CHAPTER 8

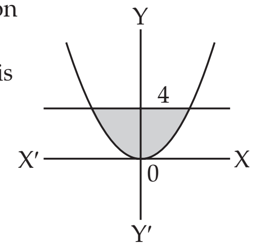
Application of The Integrals

Level - 1 CORE SUBJECTIVE QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQ) (1 Marks)

1. Option (B) is correct.

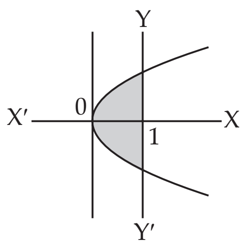
Explanation: The required region is symmetric about the y-axis. So, required area (in sq units) is



$$= \left|2\int_0^4 2\sqrt{y}dy\right| = 4\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 = \frac{64}{3}.$$

2. Option (B) is correct.

Explanation: The given curve is a rightward-opening parabola $y^2 = 4x$. The area between the curve and the X-axis from $x = 0$ to $x = 1$ is given by:



$$A = 2\int_0^1 2\sqrt{x}dx$$

$$= 2\times\left[\frac{4x^{3/2}}{3}\right]_0^1 = 2\times\frac{4}{3} = \frac{8}{3} \text{ sq. units}$$

3. Option (C) is correct.

Explanation: $y = \sqrt{x}$ as $x = y^2$, the area is:

$$A = \int_0^3 y^2 dy$$

$$= \left[\frac{y^3}{3}\right]_0^3$$

$$= \frac{3^3}{3} - \frac{0^3}{3} = \frac{27}{3} = 9 \text{ sq. units}$$

4. Option (A) is correct.

Explanation: The required area is:

$$A = \int_1^3 \log(x+1)dx$$

Using integration by parts:

$$\int \log(x+1)dx = (x+1)\log(x+1) - x$$

Evaluating from $x = 1$ to $x = 3$:

$$A = [(x + 1)\log(x + 1) - x]_1^3$$

$$= (4 \log 4 - 3) - (2 \log 2 - 1)$$

$$= 6 \log 2 - 2 \text{ sq. units}$$

ASSERTION-REASON QUESTIONS (1 Marks)

1. Option (A) is correct.

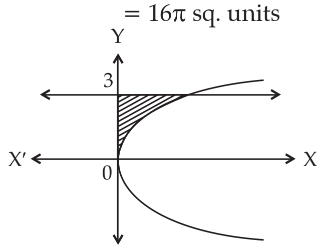
Explanation:

Required area, $A = 4\int_0^4 \sqrt{4^2 - x^2}\, dx$

$$= 4\left[\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4$$

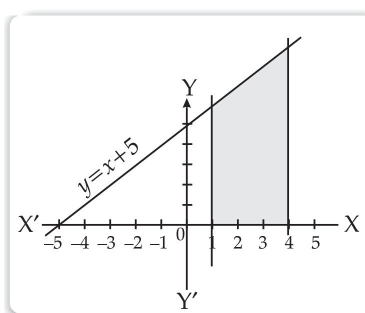
$$= 4\left[0 + 8\cdot\frac{\pi}{2}\right]$$

$= 16\pi$ sq. units



2. Option (C) is correct.

*Explanation***:** From figure, Area of shaded region,



$$A = \int_1^4 y\, dx = \int_1^4 (x+5)dx$$

$$= \left[\frac{x^2}{2} + 5x\right]_1^4$$

$$= \left(\frac{16}{2}+20\right)-\left(\frac{1}{2}+5\right)$$

$$= \frac{56}{2}-\frac{11}{2}$$

$$= \frac{45}{2} \text{ sq. units}$$

3. Option (A) is correct.

Explanation:

$$\text{Area } = \int_1^4 \frac{1}{3}(x+5)dx - \int_1^2 (4-2x)dx - \int_2^4 \frac{1}{2}(3x-6)dx$$

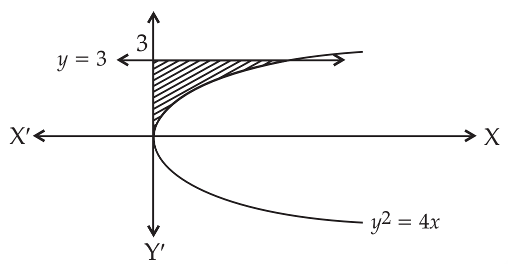
$$= \frac{1}{3}\left[\frac{(x+5)^2}{2}\right]_1^4 + 2\left[\frac{(2-x)^2}{2}\right]_1^2 - \frac{3}{2}\left[\frac{(x-2)^2}{2}\right]_2^4$$

$$= \left(\frac{81}{6}-\frac{36}{6}\right)+(0-1)-\frac{3}{4}\cdot 4$$

$$= \frac{15}{2}-1-3 = \frac{7}{2} \text{ sq. units.}$$

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1.

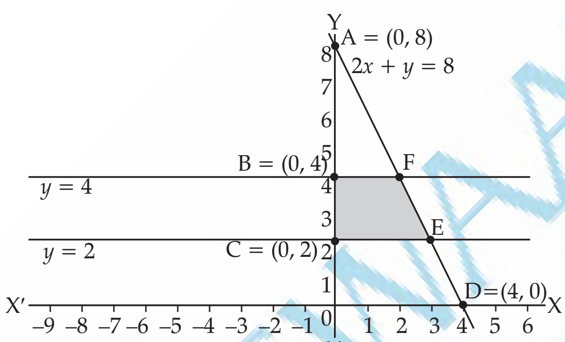


$$\text{Required area } = \int_0^3 \frac{y^2}{4}\,dy = \left[\frac{y^3}{12}\right]_0^3$$

$$= \frac{27}{12}-0$$

$$= \frac{9}{4} \text{ square units}$$

2.



$$\text{Required area } = \int_2^4 \frac{1}{2}(8-y)dy$$

$$= \frac{1}{2}\left|8y-\frac{y^2}{2}\right|_2^4$$

$= 5$ sq. units

3. Writes the expression for the displacement of the object using the velocity function as follows:

$$\int_0^3 (t^2+4t-5)\,dt$$

Evalautes the above definite integral as:

$$\left[\frac{t^3}{3}+\frac{4t^2}{2}-5t\right]_0^3$$

Applies the given limit to find the displacement as 9 + 18 – 15 = 12 km

4. Sets the integral equation as:

$$\int_0^2 (4x^3-kx^2+1)dx = 10$$

Simplifies the above equation as:

$$\left[\frac{4x^4}{4}-\frac{kx^3}{3}+x\right]_0^2 = 10$$

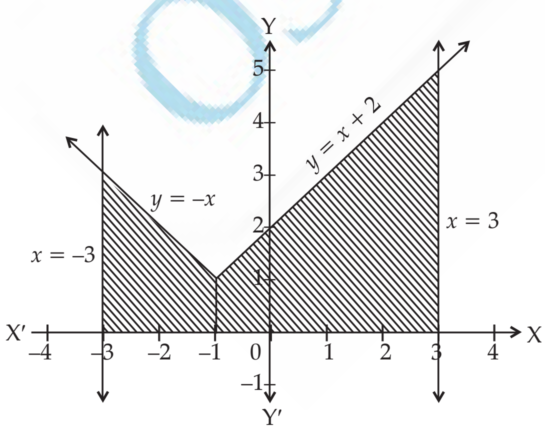
Applies the limit and simplifies the above equation as:

$$16-\frac{8k}{3}+2 = 10$$

Simplifies the above equation to get k as 3.

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1.



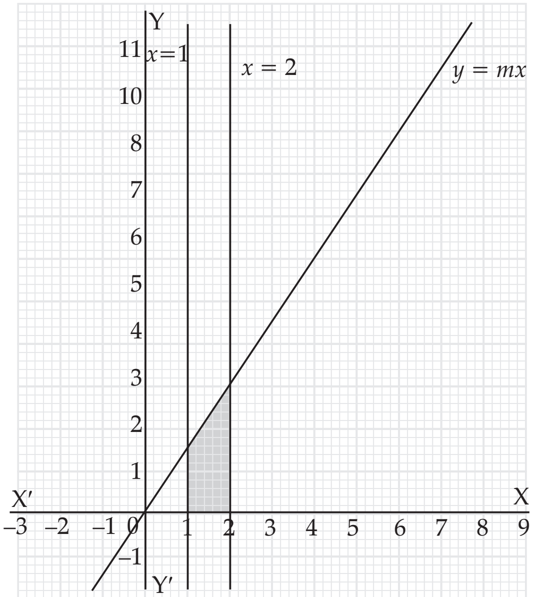
$$\text{Required area } = \int_{-3}^{-1} -x\,dx + \int_{-1}^{3} (x+2)\,dx$$

$$= \left[\frac{-x^2}{2}\right]_{-3}^{-1} + \left[\frac{x^2}{2}+2x\right]_{-1}^{3}$$

$$= \frac{-1}{2}(1-9)+\left[\left(\frac{9}{2}+6\right)-\left(\frac{1}{2}-2\right)\right]$$

$= 4 + 12 = 16$ square units

2.

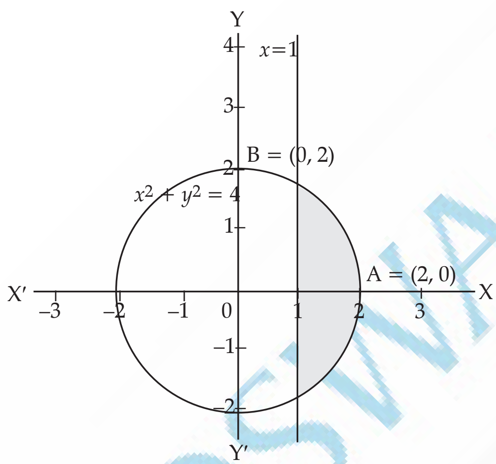


$$\text{Required area} = \int_1^2 (mx)dx$$

$$= m\left[\frac{x^2}{2}\right]_1^2$$

$$= \frac{3}{2}m \text{ sq. units}$$

3.



$$\text{Required area} = 2\int_1^2 \sqrt{4-x^2}\,dx$$

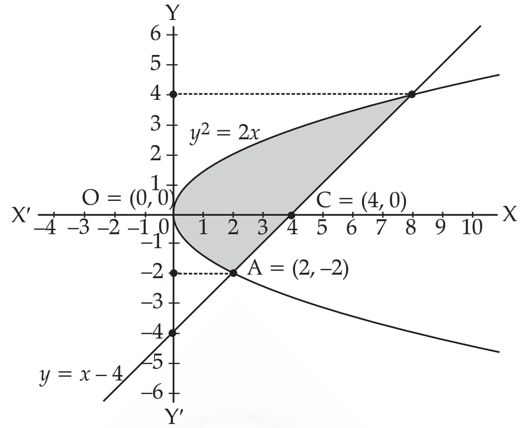
$$= 2\left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_1^2$$

$$= 2\left[\left\{0+2\left(\frac{\pi}{2}\right)\right\} - \left\{\frac{1}{2}\sqrt{3} + 2\cdot\frac{\pi}{6}\right\}\right]$$

$$= 2\left(\pi - \frac{\sqrt{3}}{2} - \frac{\pi}{3}\right)$$

$$= \left(\frac{4\pi}{3} - \sqrt{3}\right) \text{ sq. units}$$

4.



Solving $y^2 = 2x$ and $y = x - 4$, we get

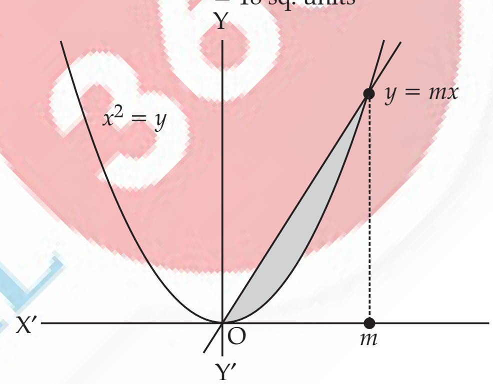
$y = 4$ or -2

$$\text{Required area} = \int_{-2}^{4}\left[(y+4) - \frac{y^2}{2}\right]dy$$

$$= \left|\frac{y^2}{2} + 4y - \frac{1}{6}y^3\right|_{-2}^{4}$$

$= 18$ sq. units

5.



x-coordinates of points of intersection are 0, m.

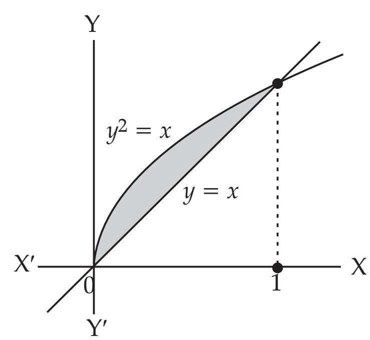
According to question, Area $= \int_0^m (mx - x^2)dx = \dfrac{32}{3}$

$$\Rightarrow \quad \left[m\frac{x^2}{2} - \frac{x^3}{3}\right]_0^m = \frac{32}{3}$$

$$\Rightarrow \frac{m^3}{6} = \frac{32}{3} \Rightarrow m^3 = 64$$

$\Rightarrow m = 4$

6.



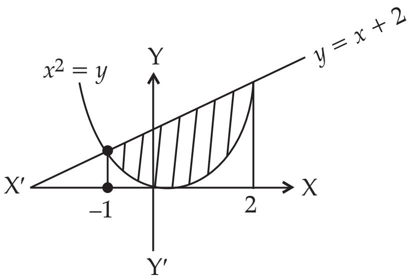
Clearly x coordinates of point of intersection are 0, 1

$$\text{Required area } = \int_0^1 (\sqrt{x} - x)dx$$

$$= \left[\frac{2x^{\frac{3}{2}}}{3} - \frac{x^2}{2}\right]_0^1$$

$$= \frac{1}{6} \text{ sq. unit}$$

7.



x-coordinates of points of intersection are –1, 2.

$$\text{Required area } = \int_{-1}^{2}[(x+2) - x^2]\, dx$$

$$= \left[\frac{(x+2)^2}{2} - \frac{x^3}{3}\right]_{-1}^{2}$$

$$= \frac{16}{3} - \frac{5}{6} = \frac{9}{2} \text{ sq. units}$$

8. Writes the integral to find the displacement of the bird in the first 40 seconds as:

$$\sqrt{10}\int_0^{40}\sqrt{t}\, dt$$

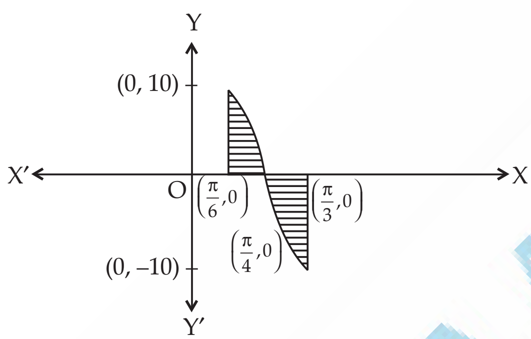
Integrates the abuove expression to get:

$$\sqrt{10}\left.\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right|_0^{40}$$

Finds the displacement of the bird in the first 40 seconds as $\frac{1600}{3}$ m or 533.33 m.

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. $y = 20\cos 2x; \left\{\frac{\pi}{6} \le x \le \frac{\pi}{3}\right\}$



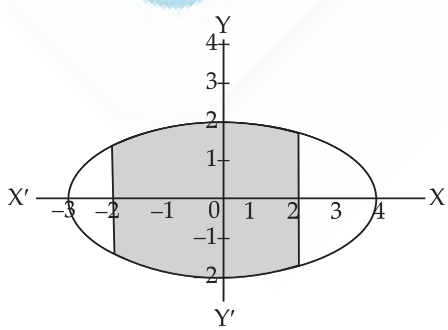
$$\text{Required area } = 20\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\cos 2x\, dx + \left|20\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\cos 2x\, dx\right|$$

$$= 20\left[\frac{\sin 2x}{2}\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}} + \left|20\left[\frac{\sin 2x}{2}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}\right|$$

$$= 10\left(1 - \frac{\sqrt{3}}{2}\right) + 10\left(1 - \frac{\sqrt{3}}{2}\right)$$

$$= 20\left(1 - \frac{\sqrt{3}}{2}\right) \text{ sq. units.}$$

2.



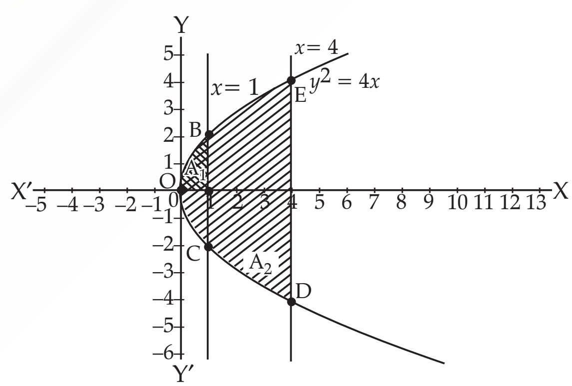
$$\text{Area } = 4\int_0^2 y\, dx$$

$$= 4\left[\frac{1}{2}\int_0^2\sqrt{4^2 - x^2}\, dx\right]$$

$$= 2\left[\frac{x}{2}\sqrt{4^2 - x^2} + 8\sin^{-1}\left(\frac{x}{4}\right)\right]_0^2$$

$$= 2\left[\sqrt{12} + \frac{8\pi}{6}\right] = 4\sqrt{3} + \frac{8\pi}{3} \text{ sq. units}$$

3.



$$A_1 = \text{Area (region OABO)}$$

$$= \int_0^1 2\sqrt{x}\, dx = 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 = \frac{4}{3} \text{ sq. units}$$

$$A_2 = \text{Area (region ODEO)}$$

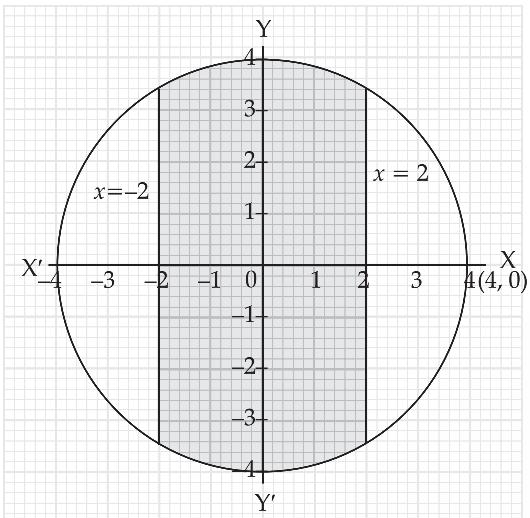
$$= 2\int_0^4 2\sqrt{x}\, dx$$

$$= 4\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$$

$$= 4 \times \frac{2}{3}[2^3] = \frac{64}{3} \text{ sq. units}$$

$$A_1 : A_2 = \frac{4}{3} : \frac{64}{3} = 1 : 16$$

4.

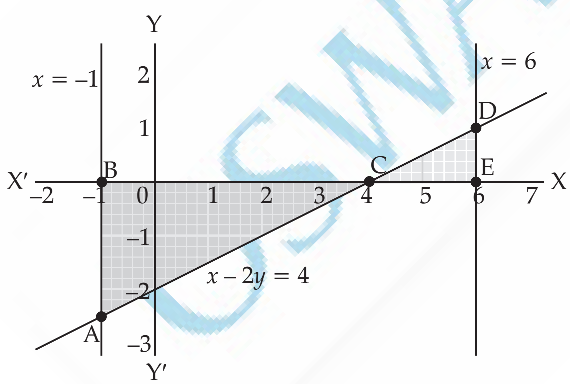


$$\text{Required area } = 4\int_0^2 \sqrt{16 - x^2}\, dx$$

$$= 4\left[\frac{x}{2}\sqrt{16 - x^2} + 8\sin^{-1}\left(\frac{x}{4}\right)\right]_0^2$$

$$= 8\sqrt{3} + \frac{16\pi}{3} \text{ sq. units}$$

5.

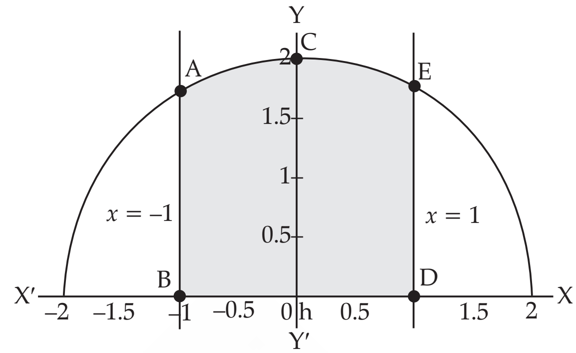


$$\text{Required area } = \left|\int_{-1}^{4}\left(\frac{x-4}{2}\right)dx\right| + \int_4^6\left(\frac{x-4}{2}\right)dx$$

$$= \left|\left[\frac{(x-4)^2}{4}\right]_{-1}^{4}\right| + \left[\frac{(x-4)^2}{4}\right]_4^6$$

$$= \frac{25}{4} + 1 = \frac{29}{4} \text{ sq. units}$$

6.

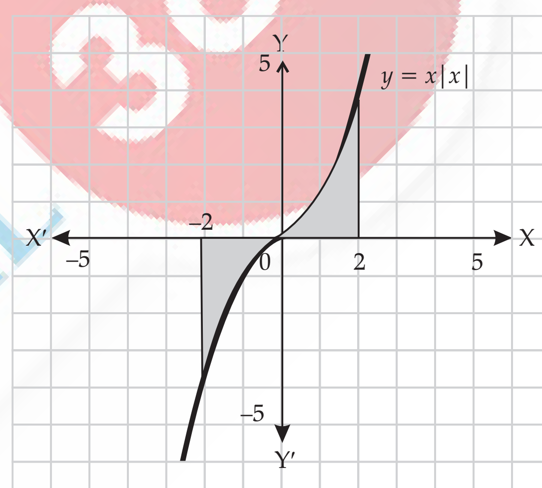


$$\text{Required area } = 2\int_0^1 \sqrt{4 - x^2}\, dx$$

$$= 2\left[\frac{x}{2}\sqrt{4 - x^2} + 2\sin^{-1}\frac{x}{2}\right]_0^1$$

$$= \sqrt{3} + \frac{2\pi}{3} \text{ sq. units}$$

7.



As, $y = x|x| = \begin{cases} -x^2, & x \le 0 \\ x^2, & x \ge 0 \end{cases}$

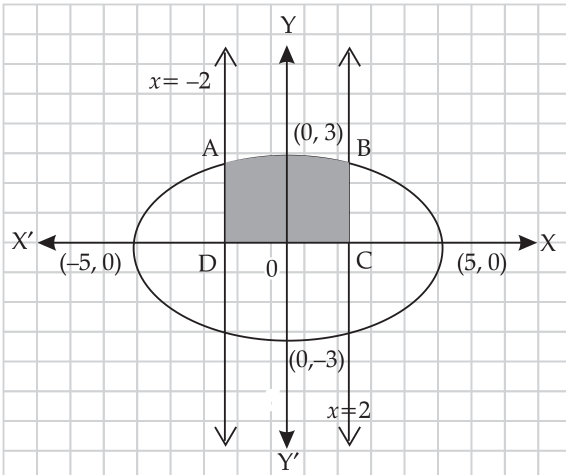
$$\text{Area of the shaded region } = \int_{-2}^{2} y\, dx$$

$$= 2\int_0^2 y\, dx = 2\int_0^2 x^2\, dx$$

$$= 2\left(\frac{x^3}{3}\right)_0^2$$

$$= 2\left(\frac{8}{3}\right) = \frac{16}{3} \text{ sq. units}$$

8.



As, $9x^2 + 25y^2 = 225 \Rightarrow y = \pm\frac{3}{5}\sqrt{5^2 - x^2}$

$$\text{Required area } = \int_{-2}^{2}\frac{3}{5}\sqrt{5^2-x^2}\,dx$$

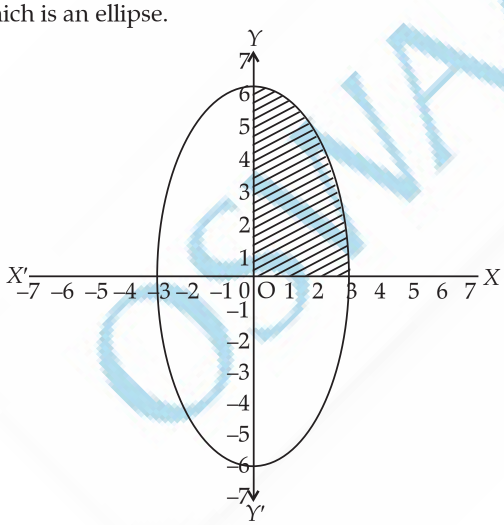
$$= \frac{6}{5}\int_0^2 \sqrt{5^2-x^2}\;dx$$

$$= \frac{6}{5}\left(\frac{x\sqrt{5^2-x^2}}{2}+\frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right)\right)_0^2$$

$$= \frac{6}{5}\left(\frac{2\sqrt{21}}{2}+\frac{25}{2}\sin^{-1}\left(\frac{2}{5}\right)\right)$$

$$= \left(\frac{6\sqrt{21}}{5}+15\sin^{-1}\left(\frac{2}{5}\right)\right) \text{ sq. units}$$

9. The given equation can be written as: $\frac{x^2}{9}+\frac{y^2}{36}=1,$ which is an ellipse.



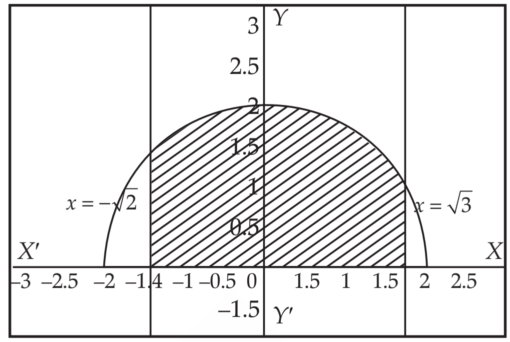
Area of the region bounded by the curve

$$= 4\times\frac{6}{3}\int_0^3\sqrt{9-x^2}\;dx$$

$$= 8\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3$$

$$= 18\pi \text{ sq. units}$$

10.



Area of the region bounded by the curve

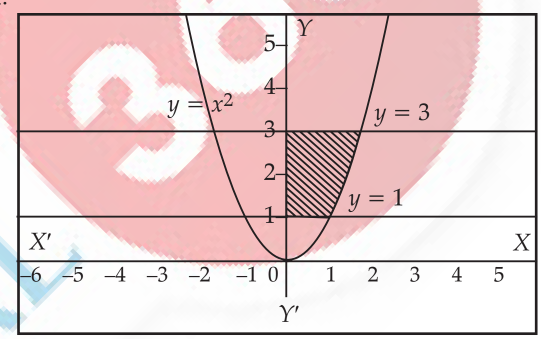
$$= \int_{-\sqrt{2}}^{\sqrt{3}}\sqrt{4-x^2}dx$$

$$= \left[\frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\frac{x}{2}\right]_{-\sqrt{2}}^{\sqrt{3}}$$

$$= \frac{\sqrt{3}}{2}+2\cdot\frac{\pi}{3}+1+2\cdot\frac{\pi}{4}$$

$$= \frac{\sqrt{3}}{2}+1+\frac{7\pi}{6} \text{ sq. units}$$

11.



Area of the region bounded by the curve

$$= \int_1^3\sqrt{y}\;dy$$

$$= \left[\frac{2y^{\frac{3}{2}}}{3}\right]_1^3 = \frac{2}{3}(3\sqrt{3}-1) \text{ sq. units}$$

12. Finds the equation of the ellipse as:

$$\frac{x^2}{36}+\frac{y^2}{16}=1$$

Express y in terms of x as:

$$y = \pm\frac{4}{6}\sqrt{36-x^2}$$

Integrates the above equation with respect to x from limit 0 to 6, that gives the area of one quarter of the ellipse. The working may look as follows:

$$\int_0^6\frac{4}{6}\sqrt{36-x^2}\;dx$$

Applies the formula of integration and simplifies as:

$$\frac{4}{6}\left[\frac{x}{2}\sqrt{6^2-x^2}+\frac{6^2}{2}\sin^{-1}\left(\frac{x}{6}\right)\right]_0^6$$

Applies the limit and solves further as:

$$\frac{4}{6}\left[\frac{6}{2}\times 0+\frac{6^2}{2}\sin^{-1}(1)-0\right]$$

Simplifies the above expression to get the area of one-quarter of the base as 6π sq feet.

Finds the area of the whole ellipse as:

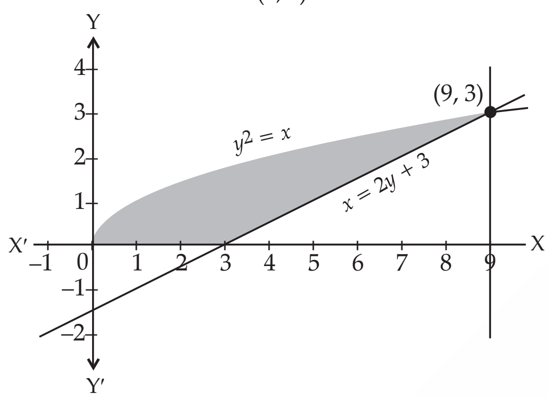
$4 \times 6\pi = 24\pi$ sq. feet.

Finds the volume of water as:

$24\pi \times 10 = 240\pi$ cubic feet

13. Given curves are $y = \sqrt{x}$ and $x = 2y + 3$

Points of intersection is (9, 3).

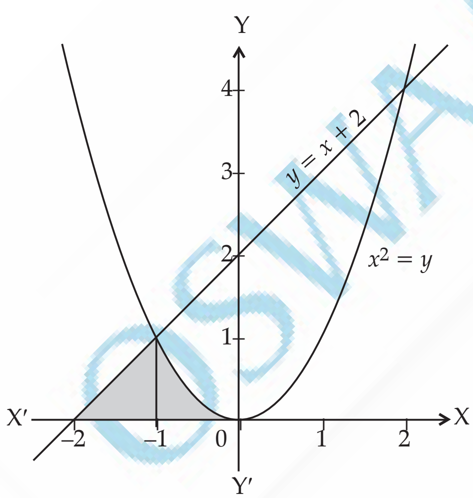


$$\text{Required area} = \int_0^9 \sqrt{x}\,dx - \int_3^9 \left(\frac{x-3}{2}\right)dx$$

$$= \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^9 - \frac{1}{4}[(x-3)^2]_3^9$$

$= 9$ sq. units

14.



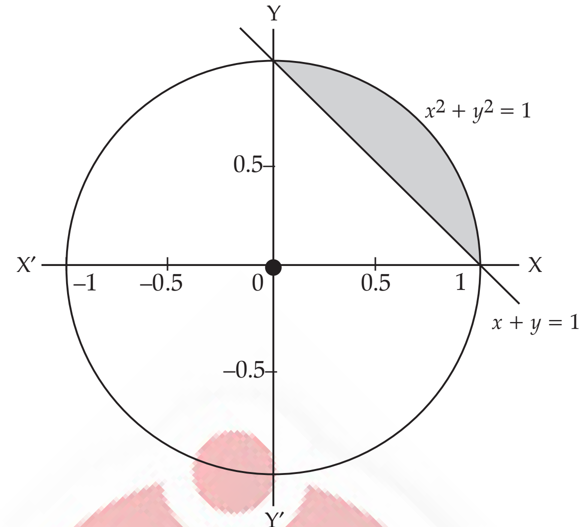
x-coordinates of point of intersection are –1, 2

$$\text{Required area} = \int_{-2}^{-1}(x+2)dx + \int_{-1}^{0} x^2\,dx$$

$$= \left.\frac{(x+2)^2}{2}\right|_{-2}^{-1} + \left.\frac{x^3}{3}\right|_{-1}^{0}$$

$$= \frac{1}{2}+\frac{1}{3}=\frac{5}{6} \text{ sq. units}$$

15. x coordinates of point of intersection are 1, 0.

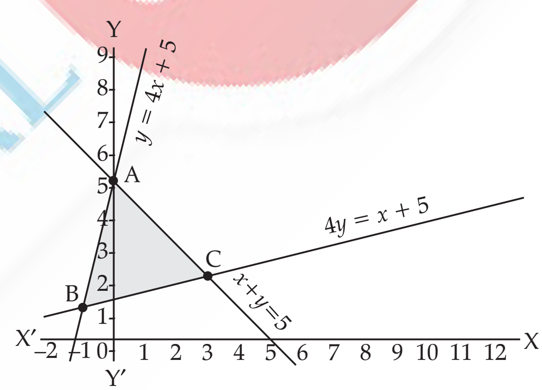


$$\text{Required area} = \int_0^1 \sqrt{1-x^2}\,dx - \int_0^1 (1-x)\,dx$$

$$= \left.\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x\right|_0^1 + \left.\frac{(1-x)^2}{2}\right|_0^1$$

$$= \frac{\pi}{4}-\frac{1}{2} \text{ sq. units}$$

16.



Vertices are A(0, 5), B(–1, 1), C(3, 2).

Required area

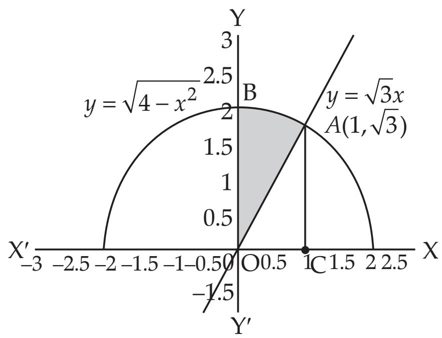
$$= \int_{-1}^{0}(4x+5)dx + \int_0^3 (5-x)dx - \int_{-1}^{3}\frac{x+5}{4}dx$$

$$= \left.\frac{(4x+5)^2}{8}\right|_{-1}^{0} - \left.\frac{(5-x)^2}{2}\right|_0^3 - \left.\frac{(x+5)^2}{8}\right|_{-1}^{3}$$

$$= \left[\frac{25}{8}-\frac{1}{8}\right]-\left[2-\frac{25}{2}\right]-[8-2]$$

$$= 3+\frac{21}{2}-6=\frac{15}{2} \text{ sq. units}$$

17.



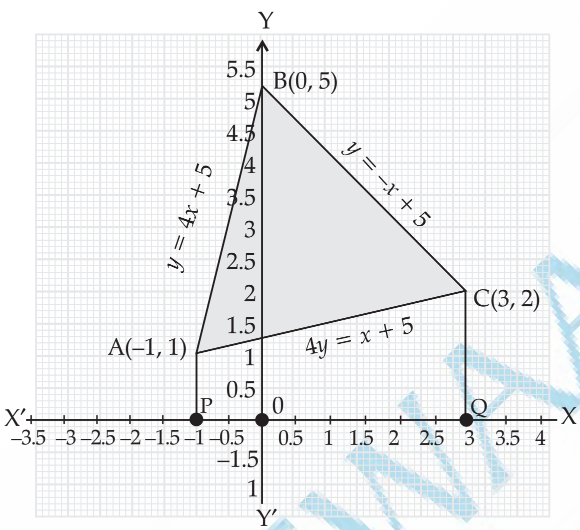
Point of intersection at $x = 1$

$$\text{ar(OAB)} = \int_0^1 \sqrt{4-x^2}\,dx - \int_0^1 \sqrt{3}x\,dx$$

$$= \left(\frac{x\sqrt{4-x^2}}{2} + 2\sin^{-1}\left(\frac{x}{2}\right)\right)\Bigg]_0^1 - \frac{\sqrt{3}}{2}x^2\Bigg]_0^1$$

$$= \frac{\sqrt{3}}{2} + 2\times\frac{\pi}{6} - \frac{\sqrt{3}}{2} = \frac{\pi}{3} \text{ sq. units}$$

18.



Equation of the lines AB, BC and AC are:

$y = 4x + 5$; $y = 5 - x$; $4y = x + 5$ respectively.

$$\text{ar}(\Delta ABC) = \int_{-1}^{0}(4x+5)dx + \int_0^3 (5-x)dx - \frac{1}{4}\int_{-1}^{3}(5+x)dx$$

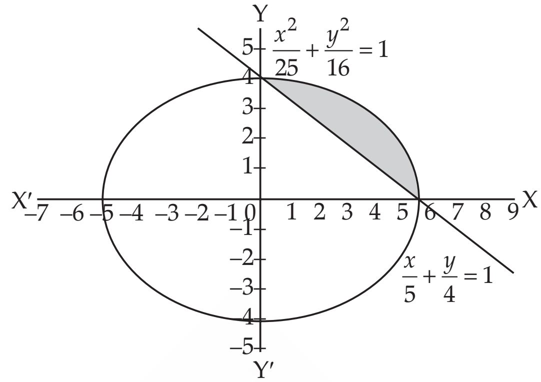
$$= (2x^2+5x]_{-1}^{0} + \left[5x - \frac{x^2}{2}\right]_0^3 - \frac{1}{4}\left[5x + \frac{x^2}{2}\right]_{-1}^{3}$$

$$= [0+0-2+5] + \left[15 - \frac{9}{2} - 0 + 0\right]$$

$$-\frac{1}{4}\left[15 + \frac{9}{2} + 5 - \frac{1}{2}\right]$$

$$= 3 + \frac{21}{2} - \frac{1}{4}(24) = \frac{21}{2} - 3 = \frac{15}{2} \text{ sq. units}$$

19.

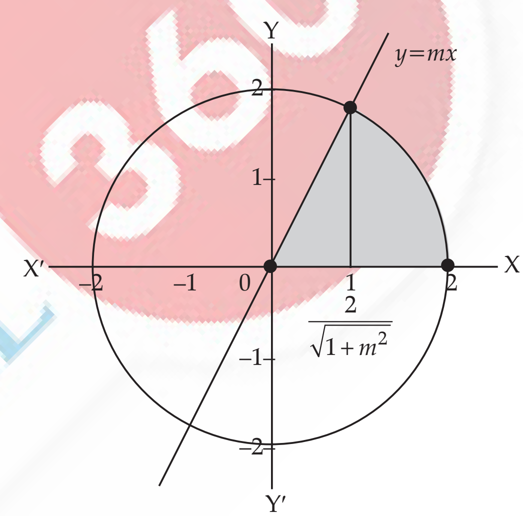


$$\text{Required area } = \frac{4}{5}\int_0^5 \sqrt{25-x^2}\,dx - \frac{4}{5}\int_0^5 (5-x)dx$$

$$= \frac{4}{5}\left(\frac{x}{2}\sqrt{25-x^2} + \frac{25}{2}\sin^{-1}\frac{x}{5}\right)\Bigg]_0^5 + \frac{2}{5}(5-x)^2\Bigg]_0^5$$

$$= \frac{4}{5}\left(\frac{25\pi}{4}\right) - 10 = 5\pi - 10 \text{ unit}^2$$

20.



$x^2 + y^2 = 4$ and $y = mx$

$$\Rightarrow x^2 + m^2x^2 = 4 \Rightarrow x = \pm\frac{2}{\sqrt{1+m^2}}$$

x-coordinate of the required point of intersection is $\frac{2}{\sqrt{1+m^2}}$.

According to question,

$$\int_0^{\frac{2}{\sqrt{1+m^2}}} mx\,dx + \int_{\frac{2}{\sqrt{1+m^2}}}^{2} \sqrt{4-x^2}\,dx = \frac{\pi}{2}$$

$$\Rightarrow m\frac{x^2}{2}\Bigg|_0^{\frac{2}{\sqrt{1+m^2}}} + \frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2}\Bigg|_{\frac{2}{\sqrt{1+m^2}}}^{2} = \frac{\pi}{2}$$

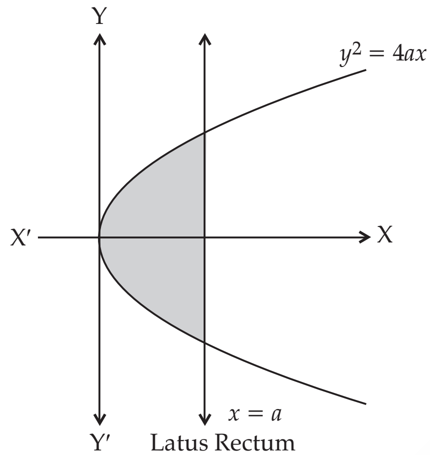
$$\Rightarrow \frac{2m}{1+m^2} + \pi - \frac{2m}{1+m^2} - 2\sin^{-1}\frac{1}{\sqrt{1+m^2}} = \frac{\pi}{2}$$

$$\Rightarrow \frac{\pi}{4} = \sin^{-1}\frac{1}{\sqrt{1+m^2}}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+m^2}} \Rightarrow m^2+1=2$$

$\Rightarrow m = 1$ (as $m > 0$)

21.



$$\text{Required area } = 2\int_0^a 2\sqrt{ax}\,dx$$

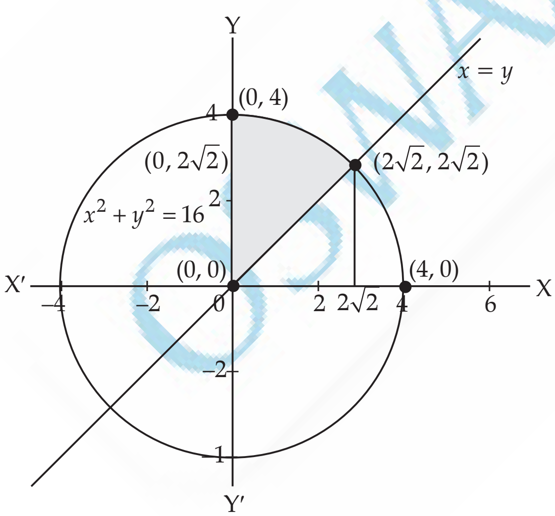
$$= 4\sqrt{a}\int_0^a \sqrt{x}\,dx$$

$$= 4\sqrt{a}\left|\frac{2x^{3/2}}{3}\right|_0^a$$

$$= \frac{8}{3}a\sqrt{a}\sqrt{a}$$

$$= \frac{8}{3}a^2 \text{ sq. units}$$

22.



$$\text{Required area } = \int_0^{2\sqrt{2}} y\,dy + \int_{2\sqrt{2}}^{4}\sqrt{16-y^2}\,dy$$

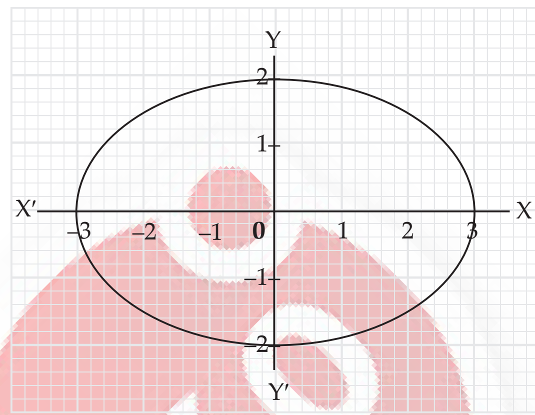
$$= \left|\frac{y^2}{2}\right|_0^{2\sqrt{2}} + \left|\frac{y}{2}\sqrt{16-y^2} + \frac{16}{2}\sin^{-1}\left(\frac{y}{4}\right)\right|_{2\sqrt{2}}^{4}$$

$$= 4 + \left\{8\sin^{-1}(1) - \sqrt{2}.\sqrt{8} - 8\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)\right\}$$

$$= 4 + 8\left(\frac{\pi}{2}\right) - 4 - 8\left(\frac{\pi}{4}\right)$$

$$= 8\left(\frac{\pi}{4}\right) \text{ or } 2\pi \text{ sq. units}$$

23.

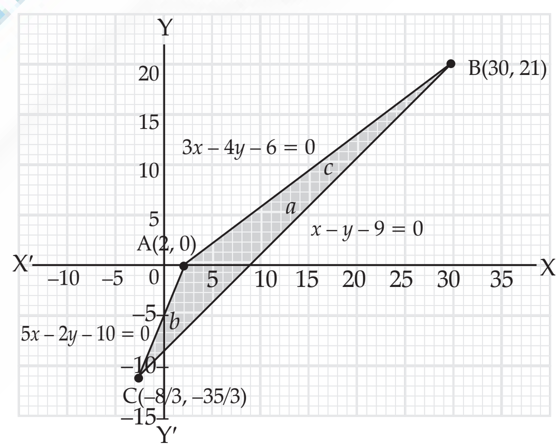


$$\text{Required area } = 4\cdot\frac{2}{3}\int_0^3\sqrt{9-x^2}\,dx$$

$$= \frac{8}{3}\left[\frac{x\sqrt{9-x^2}}{2} + \frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3$$

$$= \frac{8}{3}\left[0 + \frac{9\pi}{4}\right] = 6\pi \text{ sq. units}$$

24.



Solving the given equations, the vertcies of triangle are:

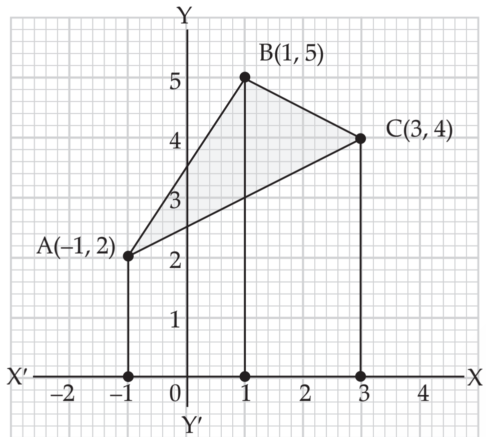
$$A(2,0),\ B(30,21) \text{ and } C\left(-\frac{8}{3}, -\frac{35}{3}\right)$$

$$\text{ar}(\Delta ABC) = \frac{3}{4}\int_2^{30}(x-2)dx - \int_9^{30}(x-9)dx + \left|\int_{-\frac{8}{3}}^{9}(x-9)dx\right| - \left|\frac{5}{2}\int_{-\frac{8}{3}}^{2}(x-2)dx\right|$$

$$= \frac{3}{8}(x-2)^2\Big]_2^{30} - \frac{1}{2}(x-9)^2\Big]_9^{30} + \left|\frac{1}{2}(x-9)^2\right|\Big]_{-\frac{8}{3}}^{9} - \left|\frac{5}{4}(x-2)^2\right|\Big]_{-\frac{8}{3}}^{2}$$

$$= 294 - \frac{441}{2} + \frac{1225}{18} - \frac{245}{9} = \frac{343}{3} \text{ sq. units}$$

25.



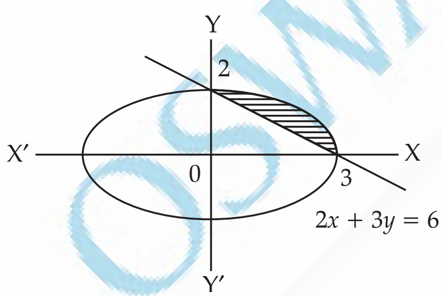
$$\text{ar (ABC)} = \int_{-1}^{1} y_{AB}dx + \int_{1}^{3} y_{BC}dx - \int_{-1}^{3} y_{AC}dx$$

$$= \int_{-1}^{1}\left(\frac{7+3x}{2}\right)dx + \int_{1}^{3}\left(\frac{11-x}{2}\right)dx - \int_{-1}^{3}\left(\frac{5+x}{2}\right)dx$$

$$= \frac{1}{2}\times\left(7x + \frac{3x^2}{2}\right)\Big]_{-1}^{1} + \frac{1}{2}\times\left(11x - \frac{x^2}{2}\right)\Big]_{1}^{3} - \frac{1}{2}\times\left(5x + \frac{x^2}{2}\right)\Big]_{-1}^{3}$$

$$= 7 + 9 - 12 = 4 \text{ sq. units}$$

26.



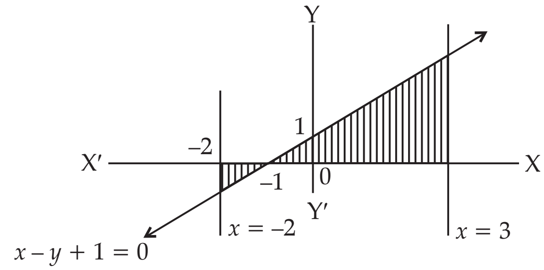
Clearly points of intersection are (3, 0) and (0, 2).

$$\text{Required area } = \frac{2}{3}\int_0^3 \sqrt{9-x^2}\,dx - \frac{2}{3}\int_0^3 (3-x)\,dx$$

$$= \frac{2}{3}\left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3} + \frac{(3-x)^2}{2}\right]_0^3$$

$$= \frac{2}{3}\left[\frac{9\pi}{4} - \frac{9}{2}\right] \text{ or } \frac{3\pi}{2} - 3 \text{ sq. units}$$

27.



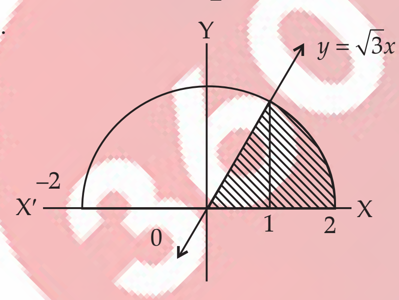
$$\text{Required area } = -\int_{-2}^{-1}(x+1)dx + \int_{-1}^{3}(x+1)\,dx$$

$$= -\frac{(x+1)^2}{2}\Big|_{-2}^{-1} + \frac{(x+1)^2}{2}\Big|_{-1}^{3}$$

$$= \frac{1}{2} + 8$$

$$= \frac{17}{2} \text{ sq. units}$$

28.



x coordinate of point of intersection is 1.

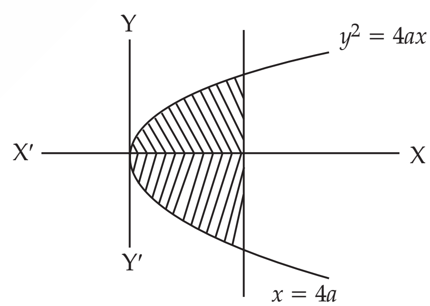
$$\text{Required area } = \int_0^1 \sqrt{3}\,x\,dx + \int_1^2 \sqrt{4-x^2}\,dx$$

$$= \sqrt{3}\,\frac{x^2}{2}\Big|_0^1 + \frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2}\Big|_1^2$$

$$= \frac{\sqrt{3}}{2} + \pi - \frac{\sqrt{3}}{2} - \frac{\pi}{3}$$

$$= \frac{2\pi}{3} \text{ unit}^2$$

29.



$$\text{Given area } = \frac{256}{3} \text{ sq. units}$$

$$\text{Area of shaded region } = 2\int_0^{4a} \sqrt{4ax}\,dx$$

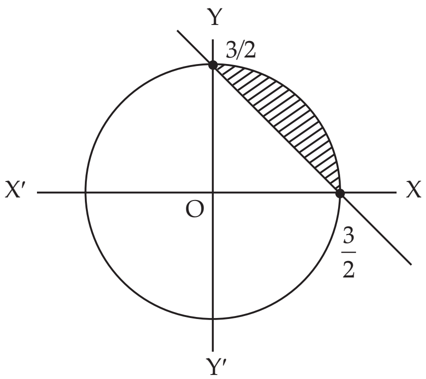
$$= 8\sqrt{a}\,\frac{x^{3/2}}{3}\Big|_0^{4a}$$

$$= \frac{64a^2}{3} \text{ sq. units}$$

Now, $\frac{64a^2}{3} = \frac{256}{3}$

$\Rightarrow a^2 = 4$ gives $a = 2$ (as $a > 0$)

30. Clearly point of intersection are $\left(\frac{3}{2}, 0\right)$ & $\left(0, \frac{3}{2}\right)$

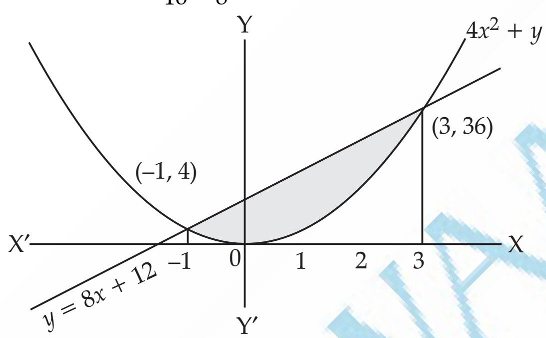


Required area $= \int_0^{3/2} \sqrt{\frac{9}{4} - x^2}\, dx - \int_0^{3/2} \left(\frac{3}{2} - x\right) dx$

$$= \frac{x}{2}\sqrt{\frac{9}{4} - x^2} + \frac{9}{8}\sin^{-1}\frac{2x}{3}\Bigg|_0^{3/2} + \frac{\left(\frac{3}{2} - x\right)^2}{2}\Bigg|_0^{3/2}$$

$$= \frac{9\pi}{16} - \frac{9}{8} \text{ sq. units}$$

31.



Points of intersection

$$4x^2 = 8x + 12$$
$$x^2 - 2x - 3 = 0$$
$$(x - 3)(x + 1) = 0$$
$$x = 3, -1$$

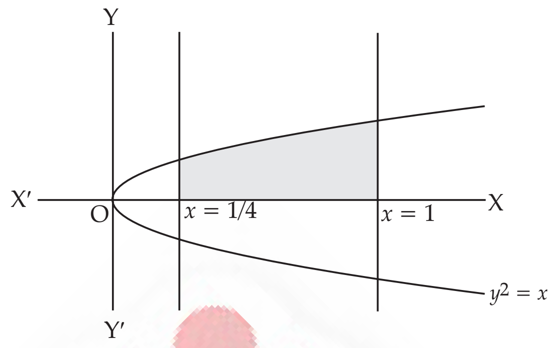
$$\text{Area} = \int_{-1}^{3} [(8x + 12) - 4x^2]\, dx$$

$$= 4x^2 + 12x - \frac{4}{3}x^3\Bigg]_{-1}^{3}$$

$$= 36 + 36 - 36 - \left(4 - 12 + \frac{4}{3}\right)$$

$$= 44 - \frac{4}{3} = \frac{128}{3} \text{ sq. units}$$

32.

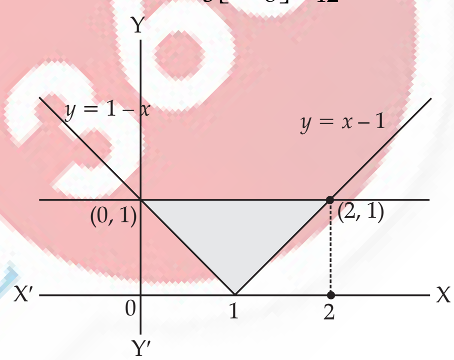


$$\text{Area} = \int_{1/4}^{1} \sqrt{x}\, dx$$

$$= \frac{2}{3}x^{3/2}\Bigg]_{1/4}^{1}$$

$$= \frac{2}{3}\left[1 - \frac{1}{8}\right] = \frac{7}{12} \text{ unit}^2$$

33.



Area of the bounded region is

$$\int_0^1 [1 - (1 - x)]dx + \int_1^2 [1 - (x - 1)]dx$$

$$= \int_0^1 x\, dx + \int_1^2 (2 - x)\, dx$$

$$= \frac{x^2}{2}\Bigg]_0^1 + \left[2x - \frac{x^2}{2}\right]_1^2$$

$$= \frac{1}{2} + \left[4 - \frac{4}{2} - 2 + \frac{1}{2}\right]$$

$$= \frac{1}{2} + \frac{1}{2} = 1 \text{ unit}^2$$

Level - 2 ADVANCED COMPETENCY FOCUSED QUESTIONS

MULTIPLE CHOICE QUESTIONS (MCQs) (1 Marks)

1. Option (D) is correct.

Explanation: The given integral:

$$\int_0^{15} R^2(t)dt$$

represents the total accumulation of the square of the rate of temperature increase over 15 minutes.

(A) Incorrect: The integral does not represent the rate but rather the accumulated squared rate.

(B) Incorrect: It does not give the average temperature increase, just the integral of the squared rate.

(C) Incorrect: The average rate would require division by the time interval (15 – 0).

(D) Correct: The integral represents the difference in accumulated squared rates over time.

2. **Option (C) is correct.**

Explanation: The given equation of the curve is:

$$-x = y^2 - 3 \Rightarrow x = 3 - y^2$$

This represents a parabola opening towards the left.

The first quadrant means $x \geq 0$ and $y \geq 0$.

Setting $x = 0$, we solve for y:

$$0 = 3 - y^2 \Rightarrow \quad y^2 = 3 \Rightarrow \quad y = \sqrt{3}$$

So, the curve extends from $y = 0$ to $y = \sqrt{3}$.

Ravi's integral:

$$\int_0^3 \sqrt{3-x}\,dx$$

Here, the limits are taken along x-axis from $x = 0$ to $x = 3$.

The function $y = \sqrt{3-x}$ is correct for this range.

This correctly represents the area under the curve in terms of x.

Ravi's integral is correct.

Kanika's integral:

$$\int_0^{\sqrt{3}} \left(3 - y^2\right) dy$$

Here, the limits are taken along y-axis from $y = 0$ to $y = \sqrt{3}$.

The function $x = 3 - y^2$ represents the horizontal length of the strip at a given y.

This correctly represents the area under the curve in terms of y.

Kanika's integral is also correct.

3. **Option (B) is correct.**

Explanation: The shaded region is enclosed between the curves $y = e^x$ and $y = e^2$, from $x = 1$ to $x = 2$.

To find the area, we use the integral:

$$\int_1^2 \left(e^2 - e^x\right) dx$$

4. **Option (A) is correct.**

Explanation:

$$\int x^2 dx = \frac{x^3}{3}$$

Applying the limits from 0 to k:

$$\left[\frac{x^3}{3}\right]_0^k = \frac{k^3}{3} - \frac{0^3}{3} = \frac{k^3}{3}$$

We are given that the area is 9:

$$\frac{k^3}{3} = 9$$

$$k^3 = 27$$

Taking the cube root:

$$k = \sqrt[3]{27} = 3$$

5. **Option (D) is correct.**

Explanation: The volume of water transferred from the overhead tank is given by the function:

$$w(t) = e^2 - e^t$$

where t is in hour and $w(t)$ is in litres.

Here we need to compute the volume transferred from $t = 0$ to $t = 2$. i.e.,

$$\text{Total volume} = \int_0^2 (e^2 - e^t)dt$$

$$= [e^2.t - e^t]_0^2$$

$$= e^2(2 - 0) - (e^2 - e^0)$$

$$= 2e^2 - e^2 + 1 = e^2 + 1$$

$$= 7.389 + 1 = 8.389 \sim 8.4 \text{ units}$$

$$= 8.4 \times 10 = 84 \text{ litres}$$

6. **Option (C) is correct.**

Explanation: The area of the shaded region is found by integrating the difference between the two functions. The given options provide integrals in terms of x and y, so we must determine the correct approach.

The given curves are $f(y)$ and $g(y)$, meaning they are functions of y, suggesting integration with respect to y.

The shaded region is bounded between $y = c$ and $y = d$.

The area is given by:

$$\int_c^d |f(y) - g(y)|\,dy$$

7. **Option (A) is correct.**

Explanation:

$$x = \sqrt{16 - y^2}$$

Since we are considering the right semicircle ($x \geq 0$), the area of the shaded region is given by integrating x with respect to y, from $y = -4$ to $y = 4$:

$$\int_{-4}^4 \sqrt{16 - y^2}\,dy$$

Since the function is symmetric about the x-axis, we can compute the integral from 0 to 4 and double the result:

$$2\int_0^4 \sqrt{16 - y^2}\,dy$$

ASSERTION-REASON QUESTIONS (1 Marks)

1. **Option (A) is correct.**

Explanation: Assertion is true. This is a fundamental concept in the Applications of Integrals. The area between curves is typically calculated as:

$$\int_a^b [f(x) - g(x)]\,dx$$

where $f(x) \geq g(x)$ over $[a, b]$

Reason is also true. The definite integral of the difference $f(x) - g(x)$ gives the net (signed) area. If $f(x) \geq g(x)$, this is equal to the actual area.

2. **Option (C) is correct.**

Explanation: Assertion is true. This is a standard interpretation in the Applications of Integrals. The definite integral of a non-negative function over an interval gives the area under the curve.

Reason is false. Differentiation, not integration, is used to find the slope of a curve. Integration is used for calculating accumulated quantities like area, volume, etc.

3. **Option (A) is correct.**

Explanation: Assertion is true. To ensure we get a positive area, we integrate $|f(x) - g(x)|$ over the interval. However, if we know one function is always above the other (say $f(x) \geq g(x)$) on $[a, b]$, then we can simply integrate $f(x) - g(x)$ without the absolute value.

Reason is also true. By definition, area cannot be negative. Hence, we take care that our integral evaluates to a non-negative value.

4. **Option (A) is correct.**

Explanation: Assertion is true.

If the curve lies below the x-axis, the definite integral $\int_a^b f(x)\,dx$ can give a negative value, which is interpreted as signed area, not actual (positive) geometric area.

Reason is also true. Definite integrals calculate area with sign, i.e., positive when the function is above the x-axis and negative when below.

VERY SHORT ANSWER TYPE QUESTIONS (2 Marks)

1. Integrates $\int_0^4 ((x-2)^3 + 2(x-2)^2)dx$ as $\left[\frac{(x-2)^4}{4}\right]_0^4 + \left[\frac{2(x-2)^3}{3}\right]_0^4$

Evaluates the above integral as $\frac{32}{3}$ and hence finds the displacement of the particle from 0 to 4 seconds as $\frac{32}{3}$ m.

2. Represents the work done by the gas as:

$$\int_{V_1}^{V_2} p dV = nRT \int_{V_1}^{V_2} \frac{1}{V} dV$$

Integrates the above expression to find the work done by the gas as:

$$nRT[\log V]_{V_1}^{V_2} = nRT[\log V_2 - \log V_1]$$
$$= nRT \log\left(\frac{V_2}{V_1}\right)$$

3. Writes the expression for the total revenue that will be generated over the last two years based on the projection as:

$\int_4^6 (e^{x-4} - 0.02)\,dx$

Solves the integral to get:

$[e^{x-4} - 0.02x]_4^6$

Simplifies the above expression as

$(e^2 - 0.02 \times 6) - (e^0 - 0.02 \times 4) = 7.39 - 0.12 - 1 + 0.08 = 6.35.$

Finds the total revenue that will be generated over the last two years based on the projection as 6.35×1000000 = ₹ 6350000.

4. Writes the expression for the total hours required to produce the units from 3600th to 10000th unit as:

$\int_{3600}^{10000} 1272x^{-\frac{1}{2}}dx$

Solves the above integral as:

$= 2544\left[x^{\frac{1}{2}}\right]_{3600}^{10000}$

Solves the above integral to find the hours required to produce the units from the 3600th to the 10000th unit as 101760 hours.

5. Writes the expression to find the area of the shaded region by integrating with respect to the x-axis as:

$$\left|\int_{-1}^{1}(x^3-1)dx\right| + \int_1^2 (x^3-1)dx$$

6. Writes an expression for the area of the shaded region as:

$$\text{Shaded area } = 25 - 2\int_1^5 \log_e x\,dx$$

7. Represents the area of the shaded region as:

$$\frac{3\pi}{2} - \int_0^{\frac{\pi}{2}} 3\cos x dx$$

Simplifies the above expression to find the area of the shaded region as follows:

$$\frac{3\pi}{2} - 3[\sin x]_0^{\pi/2} = 3\left[\frac{\pi}{2} - 1\right] \text{ sq. units}$$

8. Writes false.

States that the line $2y = x - 2$ crosses the x-axis at $x = 2$ and hence the area under the curve will have to be found as follows:

$$\left|\int_1^2 \left(\frac{1}{2}(x-1)dx\right)\right| + \int_2^4 \left(\frac{1}{2}x - 1\right)dx$$

9. Writes that the area is equivalent to the area bounded by $x^2 + y = 4$ and $x^2 - y = 4$.

Writes the following expression to calculate the bounded area:

$$\int_{-2}^{2}(4-x^2)dx + \left|\int_{-2}^{2}(x^2-4)\,dx\right|$$

10. Writes that only Swara represented the area of the shaded region correctly. Justifies the answer. For example, the parts of shaded region above and below the x-axis are equal. Hence the required area can be

computed by integrating the function with respect to y-axis from 0 to 3 and then doubling it.
Writes that Rajiv and Zaman represented the area of the shaded region incorrectly. Justifies the answer. For example, if integrated along x-axis, the limit must range from 0 to 3. But Rajiv and Zaman have integrated from (–3) to 3.

SHORT ANSWER TYPE QUESTIONS (3 Marks)

1. Equate the curves to find the limits of integration:

$$x^2 = 4 \Rightarrow x = -2 \text{ and } x = 2$$

$$\text{Area} = \int_{-2}^{2}(4-x^2)\,dx$$

$$= \left[4x - \frac{x^3}{3}\right]_{-2}^{2}$$

$$= \left(4(2) - \frac{(2)^3}{3}\right) - \left(4(-2) - \frac{(-2)^3}{3}\right)$$

$$= \left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)$$

$$= \left(\frac{24-8}{3}\right) - \left(\frac{-24+8}{3}\right)$$

$$= \frac{16}{3} + \frac{16}{3} = \frac{32}{3}$$

$$\text{Area} = \frac{32}{3} \text{ square units}$$

This area represents the land to be enclosed. While the fencing material depends on perimeter, the area helps in estimating total land enclosed, planning irrigation, seed quantity, or crop yield, and ensuring efficient use of fencing material by minimising unused space.

2. The area under the curve from $x = -4$ to $x = 4$ is given by the definite integral:

$$\text{Area} = \int_{-4}^{4}(16-x^2)\,dx$$

$$\int(16-x^2)\,dx = 16x - \frac{x^3}{3}$$

Now, evaluate from – 4 to 4:

$$= \left[16x - \frac{x^3}{3}\right]_{-4}^{4}$$

At $x = 4$:

$$16(4) - \frac{4^3}{3} = 64 - \frac{64}{3} = \frac{192-64}{3} = \frac{128}{3}$$

At $x = -4$:

$$16(-4) - \frac{(-4)^3}{3} = -64 + \frac{64}{3} = \frac{-192+64}{3} = \frac{-128}{3}$$

$$\text{Area} = \frac{128}{3} - \left(-\frac{128}{3}\right) = \frac{256}{3}\ \text{m}^2$$

3.

$$\text{Area} = \int_{-3}^{3}(9-x^2)\,dx$$

$$\int(9-x^2)\,dx = 9x - \frac{x^3}{3}$$

Now, evaluate from – 3 to 3:

$$= \left[9x - \frac{x^3}{3}\right]_{-3}^{3}$$

At $x = 3$:

$$9(3) - \frac{3^3}{3} = 27 - \frac{27}{3} = 27 - 9 = 18$$

At $x = -3$:

$$9(-3) - \frac{(-3)^3}{3} = -27 + \frac{27}{3} = -27 + 9 = -18$$

$$\text{Area} = 18 - (-18) = 36 \text{ square units}$$

CASE BASED QUESTIONS (4 Marks)

1. (i) Integrates the given marginal cost function, $16x - 1582$, with respect to x to get the total cost function as $TC(x) = 8x^2 - 1582x + k$, where k is an arbitrary constant.
At $x = 0$, the total cost is equal to the fixed cost whch is ₹ 1800. Hence, $k = 1800$.
Thus, the total cost function is $8x^2 - 1582x + 1800$.

(ii) Integrates the given marginal revenue function, $2 - 6x$, with respect to x to get the total revenue function as $TR(x) = 2x - 3x^2 + k$, where k is an arbitrary constant.
At $x = 0$, the total revenue is equal to ₹ 0. Hence, $k = 0$.
Thus, the total revenue function is $2x - 3x^2$.
Uses the total cost function found in the previous question and the above steps to find the profit function (P) as:

$$2x - 3x^2 - (8x^2 - 1582x + 1800)$$
$$= -11x^2 + 1584x - 1800$$

(iii) Finds the first derivative of the profit function found in the previous question as $\frac{dP}{dx} = -22x + 1584$.

Calculates the critical point as $x = 72$ by equating $-22x + 1584$ to 0.

Finds the second derivative as:

$$\frac{d^2x}{dP^2} = -22$$

Concludes that the maximum profit that can be earned in one day is by selling 72 units of PPE kits.

2. (i) Writes the equation to find the number of years for which Indian Collection Limited will be profitable as:

$$10 - t^{\frac{1}{2}} = 2 + 3t^{\frac{1}{3}}$$

$$\Rightarrow \quad 8 = 4t^{\frac{1}{3}}$$

$$\Rightarrow \quad t = 2^3$$

$$\Rightarrow \quad t = 8 \text{ years}$$

(ii) Writes the expression for the maximum profit that Indian Collection Limited can earn during that period as:

$$\int_0^8 [(10 - t^{\frac{1}{3}}) - (2 + 3t^{\frac{1}{3}})]dt = \int_0^8 [8 - 4t^{\frac{1}{3}}]dt$$

$$= [8t - 3t^{\frac{4}{3}}]_0^8$$

$$= 64 - 3(8)^{\frac{4}{3}}$$

$$= 64 - 48 = 16$$

Thus, the maximum profit that Indian Collection Limited can earn during that period is 16 crore rupees.

3. (i) Considers the area of the shaded region in the first quadrant as A_1.

Writes an expression to find A_1 as:

$$A_1 = \int_1^2 (x^2 - 1)\, dx$$

Considers the area of the shaded region in the fourth quadrant as A_2.

Writes an expression to find A_2 as:

$$A_2 = \left|\int_0^1 (x^2 - 1)dx\right|$$

Integrates and computes A_1 as:

$$A_1 = \left[\frac{x^3}{3} - x\right]_1^2 = \frac{4}{3} \text{ sq. units}$$

Integrates and computes A_2 as:

$$A_2 = \left|\left[\frac{x^3}{3} - x\right]_0^1\right| = \left|\frac{-2}{3}\right| = \frac{2}{3} \text{ sq units}$$

The area of the shaded region as $A_1 + A_2$ $= \frac{4}{3} + \frac{2}{3} = 2$ sq units.

(ii) An expression to find the area of the shaded region in the fourth quadrant by integrating with respect to the y-axis is given as:

$$\left|\int_{-1}^0 \sqrt{y+1}\, dy\right|$$

4. (i) Writes the expression to find the area under the curve in the first quadrant as:

$$\text{Area} = \int_0^3 \left(\frac{-3}{8}(x+1)^2 + 6\right)dx$$

Writes the solution of the above integral as:

$$\text{Area} = \left[\frac{-x^3}{8} - \frac{3x^2}{8} + \frac{45x}{8}\right]_0^3$$

Finds the area under the curve in the first quadrant as $\frac{81}{8}$ sq. units.

Finds the area of the unshaded rectangular region as $1 \times \frac{9}{2} = \frac{9}{2}$ sq. units.

Finds the area of the shaded region as $\frac{81}{2} - \frac{9}{2} = \frac{45}{8}$ sq. units.

(ii) Writes that the area of the shaded region in the second quadrant will not be the same as the first quadrant as the graph is not symmetrical about x-axis.

5. (i) Rewrites the integral of $f(x)$ as follows:

$$\int_0^{100} \left(\frac{\sqrt{x}}{100} + \frac{(x-1)^2}{100}\right)dx$$

$$= \int_0^{100} \frac{\sqrt{x}}{100}\, dx + \int_0^{100} \frac{(x-1)^2}{100}\, dx$$

Solves $\int_0^{100} \frac{\sqrt{x}}{100}\, dx = \left[\frac{2x^{\frac{3}{2}}}{300}\right]_0^{100}$

Evaluates this to find the answer as $\frac{20}{3}$.

Solves $\int_0^{100} \frac{(x^2 - 2x + 1)}{100}\, dx$

$$= \left[\frac{x^3}{300}\right]_0^{100} - \left[\frac{2x^2}{200}\right]_0^{100} + \left[\frac{x}{100}\right]_0^{100}$$

Evaluates this to find the answer as

$$\frac{10000}{3} - 100 + 1 = \frac{10000}{3} - 99.$$

Adds the values from steps 2 and 3 to get:

$$\frac{20}{3} + \frac{10000}{3} - 99 = 3241 \text{ sq. units}$$

(ii) To find $A + B$, integrates the area under $g(x) = x$ as 5000 sq units.

The integration may look as follows:

$$\int_0^{100} x\, dx = \left[\frac{x^2}{2}\right]_0^{100} = 5000$$

Finds A as (5000 – 3241) = 1759 sq. units.

Calculate G = 1759/5000 = approximately 0.35.

LONG ANSWER TYPE QUESTIONS (5 Marks)

1. Writes the total area to be a sum of two integrals are:

$$I = I_1 + I_2$$

$$= \int_0^3 \left(4 + 4\sqrt{1 - \frac{x^2}{9}}\right)dx + \int_3^6 \left(4 - 4\sqrt{1 - \frac{(x-6)^2}{9}}\right)dx$$

Applies integration by parts and writes the first integral as:

$$I_1 = 4\int_0^3 dx + \frac{4}{3}\int_0^3 \sqrt{9 - x^2}\, dx$$

$$I_1 = 4[x]_0^3 - \frac{4}{3}\left[\frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3$$

Simplifies the integral and finds the value as:

$$I_1 = 4 \times (3 - 0) + \frac{4}{3}\left[0 + \frac{9\pi}{4} - 0 - 0\right]$$

$$= 12 + \frac{4}{3}\left[\frac{9\pi}{4}\right]$$

$I_1 = 12 + 3\pi$ sq units

Applies integration by parts and writes the second integral as:

$$I_2 = 4\int_3^6 dx + \frac{4}{3}\int_3^6 \sqrt{9 - (x-6)^2}\, dx$$

$$I_2 = 4[x]_3^6 - \frac{4}{3}\left[\frac{(x-6)}{2}\sqrt{9 - (x-6)^2} + \frac{9}{2}\sin^{-1}\frac{(x-6)}{2}\right]_3^6$$

Simplifies the integral and finds the value as:

$$I_2 = 4 \times (6 - 3) - \frac{4}{3}\left[0 + 0 - 0 + \frac{9\pi}{4}\right]$$

$$= 12 - \frac{4}{3}\left[\frac{9\pi}{4}\right]$$

$I_2 = 12 - 3\pi$ sq units

Adds the two integrals to find the area under the curve as:

$I = I_1 + I_2 = 24$ sq units

2. Sets up the integral as follows:

$$\text{Area} = \int_{-3}^{-2}(x^2 - 4)dx + \left|\int_{-2}^{2}(x^2 - 4)\, dx\right| + \int_2^3 (x^2 - 4)\, dx$$

Evaluate the area from $x = -3$ to $x = -2$ as follows:

$$\int_{-3}^{-2}(x^2 - 4)dx = \left[\frac{x^3}{3} - 4x\right]_{-3}^{-2} = \frac{7}{3}$$

Evaluates the area from $x = -2$ to $x = 2$ as follows:

$$\left|\int_{-2}^{2}(x^2 - 4)\, dx\right| = \left|\left[\frac{x^3}{3} - 4x\right]_{-2}^{2}\right| = \left|\frac{-32}{3}\right| = \frac{32}{3}$$

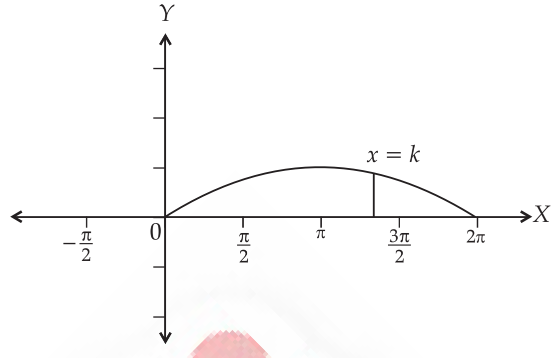
Evaluates the area from $x = 2$ to $x = 3$ as follows:

$$\int_2^3 (x^2 - 4)\, dx = \left[\frac{x^3}{3} - 4x\right]_2^3 = \frac{7}{3}$$

Adds the area found in the above 3 steps to find the area of the shaded region as:

$$\frac{7}{3} + \frac{32}{3} + \frac{7}{3} = \frac{46}{3} \text{ or } 15.33 \text{ sq units}$$

3. Draws a rough graph of $\sin\frac{x}{2}$ and $x = k$.



Writes the equation for the given situation as:

$$\int_0^k \sin\frac{x}{2}dx = 3\int_k^{2\pi} \sin\frac{x}{2}\, dx$$

Simplifies the above equation as:

$$\left[-2\cos\frac{x}{2}\right]_0^k = 3\left[-2\cos\frac{x}{2}\right]_k^{2\pi}$$

Simplifies the above equation as:

$$-2\cos\frac{k}{2} + 2\cos 0 = -6\cos\pi + 6\cos\frac{k}{2}$$

Simplifies the above equation to get k as:

$$\frac{k}{2} = \cos^{-1}\left(\frac{-1}{2}\right) = \frac{2\pi}{3}$$

$$\Rightarrow \quad k = \frac{4\pi}{3}$$

4. **(i)** The area under the curve from $x = -6$ to $x = 6$ is given by the definite integral:

$$A = \int_{-6}^{6}\left(12 - \frac{x^2}{9}\right)dx$$

$$A = \int_{-6}^{6} 12dx - \int_{-6}^{6}\frac{x^2}{9}\, dx$$

First integral:

$$\int_{-6}^{6} 12dx = 12[x]_{-6}^{6} = 12 \times (6 - (-6))$$

$$= 12 \times 12 = 144$$

Second integral:

$$\int_{-6}^{6}\frac{x^2}{9}\, dx = \frac{1}{9}\int_{-6}^{6} x^2\, dx$$

Since x^2 is an even function:

$$\int_{-6}^{6} x^2\, dx = 2\int_0^6 x^2\, dx = 2.\left[\frac{x^3}{3}\right]_0^6$$

$$= 2.\left(\frac{216}{3} - 0\right) = 2 \times 72 = 144$$

So, $\int_{-6}^{6} \frac{x^2}{9} dx = \frac{144}{9} = 16$

Total area:

$$A = 144 - 16 = 128 \text{ m}^2$$

(ii) Volume = Area × Speed

$$= 128 \times 3 = 384 \text{ m}^3/\text{s}$$

5. (i) $A = \int_{-5}^{5} (25 - x^2)\, dx$

$$A = \int_{-5}^{5} 25\, dx - \int_{-5}^{5} x^2\, dx$$

First integral:

$$\int_{-5}^{5} 25\, dx = 25[x]_{-5}^{5} = 25 \times (5 - (-5))$$

$$= 25 \times 10 = 250$$

Second integral:

Since x^2 is even:

$$\int_{-5}^{5} x^2\, dx = 2\int_{0}^{5} x^2\, dx = 2 . \left[\frac{x^3}{3}\right]_0^5$$

$$= 2 . \frac{125}{3} = \frac{250}{3}$$

$$A = 250 - \frac{250}{3} = \frac{750 - 250}{3}$$

$$= \frac{500}{3} \text{ m}^2$$

$$= 166.67 \text{ m}^2$$

(ii) Total cost $= \frac{500}{3} \times 200 = \frac{100000}{3}$

$= ₹\ 33{,}333.33$