16I = 25I$

$$x^2 = 25$$

$$x = \pm 5$$

32. Option (C) is correct.

Explanation:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$A^2 = A \times A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^n = O \text{ for } n \geq 2$$

$$A^{2023} = O$$

33. Option (B) is correct.

Explanation:

$$A = \begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix}, \; A = P + Q$$

where P is symmetric matrix and Q is skew-symmetric matrix

$$P = \frac{1}{2}\left(A + A^T\right), \; Q = \frac{1}{2}\left(A - A^T\right)$$

$$A^T = \begin{bmatrix} 2 & 5 \\ 0 & 4 \end{bmatrix}$$

$$Q = \frac{1}{2}\left(\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 5 \\ 0 & 4 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$$

34. Option (D) is correct.

Explanation: A is a 2×3 matrix

AB is defined $\Rightarrow B$ must have 3 rows (i.e., B is $3 \times n$)

AB' is defined $\Rightarrow B'$ must have 3 columns (i.e., B' is $m \times 3$)

B' is the transpose of $B \Rightarrow B$ is 3×3.

35. Option (B) is correct.

Explanation:

$$A = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}$$

$$a, b, c, d, e, f \in \{1, -1\}$$

$$\text{Total choices} = 2^6 = 64$$

36. Option (D) is correct.

Explanation:

$$A = \begin{bmatrix} 1 & 4 & x \\ z & 2 & y \\ -3 & -1 & 3 \end{bmatrix}$$

$$A^T = A$$

$$\begin{bmatrix} 1 & z & -3 \\ 4 & 2 & -1 \\ x & y & 3 \end{bmatrix} = \begin{bmatrix} 1 & 4 & x \\ z & 2 & y \\ -3 & -1 & 3 \end{bmatrix}$$

$$z = 4, \; y = -1, \; x = -3$$

$$x + y + z = -3 + (-1) + 4 = 0$$

37. Option (C) is correct.

Explanation: Given: A and B are skew – symmetric matrices of the same order.

For Skew – Symmetric Matrices

$$A^T = -A,\ B^T = -B$$

Symmetric matrix of $AB = (AB)^T$

$$(AB)^T = AB$$

Using transpose properties:

$$(AB)^T = B^T A^T = (-B)(-A) = BA$$

For AB to be symmetric:

$$AB = BA$$

38. Option (B) is correct.

Explanation:

$$A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

$$A^T = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$$

$$A + A^T = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} + \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$$

$$= \begin{bmatrix} 2\cos x & 0 \\ 0 & 2\cos x \end{bmatrix}$$

$$\therefore \quad A + A^T = \sqrt{3}I$$

$$= \begin{bmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{3} \end{bmatrix}$$

$$\therefore \quad 2\cos x = \sqrt{3}$$

$$\Rightarrow \quad \cos x = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \cos x = \cos\frac{\pi}{6}$$

$$\Rightarrow \quad x = \frac{\pi}{6}$$

39. Option (D) is correct.

Explanation: Given: A is a skew – symmetric matrix of order 3

$$A^T = -A$$

For an odd – order skew – symmetric matrix, the determinant is always zero:

$$|A| = x = 0$$

Put in for the value of x in $(2023)^x$

$$(2023)^0 = 1$$

40. Option (B) is correct.

Explanation: Given Matrices:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

Now,

$$B' = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix},\ A' = A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B'A' = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (1\cdot1+0\cdot0) & (1\cdot0+0\cdot0) \\ (1\cdot1+0\cdot0) & (1\cdot0+0\cdot0) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

41. Option (B) is correct.

Explanation:

$$x\begin{bmatrix} 1 \\ 2 \end{bmatrix} + y\begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} x+2y \\ 2x+5y \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$$

$$x + 2y = 4 \quad \ldots(i)$$

$$2x + 5y = 9 \quad \ldots(ii)$$

Multiplying eq (i) by 2,

$$2(x + 2y) = 4 \times 2$$

$$2x + 4y = 8 \quad \ldots(iii)$$

Substract (iii) from (ii)

$$(2x + 5y) - (2x + 4y) = 9 - 8$$

$$y = 1$$

From (i), $x + 2(1) = 4$

$$x = 2$$

42. Option (A) is correct.

Explanation:

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \times \begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} (a\cdot a+b\cdot b) & (a\cdot(-b)+b\cdot a) \\ (-b\cdot a+a\cdot b) & (-b\cdot(-b)+a\cdot a) \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$$

43. Option (A) is correct.

Explanation:

Given, $A^2 = A$

$$(I + A)^2 = I^2 + 2IA + A^2$$

$$= I + 2A + A$$

$$= I + 3A$$

$$(I + A)^2 - 3A = (I + 3A) - 3A$$

$$= I$$

44. Option (D) is correct.

Explanation:

$$A = [1 \quad 2 \quad 3]$$

$$A' = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$AA' = [1 \quad 2 \quad 3] \times \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$= [1(1)+2(2)+3(3)] = [1+4+9]$$

$$= [14]$$

45. Option (D) is correct.

Explanation:

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$

$$x + y + z = 6$$

$$y + z = 3$$

$$z = 2$$

$$y + 2 = 3$$

$$y = 1$$

$$x + 1 + 2 = 6$$

$$x = 3$$

$$2x + y - z = 6 + 1 - 2 = 5$$

ASSERTION-REASON QUESTIONS (1 Mark)

1. Option (D) is correct.

Explanation: Assertion (A)

We need to check whether $B' AB$ is skew – symmetric for any symmetric matrix A.

Given that A is symmetric, we have:

$$A' = A$$

Compute the transpose of $B' AB$:

$$(B' AB)' = B' A' (B')' = B' AB$$

Since $(B' AB)' = B' AB$, the matrix $B' AB$ is symmetric, not skew – symmetric.

Thus, Assertion (A) is false.

Reason (R):

A square matrix P is skew – symmetric if $P' = -P$.

Thus, Reason (R) is true.

2. Option (C) is correct.

Explanation: Every scalar matrix is a diagonal matrix.

A scalar matrix is a diagonal matrix where all diagonal elements are equal. That means:

$$A = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$$

Since every scalar matrix is also a diagonal matrix, this statement is true.

Reason (R):

In a diagonal matrix, all the non-diagonal elements are 0 and all diagonal elements are non-zero.

Thus, reason is false

3. Option (C) is correct.

Explanation:

$$A = \begin{bmatrix} 0 & -3 & 5 \\ 3 & 0 & -2 \\ -5 & 2 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & 3 & -5 \\ -3 & 0 & 2 \\ 5 & -2 & 0 \end{bmatrix}$$

$$A^T = -A$$

Assertion(A) is True.

$A^T = A$ (given in (R)) is wrong, as it defines symmetric matrices.

Reason(R) is false.

4. Option (D) is correct.

Explanation:

$$M = \begin{bmatrix} 2 & 3 \\ 6 & 4 \end{bmatrix},\ M^T = \begin{bmatrix} 2 & 6 \\ 3 & 4 \end{bmatrix}$$

$$S = \frac{M + M^T}{2} = \frac{1}{2}\begin{bmatrix} 4 & 9 \\ 9 & 8 \end{bmatrix},$$

$$K = \frac{M - M^T}{2} = \frac{1}{2}\begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$$

$$M = S + K$$

$$M \neq M^T,\ M \neq -M^T$$

Thus, assertion is false but reason is true.

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. Symmetric $= \frac{1}{2}(A + A') = \begin{bmatrix} 3 & -\frac{3}{2} \\ -\frac{3}{2} & -1 \end{bmatrix}$

Skew-symmetric $= \frac{1}{2}(A - A') = \begin{bmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$

and $A = \begin{bmatrix} 3 & -\frac{3}{2} \\ -\frac{3}{2} & -1 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

2. Here,

$$A^2 = \begin{bmatrix} 1 & -6 \\ 2 & -3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & -3 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -6 \\ 2 & -3 \end{bmatrix}$$

Now, the given equation can be written:

$$p\begin{bmatrix} 1 & -6 \\ 2 & -3 \end{bmatrix} + q\begin{bmatrix} 2 & -3 \\ 1 & 0 \end{bmatrix} + r\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

On comparing we get, $2p + q = 0$

$$\Rightarrow \left(\frac{-q}{p}\right) = 2.$$

3. Since B is a symmetric matrix, $B = B'$.

Now, $(BB')' = (B')' \times B' = BB'$

Hence, BB' is symmetric matrix.

4. Here, $D' = D$ as D is a symmetric matrix.

and $D' = -D$ as D is a skew symmetric matrix.

From above,

we get $D = -D$

$\Rightarrow$ $D = O$

$\Rightarrow$ $D = O$

Thus, D is a null matrix.

5. The given statement will be true if A is invertible or if A^{-1} exists.

Let $AB = AC$

Pre multiplying both sides by A^{-1}, we get:

$\Rightarrow$ $A^{-1}(AB) = A^{-1}(AC)$

$\Rightarrow$ $(A^{-1}A)B = (A^{-1}A)C$

$\Rightarrow \quad (I)(B) = (I)C$

$\Rightarrow \quad B = C$

6. Yes, B is a diagonal matrix.

Justification: For example, assumes B to not be a diagonal matrix, then we have

$$\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} b_{11}d_{11} & b_{12}d_{22} \\ b_{21}d_{11} & b_{22}d_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

From the above equality we observe that, as d_{11} and d_{22} are non-zero, b_{21} and b_{12} must be zero. Hence, concludes that B is a diagonal matrix.

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. Getting $A^2 = \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix}$

Getting $A^3 = \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix}$

$A^3 - 23A - 40I$

$$= \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} - \begin{bmatrix} 23 & 46 & 69 \\ 69 & -46 & 23 \\ 92 & 46 & 23 \end{bmatrix} - \begin{bmatrix} 40 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 40 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

2. (i) The number of chips packets remaining with the distributions at the end of the month in the matrix form as:

$$M - N = \begin{bmatrix} 965 & 498 \\ 872 & 689 \end{bmatrix} - \begin{bmatrix} 956 & 399 \\ 650 & 511 \end{bmatrix}$$

$$= \begin{matrix} \text{SC} & \text{CC} \\ \end{matrix}$$
$$= \begin{bmatrix} 9 & 99 \\ 222 & 178 \end{bmatrix} \begin{matrix} \longrightarrow \text{Distributor 1} \\ \longrightarrow \text{Distributor 2} \end{matrix}$$

(ii) The total cost of the chips distributed by each distributor that month using matrix multiplication is given as:

$$\begin{matrix} \text{SC} & \text{CC} \end{matrix}$$
$$\begin{bmatrix} 956 & 399 \\ 650 & 511 \end{bmatrix} \times \begin{bmatrix} 10 \\ 20 \end{bmatrix} = \begin{bmatrix} 9560 + 7980 \\ 6500 + 10220 \end{bmatrix}$$

$$= \begin{bmatrix} 17540 \\ 16720 \end{bmatrix} \begin{matrix} \rightarrow \text{Distributor 1} \\ \rightarrow \text{Distributor 2} \end{matrix}$$

3. Here,

$$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & \beta \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 0 & \beta \end{bmatrix} = \begin{bmatrix} 1 & 1+\beta \\ 0 & \beta^2 \end{bmatrix}$$

Equates A^2 to B and writes the matrix equation as:

$$\begin{bmatrix} 1 & 1+\beta \\ 0 & \beta^2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 9 \end{bmatrix}$$

Equates the corresponding elements and writes the equations $1 + \beta = 2$ and $\beta^2 = 9$.

Solves the first equation to get $\beta = 1$ and writes that $\beta = 1$ doesn't satisfy the equation $\beta^2 = 9$.

Concldes that $A^2 = B$ is not possible for any value of β.

4. By associative law of matrix multiplication, $(AB) \times C = A \times (BC)$.

Here, $A^{-1} = \frac{1}{31}\begin{bmatrix} 5 & 3 \\ -7 & 2 \end{bmatrix}$

Pre-multiplies A^{-1} on both sides of the equation $(AB) \times C = A \times (BC)$, we get

$A^{-1} \times [(AB) \times C] = A^{-1} \times [A \times (BC)] = (A^{-1}A) \times (BC) = I \times (BC) = BC$

$$BC = \frac{1}{31}\begin{bmatrix} 5 & 3 \\ -7 & 2 \end{bmatrix} \times \begin{bmatrix} -13 & 12 \\ 32 & 11 \end{bmatrix}$$

$$\Rightarrow \quad BC = \begin{bmatrix} 1 & 3 \\ 5 & -2 \end{bmatrix}$$

LONG ANSWER TYPE QUESTIONS (5 Marks)

1.

$$AB = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix} = 7I$$

Thus, $A^{-1} = \frac{1}{7}B = \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$

The, given equation can be written into a matrix equation as

$$\begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$

$$\Rightarrow \quad X = A^{-1}.C$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$$

$$= \frac{1}{7}\begin{bmatrix} 7 \\ -35 \\ -35 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ -5 \end{bmatrix}$$

$\therefore x = 1, y = -5, z = -5$

2. $$AB = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} = \begin{bmatrix} 67 & 0 & 0 \\ 0 & 67 & 0 \\ 0 & 0 & 67 \end{bmatrix} = 67I$$

Thus, $$A^{-1} = \frac{1}{67}B = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

Solution of the system of equations is given by:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}^{-1}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$= \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$= \frac{1}{67}\begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix},$$

$\therefore x = 3, y = -2, z = 1.$

3. LHS = $f(\alpha)f(-\beta)$

$$= \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha\cos\beta + \sin\alpha\sin\beta & \cos\alpha\sin\beta - \sin\alpha\cos\beta & 0 \\ \sin\alpha\cos\beta - \cos\alpha\sin\beta & \sin\alpha\sin\beta + \cos\alpha\cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\alpha-\beta) & -\sin(\alpha-\beta) & 0 \\ \sin(\alpha-\beta) & \cos(\alpha-\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$= f(\alpha - \beta)$ = RHS

4. (a) Getting, $$A^2 = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

Getting, $$A^3 = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

$$\therefore A^3 - 6A^2 + 7A + 2I = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix}$$

$$+ \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

Level - 2 ADVANCED COMPETENCY FOCUSED QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Mark)

1. Option (C) is correct.

Explanation: Element a_{22} is the element in Row 2, Column 2, which is 15. So, it represents Router R_2 on Day 2.

2. Option (A) is correct.

Explanation: The company decides to double the number of deliveries. That means every element of matrix P should be multiplied by 2:

$$2P = \begin{bmatrix} 2\times20 & 2\times30 \\ 2\times25 & 2\times35 \end{bmatrix} = \begin{bmatrix} 40 & 60 \\ 50 & 70 \end{bmatrix}$$

3. Option (A) is correct.

Explanation: Given: Advertisement hours: A = [4 6]

(4 hours on Social Media, 6 hours on TV)

Cost per hour (from the image):

$$B = \begin{bmatrix} 500 \\ 1200 \end{bmatrix}$$

(₹500/hour for Social Media, ₹1200/hour for TV)

$AB = [4\ 6].\begin{bmatrix} 500 \\ 1200 \end{bmatrix} = (4 \times 500) + (6 \times 1200) = 2000 +$

$7200 =$ ₹9200

4. Option (B) is correct.

Explanation: Q × C

$$= \begin{bmatrix} 50\times5+100\times10 \\ 60\times5+120\times10 \end{bmatrix} = \begin{bmatrix} 250+1000 \\ 300+1200 \end{bmatrix} = \begin{bmatrix} 1250 \\ 1500 \end{bmatrix}$$

5. Option (B) is correct.

Explanation: S × P = (40 × 5) + (50 × 3) + (30 × 2) = 200 + 150 + 60 = 410

ASSERTION-REASON QUESTIONS (1 Marks)

1. Option (B) is correct.

Explanation: Assertion is true. Even if both AB and BA are defined, they are not equal. Matrix multiplication is not commutative unlike real number multiplication.

Reason is also true. This is the correct condition for matrix multiplication to be defined.

Both assertion and reason are true but the reason does not explain why matrix multiplication is not commutative.

2. Option (A) is correct.

Explanation: Assertion is true. Matrices are widely used in real-life applications to represent tabular data such as these examples.

Reason is also true. This is one of the key reasons matrices are used — they efficiently handle structured numeric information.

Both assertion and reason are true and the reason is the correct explanation of assertion, as it justifies why matrices are used in such scenarios.

3. Option (A) is correct.

Explanation: Assertion is true. Matrix A: 2 rows, 3 columns (2×3)

Matrix B: 3 rows, 4 columns (3×4)

Since the number of columns of A = rows of B, multiplication AB is defined.

Resultant matrix = number of rows of A × columns of B = 2×4.

Reason is also true because this is the fundamental rule for matrix multiplication.

Both assertion and reason are true and reason is the correct explanation of assertion.

4. Option (B) is correct.

Explanation: Assertion is true. Let's take any square matrix A. Then:

$(A+A')'=A'+(A')'=A'+A=A+A'$. This means $A+A'$ is equal to its own transpose $\Rightarrow$ it is symmetric.

Reason is also true because this is the definition of the transpose.

Both assertion and reason are true but reason is not the correct explanation of assertion.

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. To find the total income, we perform matrix multiplication:

$$\text{Income} = A \times P = [15\ 20\ 10] \times \begin{bmatrix} 50 \\ 20 \\ 60 \end{bmatrix}$$

$$=(15\times50)+(20\times20)+(10\times60) = 750+400+600 = ₹1750$$

2. We have,

$$P = \begin{bmatrix} 200 & 300 \\ 150 & 250 \end{bmatrix}$$

$$\text{Cost matrix } C = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$$

Now,

$$PC = \begin{bmatrix} 200 & 300 \\ 150 & 250 \end{bmatrix}\begin{bmatrix} 10 \\ 20 \end{bmatrix}$$

$$= \begin{bmatrix} 2000+6000 \\ 1500+5000 \end{bmatrix}$$

$$= \begin{bmatrix} 8000 \\ 6500 \end{bmatrix}$$

So, profit earned by factory 1 is ₹ 8,000, and profit earned by factory 2 is ₹ 6,500.

3. We have,

$$D = \begin{bmatrix} \text{city 1} & \text{city 2} & \text{city 3} \\ 30 & 40 & 50 \\ 20 & 35 & 25 \end{bmatrix}$$

and

$$P = \begin{bmatrix} 10 \\ 12 \\ 15 \end{bmatrix}$$

Now,

$$DP = \begin{bmatrix} 30 & 40 & 50 \\ 20 & 35 & 25 \end{bmatrix}\begin{bmatrix} 10 \\ 12 \\ 15 \end{bmatrix}$$

$$= \begin{bmatrix} 300+480+750 \\ 200+420+375 \end{bmatrix}$$

$$= \begin{bmatrix} 1530 \\ 995 \end{bmatrix}$$

So, total cost = 1530 + 995 = ₹ 2525

4. We have

$$F = \begin{bmatrix} 100 & 150 \\ 80 & 120 \\ 60 & 90 \end{bmatrix}$$

Let

$$C = \begin{bmatrix} 20 \\ 20 \end{bmatrix}$$

So,

$$FC = \begin{bmatrix} 100 & 150 \\ 80 & 120 \\ 60 & 90 \end{bmatrix}\begin{bmatrix} 20 \\ 20 \end{bmatrix}$$

$$= \begin{bmatrix} 2000+3000 \\ 1600+2400 \\ 1200+1800 \end{bmatrix}$$

$$= \begin{bmatrix} 5000 \\ 4000 \\ 3000 \end{bmatrix}$$

Thus,

Total cost incurred by van 1 = ₹ 5000

Total cost incurred by van 2 = ₹ 4000

Total cost incurred by van 3 = ₹ 3000

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. Represents the quantity of sugar, wheat and rice to be purchased by Achal by the matrix:

$[2 \quad 10 \quad 5]$

Represents the prices of sugar, wheat and rice at the general store and the supermarket by the matrix:

$$\begin{bmatrix} 50 & 44 \\ 35 & 30 \\ 40 & 38 \end{bmatrix}$$

The total cost at the two places as:

$$[2 \quad 10 \quad 5]\begin{bmatrix} 50 & 44 \\ 35 & 30 \\ 40 & 38 \end{bmatrix} = [650 \quad 578]$$

Achal's total savings if he buys groceries from the supermarket = 650 – 578 – 20 = ₹ 52.

2. Represents the matrix $(P + Q)$ as shown below:

$$P + Q = \begin{bmatrix} a+3 & b-1 \\ c+1 & d-2 \end{bmatrix}$$

where $Q = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

By skew-symmetric relation $(P + Q) = -(P + Q)'$, thus, we have

$$\begin{bmatrix} a+3 & b-1 \\ c+1 & d-2 \end{bmatrix} = \begin{bmatrix} -(a+3) & -(c+1) \\ 1-b & 2-d \end{bmatrix}$$

By the equality of matrices we get the following equations:

$$a + 3 = -(a + 3) \Rightarrow a = -3$$
$$d - 2 = 2 - d \Rightarrow d = 2$$

and $$b - 1 = -(c + 1) \Rightarrow b = -c \text{ or } c = -b$$

Therefore, $$Q = \begin{bmatrix} -3 & b \\ -b & 2 \end{bmatrix} \quad (b \in \mathbb{R})$$

3. Given: Production matrix

$$P = \begin{bmatrix} 10 & 20 \\ 15 & 25 \\ 12 & 18 \end{bmatrix}$$

Each row represents the production of Gadget A and B on Day 1, Day 2, and Day 3 respectively.

Profit per unit:Gadget A: ₹50 and Gadget B: ₹80

(i) Profit per unit as a matrix can be represented as:

$$Q = \begin{bmatrix} 50 \\ 80 \end{bmatrix}$$

(ii) Profit for each day using matrix multiplication:

Profit=P×Q

$$= \begin{bmatrix} 10 & 20 \\ 15 & 25 \\ 12 & 18 \end{bmatrix} \times \begin{bmatrix} 50 \\ 80 \end{bmatrix}$$

Day 1: 10×50+20×80=500+1600=₹2100

Day 2: 15×50+25×80=750+2000=₹2750

Day 3: 12×50+18×80=600+1440=₹2040

So,

$$\text{Profit Matrix} = \begin{bmatrix} 2100 \\ 2750 \\ 2024 \end{bmatrix}$$

(iii) Total profit over all three days

= 2100+2750+2040=₹6890

4. **(i)** $C = \begin{bmatrix} 8 \\ 8 \\ 8 \end{bmatrix}$

(ii) Total Cost = E × C = $\begin{bmatrix} 5 & 4 & 3 \\ 6 & 5 & 2 \end{bmatrix} \times \begin{bmatrix} 8 \\ 8 \\ 8 \end{bmatrix}$

Day 1: 5×8+4×8+3×8=96

Day 2: 6×8+5×8+2×8=104

$$= \begin{bmatrix} 96 \\ 104 \end{bmatrix}$$

(iii) Total cost over both days:

96+104 = ₹200

5. **(i)** TC = T × C = $\begin{bmatrix} 30 & 40 & 50 \\ 20 & 25 & 35 \end{bmatrix} \times \begin{bmatrix} 10 \\ 15 \\ 20 \end{bmatrix}$

Warehouse 1 (Row 1):(30×10)+(40×15)+(50×20) =300+600+1000=1900

Warehouse 2 (Row 2):(20×10)+(25×15)+(35×20) =200+375+700=1275

$$TC = \begin{bmatrix} 1900 \\ 1275 \end{bmatrix}$$

(ii) Total cost=1900+1275=₹3175

CASE BASED QUESTIONS (4 Marks)

1. **(i)** In factory A, number of units of type I, II and III for boys are 80, 70, 65 respectively and for girls number of units of type I, II and III are 80, 75, 90 respectively.

B G

$$\therefore \quad P = \begin{bmatrix} 80 & 80 \\ 70 & 75 \\ 65 & 90 \end{bmatrix}$$

(ii) In factory B, number of units of type I, II and III for boys are 85, 65, 72 respectively and for girls number of units of types I, II and III are 50, 55, 80 respectively.

$$\therefore \quad Q = \begin{matrix} & B & G \\ & \begin{bmatrix} 85 & 50 \\ 65 & 55 \\ 72 & 80 \end{bmatrix} \end{matrix}$$

(iii) Let matrix X represent the number of units of each type produced by factory A for boys and matrix Y represents the number of units of each type produced by factory B for boys.

$$\therefore \quad X = \begin{matrix} I & II & III \\ [80 & 70 & 65] \end{matrix}$$

$$Y = \begin{matrix} I & II & III \\ [85 & 65 & 72] \end{matrix}$$

Now, total production of sports clothes of each type for boys = X + Y

$= [80\ \ 70\ \ 65] + [85\ \ 65\ \ 72]$

$= [165\ \ 135\ \ 137]$

OR

For girls, let matrix S represents the number of units of each type produced by factory A and matrix T represents the number of units of each type produced by factory B.

$$\therefore \quad S = \begin{matrix} I & II & III \\ [80 & 75 & 90] \end{matrix}$$

$$T = \begin{matrix} I & II & III \\ [50 & 55 & 80] \end{matrix}$$

Now, required matrix = S + T

$= [80\ \ 75\ \ 90] + [50\ \ 55\ \ 80]$

$= [130\ \ 130\ \ 170]$

2. (i) Let number of children = x

Amount distributed by Seema for one child = ₹ y

Now, Total money = xy

and Total money will remain the same.

Given that, if there were 8 children less, everyone would have got ₹ 10 more.

Total money now = Total money before

$(x - 8) \times (y + 10) = xy$

$\Rightarrow \quad x(y + 10) - 8(y + 10) = xy$

$\Rightarrow \quad xy + 10x - 8y - 80 = xy$

$\Rightarrow \quad 10x - 8y - 80 = 0$

$\Rightarrow \quad 10x - 8y = 80$

$\Rightarrow \quad 5x - 4y = 40$ **1**

Also, if there were 16 children more, everyone would have got ₹ 10 less.

Total money now = Total money before

$(x + 16) \times (y - 10) = xy$

$\Rightarrow \quad x(y - 10) + 16(y - 10) = xy$

$\Rightarrow \quad xy - 10x + 16y - 160 = xy$

$\Rightarrow \quad -10x + 16y - 160 = 0$

$\Rightarrow \quad 10x - 16y + 160 = 0$

$\Rightarrow \quad 5x - 8y = -80$ **1**

Thus, required equations are:

$5x - 4y = 40$ ...(i)

$5x - 8y = -80$ ...(ii)

OR

On solving eqs. (i) & (ii), we get $x = 32$ and $y = 30$.

Hence, the number of children= 32

The amount is given to each child by Seema = ₹ 30

(ii) Writing eq. (i) & eq. (ii) in matrix form, we get

$$\begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$$

(iii) Total amount = xy = 32 × 30 = ₹960

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. (i) The cost for each firm can be found as:

$$\begin{bmatrix} 30 & 10 \\ 25 & 15 \end{bmatrix} \times \begin{bmatrix} 70 \\ 100 \end{bmatrix} = \begin{bmatrix} 3100 \\ 3250 \end{bmatrix}$$

Therefore, the cost for firm 1 as ₹ 3100 and the cost for firm 2 as ₹ 3250.

Thus, firm 2 cost more to the company.

(ii) The total revenue generated by each firm from the outsourcing as:

$$300 \times \begin{bmatrix} 500 & 200 \\ 400 & 300 \end{bmatrix} = \begin{bmatrix} 150000 & 60000 \\ 120000 & 90000 \end{bmatrix}$$

So, posts from firm 1 generated 1,50,000 + 60,000 = ₹ 2,10,000 in revenue and the posts from firm 2 generated 1,20,000 + 90,000 = ₹ 2,10,000 in revenue.

The the profit generated by firm 1 as 2,10,000 – 3,100 = ₹ 206,900 and firm 2 as 2,10,000 – 3,250 = ₹ 2,06,750.

Hence, we conclude that the firm 1 was more profitable for the company than firm 2.

2. (i) Total Cost = Q × C

$$\begin{bmatrix} 120 & 200 & 150 \\ 100 & 250 & 180 \end{bmatrix} \times \begin{bmatrix} 10 \\ 15 \\ 20 \end{bmatrix}$$

$$= \begin{bmatrix} (120 \times 10) + (200 \times 15) + (150 \times 20) \\ (100 \times 10) + (250 \times 15) + (180 \times 20) \end{bmatrix} = \begin{bmatrix} 7200 \\ 8350 \end{bmatrix}$$

(ii) Total cost over 2 months=7200+8350=₹15,550

(iii) If cost of P_2 increases by ₹5, new cost vector becomes:

$$\text{in suffix} = \begin{bmatrix} 10 \\ 20 \\ 20 \end{bmatrix}$$