CHAPTER 4

Determinants

Level - 1 CORE SUBJECTIVE QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQ) (1 Mark)

1. Option (D) is correct.

Explanation: For a square matrix A of order $n \times n$, we have $A.(\text{adj } A) = |A|I_n$, where I_n is the identity matrix of order $n \times n$.

$$\text{So, } A.(\text{adj } A) = \begin{bmatrix} 2025 & 0 & 0 \\ 0 & 2025 & 0 \\ 0 & 0 & 2025 \end{bmatrix} = 2025 I_3$$

$\Rightarrow |A| = 2025$ & $|\text{adj } A| = |A|^{3-1} = (2025)^2$

$\therefore |A| + |\text{adj } A| = 2025 + (2025)^2$.

2. Option (B) is correct.

Explanation:

$$\begin{aligned} |A| = 5, |B^{-1}AB|^2 &= |(B^{-1}B).A|^2 \\ &= |I.A|^2 \\ &= |A|^2 = 5^2. \end{aligned}$$

3. Option (A) is correct.

Explanation: **Method 1: (Short cut)**

When the points (x_1, y_1), (x_2, y_2) and $(x_1 + x_2, y_1 + y_2)$ are collinear in the Cartesian plane then

$$\begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ x_1 - (x_1 + x_2) & y_1 - (y_1 + y_2) \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x_1 - x_2 & y_1 - y_2 \\ -x_2 & -y_2 \end{vmatrix} = (-x_1y_2 + x_2y_2 - x_2y_2 + x_2y_1) = 0$$

$\Rightarrow x_2y_1 = x_1y_2$.

Method 2:

When the points (x_1, y_1), (x_2, y_1) and $(x_1 + x_2, y_1 + y_2)$ are collinear in the Cartesian plane then

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_1 + x_2 & y_1 + y_2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow 1.(x_2y_1 + x_2y_2 - x_1y_2 - x_2y_2) - 1(x_1y_1 + x_1y_2 - x_1y_1 - x_2y_1) + (x_1y_2 - x_2y_1) = 0$$

[Expanding along C_3]

$\Rightarrow x_2y_1 = x_1y_2$.

4. Option (B) is correct .

Explanation:

$$\text{Let} \quad A = \begin{vmatrix} x+1 & x-1 \\ x^2+x+1 & x^2-x+1 \end{vmatrix}$$

$$= (x + 1)(x^2 - x + 1) - (x - 1)(x^2 + x + 1)$$

$$= x^3 + 1 - (x^3 - 1) = 2$$

5. Option (A) is correct.

Explanation: We have the matrix:

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$$

Now,

$$a_{11} = 2, a_{12} = -3, a_{13} = 5$$

Cofactors

$$A_{21} = (-1)^{1+2}\begin{vmatrix} -3 & 5 \\ 5 & -7 \end{vmatrix} = -[(-3)(-7) - (5)(5)] = (21 - 25) = 4$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 2 & 5 \\ 1 & -7 \end{vmatrix} = (2)(-7) - (5)(1) = -14 - 5 = -19$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 2 & -3 \\ 1 & 5 \end{vmatrix} = -[(2)(5) - (-3)(1)] = -(10 + 3) = -13$$

Now, $a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23}$

$= (2)(4) + (-3)(-19) + (5)(-13) = 8 + 57 - 65 = 0$

Shortcut trick: If the elements of one row (or column) are multiplied with co-factors of elements of any other row (or column), then their sum is zero.

6. Option (D) is correct.

Explanation:

$$D = \begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$$

Expanding along the first row:

$$D = -a\begin{vmatrix} -b & c \\ b & -c \end{vmatrix} - b\begin{vmatrix} a & c \\ a & -c \end{vmatrix} + c\begin{vmatrix} a & -b \\ a & b \end{vmatrix}$$

Thus,

$$\begin{aligned} D &= -a(0) - b(-2ac) + c(2ab) \\ &= 2abc + 2abc \\ &= 4abc \end{aligned}$$

$k = 4$

7. Option (C) is correct.

Explanation:

$$D = \begin{vmatrix} 1 & 3 & 1 \\ k & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding along the third row:

$$D = 1\begin{vmatrix} 1 & 3 \\ k & 0 \end{vmatrix}$$

$$D = 1(1(0) - 3(k)) = -3k$$

Given:

$$|D| = \pm 6$$

$$|-3k| = 6$$

$$\Rightarrow \quad 3k = \pm 6$$

$$k = \pm 2$$

8. Option (A) is correct.

Explanation: Given matrix:

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

Since A is a diagonal matrix, its inverse is obtained by taking the reciprocal of each diagonal element:

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$$

9. Option (A) is correct.

Explanation:

$$\text{Let } |A| = \begin{vmatrix} 8 & 2 & 7 \\ 12 & 3 & 5 \\ 16 & 4 & 3 \end{vmatrix}$$

$$\det(A) = \begin{vmatrix} 8 & 2 & 7 \\ 12 & 3 & 5 \\ 16 & 4 & 3 \end{vmatrix}$$

$$= 8\begin{vmatrix} 3 & 5 \\ 4 & 3 \end{vmatrix} - 2\begin{vmatrix} 12 & 5 \\ 16 & 3 \end{vmatrix} + 7\begin{vmatrix} 12 & 3 \\ 16 & 4 \end{vmatrix}$$

Now,

$$\begin{vmatrix} 3 & 5 \\ 4 & 3 \end{vmatrix} = (3 \times 3) - (5 \times 4) = 9 - 20 = -11$$

$$\begin{vmatrix} 12 & 5 \\ 16 & 3 \end{vmatrix} = (12 \times 3) - (5 \times 16) = 36 - 80 = -44$$

$$\begin{vmatrix} 12 & 3 \\ 16 & 4 \end{vmatrix} = (12 \times 4) - (3 \times 16) = 48 - 48 = 0$$

$$\det(A) = 8(-11) - 2(-44) + 7(0)$$

$$= -88 + 88 + 0 = 0$$

10. Option (B) is correct.

Explanation: We are given:

$$A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$$

We know that, $(A^{-1})^{-1} = A$

$$\because \quad (A^{-1})^{-1} = \frac{1}{\det(A^{-1})} \text{adj}(A^{-1})$$

Now, $$|A^{-1}| = \frac{4}{49} + \frac{3}{49} = \frac{7}{49} = \frac{1}{7}$$

$$\text{adj } (A^{-1}) = \begin{bmatrix} \frac{2}{7} & -\frac{1}{7} \\ \frac{3}{7} & \frac{2}{7} \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$$

11. Option (D) is correct.

Explanation:

$$A = \begin{bmatrix} -2 & 0 & 0 \\ 1 & 2 & 3 \\ 5 & 1 & -1 \end{bmatrix}$$

$$|A \cdot \text{adj}(A)| = |A|^3$$

[$\because$ If A is square matrix of order n, then $|A.\text{adj}(A)| = |A|^n$]

$$|A| = (-2)\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} - 0 + 0$$

[Expanding along R_1]

$$= (-2)[(2 \times (-1)) - (3 \times 1)]$$

$$= (-2) \times (-5) = 10$$

$$|A \cdot \text{adj}(A)| = 10^3 = 1000$$

12. Option (D) is correct.

Explanation:

$$|\text{adj}(A)| = |A|^2$$

Given:

$$|\text{adj}(A)| = 8$$

$$|A|^2 = 8$$

$$|A| = \pm\sqrt{8} = \pm 2\sqrt{2}$$

Since for any square matrix,

$$|A^T| = |A|$$

$$|A^T| = \pm 2\sqrt{2}$$

13. Option (D) is correct.

Explanation: Given that the inverse of matrix

$$A = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

We use the property:

$$A \cdot A^{-1} = I$$

Multiplying the second row of A with the second column of A^{-1}:

$$(-1 \times 3) + (1 \times \lambda) + (0 \times 3) = 1$$

$$-3 + \lambda = 1$$

$$\lambda = 4$$

14. Option (A) is correct.

Explanation: For any square matrix A of order n, the determinant property states:

$$|kA| = k^n|A|$$

Given:

$$|A| = -2, k = 5, n = 2$$
$$|5A^T| = 5^2|A|$$

Since $|A^T| = |A|$, we get:

$$|5A^T| = 25(-2) = -50$$

15. Option (D) is correct

Explanation: Given matrix:

$$A = \begin{bmatrix} 2 & -1 & 1 \\ \lambda & 2 & 0 \\ 1 & -2 & 3 \end{bmatrix}$$

$$A = 2\begin{vmatrix} 2 & 0 \\ -2 & 3 \end{vmatrix} - (-1)\begin{vmatrix} \lambda & 0 \\ 1 & 3 \end{vmatrix} + 1\begin{vmatrix} \lambda & 2 \\ 1 & -2 \end{vmatrix}$$

$$= 2((2 \times 3) - (0 \times -2)) + 1((\lambda \times 3) - (0 \times 1)) + 1((\lambda \times -2) - (2 \times 1))$$
$$= 2(6) + 3\lambda - 2\lambda - 2$$
$$= 12 + \lambda - 2$$
$$= 10 + \lambda$$

For A to be invertible:

$$10 + \lambda \neq 0$$
$$\lambda \neq -10$$

16. Option (A) is correct.

Explanation:

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 2 & 1 \\ 0 & 3 & -2 \end{bmatrix}$$

$$|A| = 0 \cdot \begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix} - 1 \cdot \begin{vmatrix} 1 & 1 \\ 0 & -2 \end{vmatrix} + (-1) \cdot \begin{vmatrix} 1 & 2 \\ 0 & 3 \end{vmatrix}$$

$$= 0 - 1\cdot(1 \times (-2) - 1 \times 0) + (-1)\cdot(1 \times 3 - 2 \times 0)$$
$$= -(-2) - (3)$$
$$= 2 - 3 = -1$$

Now,

$$|A \cdot \text{adj } (A)| = |A|^3 = (-1)^3 = -1$$

17. Option (D) is correct.

Explanation: Given:

$$A^{-1} = \frac{1}{4}B$$

We need to find the inverse of $4A$:

$$(4A)^{-1} = \frac{1}{4}A^{-1}$$

Substituting $A^{-1} = \frac{1}{4}B$:

$$(4A)^{-1} = \frac{1}{4} \times \frac{1}{4}B = \frac{1}{16}B$$

18. Option (D) is correct.

Explanation: Using the determinant property:

$$|kA| = k^n\,|A|$$

For two square matrices A and B of order 2:

$$|-3AB| = (-3)^2\,|A|\,|B|$$

Given $|A| = 2$ and $|B| = 5$:

$$|-3AB| = 9 \times 2 \times 5 = 90$$

19. Option (C) is correct.

Explanation: Using the determinant property:

$$|A^{-1}| = \frac{1}{|A|}$$

Given $|A| = 2$, we get:

$$|A^{-1}| = \frac{1}{2}$$

Now, using the property:

$$|kA^{-1}| = k^n|A^{-1}|$$

For a 2×2 matrix:

$$|4A^{-1}| = 4^2 \times \frac{1}{2}$$
$$= 16 \times \frac{1}{2} = 8$$

20. Option (D) is correct.

Explanation: Using the determinant property:

$$|\text{adj(A)}| = |A|^{n-1}$$

For a 3×3 matrix:

$$|\text{adj}(A)| = |A|^2$$

Given $|\text{adj}(A)| = 64$, we get:

$$|A|^2 = 64$$
$$|A| = \pm 8$$

21. Option (B) is correct.

Explanation:

$$A^2 - 3A + I = O$$

Multiplying both sides by A^{-1} : $A^{-1}A^2 - 3A^{-1}A = -A^{-1}I$

Since $A^{-1}A = I$, we get:

$$A - 3I = -A^{-1}$$
$$A^{-1} = 3I - A$$

Comparing

$$A^{-1} = xA + yI$$
$$3I - A = xA + yI$$
$$x = -1, y = 3$$
$$x + y = -1 + 3 = 2$$

22. Option (B) is correct.

Explanation: Using determinant properties:

$$\left|\frac{A^{-1}}{2}\right| = \frac{|A^{-1}|}{2^3} = \frac{1}{8|A|}$$

$$\left[\because |A^{-1}| = \frac{1}{|A|} \text{ and } |kA| = k^n|A|,\right.$$ k is a scalar & n is order of matrix A]

Given:

$$\frac{1}{k|A|} = \frac{1}{8|A|}$$
$$k = 8$$

23. Option (B) is correct

Explanation: A matrix is singular if its determinant is zero.

$$\begin{vmatrix} x & 2 \\ 3 & x-1 \end{vmatrix} = 0$$

$$x(x-1) - (2 \cdot 3) = 0$$
$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$
$$x = 3, -2$$

Product of values $= 3 \times (-2) = -6$

24. Option (D) is correct

Explanation: For a matrix to be non – singular, its determinant must be non – zero.

$$\begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{vmatrix} \neq 0$$

$$\Rightarrow 1\times\begin{vmatrix} 3 & 1 \\ a & 1 \end{vmatrix} - 2\times\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} + 1\times\begin{vmatrix} 2 & 3 \\ 3 & a \end{vmatrix} \neq 0$$

$\Rightarrow 1 \times (3 - a) - 2 \times(-1) + 1(2a - 9) \neq 0$

$\Rightarrow \quad 3 - a + 2 + 2a - 9 \neq 0$

$\Rightarrow \quad a - 4 \neq 0$

$\Rightarrow \quad a \neq 4$

$\therefore$ Set A is: $\mathbb{R} - \{4\}$

25. Option (B) is correct

Explanation: The determinant for the area of ΔABC is:

$$\Delta = \frac{1}{2}\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}$$

Squaring both sides:

$$\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2 = 4\Delta^2$$

26. Option (A) is correct

Explanation: Given:

$$A\cdot\text{adj}(A) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$A\cdot\text{adj}(A) = |A|I$$

$$|A| = 3$$

Also, for a 3 × 3 matrix:

$$|\text{adj}(A)| = |A|^2 = 3^2 = 9$$

$$|A| + |\text{adj}(A)| = 3 + 9 = 12$$

27. Option (A) is correct

Explanation:

Let $D = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$

Expansion along the third row

$$D = 1\times\begin{vmatrix} y+z & z+x \\ x & y \end{vmatrix} - 1\times\begin{vmatrix} x+y & z+x \\ z & y \end{vmatrix} + 1\times\begin{vmatrix} x+y & y+z \\ z & x \end{vmatrix}$$

$= (y^2 + yz - zx - x^2) - (xy + y^2 - z^2 - xz) + (x^2 + xy - yz - z^2)$

$= y^2 + yz - zx - x^2 - xy - y^2 + z^2 + xz + x^2 + xy - yz - z^2 = 0$

28. Option (D) is correct

Explanation: Area of triangle having vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given as:

$$\text{Area} = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

Given the vertices $(2, -6)$, $(5, 4)$, and $(k, 4)$, we substitute:

$$\text{Area} = \frac{1}{2}|2(4-4) + 5(4+6) + k(-6-4)|$$

$$35 = \frac{1}{2}|2(0) + 5(10) + k(-10)|$$

$$35 = \frac{1}{2}|50 - 10k|$$

$$70 = |50 - 10k|$$

Solving for k:

$50 - 10k = 70 \Rightarrow k = -2$

or $\quad 50 - 10k = -70 \Rightarrow k = 12$

29. Option (D) is correct.

Explanation:

$\because \quad |A| = |kA|$

$\therefore \quad |kA| = k^2|A|$

$|A| = k^2|A|$

$|A|(1 - k^2) = 0$

$1 - k^2 = 0 \Rightarrow k^2 = 1 \Rightarrow k = \pm 1$

Sum of all possible $k = 1 + (-1) = 0$

ASSERTION-REASON QUESTIONS (1 Mark)

1. Option (A) is correct.

Explanation:

$$A = \begin{bmatrix} 1 & \cos\theta & 1 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{bmatrix}$$

$$|A| = 2 + 2\cos^2\theta$$

Since $\cos^2\theta \in [0, 1]$, we get:

$$|A| \in [2, 4]$$

$\cos\theta \in [-1, 1]$, $\forall\theta\in[0, 2\pi]$ is always true.

Both (A) and (R) are true, and (R) correctly explains (A).

2. Option (A) is correct.

Explanation: The determinant of a square matrix follows the property:

$$\det(kA) = k^n \det(A)$$

where n is the order of the square matrix A.

The reason states that if a single row or column is multiplied by k, then the determinant is multiplied by k.

If all n rows are multiplied by k, then the determinant scales as k^n.

Hence, Assertion (A) is true, and Reason (R) is the correct explanation of (A).

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. Expands the determinant as:

   $n(n-1)! + n(n!)$

   Simplifies the above expression as:

   $n! + n(n!) = n!(n+1) = (n+1)!$

2. $|3P^2Q| = 3^4 \times 5 \times 5 \times 4 = 3^4(100) = 8100.$

3. The value of x as 1.

   Since, the value of the determinant is the same when its rows and columns are interchanged.

4. Rohan's working is correct. For example, if $A = kB$, where A and B are square matrices of order n, then $|A| = k^n|B|$, where $n = 1, 2, 3, ...$

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. $$A = \begin{bmatrix} 17 & 10 \\ 0 & -16 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ -1 & -2 \end{bmatrix}^{-1}$$

$$= -\frac{1}{8}\begin{bmatrix} 17 & 10 \\ 0 & -16 \end{bmatrix}\begin{bmatrix} -2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -5 \\ 2 & 8 \end{bmatrix}$$

Also, $A^{-1} = \frac{1}{34}\begin{bmatrix} 8 & 5 \\ -2 & 3 \end{bmatrix}$

2. $A^3 = A^{-1} \Leftrightarrow A^4 = I$

$$A^4 = A^2A^2 = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}\begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

= RHS Hence proved.

3. $$\Delta = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$$

$= x(-x^2 - 1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta)$

$= -x^3 - x + x\sin^2\theta + \sin\theta\cos\theta - \cos\theta\sin\theta + x\cos^2\theta$

$= -x^3 - x + x$

$= -x^3$, independent of θ

4. Frames an equation in p using the co-factor of element 6 as

$$A_{23} = (-1)^{2+3}\begin{vmatrix} p & -2 \\ 1 & 3 \end{vmatrix} = -3p - 2 = -11$$

On solving the above equation, we get $p = 3$

Frames an equation in q using the minor of element 3 as

$$M_{22} = \begin{vmatrix} p & -3 \\ 1 & q \end{vmatrix} = \begin{vmatrix} 3 & -3 \\ 1 & q \end{vmatrix} = 3q + 3 = 0$$

On solving the above equation, we get the value of $q = (-1)$.

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. $|A| = 1 \neq 0$ hence A^{-1} exists.

Here, $\text{adj } A = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}$

Since, $A^{-1} = \frac{\text{adj}(A)}{|A|}$

$\therefore$ $A^{-1} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}$

The given system of equations can be written as

AX = B

$\Rightarrow$ $\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

Also, $X = A^{-1}B$

$\Rightarrow$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

$$= \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

$\Rightarrow x = 0, y = -5, z = -3$

2. We know that, $AA^{-1} = I$

$$\begin{bmatrix} -1 & a & 2 \\ 1 & 2 & x \\ 3 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ b & y & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1-8a+2b & 1+7a+2y & 5-5a \\ -15+bx & 13+xy & 3x-9 \\ -5+b & 4+y & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

On comparing, we get

$-5 + b = 0 \Rightarrow b = 5, 5 - 5a = 0 \Rightarrow a = 1$

$4 + y = 0 \Rightarrow y = -4, 3x - 9 = 0 \Rightarrow x = 3$

$\therefore (a + x) - (b + y) = (1 + 3) - (5 - 4) = 3$

3. Given system of linear equations is equivalent to $AX = B$, where

$$A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix}, B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$|A| = 1200 \neq 0$

Cofactors of the elements of A are

$A_{11} = 75, A_{12} = 110, A_{13} = 72$

$A_{21} = 150, A_{22} = -100, A_{23} = 0$

$A_{31} = 75, A_{32} = 30, A_{33} = -24$

$$\text{adj } A = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj}A}{|A|} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$\therefore x = 2, y = 3, z = 5$

4. $|A| = 1 + \cot^2 x = \text{cosec}^2 x$

$$\text{adj } A = \begin{bmatrix} 1 & -\cot x \\ \cot x & 1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{\text{cosec}^2 x}\begin{bmatrix} 1 & -\cot x \\ \cot x & 1 \end{bmatrix}$$

$$A' = \begin{bmatrix} 1 & -\cot x \\ \cot x & 1 \end{bmatrix}$$

$$A'A^{-1} = \frac{1}{\text{cosec}^2 x}\begin{bmatrix} 1-\cot^2 x & -2\cot x \\ 2\cot x & 1-\cot^2 x \end{bmatrix}$$

$$= \begin{bmatrix} \sin^2 x - \cos^2 x & -2\sin x\cos x \\ 2\sin x\cos x & \sin^2 x - \cos^2 x \end{bmatrix}$$

$$= \begin{bmatrix} -\cos 2x & -\sin 2x \\ \sin 2x & -\cos 2x \end{bmatrix}$$

5. We know that $(AB)^{-1} = B^{-1}A^{-1}$

$|B^{-1}| = 1\,(16 - 9) - 3(4 - 3) + 3\,(3 - 4)$

$|B^{-1}| = 7 - 3 - 3$

$= 1$

$|A| = 5(-1) + 4(1) = -1 \neq 0$. Hence, A^{-1} exists.

Cofactors of the elements of A are:

$A_{11} = -1, A_{12} = 0, A_{13} = 1$

$A_{21} = 8, A_{22} = 1, A_{23} = -10$

$A_{31} = -12, A_{32} = -2, A_{33} = 15$

$$\text{adj } A = \begin{bmatrix} -1 & 8 & -12 \\ 0 & 1 & -2 \\ 1 & -10 & 15 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj A}}{|\text{A}|}$$

$$= \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$$

$$(AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 19 & -27 \\ -2 & 18 & -25 \\ -3 & 29 & -42 \end{bmatrix}$$

$$|(AB)^{-1}| = |B^{-1}A^{-1}| = |B^{-1}|\,|A^{-1}|$$

$$= 1 \times -1 = -1$$

6. $|A| = 1(4) - 1(2) + 1(-1) = 1 \neq 0 \therefore A^{-1}$ exists.

Cofactors of the elements of A are:

$A_{11} = 4, A_{12} = -2, A_{13} = -1$

$A_{21} = -1, A_{22} = 1, A_{23} = 0$

$A_{31} = -1, A_{32} = 0, A_{33} = 1$

$$\therefore \quad \text{adj } A = \begin{bmatrix} 4 & -1 & -1 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{|A|} = \begin{bmatrix} 4 & -1 & -1 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Given system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

$$X = A^{-1}B = \begin{bmatrix} 4 & -1 & -1 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$$

$x = -2, y = 0, z = 3$

7. For matrix $A = \begin{pmatrix} 2 & 1 & -3 \\ 3 & 2 & 1 \\ 1 & 2 & -1 \end{pmatrix}$, $|A| = -16 \neq 0$ so, A^{-1} exists.

$$\text{adj } A = \begin{pmatrix} -4 & -5 & 7 \\ 4 & 1 & -11 \\ 4 & -3 & 1 \end{pmatrix},$$

Thus, $$A^{-1} = \frac{-1}{16}\begin{pmatrix} -4 & -5 & 7 \\ 4 & 1 & -11 \\ 4 & -3 & 1 \end{pmatrix}$$

So, given equation can be written into a matrix equation as

$$AX = B$$

$$\begin{pmatrix} 2 & 1 & -3 \\ 3 & 2 & 1 \\ 1 & 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 13 \\ 4 \\ 8 \end{pmatrix} \Rightarrow X = A^{-1}.B$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{-1}{16}\begin{pmatrix} -4 & -5 & 7 \\ 4 & 1 & -11 \\ 4 & -3 & 1 \end{pmatrix}\begin{pmatrix} 13 \\ 4 \\ 8 \end{pmatrix}$$

$$= \frac{-1}{16}\begin{pmatrix}-16\\-32\\48\end{pmatrix} = \begin{pmatrix}1\\2\\-3\end{pmatrix}$$

$\Rightarrow x = 1, y = 2, z = -3$

8. For matrix $A = \begin{pmatrix}1 & 2 & 1\\2 & 3 & -1\\1 & 0 & 1\end{pmatrix}$,

Here, $|A| = -6 \neq 0$ so, A^{-1} exists.

Adjoint of Matrix A is

$$\text{adj } A = \begin{pmatrix}3 & -2 & -5\\-3 & 0 & 3\\-3 & 2 & -1\end{pmatrix},$$

Thus, $$A^{-1} = \frac{-1}{6}\begin{pmatrix}3 & -2 & -5\\-3 & 0 & 3\\-3 & 2 & -1\end{pmatrix}$$

The, given equation can be written into a matrix equation as

$$A^T X = B$$

$$\begin{pmatrix}1 & 2 & 1\\2 & 3 & 0\\1 & -1 & 1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}5\\1\\8\end{pmatrix}$$

$\Rightarrow$ $$X = (A^T)^{-1} B = X = (A^{-1})^T B$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix} = \frac{-1}{6}\begin{pmatrix}3 & -3 & -3\\-2 & 0 & 2\\-5 & 3 & -1\end{pmatrix}\begin{pmatrix}5\\1\\8\end{pmatrix}$$

$$= \frac{-1}{6}\begin{pmatrix}-12\\6\\-30\end{pmatrix} = \begin{pmatrix}2\\-1\\5\end{pmatrix}$$

$\therefore x = 2, y = -1, z = 5$

9. Here, $|A| = 1(6) - 2(3) - (4) = -12 \neq 0$, $\therefore A^{-1}$ exists

$$\text{Adj } A = \begin{bmatrix}6 & -6 & -6\\-3 & 3 & -3\\4 & 0 & -4\end{bmatrix}$$

$\therefore$ $$A^{-1} = -\frac{1}{12}\begin{bmatrix}6 & -6 & -6\\-3 & 3 & -3\\4 & 0 & -4\end{bmatrix}$$

The given system of equations can be written $AX = B$,

$$X = \begin{bmatrix}x\\y\\z\end{bmatrix}, B = \begin{bmatrix}1\\2\\3\end{bmatrix}$$

$$X = A^{-1}B$$

$\Rightarrow$ $$\begin{bmatrix}x\\y\\z\end{bmatrix} = -\frac{1}{12}\begin{bmatrix}6 & -6 & -6\\-3 & 3 & -3\\4 & 0 & -4\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{bmatrix}2\\ \frac{1}{2}\\ \frac{2}{3}\end{bmatrix}$$

$\therefore$ The solution of the given system of equations is:

$x = 2, y = \frac{1}{2}, z = \frac{2}{3}$

10. Given, $A = \begin{bmatrix}1 & 2 & -2\\-1 & 3 & 0\\0 & -2 & 1\end{bmatrix}, B^{-1} = \begin{bmatrix}3 & -1 & 1\\-15 & 6 & -5\\5 & -2 & 2\end{bmatrix}$

$(AB)^{-1} = B^{-1}A^{-1}$

$|A| = 1(3) - 2(-1) - 2(2) = 3 + 2 - 4 = 1 \neq 0$

$$\text{adj } (A) = \begin{bmatrix}3 & 2 & 6\\1 & 1 & 2\\2 & 2 & 5\end{bmatrix}$$

$$A^{-1} = \frac{1}{1}\begin{bmatrix}3 & 2 & 6\\1 & 1 & 2\\2 & 2 & 5\end{bmatrix}$$

$\therefore$ $$B^{-1}A^{-1} = \begin{bmatrix}3 & -1 & 1\\-15 & 6 & -5\\5 & -2 & 2\end{bmatrix}\begin{bmatrix}3 & 2 & 6\\1 & 1 & 2\\2 & 2 & 5\end{bmatrix}$$

$$= \begin{bmatrix}10 & 7 & 21\\-49 & -34 & -103\\17 & 12 & 36\end{bmatrix}$$

11. Given system is

$$\begin{bmatrix}1 & 2 & 3\\2 & -1 & 1\\3 & 2 & -2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}6\\2\\3\end{bmatrix}$$

$A.X = B \Rightarrow X = A^{-1}B$

$|A| = 35 \neq 0$

$A_{11} = 0 \quad A_{12} = 7 \quad A_{13} = 7$

$A_{21} = 10 \quad A_{22} = -11 \quad A_{23} = 4$

$A_{31} = 5 \quad A_{32} = 5 \quad A_{33} = -5$

$\therefore$ $$A^{-1} = \frac{1}{35}\begin{bmatrix}0 & 10 & 5\\7 & -11 & 5\\7 & 4 & -5\end{bmatrix}$$

$\Rightarrow$ $$X = \frac{1}{35}\begin{bmatrix}0 & 10 & 5\\7 & -11 & 5\\7 & 4 & -5\end{bmatrix}\begin{bmatrix}6\\2\\3\end{bmatrix}$$

$$= \frac{1}{35}\begin{bmatrix}35\\35\\35\end{bmatrix} = \begin{bmatrix}1\\1\\1\end{bmatrix}$$

$\therefore x = 1, y = 1, z = 1$

12. Here, $$\text{adj } A = \begin{bmatrix}-7 & -2\\-5 & 3\end{bmatrix}$$

$$|A| = -21 - 10 = -31$$

$$A^{-1} = \frac{-1}{31}\begin{bmatrix}-7 & -2\\-5 & 3\end{bmatrix}$$

Given system of equation is

$$\begin{bmatrix}3 & 5\\2 & -7\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}11\\-3\end{bmatrix}$$

which is $A'X = B$, where $A = \begin{bmatrix}3 & 5\\2 & -7\end{bmatrix}$, $X = \begin{bmatrix}x\\y\end{bmatrix}, B = \begin{bmatrix}11\\-3\end{bmatrix}$

$\Rightarrow$ $$X = (A')^{-1}B$$

$\Rightarrow \quad X = (A^{-1})'B$

$$= \frac{-1}{31}\begin{bmatrix} -7 & -5 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 11 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$\therefore x = 2, y = 1$

13. Given, $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$

$|A| = 1(8-6) + 1(0+9) + 2(0-6) = -1 \neq 0$

$\therefore A$ is invertible.

$$\text{adj } A = \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$$

$$\Rightarrow \quad A^{-1} = \frac{1}{|A|}\text{adj } A = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

The given system of equation can be written as $AX = B$, where

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\Rightarrow \quad X = A^{-1}B$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$\Rightarrow x = 1, y = 2, z = 1$

14. Given system of equations can be written as $AX = B$, where

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

here, $|A| = 4 \neq 0 \Rightarrow A^{-1}$ exists.

$$\therefore \quad \text{adj } A = \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

$$\Rightarrow \quad A^{-1} = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$\Rightarrow x = 2, y = 1, z = 3$

15. $|A| = 3(-2) - 4(1) + 1(5) = -5 \neq 0 \Rightarrow A^{-1}$ exists.

$A_{11} = -2, A_{12} = -1, A_{23} = 3$

$A_{21} = -1, A_{22} = 2, A_{23} = -1$

$A_{31} = 5, A_{32} = -5, A_{33} = -5$

$$\text{adj } A = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{ adj A} = -\frac{1}{5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

Given system of equations can be written as $AX = B$,

where $B = \begin{bmatrix} 2000 \\ 2500 \\ 900 \end{bmatrix}$

$X = A^{-1}B$

$$= -\frac{1}{5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}\begin{bmatrix} 2000 \\ 2500 \\ 900 \end{bmatrix}$$

$$= -\frac{1}{5}\begin{bmatrix} -2000 \\ -1500 \\ -1000 \end{bmatrix} = \begin{bmatrix} 400 \\ 300 \\ 200 \end{bmatrix}$$

$\therefore \ x = 400, y = 300$ and $z = 200$

16. Here, $|A| = (-56-56) - 6(-8-7) + 6(8-7) = -16 \neq 0$

$\Rightarrow A^{-1}$ exists

$A_{11} = -112, A_{12} = 96, A_{13} = 0$

$A_{21} = 15, A_{22} = -14, A_{23} = -1$

$A_{31} = 1, A_{32} = -2, A_{33} = 1$

$$\text{adj } A = \begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj } A = -\frac{1}{16}\begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}$$

Given system of equations can be written as $AX = B$,

where $B = \begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix}$

$X = A^{-1}B$

$$= -\frac{1}{16}\begin{bmatrix} -112 & 15 & 1 \\ 96 & -14 & -2 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 5000 \\ 35800 \\ 7000 \end{bmatrix}$$

$$= \frac{-1}{16}\begin{bmatrix} -16000 \\ -35200 \\ -27800 \end{bmatrix}$$

$\therefore x = 1000, y = 2200, z = 1800$

17. Here, $|A| = 1(-2) - 1(-5) + 1(1) = 4 \neq 0 \Rightarrow A^{-1}$ exists.

$A_{11} = -2, A_{12} = 5, A_{13} = 1$

$A_{21} = 0, A_{22} = -2, A_{23} = 2$

$A_{31} = 2, A_{32} = -1, A_{33} = -1$

$$\text{adj } A = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

Given system of equations can be written as $AX = B$,

where $B = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$

$$X = A^{-1}B$$

$$= \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 12 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$\therefore x = 3, y = 1$ and $z = 2$

Level - 2 ADVANCED COMPETENCY FOCUSED QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Mark)

1. Option (B) is correct.

Explanation: $|A| = (2)(4) - (3)(1) = 8 - 3 = 5 \neq 0$

A matrix is invertible only if its determinant $\neq 0$.

2. Option (A) is correct.

Explanation: Given,

$$\vec{a} = (3, 0) \Rightarrow a_1 = 3, a_2 = 0$$

$$\vec{b} = (0, 4) \Rightarrow b_1 = 0, b_2 = 4$$

Substituting into the determinant:

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 3 & 0 \\ 0 & 4 \end{vmatrix} = \frac{1}{2}\,|(3)(4) - (0)(0)|$$

$$= \frac{1}{2}\times 12 = 6 \text{ sq. units}$$

3. Option (C) is correct.

Explanation: Determinant $=1(5 \times 9 - 6 \times 8) - 2(4 \times 9 - 6 \times 7) + 3(4 \times 8 - 5 \times 7)$

$=1(45 - 48) - 2(36 - 42) + 3(32 - 35)$

$= 1(-3) - 2(-6) + 3(-3) = -3 + 12 - 9 = 0$

Since the determinant is zero, this means that the matrix is singular, the investment strategies (represented by rows/columns) are linearly dependent, and the matrix does not have an inverse.

4. Option (C) is correct.

Explanation: Zero determinant means the system has no unique solution – possibly inconsistent or dependent.

5. Option (A) is correct.

Explanation:

For a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant is:

$$\det(A) = ad - bc$$

Substituting values from matrix M:

$$\det(M) = (1)(k) - (2)(3) = k - 6$$

$$k - 6 = 0 \Rightarrow k = 6$$

ASSERTION-REASON QUESTIONS (1 Marks)

1. Option (C) is correct.

Explanation: Assertion is true. By definition, if the determinant of a square matrix is zero, the matrix is called singular.

Reason is false. A singular matrix does not have an inverse. Only non-singular matrices (with non-zero determinant) are invertible.

2. Option (C) is correct.

Explanation: Assertion is true. When the determinant is zero, the system is either inconsistent (no solution) or dependent (infinitely many solutions).

Reason is false because a zero determinant does not always imply inconsistency. It only means the system does not have a unique solution. It could still have infinitely many solutions (consistent and dependent system).

3. Option (A) is correct.

Explanation: Assertion is true. The determinant of any identity matrix (of any order) is always 1 because the product of diagonal elements is 1 and there are no non-diagonal contributions.

Reason is also true because that is the definition of an identity matrix.

Both assertion and reason are true and reason is the correct explanation of assertion.

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. The area of ΔPQR having base, $QR = 5$ units

Area $= \frac{1}{2}\times 5\times 4 = 10$ sq. units

Equates the area found to that using determinants as follows:

$$\frac{1}{2}\begin{vmatrix} k & -1 & 1 \\ 3 & -5 & 1 \\ 6 & -1 & 1 \end{vmatrix} = 10$$

Expands the above determinant to write the equation as:

$$\frac{1}{2}|k(-5+1)+1(3-6)+1(-3+30)|=10$$

$$\Rightarrow |-4k+24|=20$$

On solving the above equation we get the values of k as 1 or 11.

2. $$\text{Adj } A = \begin{pmatrix} 16 & 8 \\ -12 & 4 \end{pmatrix}$$

$$\text{Adj } B = \begin{pmatrix} -4 & 8 \\ -12 & -16 \end{pmatrix}$$

Now, the product of (Adj A) and (Adj B) as:

$$= \begin{bmatrix} -160 & 0 \\ 0 & -160 \end{bmatrix} \begin{bmatrix} 16 & 8 \\ -12 & 4 \end{bmatrix} \begin{bmatrix} -4 & 8 \\ -12 & -16 \end{bmatrix}$$

Thus, the matrix (Adj A) × (Adj B) is a scalar matrix.

3. We know that, for a square matrix of order n,

$$|\text{adj } B| = |B|^{(n-1)}$$

Applies the above result on (adj B) we get $|\text{adj } B| = |B|^2$.

Now, $|\text{adj } B| = (-1)(-8-3) - 1(2-2) = 11$

[Expanding along C_1]

Therefore, $|B| = \sqrt{11}$ or $(-\sqrt{11})$.

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. Assumes the cost of each adult ticket is ₹ x and each child's ticket is ₹ y. The equations that represent the given scenario as follows:

$$6x + 4y = 400$$
$$5x + 3y = 325$$

The above system of equations in the matrix form is written as $AX = B$ where:

$$A = \begin{bmatrix} 6 & 4 \\ 5 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } B = \begin{bmatrix} 400 \\ 325 \end{bmatrix}$$

Here, $|A| = 18 - 20 = -2 \neq 0$. Hence A^{-1} exists and the system has a unique solution.

Now,

$$A^{-1} = \frac{1}{|A|} \times \text{adj } A$$

$$\Rightarrow \frac{1}{-2}\begin{bmatrix} 3 & -4 \\ -5 & 6 \end{bmatrix}$$

Here, $X = A^{-1}B$ i.e.,

$$X = \frac{1}{-2}\begin{bmatrix} 3 & -4 \\ -5 & 6 \end{bmatrix}\begin{bmatrix} 400 \\ 325 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 50 \\ 25 \end{bmatrix}$$

Hence, the cost of each adult and child's ticket is ₹ 50 and ₹ 25 respectively.

2. Here,

$$\text{adj } A = \begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}$$

and

$$\text{adj } B = \begin{bmatrix} -8 & 3 \\ -4 & 2 \end{bmatrix}$$

Now,

$$\text{adj } A \times \text{adj } B = \begin{bmatrix} -20 & 6 \\ -36 & 16 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} -83 \\ -42 \end{bmatrix}$$

Also,

$$B \times A = \begin{bmatrix} 16 & -6 \\ 36 & -20 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 \\ 4 & -8 \end{bmatrix}\begin{bmatrix} 53 \\ -24 \end{bmatrix}$$

and

$$\text{adj}\,(BA) = \begin{bmatrix} -20 & 6 \\ -36 & 16 \end{bmatrix}$$

From above, we conclude that (adj A) × (adj B) = adj (BA).

3. (i) The condition for three points P(x_1, y_1), Q(x_2, y_2), R(x_3, y_3) to be collinear is:

$$\text{Area} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Substituting the given points:

$$\begin{vmatrix} 2 & 3 & 1 \\ 4 & 7 & 1 \\ k & 6 & 1 \end{vmatrix} = 0$$

(ii) Using expansion of determinant:

$= 2(7 \times 1 - 1 \times 6) - 3(4 \times 1 - 1 \times k) + 1(4 \times 6 - 7 \times k)$

$= 2(7 - 6) - 3(4 - k) + (24 - 7k)$

$= 2(1) - 3(4 - k) + (24 - 7k)$

$= 2 - 12 + 3k + 24 - 7k = (14 - 4k)$

Set determinant = 0:

$$14 - 4k = 0 \Rightarrow k = \frac{14}{4} = 3.5$$

(iii) A determinant is an efficient method for checking collinearity because:

(1) It gives a single algebraic condition that avoids the need to compute multiple slopes.

(2) It is coordinate-independent, works for any three points.

(3) It is a direct and compact approach derived from the geometric concept of area — if the area is zero, the triangle collapses into a straight line.

4. (i) $\text{Det}(A) = (2)(k) - (1)(3) = 2k - 3$

For A to be non-singular, we must have:

$$2k - 3 \neq 0 \Rightarrow k \neq \frac{3}{2}$$

(ii) When $k = 2$;

$\text{Det}(A) = 2(2) - 3 = 4 - 3 = 1$

Since $\text{Det}(A) = 1 \neq 0$, the matrix is invertible.

CASE BASED QUESTIONS (4 Marks)

1. (i) Let, No. of girl child scholarships = x

No. of meritorious achievers = y

$$x + y = 50$$

$$3000x + 4000y = 180000$$

or $$3x + 4y = 180$$

$$\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 180 \end{bmatrix}$$

(ii) $\begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix} = 1 \neq 0$

$\therefore$ System is consistent.

(iii) (a) Let $A = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 50 \\ 100 \end{bmatrix}$

$$AX = B \Rightarrow X = A^{-1}B$$

$$X = \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 50 \\ 180 \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \end{bmatrix}$$

$\Rightarrow x = 20, y = 30$

OR

(b) Required expenditure = ₹ [30(3000) + 20(4000)]

= ₹ 1,70,000

2. (i) $(x - 25)(y + 25) = xy + 625 \Rightarrow x - y = 50$

$(x - 20)(y + 10) = xy - 200 \Rightarrow x - 2y = 0$

(ii) The system of linear equations can be written in matrix form as

$$\begin{bmatrix} 1 & -1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \end{bmatrix}$$

$$\begin{vmatrix} 1 & -1 \\ 1 & -2 \end{vmatrix} = -2 + 1 = -1 \neq 0$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & -2 \end{bmatrix}^{-1}\begin{bmatrix} 50 \\ 0 \end{bmatrix}$$

$$= -\begin{bmatrix} -2 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 50 \\ 0 \end{bmatrix} = \begin{bmatrix} 100 \\ 50 \end{bmatrix}$$

$x = 100$ m, $y = 50$ m

3. (i) Matrix equation is AX = B, where

$$A = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 160 \\ 190 \\ 250 \end{bmatrix}$$

where x is the number of pens bought, y the number of bags and z the number of instrument boxes.

(ii) $|A| = 5(4 - 6) - 3(8 - 3) + 1(4 - 1) = -22$

(iii) $$\text{adj}(A) = \begin{bmatrix} -2 & -5 & +3 \\ -10 & 19 & -7 \\ 8 & -13 & -1 \end{bmatrix}^T = \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ +3 & -7 & -1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{\text{adj}(A)}{|A|}$$

$$= \frac{1}{(-22)}\begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & 13 \\ 3 & -7 & -1 \end{bmatrix}$$

OR

$$P = A^2 - 5A = \begin{bmatrix} 32 & 20 & 18 \\ 15 & 13 & 17 \\ 13 & 13 & 23 \end{bmatrix} - \begin{bmatrix} 25 & 15 & 5 \\ 10 & 5 & 15 \\ 5 & 10 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 5 & 13 \\ 5 & 8 & 2 \\ 8 & 3 & 3 \end{bmatrix}$$

4. (i) $8x + 4y = 200$; $5x + 10y = 275$

(ii) $\begin{bmatrix} 8 & 4 \\ 5 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 200 \\ 275 \end{bmatrix}$

(iii) (a) $|A| = 8 \times 10 - 5 \times 4 = 80 - 20 = 60$

OR

(b) $\text{adj } A = \begin{bmatrix} 10 & -4 \\ -5 & 8 \end{bmatrix}$

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. Given, $h(t) = pt^2 + qt + r$

At $t = 0$,

$$30 = r$$

$\therefore$ $$r = 30$$

Constructing the following equations in p and q by substituting $t = 1$ and $t = 5$ respectively:

$$p + q = 7$$

$$5p + q = 3$$

Writing the above system of equations in the matrix form using $AX = B$ as:

$$\begin{bmatrix} 1 & 1 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

Here, $|A|$ as $-4 (\neq 0)$. Hence, A^{-1} exists and the system has a unique solution.

$$\text{adj } A = \begin{bmatrix} 1 & -1 \\ -5 & 1 \end{bmatrix}$$

Finds A^{-1} using $|A|$ and adj A as:

Now, $$A^{-1} = \frac{1}{|A|} \times \text{adj A}$$

$$= \frac{1}{-4}\begin{bmatrix} 1 & -1 \\ -5 & 1 \end{bmatrix}$$

Now, $X = A^{-1}B$ where

$$X = \begin{bmatrix} p \\ q \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ 5 & 1 \end{bmatrix}, B = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 1 \\ 5 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 5 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

$$= -\frac{1}{4}\begin{bmatrix} 1 & -1 \\ -5 & 1 \end{bmatrix}\begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

$$= -\frac{1}{4}\begin{bmatrix} 4 \\ -32 \end{bmatrix} = \begin{bmatrix} -1 \\ 8 \end{bmatrix}$$

Thus, $p = -1$ and $q = 8$.
Hence, $h(t) = -t^2 + 8t + 30$.

2. Here,

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$\therefore \quad (B^{-1}A^{-1}).A = B^{-1}(A^{-1}A)$$

$$= B^{-1}.I$$

$$= B^{-1}$$

$$= \frac{1}{48}\begin{bmatrix} 0 & 0 & 16 \\ 0 & 24 & 0 \\ 3 & 0 & 0 \end{bmatrix}\begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & -\frac{1}{2} & 0 \\ \frac{-1}{8} & 0 & 0 \end{bmatrix}$$

Now,

$$\text{adj}(B^{-1}) = \begin{bmatrix} 0 & 0 & \frac{-1}{16} \\ 0 & \frac{1}{8} & 0 \\ \frac{1}{2} & 0 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{8} & 0 \\ \frac{-1}{16} & 0 & 0 \end{bmatrix}$$

Also, $\det(B^{-1}) = 0 - 0 + 0 - 0 + 1\left(\frac{-1}{16}\right) = \frac{-1}{16}$.

Therefore,

$$B = \frac{\text{adj}(B^{-1})}{|B^{-1}|}$$

$$= \frac{1}{-\frac{1}{16}}\begin{bmatrix} 0 & 0 & \frac{1}{2} \\ 0 & \frac{1}{8} & 0 \\ \frac{-1}{16} & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -8 \\ 0 & -2 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

3. Let the fixed commission payable on policies A, B and C per unit as x, y and z respectively and constructing the system of linear equations as:

$$8x + 4y + 6z = 7850$$

$$9x + 9y + 6z = 9600$$

$$12x + 9y + 12z = 15000$$

Writing the system of equations in the form of a matrix equation $AX = B$, where,

$$A = \begin{bmatrix} 8 & 4 & 6 \\ 9 & 9 & 6 \\ 12 & 9 & 12 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 7850 \\ 9600 \\ 15000 \end{bmatrix}$$

Here, $|A| = 126 \neq 0$ hence A^{-1} exists.

Now, $$\text{adj } A = \begin{bmatrix} 54 & 6 & -30 \\ -36 & 24 & 6 \\ -27 & -24 & 36 \end{bmatrix}$$

Finds A^{-1} as:

$$A^{-1} = \frac{1}{|A|} = \text{adj } A = \frac{1}{126}\begin{bmatrix} 54 & 6 & -30 \\ -36 & 24 & 6 \\ -27 & -24 & 36 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{126}\begin{bmatrix} 54 & 6 & -30 \\ -36 & 24 & 6 \\ -27 & -24 & 36 \end{bmatrix}\begin{bmatrix} 7850 \\ 9600 \\ 15000 \end{bmatrix} = \begin{bmatrix} 250 \\ 300 \\ 775 \end{bmatrix}$$

Hence, the fixed commission payable on policies A, B and C per unit are ₹ 250, ₹ 300 and ₹ 775 respectively.